Trajectory Modeling Calculation and Maneuverability Analysis of Proportional Navigation Method

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Abstract

Aiming at the ballistic characteristics and related performance indexes of a certain type of anti-aircraft missile, the differential equations of missile-target relative motion are established. Under different given initial conditions, the Runge-Kutta method is used to solve the system of micro-equations numerically. Thereby, the ballistic curve, ballistic parameters and performance index parameters of the corresponding missile interception are obtained. In the simulation, different missile/target maneuvering conditions, route shortcuts and interception methods are verified and calculated to measure the missile's ballistic characteristics and performance indicators comprehensively. The simulation results show that when the proportional guidance method is used, under different missile/target maneuvering conditions and interception methods, the greater the target maneuvering degree, the smaller the curvature radius of the missile's ballistic curve, and the greater the required normal overload and miss distance. The ballistic interception trajectory and the interception interception trajectory are relatively straight, and the required normal overload and misses are relatively small; when intercepting in the air, the radius of curvature of the ballistic trajectory is the smallest, and the required normal overload and misses are the largest.

Keywords

Ballistic characteristics, Runge-Kutta method, Proportional guidance， Normal overload， Miss distance

1. Introduction

The power of the missile mainly depends on the guidance accuracy of the missile and the number of warhead ammunition, and the guidance rules based on previous research are often referred to as conventional guidance rules. There are three main induction rules: tracking method, line-of-sight display induction method and proportional guidance method.The proportional guidance method is simple, technically easy to implement, does not require too much information, its trajectory is relatively flat, mechanical targets and low-altitude flying targets can be intercepted, the import accuracy is high, and it is widely used[1].

Generally speaking, in the guidance system of a missile, the role of the guidance law is to ensure that the missile flies and hits the target's kinematic trajectory. The guidance law also determines the flight trajectory of the missile center of mass in space.So, how to choose a suitable guidance method to improve the guidance performance of the missile, improve the hit accuracy of the missile, and reduce the amount of misses, has always been our special attention[2-3]. Some scholars briefly introduced the commonly used missile guidance laws in their articles, took the proportional guidance method as an example to conduct a two-dimensional plane simulation of the guidance ballistic, and analyzed the proportional guidance coefficient, the ratio of the missile and the target speed, etc. The influence of the

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change of the main ballistic parameters on the ballistics, some suggestions for improving the missile guidance and control system are put forward[4].

2. Vertical plane trajectory model

This paper mainly studies the motion characteristics of the missile in the vertical plane. First, the missile's full ballistic equation is decomposed on the vertical plane, and the dynamic mathematical models of 10 vertical planes are obtained. Secondly, the dynamic mathematical model of the vertical plane is replaced by the kinematic mathematical model of the vertical plane, that is, the change of the tangential velocity and the normal maneuvering law of the missile are replaced by the known variation law of the missile velocity movement. Then, the Runge-Kutta method is used to solve the two differential equations about the sight angle of the target and the distance of the projectile, and the simulation model of the ballistic trajectory is normalized.

2.1. Dynamic mathematical model

Usually the movement of the missile can be decomposed into the movement of the missile's vertical plane and the movement of the missile's horizontal plane. If we require a missile to fly at high altitude and high speed in the vertical plane, then a horizontal lateral force in the horizontal direction should be equivalent to at zero. If we choose the ground coordinate system Ax axis is located in the vertical plane, at this time, β, γ, etc. are correspondingly regarded as zero. When a particle in the vertical plane moves, the missile only performs a parallel motion and a rotational motion along the axis and the OZ_1 axis in the rotational direction of the center of the inner particle on the vertical plane. At this time, z=0 , $\omega_x = 0, \omega_y = 0$. When the missile moves along the vertical plane, the external forces of the missile are only: the thrust P of the engine, the air resistance X, the lift Y, and the gravity G.Such as Formula 1.

$$\begin{cases} m\frac{dV}{dt} = P\,cos\alpha - X - mg\,sin\theta \\ mV\frac{d\theta}{dt} = Psin\alpha + Y - mgcos\theta \\ J_z\frac{d\omega_z}{dt} = M_z \\ \frac{dx}{dt} = V\,cos\theta \\ \frac{dy}{dt} = V\,sin\theta \\ \frac{d\vartheta}{dt} = \omega_z \\ \frac{dm}{dt} = -m_c \\ \alpha = \vartheta - \theta \\ \phi_1 = 0 \\ \phi_4 = 0 \end{cases} \qquad (1)$$

2.2. Kinematic mathematical model

In the preliminary research stage of our anti-aircraft missile and guidance system technology, in order to further simplify the research, we assume that the missile, the target and the guidance station always move in the same fixed plane, that is, on the attack plane, which may be a vertical plane, or a may be horizontal or inclined.

We choose the attack plane as the plumb plane for research.According to the kinematic analysis method of guided ballistics, it is assumed that a relative motion parameter equation between the missile and the attack target can be defined by the motion parameters such as r, q in polar coordinates in a plane called the missile attack force. The relative transformation motion law of , etc. can be used to describe it accurately.

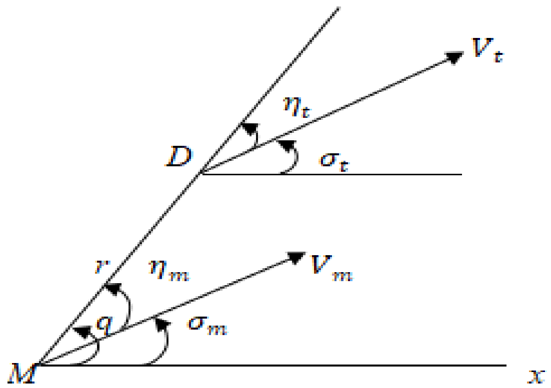

Figure 1: Geometric relation of relative motion

The relative motion equation of automatic aiming guidance refers to the equation describing the relative distance r and the rate of change of the target line angle q. According to the relative motion relationship between the missile and the target shown in Figure 1, the relative motion equation can be established directly. The missile velocity vector V_M and the target velocity vector V_T are decomposed along the target line direction and the normal line direction, respectively. The component $V_M cos\eta_M$ points to the target along the target line, which reduces the relative distance r. The component $V_T cos\eta_T$ deviates from the missile, which increases the relative distance r. The normal component along the target line, $V_M sin\eta_M$, rotates the target line counterclockwise around the origin where the target is located to increase the target line angle q. The component $V_T sin\eta_T$ rotates the target line clockwise around the missile's position as the origin, thereby reducing the target line angle q.

We also consider the relative geometric motion relationship between the two motion angles shown in Figure 1. When guided proportionally, when the attack plane is a vertical plane, σ is the ballistic inclination angle θ. Therefore, the mathematical model of the kinematics of the missile in the vertical plane can be written as Formula 2.

Some parameters in the ballistic dynamics mathematical model of the vertical plane are very confidential, so it is difficult for us to obtain these parameters. For example, the first two equations of Formula 1 can know the lift coefficient and drag coefficient in the lift Y and the drag X, so the ballistic is used. It is difficult to directly study the motion law of the missile with the dynamic mathematical model. Therefore, we will study the kinematic mathematical model of the vertical plane ballistic trajectory, and use the change law of the missile velocity movement in the kinematic model to replace the change law of the tangential velocity of the first two equations of the equation group in the vertical plane ballistic dynamic mathematical model. and the law of motion in the normal direction.

$$\begin{cases} \frac{dr}{dt} = V_T cos\eta_T - V_M cos\eta_M \\ \frac{dq}{dt} = \frac{1}{r}(V_M cos\eta_M - V_T cos\eta_T) \\ q = \sigma_M + \eta_M \\ q = \sigma_T + \eta_T \\ \frac{d\sigma}{dt} = K\frac{dq}{dt} \end{cases} \quad (2)$$

In general, since the right side of the equation is a nonlinear function of the motion parameters, the missile motion equations are nonlinear first-order ordinary differential equations. In this paper, the Runge-Kutta method is used to carry out high-precision calculations. This algorithm is a high-precision program that is easy to edit and calculate. It is also possible to quickly change the calculation step size.

By normalizing the kinematic mathematical model of the vertical plane trajectory, we can design the simulation algorithm according to the simulation model and initial conditions, and then carry out the simulation. After obtaining the trajectory and the trajectory of the target, the final analysis of the results is carried out, including the calculation of the missed target amount and the normal overload.

2.3. Simulation analysis

2.3.1. Simulation example

The target moves in a straight line with horizontal uniform acceleration, and the acceleration is a_T.The missile moves at a uniform acceleration of a_M .

An example of missile head-on interception: the initial target speed is 300m/s, the initial missile speed is 600m/s, the initial missile target distance is 7000m, the initial target line of sight angle is 50 °, the initial lead angle is -3 °and the relevant parameters are shown in Table 1.

Table 1

Missile head-on interception

example	a_T	a_M
A1	0	0
B1	0	15
C1	6	15

An example of missile tail pursuit and interception: the initial target speed is 300m/s, the initial missile speed is 600m/s, the initial missile target distance is 2000m, the initial target line of sight angle is 50 °, and the initial lead angle is 3 °and the relevant parameters are shown in Table 2.In addition, we also assume that in the process of missile flying to the target, the rotation of the earth, the curvature of the earth and the influence of wind are not considered, and the missile also has an ideal control and guidance system.

Table 2

Missile tail pursuit interception

example	a_T	a_M
A2	0	0
B2	0	15
C2	5	15

2.3.2. Simulation result

When the scale factor is 3, the flight time and miss distance of the three calculation examples of the missile intercepting the target head-on are shown in the Table 3 below.The last three are data with a scale factor of 5.The data of tail pursuit interception are shown in Table 4.

From the simulation results, we know that in the case of only increasing the flight speed of the missile, the flight time of the missile decreases with the increase of the speed of the missile to a certain extent. The proportional coefficient K also has a certain effect on the flight time, but the effect is not as great as the effect of increasing the missile flight speed on the flight time. In the case of only increasing the target flight speed, the flight time decreases with the increase of the target speed to a certain extent.

Table 3

Results of head-on interception of targets

example	flight time(s)	Miss distance(s)
A1	9.55	3.16
B1	8.75	2.70
C1	8.65	4.82
A11	9.45	4.65
B11	8.60	2.78
C11	8.55	6.34

Table 4
Results of tail pursuit interception of targets

example	flight time(s)	Miss distance(s)
A2	2.80	4.73
B2	2.70	6.20
C2	2.65	8.21
A22	2.75	5.36
B22	2.65	4.42
C22	2.55	7.38

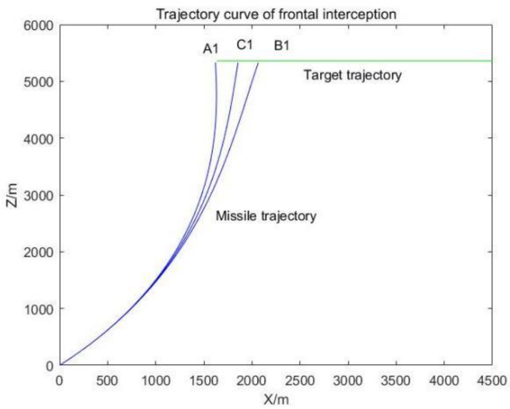

Figure2: Trajectory of A1,B1,C1

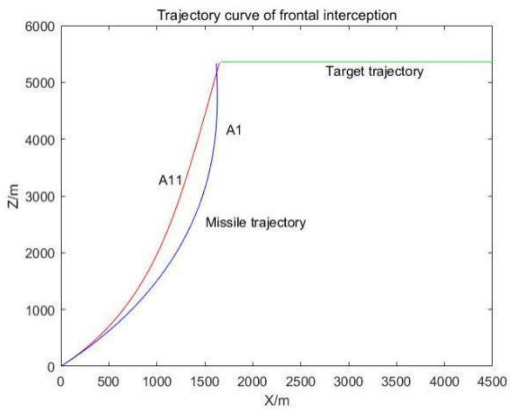

Figure 3: Trajectory of A1,A11

As the speed of the missile increases, the trajectory of the missile will be straighter, and the radius of curvature will be larger; as the speed of the target increases, the trajectory of the missile will be more curved, and the radius of curvature will be smaller; as the proportional coefficient increases, the trajectory of the missile will be more Straight, the radius of curvature will be larger. In addition, a relatively curved front section of the ballistic trajectory can be obtained. In the front section of the missile trajectory, the proportional guidance method can reasonably and fully utilize the maneuverability of the missile.

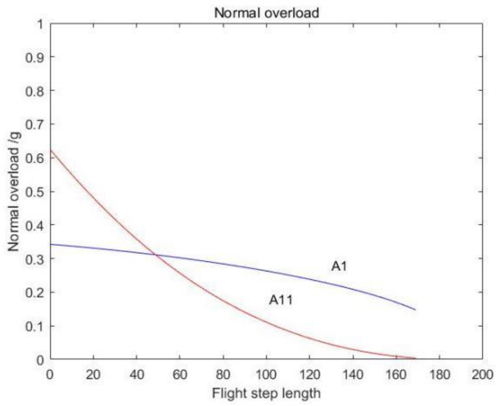

Figure 4: Normal overload of A1,A11

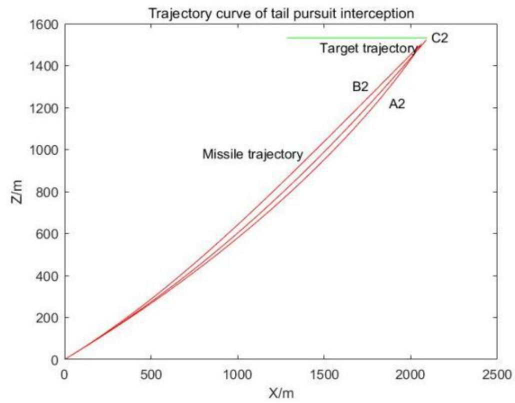

Figure 5: Trajectory of A2,B2,C2

When the missile speed is constant, the normal overload will decrease to a certain extent with the increase of the proportional coefficient K. As shown in Figure 4.At the same time, the miss distance is also an important index for us to study whether the missile can hit the target accurately. With the increase of target maneuvering acceleration, the miss distance will also increase, and the missile maneuvering acceleration and scale coefficient will also have different effects on the miss distance.The simulation data of miss distance are shown in Table 3 and Table 4.

The simulation diagram of the relevant tail chasing intercepting target is similar to the simulation diagram of the head-on intercepting target, which will not be explained here.

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4. Conclusion

From the comprehensive point of view of the paper, the guidance law of the missile determines whether the missile can accurately hit the target. In the process of attacking the target, the missile needs to comprehensively consider the position state of the missile attack target, and select the appropriate proportional coefficient K to determine whether the missile trajectory can be carried out. Reasonable control, etc., and the process of missile attacking the target is a very complex process, and there are many factors that affect it. In the process of modeling in this paper, we have simplified the model to facilitate the analysis of various problems. Many factors We have not considered it, and this is also the direction of our next research.

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