Causes for Changing Profiles (Extended Abstract)*

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Abstract
User profiles are an essential Knowledge Representation tool in several areas of information technology. In a recent paper, Fermé *et al.* presented a formal framework for representing user profiles and profile revision operators defined through a Knowledge-Driven perspective. In this paper, we analyse the possibility of going from one given user profile to another by means of a profile revision operator. More precisely, given two profiles P and Q we present some conditions which ensure that there is a profile revision operator $\odot$ on P and a sentence α such that $P \odot \alpha = Q$. Furthermore, considering a fixed operator $\odot$, we characterize the *change formulas* α which are such that $P \odot \alpha = Q$, by identifying upper and lower bounds for their sets of models. Analogous results are obtained for the case of a "system of equations" $P_i \odot_i \alpha = Q_i$ for every $i \in \{1, \dots, m\}$. Furthermore, a similar study is carried out considering profile revision operators defined on sets of profiles (which take sets of profiles to sets profiles rather than a single profile to a single profile).

Keywords
Belief change, AGM belief revision, profile dynamics, personalized systems

1. Introduction

The study of user profiles and their dynamics over time has gained increasing attention in the field of information technology [2, 3, 4, 5, 6]. In this paper, we expand the work presented in [7] by investigating the existence of profile revision operators $\odot_i$ and of a change formula α such that a list of one or more equations of the form $\mathtt{G}_i \odot_i \alpha = \mathtt{F}_i$ is satisfied, where $\mathtt{G}_i$ and $\mathtt{F}_i$ are, respectively, the initial and final sets of profiles or single profiles of an agent.

2. Background

2.1. Formal Preliminaries

Given a set S, we will denote by $\mathcal{P}(S)$ the power set of S, *i.e.* the set of all subsets of S. Given a set A a binary relation $\preceq$ on A is:
- reflexive if and only if $\alpha \preceq \alpha$ for all $\alpha \in A$;
- transitive if and only if it holds that if $\alpha \preceq \beta$ and $\beta \preceq \delta$, then $\alpha \preceq \delta$, for all $\alpha, \beta, \delta \in A$;
- antisymmetric if and only if it holds that if $\alpha \preceq \beta$ and $\beta \preceq \alpha$, then $\alpha = \beta$, for all $\alpha, \beta \in A$.
- total if and only if $\alpha \preceq \beta$ or $\beta \preceq \alpha$ for all $\alpha, \beta \in A$.
- irreflexive if and only if $\alpha \not\prec \alpha$, for all $\alpha \in A$.
- a pre-order if it is reflexive and transitive.
- an order if it is a pre-order which is also antisymmetric.
- a strict order if it is irreflexive and transitive.[1]
- a total strict order $\prec$ on A if and only if it is a strict order and it holds that if $\alpha \neq \beta$, then $\alpha \prec \beta$ or $\beta \prec \alpha$, for all $\alpha, \beta \in A$.

Given a pre-order $\preceq$ on a set A, the associated strict part $\prec$ is defined by $\alpha \prec \beta$ if and only if $\alpha \preceq \beta$ and $\beta \not\preceq \alpha$, for all $\alpha, \beta \in A$. $\alpha \simeq \beta$ will be used to denote that $\alpha \preceq \beta$ and $\beta \preceq \alpha$.

Let A be a set and Γ be a finite subset of A. Given a total strict order $\prec$ on A, the minimum of Γ with respect to $\prec$ is denoted by $min(\Gamma, \prec)$ and is defined as follows:
$P = min(\Gamma, \prec)$ iff
$P \in \Gamma$ and $P \prec Q$ for all $Q \in \Gamma \setminus \{P\}$.

Given a pre-order $\preceq$ on A, the set of minimal elements of Γ with respect to $\preceq$ is denoted by $Min(\Gamma, \preceq)$ and is defined as follows: $Min(\Gamma, \preceq) = \{P \in \Gamma : Q \not\prec P, \text{ for all } Q \in \Gamma\}$.

We note that if $\preceq$ is a total pre-order, then
$Min(\Gamma, \preceq) = \{P \in \Gamma : P \preceq Q, \text{ for all } Q \in \Gamma\}$.

2.2. Profiles Definition

Definition 1. *[7] Let $\mathbb{L} = \ll L_1, L_2, ..., L_n \gg$ be a tuple of labels. For each $i \in \{1, \dots, n\}$ let D_i be a finite set associated with label L_i, that we will designate by the domain of L_i. A* profile, *associated with $\mathbb{L}$, denoted by $P_{\mathbb{L}}$ (or simply by P if the tuple of labels is clear from the context), is an element of $D_1 \times D_2 \times \ldots \times D_n$. The set of all profiles associated with $\mathbb{L}$ will be denoted by $\mathbb{P}_{\mathbb{L}}$ (or simply by $\mathbb{P}$ if the tuple of labels is clear from the*

[1] Every irreflexive and transitive relation on a set A is also antisymmetric.

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context).[2]

Definition 2. *[7] Given a tuple of labels $\mathbb{L} = \ll L_1, L_2, ..., L_n \gg$, for each $i \in \{1, ..., n\}$, let D_i be the domain associated with the label L_i. The alphabet of symbols of the language $\mathcal{L}_{\mathbb{L}}$ (or simply $\mathcal{L}$) associated with $\mathbb{L}$ that we will consider is:*
1. $L_1, L_2, ..., L_n$ (labels); 2. (symbol of equality); 3. (,) (punctuation symbols); 4. $a, b, ...$ (elements of $\bigcup_{i=1}^{n} D_i$); 5. $\perp$ (symbol to represent a contradiction); 6. $\neg$, $\wedge$, $\vee$, $\rightarrow$, $\leftrightarrow$ (symbols of connectives).

Definition 3. *[7] Let $\mathbb{L} = \ll L_1, L_2, ..., L_n \gg$ be a tuple of labels. For each $i \in \{1, ..., n\}$, let D_i be the domain associated with the label L_i.*
An atomic formula in $\mathcal{L}_{\mathbb{L}}$ is defined by: if L_i is a label occurring in $\mathbb{L}$ and $a \in D_i$, then $L_i = a$ is an atomic formula of $\mathcal{L}_{\mathbb{L}}$.
A well-formed formula (wff) of $\mathcal{L}_{\mathbb{L}}$ is defined by:
1. Every atomic formula of $\mathcal{L}_{\mathbb{L}}$ is a wff of $\mathcal{L}_{\mathbb{L}}$.
2. If A and B are wffs of $\mathcal{L}_{\mathbb{L}}$, so are $(\neg A)$, $(A \wedge B)$, $(A \vee B)$, $(A \rightarrow B)$ and $(A \leftrightarrow B)$.

Definition 4. *[7] Let $\mathbb{L} = \ll L_1, L_2, ..., L_k \gg$ be a set of labels. For each $i \in \{1, ..., k\}$, let D_i be the domain associated with the label L_i. A profile $P = \langle P_1, P_2, ..., P_k \rangle$ is said to satisfy a formula α, denoted by $P \models \alpha$, if it can be shown inductively to do so under the following conditions:*
1. $P \models (L_i = a)$ iff $P_i = a$; 2. $P \models (\neg\beta)$ iff $P \not\models \beta$; 3. $P \models (\beta \wedge \delta)$ iff $P \models \beta$ and $P \models \delta$; 4. $P \models (\beta \vee \delta)$ iff $P \models \beta$ or $P \models \delta$; 5. $P \models (\beta \rightarrow \delta)$ iff $P \not\models \beta$ or $P \models \delta$; 6. $P \models (\beta \leftrightarrow \delta)$ iff ($P \models \beta$ iff $P \models \delta$).

We say that P is a model of α if and only if $P \models \alpha$. The set of models of α is denoted by $\|\alpha\|$. It holds that $\| \perp \| = \emptyset$. A set of profiles Γ is said to satisfy α if and only if every profile in Γ is a model of α. We say that α is a tautology if and only if $\|\alpha\| = \mathbb{P}_{\mathbb{L}}$. We will use $\models \alpha$ to denote that α is a tautology.

The following definition introduces the notion of Γ-faithful binary relation on $\mathbb{P}_{\mathbb{L}}$.

Definition 5. *[7] Let $\mathbb{L}$ be a tuple of labels and Γ be a non-empty subset of $\mathbb{P}_{\mathbb{L}}$. A binary relation $\preceq_\Gamma$ on $\mathbb{P}_{\mathbb{L}}$ is Γ-faithful if it satisfies:*
1. If $P_i \in \Gamma$ and $P_j \in \Gamma$, then $P_i \prec_\Gamma P_j$ does not hold.
2. If $P_i \in \Gamma$ and $P_j \in \mathbb{P}_{\mathbb{L}} \setminus \Gamma$, then $P_i \prec_\Gamma P_j$.

We will use $\preceq_P$ as an abbreviations of $\preceq_{\{P\}}$. We will also write P-faithful instead of $\{P\}$-faithful. Note that if $\prec_P$ is a strict order on $\mathbb{P}_{\mathbb{L}}$, then the first condition of Definition 5 follows trivially, since $\prec_P$ is irreflexive, and the second condition can be rewritten as $P \prec_P P_i$ for all $P_i \in \mathbb{P}_{\mathbb{L}} \setminus \{P\}$.

[2]Note that $\mathbb{P}_{\mathbb{L}}$ is finite.

2.2.1. Model 1. From One Profile to One Profile

In this subsection we present the first model for profile dynamics. In this model, we revise a profile by a formula of the language obtaining as output a profile.

Definition 6. *[7] Let $\mathbb{L} = \ll L_1, L_2, ..., L_n \gg$ be a tuple of labels and let P be a profile associated with $\mathbb{L}$. An operator $\odot$ is a PtoP profile revision on P if and only if there is a P-faithful total strict order $\prec_P$ on $\mathbb{P}_{\mathbb{L}}$, , such that for all sentences α:*

$$P \odot \alpha = \begin{cases} min(\|\alpha\|, \prec_P) & \textit{if } \|\alpha\| \neq \emptyset \\ P & \textit{otherwise} \end{cases}$$

The operator $\odot$ defined as presented above will be denoted by $\odot_{\prec_P}$.

An axiomatic characterizations for PtoP profile revision operators has been presented in [7]. That axiomatic characterizations includes, among others, the following postulates, which are based on the modified version of the AGM revision postulates and the update postulates proposed by Katsuno and Mendelzon [8, 9].

(P1) If $\|\alpha\| \neq \emptyset$, then $P \odot \alpha \models \alpha$.
(P2) If $\|\alpha\| = \emptyset$, then $P \odot \alpha = P$.
(P3) If $P \models \alpha$, then $P \odot \alpha = P$.

2.2.2. Model 2. From a Set of Profiles to a Set of Profiles

We now present a model which addresses the problem of revising a set of profiles.

Definition 7. *[7] Let $\mathbb{L} = \ll L_1, L_2, ..., L_n \gg$ be a tuple of labels and let Γ be a non-empty subset of $\mathbb{P}_{\mathbb{L}}$. An operator $\odot$ is a SPtoSP profile revision on Γ if and only if there exists a Γ-faithful pre-order $\preceq_\Gamma$ on $\mathbb{P}_{\mathbb{L}}$, such that, for all sentences α:*

$$\Gamma \odot \alpha = \begin{cases} Min(\|\alpha\|, \preceq_\Gamma) & \textit{if } \|\alpha\| \neq \emptyset \\ \Gamma & \textit{otherwise} \end{cases}$$

The operator $\odot$ defined as presented above will be denoted by $\odot_{\preceq_\Gamma}$.

In [7], an axiomatic characterizations for SPtoSP profile revision operators has been presented which contains, among others, the following postulates:

(SP1) If $\|\alpha\| \neq \emptyset$, then $\Gamma \odot \alpha \subseteq \|\alpha\|$.
(SP2) If $\|\alpha\| = \emptyset$, then $\Gamma \odot \alpha = \Gamma$.
(SP3) $\Gamma \odot \alpha \neq \emptyset$.
(SP5) If $\Gamma \cap \|\alpha\| \neq \emptyset$ then $\Gamma \odot \alpha = \Gamma \cap \|\alpha\|$.

3. Profiles Dynamics

3.1. Model 1. From One Profile to One Profile

In this subsection we characterize operators $\odot_i$ and formulas α which satisfy certain sets of one or more equations of the form $P_i \odot_i \alpha = Q_i$ where P_i and Q_i are, respectively, the inicial and the final profiles of an agent.

Observation 1. *Let $\mathbb{L} = \ll L_1, L_2, ..., L_n \gg$ be a tuple of labels and let $P \in \mathbb{P}_{\mathbb{L}}$. Let $\odot : \mathcal{L}_{\mathbb{L}} \to \mathbb{P}_{\mathbb{L}}$ be a profile revision operator on P that satisfies (P1), (P2) and (P3). It holds that $P \odot \alpha = P$ if and only if $\|\alpha\| = \emptyset$ or $P \in \|\alpha\|$.*

Observation 2. *Let $\mathbb{L} = \ll L_1, L_2, ..., L_n \gg$ be a tuple of labels, $m \in \mathbb{N}_1$, and $\{P_1, \ldots, P_m\} \cup \{Q_1, \ldots, Q_m\} \subseteq \mathbb{P}_{\mathbb{L}}$. For each $i \in \{1, \ldots, m\}$ let $\odot_i : \mathcal{L}_{\mathbb{L}} \to \mathbb{P}_{\mathbb{L}}$ be profile revision operators on P_i that satisfy (P1), (P2) and (P3). If for all $i \in \{1, \ldots, m\}$ it holds that $P_i \odot_i \alpha = Q_i$ and $P_i \neq Q_i$, then $\{Q_1, \ldots, Q_m\} \subseteq \|\alpha\| \subseteq \mathbb{P}_{\mathbb{L}} \setminus \{P_1, \ldots, P_m\}$.*

Observation 3. *Let $\mathbb{L} = \ll L_1, L_2, ..., L_n \gg$ be a tuple of labels, $m \in \mathbb{N}_1$, and $\{P_1, \ldots, P_m\} \subseteq \mathbb{P}_{\mathbb{L}}$. If $\{Q_1, \ldots, Q_m\} \subseteq \|\alpha\| \subseteq \mathbb{P}_{\mathbb{L}} \setminus \{P_1, \ldots, P_m\}$. Then for all $i \in \{1, \ldots, m\}$ there exists a PtoP profile revision operator $\odot_i$ on P_i such that $P_i \odot_i \alpha = Q_i$.*

Observation 4. *Let $\mathbb{L} = \ll L_1, L_2, ..., L_n \gg$ be a tuple of labels, $m \in \mathbb{N}_1$, and $\{P_1, \ldots, P_m\} \cup \{Q_1, \ldots, Q_m\} \subseteq \mathbb{P}_{\mathbb{L}}$ be such that $P_i \neq Q_i$ for all $i \in \{1, \ldots, m\}$.*

For each $i \in \{1, \ldots, m\}$ let $\odot_{\prec_{P_i}}$ be a PtoP profile revision operator on P_i. It holds that,

$$\forall i \in \{1, \ldots, m\}\ P_i \odot_{\prec_{P_i}} \alpha = Q_i$$
$$\textit{iff}$$
$$\{Q_1, \ldots, Q_m\} \subseteq \|\alpha\| \subseteq \bigcap_{k=1}^{m} (\Omega_k \cup \{Q_k\}).$$

Where $\Omega_k = \{P_j \in \mathbb{P}_{\mathbb{L}} : Q \prec_{P_k} P_j\}$.

3.2. Model 2. From a Set of Profiles to a Set of Profiles

In this subsection we present a study similar to the one carried in the previous subsection, but concerning profile revision operators on sets of profiles (rather than on single profiles).

Observation 5. *Let $\mathbb{L} = \ll L_1, L_2, ..., L_n \gg$ be a tuple of labels and let Γ be a non-empty subset of $\mathbb{P}_{\mathbb{L}}$. Let $\odot : \mathcal{L}_{\mathbb{L}} \to \mathcal{P}(\mathbb{P}_{\mathbb{L}})$ be a profile revision operator on Γ that satisfies (SP1), (SP2) and (SP5). It holds that $\Gamma \odot \alpha = \Gamma$ if and only if $\|\alpha\| = \emptyset$ or $\Gamma \subseteq \|\alpha\|$.*

Observation 6. *Let $\mathbb{L} = \ll L_1, L_2, ..., L_n \gg$ be a tuple of labels and let Γ be a non-empty subset of $\mathbb{P}_{\mathbb{L}}$, and $\Phi \subset \Gamma$. Let $\odot : \mathcal{L}_{\mathbb{L}} \to \mathcal{P}(\mathbb{P}_{\mathbb{L}})$ be a profile revision operator on Γ that satisfies (SP1), (SP2), (SP3) and (SP5). It holds that $\Gamma \odot \alpha = \Phi$ if and only if $\Gamma \cap \|\alpha\| = \Phi$ and $\Phi \neq \emptyset$.*

Observation 7. *Let $\mathbb{L} = \ll L_1, L_2, ..., L_n \gg$ be a tuple of labels. Let $m \in \mathbb{N}_1$ and for each $i \in \{1, \ldots, m\}$ let Γ_i be a non-empty subset of $\mathbb{P}_{\mathbb{L}}$ and $\odot_i : \mathcal{L}_{\mathbb{L}} \to \mathcal{P}(\mathbb{P}_{\mathbb{L}})$ be a profile revision operator on Γ_i that satisfies (SP1), (SP2) and (SP5). If, for all $i \in \{1, \ldots, m\}$, it holds that $\Gamma_i \odot_i \alpha = \Phi_i$ and $\Phi_i \not\subseteq \Gamma_i$, then $\bigcup_{i=1}^{m} \Phi_i \subseteq \|\alpha\| \subseteq \mathbb{P}_{\mathbb{L}} \setminus \bigcup_{i=1}^{m} \Gamma_i$.*

Observation 8. *Let $\mathbb{L} = \ll L_1, L_2, ..., L_n \gg$ be a tuple of labels. Let $m \in \mathbb{N}_1$ and for all $i \in \{1, \ldots, m\}$ let Γ_i and Φ_i be non-empty subsets of $\mathbb{P}_{\mathbb{L}}$ such that it holds that either $\Gamma_i \cap \Phi_i = \emptyset$ or $\Phi_i \subseteq \Gamma_i$.*
If $\bigcup_{i=1}^{m} \Phi_i \subseteq \|\alpha\| \subseteq \bigcap_{i=1}^{m} (\Phi_i \cup \mathbb{P}_{\mathbb{L}} \setminus \Gamma_i)$, then, for all $i \in \{1, \ldots, m\}$, there exists a SPtoSp profile revision operator $\odot_i$ on Γ_i such that $\Gamma_i \odot_i \alpha = \Phi_i$.

Observation 9. *Let $\mathbb{L} = \ll L_1, L_2, ..., L_n \gg$ be a tuple of labels, Γ and Φ be non-empty subsets of $\mathbb{P}_{\mathbb{L}}$ and $\Phi \neq \mathbb{P}_{\mathbb{L}}$.It holds that either (i) there exists a formula α and a SPtoSP profile revision operator $\odot$ such that $\Gamma \odot \alpha = \Phi$, or (ii) there exist formulas α_1 and α_2 and SPtoSP profile revision operators $\odot_1$ and $\odot_2$ such that $(\Gamma \odot_1 \alpha_1) \odot_2 \alpha_2 = \Phi$.*

Observation 10. *Let $\mathbb{L} = \ll L_1, L_2, ..., L_n \gg$ be a tuple of labels. Let $m \in \mathbb{N}_1$ and for $i \in \{1, \ldots, m\}$ let Γ_i and Φ_i be two distinct non-empty subsets of $\mathbb{P}_{\mathbb{L}}$. For $i \in \{1, \ldots, m\}$ let $\preceq_{\Gamma_i}$ be a Γ_i-faithfull pre-order on Γ_i such that it holds that $P_j \not\prec_{\Gamma_i} P_k$, for all $P_j, P_k \in \Phi_i$. Let $\odot_{\preceq_{\Gamma_i}}$ be an SPtoSP profile revision operator on Γ_i. It holds that,*

$$\forall i \in \{1, \ldots, m\}\ \Gamma_i \odot_{\preceq_{\Gamma_i}} \alpha = \Phi_i$$
$$\textit{iff}$$
$$\bigcup_{i=1}^{m} \Phi_i \subseteq \|\alpha\| \subseteq \bigcap_{i=1}^{m} (\Omega_i \cup \Phi_i)$$

where $\Omega_i = \{P_k \in \mathbb{P}_{\mathbb{L}} : P_n \prec_{\Gamma_i} P_k$, for some $P_n \in \Phi_i\}$.

4. Conclusion

User profiles are important tools in several areas of information technology. Given a profile, sometimes it is necessary to determine a set of tasks, or pieces of training which can transform the profile of a user into a target profile. In this paper, we have characterized the solutions of “systems of equations" (or single equations) of the form $\mathtt{G}_i \odot_i \alpha = \mathtt{F}_i$, where $\mathtt{G}_i$ and $\mathtt{F}_i$ are, respectively, the initial and final sets of profiles or single profiles of an agent and $\odot_i$ and α are the “unknowns" (that we wish to determine).

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References

[1] E. Fermé, M. Garapa, M. D. L. Reis, Causes for changing profiles (preliminary report), in: 1st Workshop on AI-driven heterogeneous data management: Completing, merging, handling inconsistencies and query-answering (ENIGMA 2023), CEUR Workshop Proceedings (CEUR-WS.org), 2023.

[2] J. Bobadilla, F. Ortega, A. Hernando, A. Gutiérrez, Recommender systems survey, Knowledge-based systems 46 (2013) 109–132.

[3] S. Zhang, L. Yao, A. Sun, Y. Tay, Deep learning based recommender system: A survey and new perspectives, ACM Computing Surveys (CSUR) 52 (2019) 1–38.

[4] D. Browne, Adaptive user interfaces, Elsevier, 2016.

[5] P. Lops, C. Musto, F. Narducci, G. Semeraro, Semantics in Adaptive and Personalised Systems, Springer, 2019.

[6] W. Deng, I. Papavasileiou, Z. Qiao, W. Zhang, K.-Y. Lam, S. Han, Advances in automation technologies for lower extremity neurorehabilitation: A review and future challenges, IEEE reviews in biomedical engineering 11 (2018) 289–305.

[7] E. Fermé, M. Garapa, M. D. L. Reis, Y. Almeida, T. Paulino, M. Rodrigues, Knowledge-driven profile dynamics (2022). Submitted manuscript.

[8] H. Katsuno, A. Mendelzon, Propositional knowledge base revision and minimal change, Journal of Artificial Intelligence 52 (1991) 263–294.

[9] H. Katsuno, A. Mendelzon, On the difference between updating a knowledge base and revising it, in: P. Gärdenfors (Ed.), Belief Revision, number 29 in Cambridge Tracts in Theoretical Computer Science, Cambridge University Press, 1992, pp. 183–203.