

# Modeling of Negotiations Between Supply Chain Participants Based on a Multi-Agent System

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## Abstract

Currently, to solve the problem of modeling negotiations between supply chain participants, multi-agent systems with more than two agents are used. To formalize the negotiation process between agents, the monotonic concession protocol was used, which was modified to allow the use of more than two agents. As a strategy, the Zeuthen strategy for the protocol was used, which was modified to allow the use of more than two agents, and also the choice of the agreement option for the conceding agent for this strategy was formalized. The agreement, the utility function and the risk of going into conflict for the protocol were formalized, which allows us to consider the distribution of tasks between agents as a modified assignment problem, in which all agents should receive approximately the same income from solving problems. The proposed approach can be used in various multi-agent systems for modeling negotiations.

## Keywords

Multi-agent systems, negotiation modeling, supply chains, monotonic concession protocol, Zeuthen strategy

## 1. Introduction

Currently, one of the areas of logistics is supply chain management [1-2]. In this area, artificial intelligence methods are actively used for forecasting [3-4], optimal placement and movement of products [5]. Also, for the supply chain management process, an important role is played by the interaction between its participants [6-8], which can be modeled using a multi-agent system [9-11].

In a multi-agent system, agents can be altruistic or selfish.

Altruistic agents share a common goal and therefore there is no potential conflict of interest. The use of altruistic agents greatly simplifies the process of designing a multi-agent system, since for all agents it is possible to predetermine the tasks they solve without the need to model negotiations between them. For example, the assignment problem can be solved through altruistic agents. The use of altruistic agents is unacceptable if it is necessary to simulate the behavior of people or organizations that pursue their own interests.

Selfish agents do not share a common goal, so there is a potential conflict of interest. The use of selfish agents requires modeling of the negotiations between them so that all agents can achieve their goals. For example, the assignment problem can be modified so that all agents receive approximately the same income by negotiating the distribution of tasks to be performed between them.

In this paper, we will limit ourselves to considering selfish agents, since negotiations should be modeled only between them.

Negotiations between selfish agents continue in a series of rounds in which agents act sequentially [12-14].

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To model negotiations [15-17] the following are used:

1. Approach of the Nash. It is assumed that two prerequisites are met: in case of refusal to negotiate, the parties receive zero utility and both agents know each other's utility functions. The goal of negotiations should be to maximize the product of the utilities of both agents.
2. Approach of the Rubinstein. A negotiation protocol is followed and the behavior of agents, who act according to this protocol, is analyzed. Negotiations can last an unlimited number of rounds, but in order to reach an agreement as soon as possible, it is assumed that in each round of negotiations, agents bear fixed costs and the usefulness of the result obtained decreases with each round.

For both approaches, a common disadvantage is that there can be only two agents [18]. Further improvements of these approaches [19-21], increasing the number of agents, led to high computational complexity.

In studies devoted to the negotiation problem, two types of strategies can be found: time-dependent [7, 22, 23], and behavior-dependent [24-25].

Time-dependent strategies are divided into:

- conservative (the agent continues to insist on his initial offer until the deadline, and then quickly makes concessions up to the minimum acceptable price);
- linear (the agent has a linear utility function);
- compliant (at the beginning of negotiations, the agent quickly makes concessions up to the minimum acceptable price).

In time-dependent strategies, the behavior of the agent does not depend on the actions of the second agent. The disadvantage of these strategies is that agents, like people, usually come to an agreement gradually.

Behavior-dependent strategies are based on the behavior of the second agent. These strategies require knowledge of the utility function of the second agent. The disadvantage of these strategies is the need to analyze the sequence of actions of the agent.

For strategies that depend on time or behavior, a common disadvantage is that there can be only two agents. In this regard, the modeling of negotiations, which will eliminate the indicated drawback, is relevant.

The aim of the work is to model negotiations between supply chain participants based on a multi-agent system. To achieve the goal, the following tasks were set and solved:

- propose a protocol defining the rules for negotiations between agents;
- propose a strategy based on the developed protocol that agents use during negotiations;
- perform numerical studies.

## 2. Materials and methods

### 2.1. Formalization of concepts used in protocol and strategy for negotiation

Let the finite set of admissible variants of agreements on how agents distribute the tasks to be solved among themselves be denoted as  $\Delta$ . Depending on the problem, one agent can solve both a single task and several tasks, both individually and jointly.

In this paper, a variant of the agreement  $\delta_l^t$ ,  $\delta_l^t \in \Delta$ , which was proposed by the  $l^{\text{th}}$  agent in the  $t^{\text{th}}$  round of negotiations on the distribution of tasks to be solved, is presented in the form

$$\delta_l^t = [x_{ij}^t], x_{ij}^t \in \{0,1\}, i \in \{1,\dots,m\}, j \in \{1,\dots,n\}, \quad (1)$$

where  $n$  – number of tasks;

$m$  – number of agents;

$x_{ij}^t$  indicates that the  $i^{\text{th}}$  agent solves or does not solve the  $j^{\text{th}}$  problem.

Let the conflicting agreement be defined as  $\delta^c$ ,  $\delta^c \in \Delta$ .

In this paper, the utility function of the variant of the agreement, which is proposed by the  $l^{\text{th}}$  agent in the  $t^{\text{th}}$  round of negotiations, for the  $i^{\text{th}}$  agent is defined as income from the performance of the tasks by this agent

$$u_i(\delta_i^t) = \sum_{j=1}^n w_{ij} x_{ij}^t, \quad (2)$$

where  $w_{ij}$  – income from the solution by the  $i^{\text{th}}$  agent of the  $j^{\text{th}}$  task.

Such utility function satisfies two constraints:

1. The function is monotonous, that is, adding the tasks to be solved never diminishes the utility.
2. If there are no tasks to be solved, then the utility is zero.

In this paper, the risk of making a conflict (unwillingness to make concessions) for the  $i^{\text{th}}$  agent in the  $t^{\text{th}}$  round of negotiations is defined as

$$r_i^t = \sum_k u_k(\delta_i^t), \quad k \in \{1, \dots, m\} / \{i\}. \quad (3)$$

## 2.2. Extended monotonic concession protocol for multiple agents

In this paper, we propose an extended monotonic concession protocol for multiple agents.

1. Negotiations continue in a series of rounds.
2. In the first round of negotiations, all agents simultaneously propose agreement options  $\delta_1^1, \dots, \delta_m^1$ , respectively (the agent chooses an agreement option from a finite set of agreement options  $\Delta$ ).
3. If in the  $t^{\text{th}}$  round of negotiations the agents agree, i.e.  $\exists i : \forall j \in \overline{1, m} u_j(\delta_i^t) \geq u_j(\delta_j^t)$ , then:
  - if  $\exists i : \forall j \in \overline{1, m} u_j(\delta_i^t) = u_j(\delta_j^t)$ , then negotiations conclude and a variant of the agreement will be randomly selected from the set of  $\{\delta_j^t\}$ ;
  - if  $\neg \exists i : \forall j \in \overline{1, m} u_j(\delta_i^t) = u_j(\delta_j^t)$ , then negotiations conclude and the option of the agreement will be selected  $\delta_i^t$ .
4. If in the  $t^{\text{th}}$  round the agents did not agree, i.e.  $\neg \exists i : \forall j \in \overline{1, m} u_j(\delta_i^t) \geq u_j(\delta_j^t)$ , but there is an agent  $i^*$  willing to make a concession, then negotiations continue. In round  $t+1$  the conceding agent  $i^*$  can offer other agents only such a variant of the agreement  $\delta_{i^*}^{t+1}$ , for which the concession condition is satisfied.
5. If in the  $t^{\text{th}}$  round the agents did not agree, i.e.  $\neg \exists i : \forall j \in \overline{1, m} u_j(\delta_i^t) \geq u_j(\delta_j^t)$ , and there is no agent willing to make a concession, then negotiations end in a conflicting agreement  $\delta^c$ .

## 2.3. Zeuthen extended strategy for multiple agents

In this paper, we propose an extended Zeuthen strategy for multiple agents.

1. In the first round of negotiations, all agents simultaneously propose the most preferred agreement options for them, i.e.

$$\delta_i^1 = \arg \max_{\delta \in \Delta} u_i(\delta), \quad i \in \overline{1, m}. \quad (4)$$

2. If in the  $t^{\text{th}}$  round of negotiations the agents did not agree, but the concession of one of them is possible, then the conceding agent is defined as the agent who has the least risk of going into a conflict, i.e.

$$3. \quad i^* = \arg \min_{i \in \overline{1, m}} r_i^t. \quad (5)$$

If there are several agents with the same minimal risk  $r_i^t$ , then agent  $i^*$  is randomly selected from among them.

For all agents except agent  $i^*$ ,  $\delta_j^{t+1} = \delta_j^t$ .

4. The conceding agent  $i^*$  proposes a variant of the agreement  $\delta_{i^*}^{t+1}$  in the round  $t+1$ . The following choice of the variant of the agreement  $\delta_{i^*}^{t+1}$  is proposed, which will increase the risk for the agent  $i^*$  to enter into a conflict, that is

$$\Delta_{i^*}^t = \left\{ \delta \mid \sum_k u_k(\delta) > r_{i^*}^t, \delta \in \Delta, k \in \{1, \dots, m\} / \{i^*\} \right\}, \quad (6)$$

$$\tilde{\Delta}_{i^*}^t = \arg \max_{\delta \in \Delta_{i^*}^t} u_{i^*}(\delta), \quad (7)$$

$$\delta_{i^*}^{t+1} = \arg \max_{\delta \in \tilde{\Delta}_{i^*}^t} \sum_k u_k(\delta), k \in \{1, \dots, m\} / \{i^*\}. \quad (8)$$

If there are several options for agreements, then option  $\delta_{i^*}^{t+1}$  is randomly selected from them.

### 3. Numerical study

#### 3.1. Extended monotonic concession protocol and Zeuthen strategy in the case of two agents

Let the number of tasks be  $n=4$ .

Let the number of agents  $m=2$ .

Let the task can be solved by only one agent, but one agent can solve several tasks.

Let the finite set of admissible variants of agreements be defined as

$$\Delta = \left\{ \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}, \right. \\ \left. \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \end{bmatrix}, \right. \\ \left. \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix} \right\}.$$

Let the conflicting agreement be defined as

$$\delta^c = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}.$$

Let the income matrix from solving tasks by agents be defined as

$$W = \begin{bmatrix} 10 & 20 & 10 & 20 \\ 20 & 10 & 20 & 10 \end{bmatrix}.$$

Next, we will step by step execute the extended monotonic concession protocol.

1. In the first round of negotiations, both agents simultaneously propose the most preferable agreement options for them, i.e.

$$\delta_1^1 = \arg \max_{\delta \in \Delta} u_1(\delta) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\delta_2^1 = \arg \max_{\delta \in \Delta} u_2(\delta) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}.$$

2. In the first round of negotiations, both agents did not agree, since  $\neg \exists i : \forall j \in \overline{1,2} u_j(\delta_i^1) \geq u_j(\delta_j^1)$ , where  $u_1(\delta_1^1) = 60$ ,  $u_2(\delta_1^1) = 0$ ,  $u_2(\delta_2^1) = 60$ ,  $u_1(\delta_2^1) = 0$ .

Therefore, the concession of one of them is possible. To determine the conceding agent, the risks of going into conflict are calculated, i.e.

$$r_1^1 = u_2(\delta_1^1) = 0, \quad r_2^1 = u_1(\delta_2^1) = 0,$$

and the agent with the lowest risk is determined, i.e.

$$\arg \min_{i \in \{1,2\}} r_i^1 = \{1,2\}.$$

Since there are two such agents, the conceding agent  $i^*$  will be randomly selected from among them. For example, let  $i^* = 1$ , then  $\delta_2^2 = \delta_2^1$ .

3. In the second round of negotiations, the conceding agent 1 randomly chooses an agreement option from the set  $\tilde{\Delta}_1^1 = \left\{ \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \right\}$ . For example, let the agreement option

$$\delta_1^2 = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix} \text{ be chosen.}$$

4. In the second round of negotiations, both agents did not agree, since  $\neg \exists i : \forall j \in \overline{1,2} \ u_j(\delta_i^2) \geq u_j(\delta_j^2)$ , where  $u_1(\delta_1^2) = 50$ ,  $u_2(\delta_1^2) = 20$ ,  $u_2(\delta_2^2) = 60$ ,  $u_1(\delta_2^2) = 0$ .

Therefore, the concession of one of them is possible. To determine the conceding agent, the risks of going into conflict are calculated, i.e.

$$r_1^2 = u_2(\delta_1^2) = 20, \quad r_2^2 = u_1(\delta_2^2) = 0,$$

and the agent with the lowest risk is determined, i.e.

$$i^* = \arg \min_{i \in \{1,2\}} r_i^2 = 2.$$

Then  $\delta_1^3 = \delta_1^2$ .

5. In the third round of negotiations, the conceding agent 2 randomly chooses an agreement option from the set  $\tilde{\Delta}_2^2 = \left\{ \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix} \right\}$ . For example, let the agreement option

$$\delta_2^3 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 \end{bmatrix} \text{ be chosen.}$$

6. In the third round of negotiations, both agents did not agree, since  $\neg \exists i : \forall j \in \overline{1,2} \ u_j(\delta_i^3) \geq u_j(\delta_j^3)$ , where  $u_1(\delta_1^3) = 50$ ,  $u_2(\delta_1^3) = 20$ ,  $u_2(\delta_2^3) = 50$ ,  $u_1(\delta_2^3) = 20$ .

Therefore, a concession of one of them is possible. To determine the conceding agent, the risks of conflict are calculated, i.e.

$$r_1^3 = u_2(\delta_1^3) = 20, \quad r_2^3 = u_1(\delta_2^3) = 20,$$

and the agent with the lowest risk is determined, i.e.

$$\arg \min_{i \in \{1,2\}} r_i^3 = \{1,2\}.$$

Since there are two such agents, the conceding agent  $i^*$  will be randomly selected from among them. For example, let  $i^* = 1$ , then  $\delta_2^4 = \delta_2^3$ .

7. In the fourth round of negotiations, the conceding agent 1 chooses the only agreement option

$$\delta_1^4 = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix} \text{ from the set } \tilde{\Delta}_1^3 = \left\{ \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix} \right\}.$$

8. In the fourth round of negotiations, both agents did not agree, since  $\neg \exists i : \forall j \in \overline{1,2} \ u_j(\delta_i^4) \geq u_j(\delta_j^4)$ , where  $u_1(\delta_1^4) = 40$ ,  $u_2(\delta_1^4) = 40$ ,  $u_2(\delta_2^4) = 50$ ,  $u_1(\delta_2^4) = 20$ .

Therefore, a concession of one of them is possible. To determine the conceding agent, the risks of conflict are calculated, i.e.

$$r_1^4 = u_2(\delta_1^4) = 40, \quad r_2^4 = u_1(\delta_2^4) = 20,$$

and the agent with the lowest risk is determined, i.e.

$$i^* = \arg \min_{i \in \{1,2\}} r_i^4 = 2.$$

Then  $\delta_1^5 = \delta_1^4$ .

9. In the fifth round of negotiations, the conceding agent 2 chooses the only agreement option

$$\delta_2^5 = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix} \text{ from the set } \tilde{\Delta}_2^4 = \left\{ \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix} \right\}.$$

10. In the fifth round of negotiations, both agents agreed, since  $\exists i : \forall j \in \overline{1,2} \ u_j(\delta_i^5) = u_j(\delta_j^5)$ , where  $u_1(\delta_1^5) = 40$ ,  $u_2(\delta_1^5) = 40$ ,  $u_2(\delta_2^5) = 40$ ,  $u_1(\delta_2^5) = 40$ . Therefore, the negotiations are completed and a variant of the agreement will be randomly selected from the set  $\{\delta_j^5\}$ . For example, let it be  $\delta_1^5$ .

The resulting agreement  $\delta_1^5$  will be the maximum point on the discrete set  $\Delta$  of the discrete function  $\Phi(k) = (u_1(\delta_k) - u_1(\delta^c))(u_2(\delta_k) - u_2(\delta^c))$ ,  $k = \{1, \dots, |\Delta|\}$ ,  $\delta_k \in \Delta$ , and is the Nash solution of the negotiation task.

### 3.2. Extended monotonic concession protocol and Zeuthen strategy in the case of three agents

Let the number of tasks be  $n=3$ .

Let the number of agents  $m=3$ .

Let the task can be solved by only one agent, but one agent can solve several tasks.

Let the finite set of admissible variants of agreements be defined as

$$\Delta = \left\{ \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 \\ 1 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 1 \end{bmatrix}, \right. \\ \left. \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \right. \\ \left. \begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}, \right. \\ \left. \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \right\}.$$

Let the conflicting agreement be defined as

$$\delta^c = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}.$$

Let the income matrix from solving tasks by agents be defined as

$$W = \begin{bmatrix} 5 & 5 & 20 \\ 5 & 20 & 5 \\ 20 & 5 & 5 \end{bmatrix}.$$

Next, we will step by step execute the extended monotonic concession protocol.

1. In the first round of negotiations, three agents simultaneously offer their preferred options for agreements, i.e.

$$\begin{aligned}\delta_1^1 &= \arg \max_{\delta \in \Delta} u_1(\delta) = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \\ \delta_2^1 &= \arg \max_{\delta \in \Delta} u_2(\delta) = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}, \\ \delta_3^1 &= \arg \max_{\delta \in \Delta} u_3(\delta) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}.\end{aligned}$$

2. In the first round of negotiations, three agents did not agree, because  $\neg \exists i : \forall j \in \overline{1,3} u_j(\delta_i^1) \geq u_j(\delta_j^1)$ , where  $u_1(\delta_1^1) = 30$ ,  $u_2(\delta_1^1) = 0$ ,  $u_3(\delta_1^1) = 0$ ,  $u_2(\delta_2^1) = 30$ ,  $u_1(\delta_2^1) = 0$ ,  $u_3(\delta_2^1) = 0$ ,  $u_3(\delta_3^1) = 30$ ,  $u_1(\delta_3^1) = 0$ ,  $u_2(\delta_3^1) = 0$ . Therefore, a concession of one of them is possible. To determine the conceding agent, the risks of conflict are calculated, i.e.

$$r_1^1 = u_2(\delta_1^1) + u_3(\delta_1^1) = 0, \quad r_2^1 = u_1(\delta_2^1) + u_3(\delta_2^1) = 0, \quad r_3^1 = u_1(\delta_3^1) + u_2(\delta_3^1) = 0,$$

and the agent with the lowest risk is determined, i.e.

$$\arg \min_{i \in \overline{1,3}} r_i^1 = \{1, 2, 3\}.$$

Since there are three such agents, a conceding agent  $i^*$  will be randomly selected from among them. For example, let  $i^* = 1$ , then  $\delta_2^2 = \delta_2^1$ ,  $\delta_3^2 = \delta_3^1$ .

3. In the second round of negotiations, the conceding agent 1 randomly chooses an agreement

option from the set  $\tilde{\Delta}_1^1 = \left\{ \begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \right\}$ . For example, let the agreement option

$$\delta_1^2 = \begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} \text{ be chosen.}$$

4. In the second round of negotiations, three agents did not agree, because  $\neg \exists i : \forall j \in \overline{1,3} u_j(\delta_i^2) \geq u_j(\delta_j^2)$ , where  $u_1(\delta_1^2) = 25$ ,  $u_2(\delta_1^2) = 0$ ,  $u_3(\delta_1^2) = 20$ ,  $u_2(\delta_2^2) = 30$ ,  $u_1(\delta_2^2) = 0$ ,  $u_3(\delta_2^2) = 0$ ,  $u_3(\delta_3^2) = 30$ ,  $u_1(\delta_3^2) = 0$ ,  $u_2(\delta_3^2) = 0$ . Therefore, a concession of one of them is possible. To determine the conceding agent, the risks of conflict are calculated, i.e.

$$r_1^2 = u_2(\delta_1^2) + u_3(\delta_1^2) = 20, \quad r_2^2 = u_1(\delta_2^2) + u_3(\delta_2^2) = 0, \quad r_3^2 = u_1(\delta_3^2) + u_2(\delta_3^2) = 0,$$

and the agent with the lowest risk is determined, i.e.

$$\arg \min_{i \in \overline{1,3}} r_i^2 = \{2, 3\}.$$

Since there are two such agents, a conceding agent  $i^*$  will be randomly selected from among them. For example, let  $i^* = 2$ , then  $\delta_1^3 = \delta_1^2$ ,  $\delta_3^3 = \delta_3^2$ .

5. In the third round of negotiations, the conceding agent 2 randomly chooses an agreement

option from the set  $\tilde{\Delta}_2^2 = \left\{ \begin{bmatrix} 0 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix} \right\}$ . For example, let the agreement option

$$\delta_2^3 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix} \text{ be chosen.}$$

6. In the third round of negotiations, three agents did not agree, because  $\neg\exists i : \forall j \in \overline{1,3} u_j(\delta_i^3) \geq u_j(\delta_j^3)$ , where  $u_1(\delta_1^3) = 25$ ,  $u_2(\delta_1^3) = 0$ ,  $u_3(\delta_1^3) = 20$ ,  $u_2(\delta_2^3) = 25$ ,  $u_1(\delta_2^3) = 0$ ,  $u_3(\delta_2^3) = 20$ ,  $u_3(\delta_3^3) = 30$ ,  $u_1(\delta_3^3) = 0$ ,  $u_2(\delta_3^3) = 0$ . Therefore, a concession of one of them is possible. To determine the conceding agent, the risks of conflict are calculated, i.e.

$$r_1^3 = u_2(\delta_1^3) + u_3(\delta_1^3) = 20, r_2^3 = u_1(\delta_2^3) + u_3(\delta_2^3) = 20, r_3^3 = u_1(\delta_3^3) + u_2(\delta_3^3) = 0,$$

and the agent with the lowest risk is determined, i.e.

$$i^* = \arg \min_{i \in \overline{1,3}} r_i^3 = 3.$$

Then  $\delta_1^4 = \delta_1^3$ ,  $\delta_2^4 = \delta_2^3$ .

7. In the fourth round of negotiations, the conceding agent 3 randomly chooses an agreement

option from the set  $\tilde{\Delta}_3^3 = \left\{ \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \right\}$ . For example, let the agreement option

$$\delta_3^4 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix} \text{ be chosen.}$$

8. In the fourth round of negotiations, three agents did not agree, since  $\neg\exists i : \forall j \in \overline{1,3} u_j(\delta_i^4) \geq u_j(\delta_j^4)$ , where  $u_1(\delta_1^4) = 25$ ,  $u_2(\delta_1^4) = 0$ ,  $u_3(\delta_1^4) = 20$ ,  $u_2(\delta_2^4) = 25$ ,  $u_1(\delta_2^4) = 0$ ,  $u_3(\delta_2^4) = 20$ ,  $u_3(\delta_3^4) = 25$ ,  $u_1(\delta_3^4) = 20$ ,  $u_2(\delta_3^4) = 0$ . Therefore, a concession of one of them is possible. To determine the conceding agent, the risks of conflict are calculated, i.e.

$$r_1^4 = u_2(\delta_1^4) + u_3(\delta_1^4) = 20, r_2^4 = u_1(\delta_2^4) + u_3(\delta_2^4) = 20, r_3^4 = u_1(\delta_3^4) + u_2(\delta_3^4) = 20,$$

and the agent with the lowest risk is determined, i.e.

$$\arg \min_{i \in \overline{1,3}} r_i^4 = \{1,2,3\}.$$

Since there are three such agents, a conceding agent  $i^*$  will be randomly selected from among them. For example, let  $i^* = 1$ , then  $\delta_2^5 = \delta_2^4$ ,  $\delta_3^5 = \delta_3^4$ .

9. In the fifth round of negotiations, the conceding agent 1 chooses the only agreement option

$$\delta_1^5 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \text{ from the set } \tilde{\Delta}_1^4 = \left\{ \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \right\}.$$

10. In the fifth round of negotiations, three agents did not agree, since  $\neg\exists i : \forall j \in \overline{1,3} u_j(\delta_i^5) \geq u_j(\delta_j^5)$ , where  $u_1(\delta_1^5) = 20$ ,  $u_2(\delta_1^5) = 20$ ,  $u_3(\delta_1^5) = 20$ ,  $u_2(\delta_2^5) = 25$ ,  $u_1(\delta_2^5) = 0$ ,  $u_3(\delta_2^5) = 20$ ,  $u_3(\delta_3^5) = 25$ ,  $u_1(\delta_3^5) = 20$ ,  $u_2(\delta_3^5) = 0$ . Therefore, a concession of one of them is possible. To determine the conceding agent, the risks of conflict are calculated, i.e.

$$r_1^5 = u_2(\delta_1^5) + u_3(\delta_1^5) = 40, r_2^5 = u_1(\delta_2^5) + u_3(\delta_2^5) = 20, r_3^5 = u_1(\delta_3^5) + u_2(\delta_3^5) = 20,$$

and the agent with the lowest risk is determined, i.e.

$$\arg \min_{i \in \overline{1,3}} r_i^5 = \{2,3\}.$$

Since there are two such agents, a conceding agent  $i^*$  will be randomly selected from among them. For example, let  $i^* = 2$ , then  $\delta_1^6 = \delta_1^5$ ,  $\delta_3^6 = \delta_3^5$ .

11. In the sixth round of negotiations, the conceding agent 2 chooses the only agreement option

$$\delta_2^6 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \text{ from the set } \tilde{\Delta}_2^5 = \left\{ \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \right\}.$$



12. In the sixth round of negotiations, three agents did not agree, since  $\neg \exists i : \forall j \in \overline{1,3} u_j(\delta_i^6) \geq u_j(\delta_j^6)$ , where  $u_1(\delta_1^6) = 20$ ,  $u_2(\delta_1^6) = 20$ ,  $u_3(\delta_1^6) = 20$ ,  $u_2(\delta_2^6) = 20$ ,  $u_1(\delta_2^6) = 20$ ,  $u_3(\delta_2^6) = 20$ ,  $u_3(\delta_3^6) = 25$ ,  $u_1(\delta_3^6) = 20$ ,  $u_2(\delta_3^6) = 0$ . Therefore, a concession of one of them is possible. To determine the conceding agent, the risks of conflict are calculated, i.e.

$$r_1^6 = u_2(\delta_1^6) + u_3(\delta_1^6) = 40, \quad r_2^6 = u_1(\delta_2^6) + u_3(\delta_2^6) = 40, \quad r_3^6 = u_1(\delta_3^6) + u_2(\delta_3^6) = 20,$$

and the agent with the lowest risk is determined, i.e.

$$i^* = \arg \min_{i \in \overline{1,3}} r_i^6 = 3.$$

Then  $\delta_1^7 = \delta_1^6$ ,  $\delta_2^7 = \delta_2^6$ .

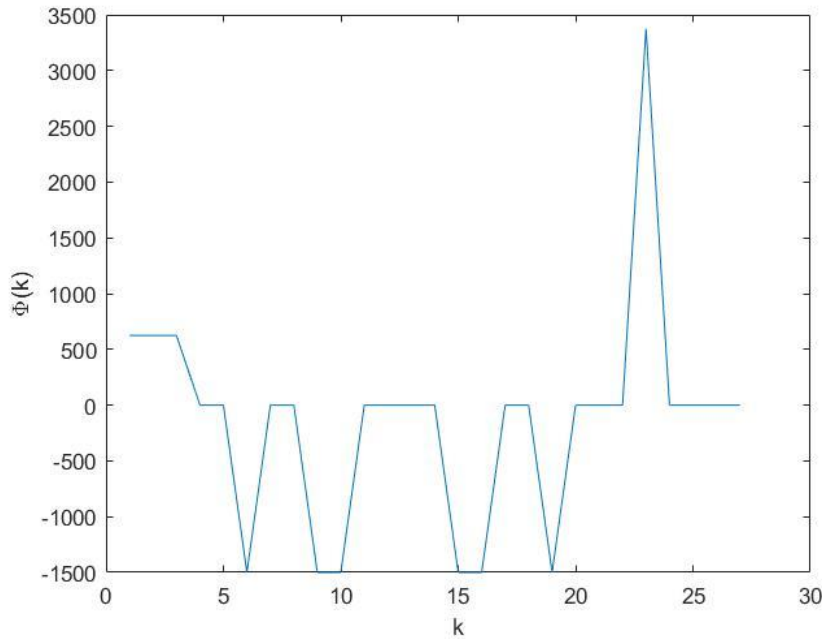
13. In the seventh round of negotiations, the conceding agent 3 chooses the only agreement option

$$\delta_3^7 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \text{ from the set } \tilde{\Delta}_3^6 = \left\{ \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \right\}.$$

14. In the seventh round of negotiations, the three agents agreed because  $\exists i : \forall j \in \overline{1,3} u_j(\delta_i^7) = u_j(\delta_j^7)$ , where  $u_1(\delta_1^7) = 20$ ,  $u_2(\delta_1^7) = 20$ ,  $u_3(\delta_1^7) = 20$ ,  $u_2(\delta_2^7) = 20$ ,  $u_1(\delta_2^7) = 20$ ,  $u_3(\delta_2^7) = 20$ ,  $u_3(\delta_3^7) = 20$ ,  $u_1(\delta_3^7) = 20$ ,  $u_2(\delta_3^7) = 20$ . Therefore, negotiations are completed and an agreement variant from the set  $\{\delta_j^7\}$  will be randomly selected. For example, let it be  $\delta_1^7$ .

The resulting agreement  $\delta_1^7$  will be the maximum point on the discrete set  $\Delta$  of the discrete function  $\Phi(k) = (u_1(\delta_k) - u_1(\delta^c))(u_2(\delta_k) - u_2(\delta^c))(u_3(\delta_k) - u_3(\delta^c))$ ,  $k = \{1, \dots, |\Delta|\}$ ,  $\delta_k \in \Delta$ , and is the Nash solution of the negotiation task.

Figure 1 shows the dependence of the discrete function  $\Phi(k)$  on the number of acceptable variants of the agreement  $k$ . The resulting agreement  $\delta_1^7$  corresponds to the number of the acceptable variant of the agreement  $k=23$ .



**Figure 1:** Dependence of the discrete function  $\Phi(k)$  on the number of acceptable variants of the agreement  $k$

## 4. Conclusions

To solve the problem of modeling negotiations between supply chain participants, the existing approaches to modeling negotiations were investigated. These studies have shown that the use of multi-agent systems with more than two agents is currently relevant.

The proposed formalization of the modified agreement, the utility function and the risk of going into conflict made it possible to formalize the modified protocol of monotonous concessions and the Zeuthen strategy for negotiating and expand the problem of modeling negotiations between two selfish agents to many agents, which allows solving a wider range of problems.

The proposed approach has the following features:

- the utility functions of a set of selfish agents are compared (one-to-many comparison) to determine an agreement that satisfies all agents. In contrast to the classical assignment problem, the main objective is not the achievement of the extremum of the sum of the values of the utility functions of all agents (that are altruistic), but the proximity of the values of the utility functions of all selfish agents;
- the agreement variant matrix reflects the distribution of tasks between a set of selfish agents and is similar to the variant of the task distribution matrix between a set of agents (that are altruistic) for the classical assignment problem;
- the yielding agent (the agent with the least risk of conflict) is selected from a set of agents. In contrast to the classical assignment problem, the main objective is not the achievement of an extremum of the sum of the values of the utility functions of all agents (that are altruistic), but the minimum deterioration in the value of the utility function of the yielding agent.

This allows us to consider the problem of modeling negotiations between participants in supply chains as a variant of the assignment problem in the case of selfish agents.

The proposed approach can be used in various multi-agent systems for modeling negotiations (for example, the manufacture of goods by different firms or employees, the delivery of goods by different firms or drivers, the distribution of goods between bases, etc.).

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