

Intelligent system for planning treatment of retinopathy with laser technology

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Abstract

The novelty of this work lies in the development of a computer intelligent system, utilizing an algorithm grounded in mathematical programming, to optimize the placement of photocoagulates. The research considers effective treatment strategies for diabetic retinopathy, a serious disease that can lead to vision impairment or blindness. Retinal laser photocoagulation, which allows for the destruction of abnormal blood vessels, is identified as a key treatment method. The study highlights the importance of evenly distributing photocoagulation spots within the edematous area, avoiding contact with blood vessels. The potential of this approach is further enhanced by the development of an algorithm for optimal photocoagulate placement, based on mathematical programming. A numerical example is given. Future research will focus on a more precise formal description of the photocoagulation domain, giving opportunities for even more accurate and effective treatments. The continuous refinement of these methods is crucial for advancing the field and improving patient outcomes.

Keywords

Intelligent system, modelling, optimization, IPOPT, retinal laser photocoagulation, circle, mathematical programming

1. Introduction

In the ever-evolving landscape of medical science, the fusion of intelligence and modern computer technology has paved the way for transformative breakthroughs in various domains. Ophthalmology, with its intricate focus on visual health, has been significantly influenced by these advancements. Among the myriad ocular challenges, diabetic retinopathy stands as a formidable adversary, necessitating innovative solutions for both diagnosis and treatment. The crucial role of applying cutting-edge intelligent computer technologies in addressing these challenges cannot be overstated, as they play a pivotal role in shaping the future of diagnostics and therapeutic interventions in the field.

Diabetic retinopathy, a complication stemming from diabetes, poses a substantial global health burden. Its progressive nature, coupled with the potential for severe vision impairment, underscores the urgency for precise and effective therapeutic strategies. Traditional treatment approaches, such as laser photocoagulation, have been pivotal in managing this condition. However, the landscape is rapidly evolving, with a paradigm shift towards intelligent systems that harness the power of artificial intelligence (AI), image processing, and advanced laser technology.

The application of intelligent technologies in the context of diabetic retinopathy management signifies a leap forward in our capability to tailor treatments with unprecedented precision. The

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integration of AI-driven algorithms addresses the inherent complexity of retinal imaging data. These algorithms excel in discerning subtle anomalies indicative of diabetic retinopathy, from microaneurysms to neovascularization, thus aiding in swift and accurate diagnosis.

Moreover, the dynamic nature of diabetic retinopathy demands not only accurate diagnostics but also personalized treatment plans. This is where intelligent systems shine, offering meticulous analysis of patient-specific data to formulate targeted strategies. Through machine learning, these systems adapt and evolve, continuously refining their understanding of the disease and optimizing treatment recommendations.

The advent of laser technology has been a cornerstone in retinal therapy, and its synergy with intelligent systems enhances its efficacy. Laser photocoagulation, traditionally applied uniformly, can now be precisely tailored through intelligent planning. This not only ensures that affected areas are accurately targeted but also minimizes collateral damage to healthy tissue.

The intricacies of diabetic retinopathy treatment underscore the need for advanced technologies. The condition's multifaceted nature, varying from patient to patient, demands a nuanced and personalized approach. Conventional methods, while effective to a certain extent, often fall short in delivering the precision required for optimal outcomes.

The integration of intelligent systems with laser technology marks a paradigm shift towards precision medicine in diabetic retinopathy management. By leveraging the power of machine learning, these systems not only aid in diagnosis but also empower clinicians with insights into disease progression and response to treatment.

As we delve deeper into the intricate interplay between intelligent systems and laser technology for diabetic retinopathy, this article aims to explore the current landscape, highlight recent advancements, and envision the future trajectory of this burgeoning field. Through a comprehensive examination of the challenges, opportunities, and ethical considerations, we endeavor to provide a holistic understanding of the transformative potential these technologies hold in reshaping the landscape of diabetic retinopathy management.

Retinal laser photocoagulation is a crucial procedure used to treat various retinal conditions, including diabetic retinopathy [1]. This procedure uses a laser to generate heat and create a burn, which develops into scar tissue in a targeted area of the retina. The scar tissue helps to seal off abnormal or leaky blood vessels that can develop in certain retinal diseases, such as diabetic retinopathy. This can help to stabilize vision and prevent further vision loss.

The primary thrust of this research is directed towards the meticulous delineation of the photocoagulation domain through a precise and formal description. This involves delving into the intricate details of the mathematical methods employed for the strategic placement of photocoagulators within a designated area, especially one characterized by irregular shapes. The latest advancements in the domain of dense packing of circles, as depicted in Fig. 1, serve as a foundational inspiration for the development and refinement of these mathematical methodologies.

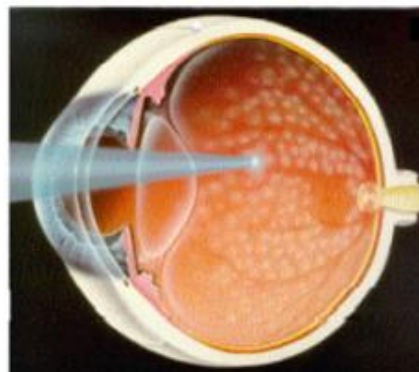


Figure 1: Retinal laser photocoagulation

In essence, the research endeavors to establish a comprehensive understanding of the photocoagulation domain, aiming for precision in both its definition and application. This entails a profound exploration of the geometric intricacies associated with irregular shapes, and the utilization of cutting-edge mathematical techniques to optimize the arrangement of photocoagulators within such complex spatial configurations.

The focal point lies in leveraging the advancements in dense packing of circles, as showcased in Fig. 1, to inform the development of methodologies that ensure an efficient and effective distribution of photocoagulation points. This not only entails addressing the challenges posed by irregular shapes but also delves into maximizing the coverage and minimizing the occurrence of untreated areas within the designated region.

The utilization of mathematical methods becomes crucial in this context, as they provide a systematic and algorithmic approach to handle the complexities inherent in the irregular shape of the treatment area. The research aspires to contribute to the refinement of these methods, ensuring their adaptability to real-world scenarios and their applicability in optimizing the outcomes of photocoagulation treatments.

The management of diabetic macular edema involves a sophisticated approach incorporating both conservative measures and advanced laser surgical techniques. Laser photocoagulation targeting the macular region of the retina stands out as the primary therapeutic approach for diabetic macular edema [2]. This intricate procedure entails the precise application of controlled microburns, known as laser photocoagulates, within the edematous region of the retina. These photocoagulates are administered individually or in a carefully designed pattern, with predetermined positioning, followed by the implementation of the generated plan onto the retinal image.

A relevant strategy involves uniformly distributing photocoagulation spots, ensuring their placement at maximum, evenly spaced intervals within the edematous area, while carefully avoiding contact with blood vessels. The application of these photocoagulates with premeditated planning is executed under the supervision of an automatic laser beam positioning system, ensuring meticulous and precise treatment. Nevertheless, a notable challenge in the photocoagulation procedure often arises from the manual assignment of the photocoagulation map, leading to suboptimal placement or inadvertent application within restricted regions of the fundus (macular zones). Additionally, manual placement of the photocoagulates significantly prolongs the duration of the operation.

2. Related papers

Artificial Intelligence and Machine Learning (ML) have become important tools in modern medicine, including ophthalmology. Many studies confirm the effectiveness of using AI and ML for analyzing retinal images and diagnosing various eye diseases.

Photocoagulation is a standard method of treating retinopathy. However, the effectiveness of this method largely depends on the accuracy of photocoagulation placement planning.

Several studies have been conducted to develop systems that use AI for photocoagulation planning. These systems have shown promising results but still require further research and improvements.

The first experiments on light coagulation were conducted in 1946. In 1949, the first successful treatment of retinal detachment using a light beam (light coagulation) was carried out with a device constructed by Gerhard Meyer-Schwickerath.

The Early Treatment Diabetic Retinopathy Study (ETDRS), published in 1985, was a significant milestone in the treatment of diabetic macular edema. This randomized, prospective, multicenter, clinical trial was designed to evaluate the role of aspirin and argon laser photocoagulation in patients with nonproliferative and early proliferative diabetic retinopathy [3, 4]. Conventional macular focal and grid laser photocoagulation were established as the treatment of choice for diabetic macular edema based on the findings of the ETDRS.

Pattern SCanning Laser is the first laser device for ophthalmology with automated positioning of laser pulses based on high-speed mirrors and a set of templates. The system allows for the pre-positioning and stabilization of the laser beam on the retina, in threshold and subthreshold treatments [5].

NAVILAS (Navigation Laser) is a technology for digital navigation on the retina [6]. Based on the same high-speed mirrors as the pattern scanning technology it also includes retinal photography, digital operation planning, and continuous tracking of the retinal position during the operation to ensure safety and accuracy of laser pulse application

The paper [7] proposes a new information technology for creating a macular photocoagulation map in the laser treatment of diabetic retinopathy and evaluating the treatment strategy's effectiveness.

Paper [8] proposes the development of a computer system for high-tech medical use in ophthalmology. It presents an overview of the main methods and algorithms that formed the basis of the coagulation planning system

The article [9] is a comprehensive review of recent advancements in the detection of diabetic retinopathy using deep learning. The paper conducts an in-depth investigation into several recently proposed frameworks based on machine learning and deep learning networks to classify non-proliferative diabetic retinopathy, exudates, hemorrhages, and micro aneurysms. It discusses the use of several promising pre-trained deep learning models to classify stages of diabetic retinopathy. The authors also explore the use of transfer learning on pre-trained models like GoogLeNet, AlexNet, VGG, etc. They note that almost all public and private datasets widely available for research are imbalanced¹. To address these issues, generative adversarial networks (GANs) and their variants were used to generate label-preserving data.

The article [10] presents the main points of applying artificial intelligence in ophthalmology for forming a coagulate map to carry out laser eye treatment, exemplified by the development of a computer system for personalizing retinal laser photocoagulation.

Currently existing software packages predominantly concentrate on the utilization of predefined templates (patterns) for placing photocoagulates [11]. As a result, the effectiveness of photocoagulate coverage within the edematous area is sacrificed to accelerate the procedure. The use of templates leads to an uneven distribution of photocoagulates. Consequently, the creation of an algorithm that ensures optimal photocoagulate placement while minimizing the procedural time continues to be a significant challenge.

The process of administering a sequence of photocoagulates to the eye's fundus is regulated by specific conditions. It is essential to avoid overlap with the macular area, major blood vessels, and healthy regions. The coagulation zone should cover the maximum area of the exudate region.

Additionally, it is crucial to prevent the overlap of photocoagulate regions to ensure that the microburn dose remains within acceptable limits.

The majority of current approaches addressing the challenge of densely packing circles are tailored for regions with fixed shapes [12]. Conversely, methods specifically developed for arbitrary regions tend to exhibit high computational complexity, leading to procedure times that surpass acceptable limits.

A variety of heuristic algorithms, with the goal of optimizing the placement of photocoagulates, were introduced in [13]. These algorithms offer "pseudo-optimal" solutions. Unlike the challenge of densely packing circles, the problem of determining photocoagulate placement does not assure an optimal solution. This means there is no guarantee for achieving the maximum number of photocoagulates in the area of interest. The first algorithm involves the random filling of a designated area with circles. The second algorithm employs an ordered approach, using a lattice placement for the circles. The remaining five algorithms are adaptations of a sequential single placement strategy incorporating heuristic rules. To assess the effectiveness of the placement, the authors examine various criteria, with a key focus on the number of photocoagulates placed. This parameter serves as a crucial measure, determining the density of filling the affected area.

The literature confirms the potential of using AI in ophthalmology and, in particular, for photocoagulation planning [14-16]. However, further research is needed to optimize these systems and implement them in clinical practice.

A scrutiny of prevailing laser coagulation techniques in the context of treating diabetic retinopathy reveals that contemporary methods fall short of delivering the requisite efficacy in retinal laser coagulation treatment. This deficiency arises from the nonuniform distribution of laser energy across the pigment epithelium, potentially resulting in an excessive impact on certain areas of the retina and its anatomical components. The findings from the overview emphasize that the effectiveness of retinal laser coagulation for diabetic retinopathy hinges on the relative positioning of coagulates and the parameters governing laser exposure.

The delineated stages in the processing of diagnostic data for the formulation and intellectual analysis of a preliminary coagulation plan include the selection of the laser exposure zone, assessment of safe parameters for the laser pulse reaching the fundus, creation of a coagulation plan within the laser exposure zone, and a subsequent analysis of the generated plan to forecast the therapeutic effect.

Our main objective in this study is to devise an algorithm that addresses the challenge of densely arranging circles within a polygonal domain, taking into account restricted areas through the utilization of mathematical programming-based packing methods. The goal is to maximize the packing factor.

3. Mathematical description of the coagulant placement

One of the predominant limitations inherent in established algorithms designed for the placement of photocoagulants lies in their tendency to generate sparse or pre-defined pattern solutions. Recognizing the imperative need to enhance the coverage of photocoagulants on the fundus, we advocate for a novel approach that leverages the principles of mathematical programming.

This innovative method aims to overcome the drawbacks of conventional algorithms by adopting a mathematical programming framework. Unlike existing solutions that often result in sparse patterns, our proposed method seeks to optimize the distribution of photocoagulants on the fundus through a systematic application of mathematical programming principles. This strategic utilization of mathematical programming not only addresses the inadequacies of current algorithms but also introduces a more dynamic and adaptable framework for achieving comprehensive coverage during photocoagulation procedures.

By incorporating mathematical programming principles, our proposed method endeavors to introduce a level of flexibility and precision that is instrumental in avoiding the generation of sparse or predetermined patterns. This shift in approach holds the promise of significantly improving the efficacy of photocoagulation treatments, ensuring a more uniform and effective distribution of therapeutic interventions on the fundus. The formal description of photocoagulant placement in the treatment area employs the well-known method of phi-functions. The placement region has, in general, an irregular shape, which can be described as the union of non-intersecting non-convex polygons, separated by vessel walls (Fig.2).

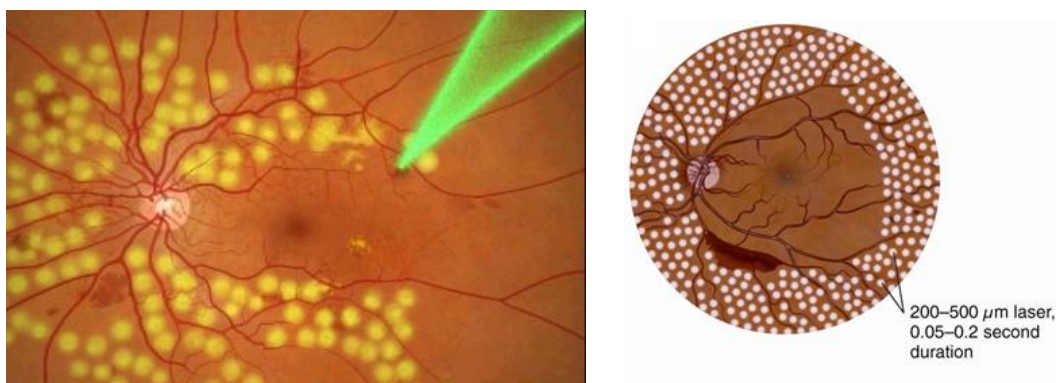


Figure 2: Treatment area

Each non-convex polygon is represented as a convex polygon from which a finite number of convex polygons have been removed.

Segments are used to model the boundaries of non-penetrating vessels, i.e., vessels whose ends lie within the polygons (Fig.3).

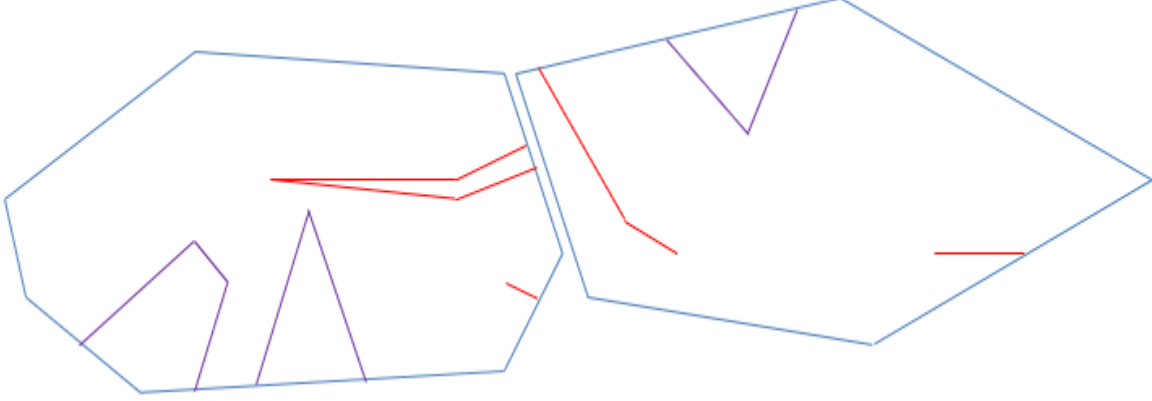


Figure 3: Segments modeling the boundaries of non-penetrating vessels

The treatment domain D is represented as a union of non-convex polygons:

$$D = \bigcup_{k=1}^p D_k, \text{ int } D_k \cap \text{int } D_l = \emptyset. \quad (1)$$

In turns,

$$D_k = P_k \setminus \left(\bigcup_{l \in Q} q_l \cup \bigcup_{m \in S} s_m \right), \quad (2)$$

where q_l are convex polygons, s_m are segments. This enables decomposition, meaning the original problem can be subdivided into independent subproblems for each polygon.

We assume burns are modeled as circles C_i with radius r . Let (x_i, y_i) , $i = 1, 2, \dots, n$ be centers of circles and δ_1 be a minimal distance between each pair of circles. Let δ_2 be a minimal distance between circles C_i and the frontiers of D_k .

The condition of placing of circles C_i and C_j taking into account the distance δ_1 is presented as

$$\Phi_{ij}(x_i, y_i, x_j, y_j) = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2} - 2r \geq \delta_1, \quad i < j \in \{1, 2, \dots, n\} \quad (3)$$

while packing C_i into D_k , $k = 1, 2, \dots, p$, to meet the distance limitation δ_2 is defined as

$$\Phi_i(x_i, y_i) \geq \delta_2, \quad i \in \{1, 2, \dots, n\}. \quad (4)$$

Here, $\Phi_{ij}(x_i, y_i, x_j, y_j)$ and $\Phi_i(x_i, y_i)$ are normalized phi-functions.

According to the phi-function method [17-19] $\Phi_i(x_i, y_i)$ can be represented as:

$$\Phi_i(x_i, y_i) = \min\{\Psi_{1t}(x_i, y_i), \Psi_{2l}(x_i, y_i), \Psi_{3m}(x_i, y_i), t \in T, l \in Q, m \in S\}$$

where $\Psi_{1i}(x_i, y_i)$ is a normalized phi-function of C_i and half-plane defined by an edge of P_k , $\Psi_{2i}(x_i, y_i)$ is a normalized phi-function of C_i and q_i , $\Psi_{3m}(x_i, y_i)$ is a normalized phi-function of C_i and s_m .

A general problem of placement of coagulants (GPPC) is stated as follows .

Problem GPPC.

Place the maximal number $n = n^*$ of circles with radius r in the treatment domain D such that the placement conditions (3), (4) are satisfied.

4. Two-stage decomposition solving approach

To tackle the issue, we propose a two-stage approach. Firstly, we estimate circles that are ensured to lie within a specified area. Then, in the second stage, we work through a series of problems to determine the maximum number of circles that can fit within the given area, while considering all constraints.

Taking into account the partition (1) for the treatment domain D , problem GPPC is decomposed into p independent subproblems PPC-k for each subset D_k .

Let's denote n_k^* as the maximum value of circles for each subtask. As is known, the maximum packing density of circles of the same radius is about 90%. Taking this value into account, one can obtain the upper bound of the number of circles:

$$\hat{n}_k^* = \left\lfloor \frac{0.9S(D_k)}{\pi(r + \delta_1)^2} \right\rfloor,$$

where A is the area of the polygon P .

To find the value of n_k^* , the method of dichotomy is used on the interval $[a, b]$. The initial value of $a = 0$ corresponds to the situation when it is impossible to place a single circle of radius in the region D_k , whereas the initial value of $b = \hat{n}_k^*$ corresponds to the unattainable in practice situation when the solution to the problem will be the placement of all circles \hat{n}_k^* .

To determine whether it is possible to place $n = \lceil (a + b) / 2 \rceil$ circles $C_i, i = 1, 2, \dots, n$ in D_k , the following problem is solved:

$$\max \sum_{i=1}^n r_i \quad (5)$$

considering

$$\Phi_{ij}(x_i, y_i, x_j, y_j, r_i, r_j) = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2} - (r_i + r_j) \geq \delta_1, \Phi_i(x_i, y_i, r_i) \geq \delta_2. \quad (6)$$

If $\max \sum_{i=1}^n r_i^* < nr$, then circles $C_i, i = 1, 2, \dots, n$, are not packed in D_k . In this case, a new value $b = n$ is set.

Otherwise, if $\max \sum_{i=1}^n r_i^* = nr$, then a placement of $C_i, i = 1, 2, \dots, n$, are placed in D_k . Value $a = n$ is then considered and $n = \lceil (a + b) / 2 \rceil$ is updated.

The process continues until $a = n$ and $b = n + 1$. In so doing $a = n$ is a solution of PPC-k.

In light of formulating the problem within the realm of mathematical programming, its resolution stands amenable to contemporary methodologies in nonlinear optimization. This can be effectively realized through the deployment of robust solvers, sophisticated computational tools specifically designed for navigating the intricacies of nonlinear optimization challenges. These solvers, characterized by their advanced algorithmic frameworks, contribute to the efficiency and accuracy of the optimization process.

Moreover, the integration of intelligent systems further amplifies the problem-solving capacity. By incorporating advanced algorithms and heuristic approaches, these intellectual frameworks enhance the optimization process, fostering adaptability and agility in addressing complex mathematical programming tasks. The synergy between cutting-edge solvers and intelligent systems not only elevates the analytical capabilities but also augments the overall efficacy in attaining optimal solutions for the given mathematical programming problem.

The algorithm proposed in the study provides the capability for collective repositioning of circles, enhancing the optimal coverage of the treatment area with photocoagulants. It is important to note that, although the algorithm does not guarantee a universally applicable solution to the problem, the application of this method has the potential to significantly improve the filling ratio of the considered area.

The presented algorithm not only serves as a tool for the efficient distribution of circles during photocoagulation but also implies that the best coverage of the treatment area is achieved through the collective interaction of elements. It is noteworthy that, despite the absence of a guaranteed global solution to the problem, this method opens up the possibility of substantial enhancement in the filling of the therapeutic area.

5. Computational experiments and discussions results

The intelligent system was meticulously architected using the C++ programming language, renowned for its computational efficiency and versatility. The choice of C++ provided a robust foundation, offering low-level memory control and high-performance features requisite for the intricate algorithms inherent in intelligent systems.

A cornerstone of this development was the strategic integration of the IPOPT library [20], an open-source software package tailored for large-scale nonlinear optimization challenges. IPOPT's adaptability positions it as an optimal choice for constructing intelligent systems where optimization constitutes a central facet.

IPOPT employs an interior-point method for optimization, excelling in solving complex nonlinear optimization problems with continuous variables. This method empowers the system to navigate intricate optimization landscapes, handling intricate constraints and nonlinear objectives endemic to intelligent systems.

The seamless integration of IPOPT into the C++ codebase facilitated the articulation and resolution of optimization problems central to the intelligent system's functionality. The library offered a comprehensive suite of tools for articulating problem constraints, objectives, and variables, with its interior-point optimization algorithm ensuring efficiency and precision in solutions.

In this study, the efficiency of an algorithm for convex polygons was investigated. An area which is a convex approximation of a polygon that is part of the treatment area considered in [13] (Fig. 4), was chosen for testing.

The radius of circles is $r = 1.75$, the distance is $\delta = 0.25$.



Figure 4: The treatment domain [13]

The placement of $n = 135$ circles was obtained (Fig. 5) improving 133 circles in Fig. 4.

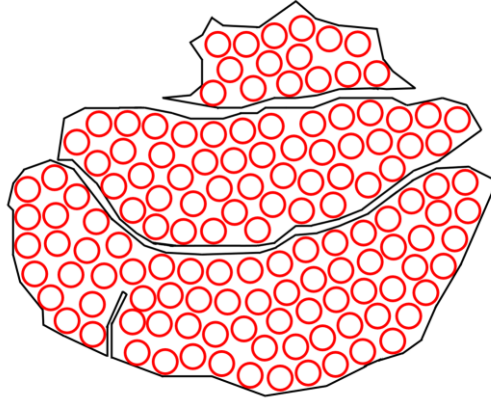


Figure 5: Placement of 135 circles

Despite the inherent approximation in selecting data, which was based on scanning sizes from Fig. 5, the resulting placement structure exhibits a notable divergence. Remarkably, this alternative arrangement successfully mitigates the presence of substantial uncovered areas. This not only underscores the adaptability of the developed algorithm but also signals its potential viability in practical applications related to the compilation of treatment plans.

The utilization of approximate data, while scanning the dimensions from Fig. 5, was a necessary compromise. However, the consequential placement structure, distinguished by its absence of large uncovered areas, introduces an intriguing paradigm shift. The implications of this outcome extend far beyond a mere algorithmic success; they point towards a promising avenue for real-world applications in the intricate task of compiling treatment plans.

This fortuitous departure from the initially chosen data configuration not only demonstrates the algorithm's resilience but also accentuates its versatility. The absence of significant uncovered areas suggests a higher degree of efficacy in practical scenarios, where precision and comprehensive coverage are paramount.

The emergent structure, as revealed through this altered placement arrangement, instills confidence in the algorithm's adaptability to the complexities of real-world challenges associated with treatment plan compilation. The potential implications for clinical applications are particularly compelling, as the algorithm showcases an ability to navigate and optimize placement structures to achieve more robust and comprehensive outcomes.

6. Conclusion

The work underscores the critical need for developing efficient treatment strategies tailored to diabetic retinopathy, a serious condition demanding effective therapeutic interventions. Among the most impactful techniques stands retinal laser photocoagulation, facilitating the sealing or elimination of abnormal blood vessels to prevent further damage to the retina. The significance of uniformly distributing photocoagulation spots, ensuring optimal placement at evenly spaced intervals within the edematous region while avoiding contact with blood vessels, is emphasized.

In this context, the utilization of intelligent systems has exhibited substantial promise. Powered by advanced machine learning algorithms and artificial intelligence, these systems can analyze retinal images to pinpoint optimal photocoagulation points with exceptional precision. This not only enhances treatment accuracy but also reduces planning time, thereby increasing

overall treatment efficiency. Furthermore, the integration of patient-specific data, such as retinal morphology and disease progression, into these systems enhances their ability to tailor treatment plans to individual needs, further optimizing outcomes.

Moreover, intelligent systems offer the potential for continuous learning and improvement. Through exposure to extensive datasets and diverse cases, these systems can adapt and refine their performance over time. This adaptability enables them to stay abreast of evolving medical knowledge and technological advancements, ensuring they remain at the forefront of diabetic retinopathy treatment.

Additionally, future research endeavors will include a more precise formal delineation of the photocoagulation domain. This will enable even more accurate and effective treatments, further maximizing the algorithm's potential in practical applications. The ongoing enhancement and refinement of these methodologies, driven by advancements in artificial intelligence and machine learning, are crucial for advancing the field and delivering optimal outcomes for patients with diabetic retinopathy.

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