

On the instantiation of argument-incomplete argumentation frameworks

Antonio Yuste-Ginel¹, Carlo Proietti^{2,*}

¹Universidad Complutense de Madrid, Departamento de Lógica y Filosofía Teórica, Spain

²Consiglio Nazionale delle Ricerche, Istituto di Linguistica Computazionale «A. Zampolli» (CNR-ILC), Italy

Abstract

Argument-incomplete argumentation frameworks provide an intuitive way of representing uncertainty in argumentative contexts. It is however possible that, taking structured argumentation as a reference point, the general assumptions of these models present the same risks of hasty generalization attributed to some abstract argumentation models, as they do not have a structured counterpart. Here, we focus on a specific instantiation of argument-incomplete argumentation frameworks: rooting the uncertainty about arguments in the uncertainty about the application of ASPIC⁺-inference rules. We show (Proposition 1) that the abovementioned risk is concrete. Therefore a more fine-grained representation of uncertainty at the abstract level is needed, which we provide with implicative argument-incomplete argumentation frameworks and prove to work (Theorem 1).

Keywords

Abstract argumentation, Structured argumentation, Uncertainty, Incompleteness

1. Introduction and motivation

Encoding uncertainty about arguments and attacks is key for applying formal argumentation in several contexts, including strategic ones such as modelling opponents in a debate [1]. Recent literature on abstract argumentation witnesses models of different inspiration [2, 3, 4, 5, 6]. Yet, one question is whether such abstract models are adequate to capture proper argumentative uncertainty. This echoes more general concerns about abstract argumentation models, insofar as they open to assumptions and generalisations that are unjustified or meaningless at the structured level (as shown in [7, 8, 9]).

To address this question in more precise terms, we consider *argument-incomplete abstract argumentation frameworks* (arg-IAAFs) [10, 4, 11] as our abstract model of qualitative uncertainty. Further, we take ASPIC⁺ [7], with its notion of *structured argumentation frameworks* (SAFs), as our underlying formalism for structured argumentation. Following the suggestions by [11], uncertainty can be generated either by uncertain inference rules or by incomplete preference profiles. Here we focus on the first option: the uncertainty of an argument is explained by the uncertainty of whether one or more inference rules of this argument must be applied.

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*Corresponding author.

✉ antoyust@ucm.es (A. Yuste-Ginel); carlo.proietti@ilc.cnr.it (C. Proietti)

🆔 0000-0002-4380-3095 (A. Yuste-Ginel); 0000-0001-9044-9642 (C. Proietti)



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More procedurally, we define *rule-incomplete structured argumentation frameworks* (rul-ISAFs) as incomplete extensions of SAFs, and as natural counterparts of arg-IAAFs in abstract argumentation. As a first negative result (Proposition 1), we show that there are rul-ISAFs that cannot be represented abstractly as arg-IAAFs. However, correspondence is retrieved (Theorem 1) if we instead consider a refinement of arg-IAAFs, that we name *implicative argument-incomplete abstract argumentation framework* (imp-arg-IAAF).

Section 2 provides the necessary formal background on abstract and structured argumentation, as well as the newly defined notions. In Section 3 we prove our main results. Section 4 sketches our current research directions.

2. Background

A (Dung) **abstract argumentation framework** (AAF) is a directed graph $\langle \text{Arg}, \text{Def} \rangle$ where Arg is a set of *arguments* and $\text{Def} \subseteq \text{Arg} \times \text{Arg}$ is a *defeat relation* among them.

An **argument-incomplete abstract argumentation framework** (arg-IAAF) is a tuple $\langle \text{Arg}^F, \text{Arg}^?, \text{Def} \rangle$ where Arg^F and $\text{Arg}^?$ are two pairwise disjoint sets of arguments and $\text{Def} \subseteq (\text{Arg}^F \cup \text{Arg}^?) \times (\text{Arg}^F \cup \text{Arg}^?)$.

A **completion** of $\langle \text{Arg}^F, \text{Arg}^?, \text{Def} \rangle$ is any AAF $\langle \text{Arg}^*, \text{Def}^* \rangle$ s.t.:

- $\text{Arg}^F \subseteq \text{Arg}^* \subseteq \text{Arg}^F \cup \text{Arg}^?$.
- $\text{Def}^* = \text{Def} \upharpoonright_{\text{Arg}^*}$.

As announced, arg-IAAFs will fail to be expressive enough for capturing uncertain inference rules. That's why we need a more refined formalism. An **implicative argument-incomplete abstract argumentation framework** (imp-arg-IAAF) is $\langle \text{Arg}^F, \text{Arg}^?, \text{Def}, \Delta \rangle$ where $\langle \text{Arg}^F, \text{Arg}^?, \text{Def} \rangle$ is an argument-incomplete abstract argumentation framework and $\Delta \subseteq \text{Arg}^? \times \text{Arg}^?$ is a set of implicative dependencies. Informally, $\langle A, B \rangle \in \Delta$ means that B appears in a completion whenever A does. Formally, a **completion** of $\langle \text{Arg}^F, \text{Arg}^?, \text{Def}, \Delta \rangle$ is any AAF $\langle \text{Arg}^*, \text{Def}^* \rangle$ s.t.:

- $\langle \text{Arg}^*, \text{Def}^* \rangle$ is a completion of $\langle \text{Arg}^F, \text{Arg}^?, \text{Def} \rangle$.
- For all $\langle X, Y \rangle \in \Delta$ if $X \in \text{Arg}^*$, then $Y \in \text{Arg}^*$.

Note that imp-arg-IAAFs can be seen as a restricted class of constrained incomplete AFs [12, 13] or as a restricted class IAFs with correlations [14].

Finally, we introduce the ASPIC⁺ notions [15] that are relevant to our study. A **structured argumentation framework** (SAF) is a tuple $\text{SAF} = \langle \text{L}, \bar{\cdot}, \text{R}, \mathfrak{n}, \text{K}, \text{Arg}, \text{Att}, \preceq \rangle$ where each component is defined as follows:

- L is a **formal language**.
- $\bar{\cdot} : \text{L} \rightarrow 2^{\text{L}}$ is a **contrary function**. We say that:
 - φ is contrary of ψ iff $\psi \in \bar{\varphi}$ but $\varphi \notin \bar{\psi}$.
 - φ is contradictory of ψ (noted $\varphi = -\psi$) iff $\psi \in \bar{\varphi}$ and $\varphi \in \bar{\psi}$.

Each $\varphi \in \text{L}$ is assumed to have at least one contradictory.

- $R = R_s \cup R_d$ with $R_s \cap R_d = \emptyset$ is a set of **inference rules** (sequences of elements of L). R_s represents strict rules while R_d represents defeasible rules.
- $n : R_d \rightarrow L$ is a partial **naming function** for defeasible rules.
- $K \subseteq L$ is a **knowledge base**, assumed to be split into two disjoint subsets K_n (axioms) and K_p (ordinary premises).
- Arg is the set of **arguments** of SAF, which is defined inductively together with some auxiliary functions: $\text{Sub}(\cdot)$ (returns the **subarguments** of an argument), $\text{Prem}(\cdot)$ (returns the **premises** of an argument), $\text{Conc}(\cdot)$ (returns the **conclusion** of an argument), and $\text{TopRule}(\cdot)$ (returns the **last rule** employed in the construction of an argument). We have that $A \in \text{Arg}$ iff:
 - $A = [\varphi]$ if $\varphi \in K$, with $\text{Prem}(A) = \text{Conc}(A) = \{\varphi\}$, $\text{Sub}(A) = \{[\varphi]\}$ and $\text{TopRule}(A)$ is left undefined.
 - $A = [A_1, \dots, A_n \rightarrow \varphi]$ if A_i is an argument for $1 \leq i \leq n$ and $(\text{Conc}(A_1), \dots, \text{Conc}(A_n), \varphi) \in R_s$, with $\text{Prem}(A) = \bigcup_{1 \leq i \leq n} \text{Prem}(A_i)$, $\text{Conc}(A) = \varphi$, $\text{Sub}(A) = \{A\} \cup \bigcup_{1 \leq i \leq n} \text{Sub}(A_i)$, $\text{TopRule}(A) = (\text{Conc}(A_1), \dots, \text{Conc}(A_n), \varphi)$.
 - $A = [A_1, \dots, A_n \Rightarrow \varphi]$ if A_i is an argument for $1 \leq i \leq n$ and $(\text{Conc}(A_1), \dots, \text{Conc}(A_n), \varphi) \in R_d$, with $\text{Prem}(A) = \bigcup_{1 \leq i \leq n} \text{Prem}(A_i)$, $\text{Conc}(A) = \varphi$, $\text{Sub}(A) = \{A\} \cup \bigcup_{1 \leq i \leq n} \text{Sub}(A_i)$, $\text{TopRule}(A) = (\text{Conc}(A_1), \dots, \text{Conc}(A_n), \varphi)$.
- $\text{Att} \subseteq \text{Arg} \times \text{Arg}$ is the **attack relation** of SAF. ASPIC⁺ allows for three kinds of attacks. We say that A **attacks** B (i.e., $\langle A, B \rangle \in \text{Att}$) iff A undermines, rebuts or undercuts B , where:
 - A **undermines** B (on B') iff $\text{Conc}(A) \in \bar{\varphi}$ for some $B' = \varphi \in \text{Prem}(B)$ and $\varphi \in K_p$.
 - A **rebuts** B (on B') iff $\text{Conc}(A) \in \bar{\varphi}$ for some $B' = \text{Sub}(B)$ of the form $B'_1, \dots, B'_n \Rightarrow \varphi$.
 - A **undercuts** B (on B') iff $\text{Conc}(A) \in \overline{n(\text{TopRule}(B'))}$ for some $B' = \text{Sub}(B)$ with $\text{TopRule}(B') \in R_d$.
- $\preceq \subseteq \text{Arg} \times \text{Arg}$ is a preference relation among arguments, with $\prec = \preceq \setminus \preceq^{-1}$ its strict counter-part.

Given $\text{SAF} = \langle L, \bar{\cdot}, R, n, K, \text{Arg}, \text{Att}, \preceq \rangle$, we use $L(\text{SAF})$ to denote L and apply the same convention for the rest of the components. Let $A, B \in \text{Arg}(\text{SAF})$, we say that A **defeats** B iff: (i) A undercuts B , or (ii) A undermines/rebuts B (on B') and $A \not\prec B'$. The set of all defeats for a given SAF is denoted $\text{Def}(\text{SAF})$. Finally, given SAF, the **Dung's argumentation framework associated to SAF** is just $\text{DAF}(\text{SAF}) = (\text{Arg}(\text{SAF}), \text{Def}(\text{SAF}))$.

3. Structured frameworks with uncertain inference rules

As mentioned, here we consider the set of rules of a given structured argumentation framework as a source of uncertainty for the presence of arguments. In the same spirit of IAFs, one can define a **rule-incomplete structured argumentation framework** as a tuple $\text{rul-ISAF} = \langle L, \bar{\cdot}, R, n, K, \text{Arg}, \text{Att}, \preceq \rangle$, where every component is just as in a SAF except from the set of rules R , which is split into four pairwise disjoint subsets $R = R_s^F \cup R_s^? \cup R_d^F \cup R_d^?$, representing respectively certain strict rules, uncertain strict rules, certain defeasible rules and uncertain defeasible rules. Then, a **rule-completion of** rul-ISAF is any $\text{SAF}^* = \langle L, \bar{\cdot}, R^*, n, K, \text{Arg}^*, \text{Att}^*, \preceq^* \rangle$ where:

- $R^* = R_s^* \cup R_d^*$ is s.t.:
 - $R_s^F \subseteq R_s^* \subseteq (R_s^F \cup R_s^?)$;
 - $R_d^F \subseteq R_d^* \subseteq (R_d^F \cup R_d^?)$.
- Arg^* and Att^* are the set of arguments and attacks generated by R^* .
- $\preceq^* = \preceq_{\uparrow \text{Arg}^*}$.

We denote by $\text{rul-completions}(\text{ISAF})$ the set of rule-completions of rul-ISAF .
Finally, let rul-ISAF be given, its set of **completions** is simply defined as:

$$\text{completions}(\text{rul-ISAF}) = \{\text{DAF}(\text{SAF}^*) \mid \text{SAF}^* \in \text{rul-completions}(\text{ISAF})\}.$$

Our research question can be now put in more precise terms: is the set of completions of a rul-ISAF always equal to the set of completions of some arg-IAAF ? The answer is negative:

Proposition 1. *Given rul-ISAF , it is not necessarily the case that there is arg-IAAF s.t.: $\text{completions}(\text{rul-ISAF}) = \text{completions}(\text{arg-IAAF})$.*

Proof. A simple counterexample is provided by considering any rul-ISAF where L is the language of propositional logic containing atoms p, q and r , $\bar{\cdot}$ is classical negation, with $R_s^F = R_s^? = \emptyset$, $R_d^F = \{(q, r)\}$, and $R_d^? = \{(p, q)\}$, $K_s = \emptyset$, $K_p = \{p\}$, and $\preceq = \emptyset$. Then, $\text{completions}(\text{rul-ISAF})$ has two members, namely $\langle \{[p]\}, \emptyset \rangle$ and $\langle \{[p], [[p] \Rightarrow q], [[[p] \Rightarrow q] \Rightarrow r]\}, \emptyset \rangle$. Clearly, for basic cardinality reasons, no arg-IAAF has an isomorphic set of completions, since a completion with one argument and a completion with three arguments would force the presence of another completion with two arguments. \square

As mentioned, one way to interpret this result is that nothing guarantees that the completion of an $(\text{arg-})\text{IAAF}$ is subargument closed, e.g., that $[[[p] \Rightarrow q] \Rightarrow r]$ forces the presence of $[[p] \Rightarrow q]$, as in the proof of Proposition 1.

Theorem 1. *Let rul-ISAF be a rul-ISAF , there exists an imp-arg-IAAF s.t.*

$$\text{completions}(\text{rul-ISAF}) = \text{completions}(\text{imp-arg-IAAF}).$$

Sketch of the proof. Let $\text{rul-ISAF} = \langle L, \bar{\cdot}, R, n, K, \text{Arg}, \text{Att}, \preceq \rangle$ be a rul-ISAF , we will use two of its completions in the proof:

- SAF^F is the rule-completion whose set of rules is $R^F = R_s^F \cup R_d^F$.
- SAF^{max} is the rule-completion whose set of rules is $R^{\text{max}} = R_s^F \cup R_d^F \cup R_s^? \cup R_d^?$.

Now, we are going to build the target $\text{imp-arg-IAAF} \langle \text{Arg}^F, \text{Arg}^?, \text{Def}, \Delta \rangle$:

- $\text{Arg}^F = \text{Arg}(\text{SAF}^F)$.
- $\text{Arg}^? = \text{Arg}(\text{SAF}^{\text{max}}) \setminus \text{Arg}(\text{SAF}^F)$.
- $\text{Def} = \text{Def}(\text{SAF}^{\text{max}})$.
- $\Delta = \{\langle X, Y \rangle \in \text{Arg}^? \times \text{Arg}^? \mid Y \in \text{Sub}(X)\}$.

We show that both directions of the equality $\text{completions}(\text{rul-ISAF}) = \text{completions}(\langle \text{Arg}^F, \text{Arg}^?, \text{Def}, \Delta \rangle)$ hold:

[\subseteq] Suppose $\langle \text{Arg}^*, \text{Def}^* \rangle \in \text{completions}(\text{rul-ISAF})$, which amounts by definition of completions to

(H1) $\langle \text{Arg}^*, \text{Def}^* \rangle = \text{DAF}(\text{SAF}^*)$ for some $\text{SAF}^* \in \text{rul-completions}(\text{ISAF})$

Hence, we just need to check that $\langle \text{Arg}^*, \text{Def}^* \rangle$ satisfies the conditions for being a completion of $\langle \text{Arg}^F, \text{Arg}^?, \text{Def}, \Delta \rangle$, namely: (a) $\text{Arg}^F \subseteq \text{Arg}^* \subseteq \text{Arg}^F \cup \text{Arg}^?$; (b) $\text{Def}^* = \text{Def}_{\uparrow \text{Arg}^*}$; and (c) for all $\langle X, Y \rangle \in \Delta$ if $X \in \text{Arg}^*$, then $Y \in \text{Arg}^*$.

To do so, we need to establish the following claims, whose proof we omit:

Lemma 1. *Let SAF and SAF' only differ in their set of inference rules. Then:*

1. $R(\text{SAF}) \subseteq R(\text{SAF}')$ implies $\text{Arg}(\text{SAF}) \subseteq \text{Arg}(\text{SAF}')$.
2. $R(\text{SAF}) \subseteq R(\text{SAF}')$ implies $\text{Def}(\text{SAF}) = \text{Def}(\text{SAF}')_{\uparrow \text{Arg}(\text{SAF})}$.
3. $\text{Arg}(\text{SAF})$ is closed under subarguments (i.e., $X \in \text{Arg}(\text{SAF})$ implies $\text{Sub}(X) \subseteq \text{Arg}(\text{SAF})$).

Based on how we defined our target imp-arg-IAAF and H1, condition 1 of Lemma 1 entails (a), since $R(\text{SAF}^F) \subseteq R(\text{SAF}^*) \subseteq R(\text{SAF}^{\text{max}})$. For the same reason, (b) follows from condition 2 by $R(\text{SAF}^*) \subseteq R(\text{SAF}^{\text{max}})$. Finally, condition 3 entails (c) by how we set Δ .

[\supseteq] Suppose $\langle \text{Arg}^*, \text{Def}^* \rangle \in \text{completions}(\langle \text{Arg}^F, \text{Arg}^?, \text{Def}, \Delta \rangle)$. Then conditions (a)-(c) above are satisfied. We need to show that $\langle \text{Arg}^*, \text{Def}^* \rangle \in \text{completions}(\text{rul-ISAF})$, which means, by definition, that for some $\text{SAF} \in \text{rul-completions}(\text{ISAF})$ we have $\text{DAF}(\text{SAF}) = \langle \text{Arg}^*, \text{Def}^* \rangle$. Now, let us consider $\text{SAF}' = \langle L, \bar{\cdot}, R', n, K, \text{Arg}', \text{Att}', \preceq' \rangle$ where $R' = \{\text{TopRule}(X) \mid X \in \text{Arg}^*\}$, $\text{Arg}' = \text{Arg}^*$, $\text{Att}' = \text{Att}_{\uparrow \text{Arg}'}$ and $\preceq' = \preceq_{\uparrow \text{Arg}'}$ (it is easy to check that SAF' is actually a SAF). Using conditions (a)-(c) above, one can show that either SAF' is the rule-completion of ISAF we are looking for, or that there is one rule-completion SAF^* with $R(\text{SAF}') \subseteq R(\text{SAF}^*)$ s.t. $\text{DAF}(\text{SAF}^*) = \text{DAF}(\text{SAF}')$. \square

4. Future work

A natural completion of the present work is to investigate whether the existential claim of Theorem 1 holds in the other direction as well, i.e. for every imp-arg-IAAF there is a rul-ISAF with an isomorphic set of completions. Together with Theorem 1, this would amount to a characterization result for the class of rul-ISAF s. We conjecture that this is in fact the case, but the proof is not as direct as that of Theorem 1 and we leave it for future work.

Another task is to investigate the second root of uncertainty mentioned by [11, Section 8.1.]: incomplete defeats based on incomplete preference profiles. Here again, we think a negative result, analogous to Proposition 1, obtains. As for the analogous to Theorem 1, we think that a restricted version of attack-IAAF s ([16]) with correlations could work at the abstract level.

Finally, a third pending task is to accomplish a detailed comparison of our work with the very recent paper [17], where incomplete theories rooted in incomplete knowledge bases for a special instance of ASPIC^+ are investigated. Roughly, and besides the clear contrast between uncertain inference rules vs. uncertain knowledge bases, [17] focuses on so-called stability and relevance problems, while our focus is on expressivity of uncertainty, so both approaches can be seen as complementary in these two senses.

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