TGTS Based Argumentation

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Abstract

Suppose two discussants hold opposite views about the level of pollution in Pinarella, and start arguing about that. Then, one of them says something like "The winner of the next UEFA Champions League final match will be Hellas Verona", which is clearly out of topic with respect to the discussion topic. How do we model such a situation in an argumentation process? Our aim here is to provide a framework capable of handling such a phenomenon, namely a situation where one of the discussants in an argumentation process goes out of topic and gives rise to a certain reaction from the other. The ingredients of such a model are: a game-theoretical-semantics with a verifier and a falsifier, a discussion and a discussion topic. We develop our framework using a Paraconsistent Weak Kleene logic (PWK), with the off-topic reading of its non-classical value, and a topic-game-theoretical-semantics.

Keywords

Topic, Weak Kleene Logic, Game-Theoretical Semantics, Topic Game-Theoretical Semantics, Argumentation theory, Argumentation Process

1. Introduction

Imagine the following situation: there are two discussants holding an opposite view about the level of pollution in Pinarella. They start arguing about that. Then, one of the two says: "Pinarella is a sweet prime number", which is clearly out of topic with respect to the discussion topic, the level of pollution in Pinarella. Suppose we need a framework capable of handling such a phenomenon, namely a situation where one of the discussants in an argumentation process goes out-of-topic. In this short paper we sketch a model to give rise of this kind of situations in an argumentation process. We first briefly introduce the paraconsistent Weak Kleene logic (PWK), which belongs to the family of the Weak Kleene logics (WK3), where the third value, **u** – traditionally understood as *nonsense*, *meaninglessness* or *undefined*[1] – has been recently interpreted as *off-topic* [4]. Then, we present a new semantics for such a logic, the *topic*

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[1]See e.g. Bochvar and Bergmann [1], Halldén [2], and Ciuni and Carrara [3].

game-theoretical semantics (TGTS), consisting of the following parts: (i) a set of discussants; (ii) a set of on-topic sentences with respect to the discussion topic; (iii) a set of discussion rules; and (iv) a strategy for the discussion capable of allowing us to distinguish on-topic and off-topic discussions.

2. PWK and the Off-topic Interpretation

Paraconsistent Weak Kleene (PWK) belongs to the family of the Weak Kleene logics (WK3).[2] The language of PWK is the standard propositional language, $\mathcal{L}$. Given a nonempty countable set $\mathsf{Var} = \{p, q, r, \dots\}$ of atomic propositions, the language is defined by the following Backus-Naur Form:

$$\Phi_{\mathcal{L}} ::= p \mid \neg\phi \mid \phi \vee \psi \mid \phi \wedge \psi \mid \phi \supset \psi$$

We use $\phi, \psi, \gamma, \delta \dots$ to denote arbitrary formulas, $p, q, r, \dots$ for atomic formulas, and $\Gamma, \Phi, \Psi, \Sigma, \dots$ for sets of formulas. Propositional variables are interpreted by a valuation function $V_a : \mathsf{Var} \longmapsto \{\mathbf{t}, \mathbf{u}, \mathbf{f}\}$ that assigns one out of three values to each $p \in \mathsf{Var}$. The valuation extends to arbitrary formulas according to the following definition:

Definition 2.1 (Valuation). *A valuation* $V : \Phi_{\mathcal{L}} \longmapsto \{\mathbf{t}, \mathbf{u}, \mathbf{f}\}$ *is the unique extension of a mapping* $V_a : \mathsf{Var} \longmapsto \{\mathbf{t}, \mathbf{u}, \mathbf{f}\}$ *that is induced by the tables from Table 1.*

ϕ	$\neg\phi$
t	**f**
u	**u**
f	**t**

$\phi \vee \psi$	**t**	**u**	**f**
t	**t**	**u**	**t**
u	**u**	**u**	**u**
f	**t**	**u**	**f**

$\phi \wedge \psi$	**t**	**u**	**f**
t	**t**	**u**	**f**
u	**u**	**u**	**u**
f	**f**	**u**	**f**

Table 1
Weak tables for logical connectives in $\mathcal{L}$

Table 1 provides the full *weak tables* from Kleene et al. [5, §64]. As usual, $\phi \supset \psi =_{def} \neg\phi \vee \psi$, and its table follows accordingly. The way **u** transmits is called *contamination*, since for all formulas ϕ in $\mathcal{L}$ and any valuation V, $V(\phi) = \mathbf{u}$ iff $V_a(p) = \mathbf{u}$ for some $p \in var(\phi)$, where $var(\phi)$ is the set of all and only the atomic propositions occurring in ϕ.

The logical consequence relation of PWK is defined as preservation of non-false values – i.e., the designated values are both **u** and **t**.

The third value, **u** has been traditionally understood as *nonsense*, *meaninglessness* or *undefined*. Recently, [4] gives a new interpretation of it. He proposes to "[...] read the value 1 not simply as *true* but rather as *true and on-topic*, and similarly 0 as *false and on-topic*. Finally, read the third value 0.5 as *off-topic*" [4, p. 140].[3] Unfortunately, Beall [4] is silent about *what a topic is*. But we can make some assumptions and develop his proposal in order to make it complete and suitable for our purposes.

[2] On these systems see e.g. [5], [1], [6], and [7].

[3] Of course, 1, 0 and 0.5 correspond to **t**, **f** and **u**, respectively.

We assume that topics can be represented by sets. We use bold letters for topics, such as **s**, **t**, etc. $\subseteq$ is the inclusion relation between topics, so that $\mathbf{s} \subseteq \mathbf{t}$ expresses that **s** is included into (or is a subtopic of) **t**.[4] Given that, we define a *degenerate* topic as one that is included in every topic. Also, we define the overlap relation between topics as follows: $\mathbf{s} \cap \mathbf{t}$ iff there exists a non-degenerate topic **u** such that $\mathbf{u} \subseteq \mathbf{s}$ and $\mathbf{u} \subseteq \mathbf{t}$. Further, it is assumed that every meaningful sentence α comes with a *least* subject matter, represented by $\tau(\alpha)$. $\tau(\alpha)$ is the unique topic which α is about, such that for every topic α is about, $\tau(\alpha)$ is included into it. Thus, we say that α is *exactly* about $\tau(\alpha)$. But α can also be *partly* or *entirely* about other topics: α is entirely about **t** iff $\tau(\alpha) \subseteq \mathbf{t}$, whereas α is partly about **t** iff $\tau(\alpha) \cap \mathbf{t}$.

Next, we assume the following conditions concerning how topics behave with respect to the logical connectives:

1. $\tau(\phi \wedge \psi) = \tau(\phi) \cup \tau(\psi)$.
2. $\tau(\phi \vee \psi) = \tau(\phi) \cup \tau(\psi)$.
3. $\tau(\neg\phi) = \tau(\phi)$.

As shown in Carrara et al. [8, §2], from these assumptions we can also prove that the topic of a complex sentence boils down to the union of the topics of its atomic components.

Further, not only do sentences have a topic, but also sets of sentences do. More in detail, we have the following:

Definition 2.2. *Given a set S of sentences of $\mathcal{L}$, i.e. $S \subseteq \Phi_{\mathcal{L}}$, the topic of S, that is $\tau(S)$, is such that $\tau(S) = \bigcup\{\tau(\phi) \mid \phi \in S\}$.*

Then, since both theories and arguments can be represented by sets of sentences, we can legitimately speak about their topics. Moreover, as shown by Carrara et al. [8, Corollary 2.2], what a set of sentences S is about boils down to the union of what the atomic components of each claims in S are about: that is, $\tau(S) = \bigcup\{\tau(p) \mid p \in var(S)\}$, where $var(S)$ is the set of all and only the atomic variables occurring in the sentences that belong to S.

Finally, let us set a reference (or discourse) topic, τ_R, that is the topic that one or more agents discuss/argue about. Then, a sentence ϕ, or an argument A, or a theory T[5] are off-topic with respect to τ_R iff $\tau(\phi), \tau(A), \tau(T) \nsubseteq \tau_R$ – i.e. iff ϕ, A and T are not entirely about τ_R. Given such a regimentation of the notion of topic and Beall's off-topic interpretation of **u**, our aim now is to use them to get an argumentation framework based on PWK.

3. TGTS

In this section we present a new PWK semantics, the *topic game-theoretical semantics* (TGTS), which is based on Hintikka's game-theoretical semantics (GTS). TGTS consists of the following parts: (i) a set of discussants, I = {verifier, falsifier}; (ii) a set Φ_R of on-topic sentences with respect to the discussion topic (τ_R), that essentially depends on the set of the on-topic atomic

[4]The inclusion relation, $\subseteq$, is usually taken to be reflexive, so that every topic includes itself.
[5]Here, arguments and theories are taken to be sets of sentences.

propositions, Var_R; (iii) a set of discussion rules, $\{R_0, R_1\}$; and (iv) a non-losing strategy condition.

Let us now discuss these parts in more details. At the beginning of the discussion, the verifier and the falsifier hold opposite (classical) opinions – i.e., **t** and **f** – about a given proposition, say ϕ. We denote such a discussion with $D(\phi)$. Thus, assume there is a reference/discussion topic, τ_R, which generates a partition on $\Phi_\mathcal{L}$ that separates all the on-topic sentences from the off-topic ones. As before, any sentence $\psi \in \Phi_\mathcal{L}$ is on-topic with respect to the discussion topic iff $\tau(\psi) \subseteq \tau_R$. If this is not the case, then ψ is off-topic. We call Φ_R the set of all and only the on-topic sentences with respect to τ_R. Then, $\Phi_\mathcal{L} \smallsetminus \Phi_R$ is the set of all and only the off-topic sentences. Moreover, also the set of atomic propositions, Var, divides into the set of on-topic atomic propositions, Var_R, and the set of the off-topic ones, $\mathsf{Var} \smallsetminus \mathsf{Var}_R$.

The following rules constrain how the discussion is made:

Definition 3.1 (Discussion Rules). *For any $\phi \in \Phi_\mathcal{L}$, the discussion $D(\phi)$ is divided into two sub-discussions, $D_0(\phi)$ and $D_1(\phi)$, which will each take place in turn.*

The rules for $D_0(\phi)$ are as follows:

- *($R_0.At$) If $p \notin \mathsf{Var}_R$, then the two discussants reach a draw and close the discussion. Otherwise, the two discussants move on to discussion $D_1(p)$.*
- *($R_0. \circledast (\phi, \psi)$) If $\circledast(\phi, \psi) \notin \Phi_R$, then the two discussants reach a draw and close the discussion. Otherwise, the two discussants move on to discussion $D_1(\circledast(\phi, \psi))$ (here, $\circledast$ is any well-formed formula which combines $\phi, \psi, \neg, \vee, \wedge, \supset$).*

The rules for $D_1(\phi)$ are as follows:

- *($R_1.At$) If p is true, the verifier wins $D(p)$ and the falsifier loses. If p is false, the falsifier wins $D(p)$ and the verifier loses it.*
- *($R_1.\neg$) $D_1(\neg\phi)$ is like $D_1(\phi)$, except that the roles of the two players (as defined by these rules) are interchanged.*
- *($R_1.\vee$) $D_1(\phi \vee \psi)$ begins with the choice by the verifier of δ (δ is either ϕ or ψ). The rest of the discussion is as in $D_1(\delta)$.*
- *($R_1.\wedge$) $D_1(\phi \wedge \psi)$ begins with the choice by the falsifier of δ (δ is either ϕ or ψ). The rest of the discussion is as in $D_1(\delta)$.*
- *($R_1. \supset$) $D_1(\phi \supset \psi)$ is the same as $D_1(\neg\phi \vee \psi)$.*

Based on the notion of a winning strategy in GTS, a non-losing strategy in TGTS for PWK is defined as follows:

Definition 3.2 (Non-losing Strategy). *The initial verifier (falsifier) has a non-losing strategy in $D(\phi)$ if either the discussants reach a draw in $D_0(\phi)$, or the initial verifier (falsifier) has a winning strategy in $D_1(\phi)$.*

From this definition, two facts follow immediately:

Fact 3.1. *Both of the initial verifier and falsifier have a non-losing strategy in $D(\phi)$ if and only if the discussants reach a draw in $D(\phi)$.*

Fact 3.2. *Only one of the two initial discussants has a non-losing strategy in $D(\phi)$ if and only if one discussant has a winning strategy in $D_1(\phi)$.*

4. Suggestions and Concluding Remarks

What is the relation between argumentation and TGTS? According to McBurney and Parsons [9], McBurney et al. [10], "game-theoretical semantics have also been used to study the properties of formal argumentation systems and dialogue protocols, such as their computational complexity, or the extent of truth-convergence under an inquiry dialogue protocol, and to identify acceptable sets of arguments in argument frameworks." [9, p. 272]. As [11] suggests, there are a number of mainstream argumentation semantics developed by means of structured discussion. Consider, for example, the dialogical argumentation: it emphasizes the exchange of arguments and counterarguments between agents, which includes consideration of protocols and strategies for the agents to follow. [12] proposes the dialogue-based (or dialetical) approach to logic and argumentation theory, namely dialogue logic. The same proposal has been summarized in [13], where the proof theoretical approach of Lorenzen and Lorenz [14] and the model theoretical approach of Hintikka, GTS, are included.[6] If we follow Hintikka's idea "to consider all reasoning and argumentation as a question-answer sequence, intersperse by logical (deductive inferences)" [17, pp. 307–308], we can consider a topic based discussion on a sentence as a sequence for answering a "yes or no or off-topic" question about a sentence. Here we propose that TGTS can be regarded as a type of argumentation semantics that is able to deal with the off-topic phenomenon. As we have introduced in the previous sections, the ingredients of TGTS are a discussion topic, two discussants, some specific discussion rules, and a non-losing strategy. Not only it can deal with the off-topic phenomenon, it is also able to account for the existence of a non-losing strategy in such a type of argumentation. We believe this will provide an innovative understanding of a particular class of argumentation processes, and this might set a new trend in formal argumentation.

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References

D. A. Bochvar, M. Bergmann, On a three-valued logical calculus and its application to the analysis of the paradoxes of the classical extended functional calculus, History and Philosophy of Logic 2 (1981) 87–112.

S. Halldén, The Logic of Nonsense, Uppsala Universitets Arsskrift, Uppsala, 1949.

[6] See also Saarinen [15], Rahman and Tulenheimo [16] for comparison.

R. Ciuni, M. Carrara, Semantical analysis of weak kleene logics, Journal of Applied Non-Classical Logics 29 (2019) 1–36.

J. Beall, Off-topic: A new interpretation of weak-kleene logic, The Australasian Journal of Logic 13 (2016) 136–142.

S. C. Kleene, N. de Bruijn, J. de Groot, A. C. Zaanen, Introduction to Metamathematics, volume 483, van Nostrand, New York, 1952.

S. Bonzio, Gil-Ferez, J., Paoli, F., Peruzzi, L., On paraconsistent weak Kleene logic, Studia Logica 105 (2017) 253–297.

F. Paoli, M. Pra Baldi, Proof theory of paraconsistent weak kleene logic, Studia Logica 108 (2020) 779–802.

M. Carrara, F. Mancini, W. Zhu, A topic game theoretical semantics (tgts) for $\vDash_{\mathsf{pwk}}$, manuscript (2022) 1–26.

P. McBurney, S. Parsons, Dialogue games for agent argumentation, in: Argumentation in artificial intelligence, Springer, 2009, pp. 261–280.

P. McBurney, S. Parsons, K. Atkinson, H. Prakken, A. Wyner, Talking about doing, in: From Knowledge Representation to Argumentation in AI, Law and Policy Making, College Publications, 2013, pp. 151–166.

M. Caminada, Argumentation semantics as formal discussion (2017).

P. Besnard, A. Garcia, A. Hunter, S. Modgil, H. Prakken, G. Simari, F. Toni, Introduction to structured argumentation, Argument & Computation 5 (2014) 1–4.

N. Clerbout, Z. McConaughey, Dialogical Logic, in: E. N. Zalta, U. Nodelman (Eds.), The Stanford Encyclopedia of Philosophy, ????

P. Lorenzen, K. Lorenz, Dialogische logik, Wissenschaftliche Buchgesellschaft, 1978.

E. Saarinen, Dialogue semantics versus game-theoretical semantics, in: PSA: Proceedings of the Biennial Meeting of the Philosophy of Science Association, volume 1978, 1978, pp. 41–59.

S. Rahman, T. Tulenheimo, From games to dialogues and back, in: Games: unifying logic, language, and philosophy, Springer, 2009, pp. 153–208.

J. Hintikka, Strategic thinking in argumentation and argumentation theory, Revue Internationale de Philosophie 50 (1996) 307–324.