How Risk Preferences Shape City-State Success: An Agent-Based Model of Resource Management*

Andrea Piras[1,†], Francesco Bertolotti[1,*,†]

[1]School of Industrial Engineering, LIUC – Carlo Cattaneo University, Castellanza Varese, Italy

Abstract

This paper presents an agent-based model to study the dynamics of city-state systems, focusing on the interaction between military and economic actions in a closed environment, with the aim of drawing more general conclusions about risky behaviour with limited resources in a competitive environment. The model includes three types of agents: city-states, villages, and battalions, where city-states are the primary decision-makers that can establish villages for food and recruit battalions for defence and aggression. Simulation data was generated using grid sampling, and analysis suggests that a risk-seeking strategy is more effective in high-cost scenarios if the production rate is sufficiently high. Future work could include memory and trading behaviour to improve the relevance of the model and the generalisability of the results.

Keywords

agent-based modelling, risk preferences, risk aversion, city-states, computational history,

1. Introduction

Social science has a long-standing tradition of using computational methods [1, 2], especially agent-based models (ABMs) [3, 4, 5]. This interdisciplinary approach leverages computational tools and large-scale data collected from various sources to uncover insights into individual and collective human behavior [6]. In this context, multi-agent simulation models are considered to have the capacity to lead to a "generative" approach [4, 7, 8] and to embody an evolutionary perspective [9, 10]. Thus, in this field, they are considered both a means to perform prediction [11, 12, 13] and to enhance understanding [14, 15, 16] of a phenomenon.

Recently, there has been a growing interest in using computational methods to understand historical phenomena [17, 18, 19, 20]. Archaeologists have employed multi-agent simulation models to validate their hypotheses regarding excavations [21, 22, 23, 24]. Additionally, various fields, such as the emergence of trading networks in specific areas [25] and the effects of climate change on societal outcomes [26], have utilized this methodology, typically with long simulation time-steps.

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*Corresponding author.

†These authors contributed equally.

an17.piras@stud.liuc.it (A. Piras); fbertolotti@liuc.it (F. Bertolotti)

0009-0005-4270-1467 (A. Piras); 0000-0003-1274-9628 (F. Bertolotti)

Given that war systems have long been considered complex systems [27], agent-based modeling has a tradition of being used to study strategies and action consequences of different kinds of combat systems [28, 29], including real-world armies [30]. Due to its flexibility, it has been applied to both small military units [31] and battles involving tens of thousands of units to assess potential alternative outcomes [32]. Although these models include and analyze tactics to defeat the enemy on the field, this type of competition is tactical rather than strategic, as it omits long-term decisions regarding resources. Walbert et al. [33] present an agent-based simulation model based on empirical data to assess how and why states start a war, considering their network of relationships and wealth accumulation.

In this paper, we present an ABM of a city-states system, where cities can perform both military and economic actions [34]. Specifically, there are three kinds of agents: cities, villages, and battalions. The primary decision-making agents are the cities, which can generate villages to produce food and battalions for defense and aggression. Cities consume food to maintain their population level and can generate wealth that can be invested in technological developments. These developments can enhance the efficiency of battalions, food production, or wealth generation.

In light of the preceding description, it is possible to categorise the strategic attitudes of cities into two broad categories: expansive and conservative. These two scenarios are linked to different risk predispositions that each city decides to adopt. It is reasonable to posit that cities with greater resource availability will adopt more expansive strategies, simply because they are the only ones that can afford to do so. Conversely, we anticipate that cities with limited resources will adopt a more cautious approach, attempting to minimise their exposure to potential external threats.

The results of the paper are counter-intuitive and of significant interest for decision-making. City-states can be seen as black boxes that generate resources to buy goods, where resources are food and gold, and the goods are military units and technological investments that increase production rates. Given a fixed resource generation rate, we would expect that if the cost of production is low, the best strategy would be to produce as much as possible, and vice versa for high production costs. However, the results indicate the opposite. We explain this observation by considering the higher value of individual units. When producing and investing are more expensive, each unit has a greater marginal advantage. Therefore, producing more is rewarded with a higher chance of survival. However, if the production rate is too low, this advantage no longer applies because there are insufficient means to achieve adequate production. In behavioral terms, this suggests that a risk-seeking strategy is preferable when the cost of investment is high, but this does not apply if the production rate is too low.

The paper is structured as follows. First, the methodology is explained, including a detailed description of the agent-based model and the experimental design. Next, the results are presented and discussed. Finally, conclusions are drawn.

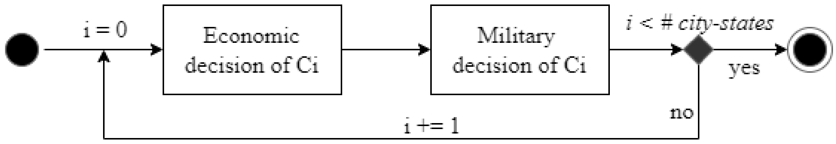


Figure 1: Model scheduling for a single time-step for a city-state C_i

2. Methodology

2.1. Agent-based model

This research paper presents an agent-based model (ABM) that examines the interactions between city-states located within a landscape. ABM is a computational methodology that simulates the behavior of systems by modeling the behavior and interactions of their composing entities [35, 36].
The model depicts a bi-dimensional and topological explicit, i.e. the fact that agents have a certain position in a space, in this case bi-dimensional, system where a limited amount of city-states are competing for space, and where no new city-states can be founded. City-states can produce food by means of villages, and this overall affect the growth level of the population, which has a positive effect on every economical aspects. Also, city-states and can attack each other with military units. No other kind of interaction has been inserted in the model. The purpose of the model is then to observe which kind of cities survives in different environmental setting, and try to draw a more general understanding regarding competition in a close environment with scarce resources.

So, the model assesses the different paths that each city-state can take in terms of economic development, military strategy, and resource management to achieve survival and prosperity over a specific period. In the model, three types of agents are present: city-states C_i, villages V_i, and battalions B_i. City-states are the primary decision-makers that undertake various actions. Figure 1 depict their scheduling for a single time-step. Villages are the food producers and provide the necessary supplies to sustain the population of the city-states, which is the driver for the whole economics of the city-state, as depicted in Figure 3. Battalions are recruited by the city-states to defend against external threats or to launch military campaigns against other city-states. Each agent type plays a distinct role in the simulation, contributing to the overall dynamics of resource management, economic growth, and military strategy.
Each city-state (C_i) possesses the state variables depicted in 1, which can undergo both endogenous changes, such as the population stock $p_i(t)$, which represents the number of tax-generating citizens in the city who are also available for enrolment, that increases when a certain amount of food $f_i(t)$ is available in the city to cover the food needs of the citizens and soldiers stationed to defend the city, and exogenous changes, related to the interaction processes between city-states. The gold $g_i(t)$ of the city-state (C_i) is linked to the population by a positive dependence on the fact that this increases with the collection of taxes in direct proportion to the number of citizens in the city. The variables $w_i(t)$, $ct_i(t)$, $mt_i(t)$, and $cd_i(t)$, which respectively represent the general wealth level of the population, the technological level in the civil field, the technological level in

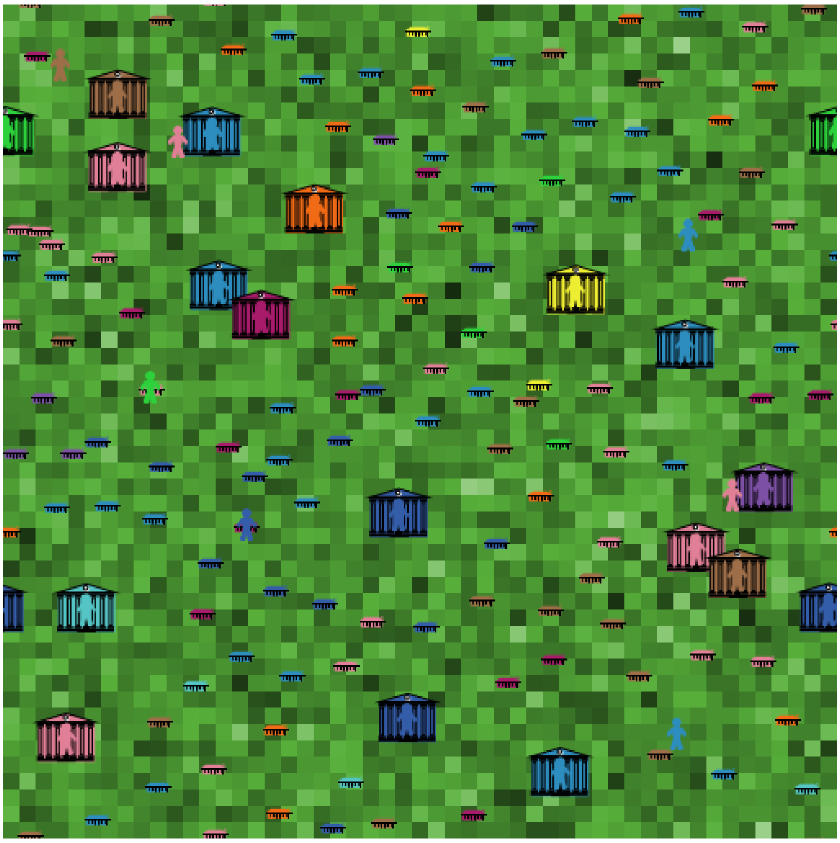


Figure 2: Graphical interface of the model

Name	Description
$g_i(t)$	Gold in the city
$f_i(t)$	Food in the city
$p_i(t)$	Population of the city
$w_i(t)$	Wealth of the the city
$ct_i(t)$	Civil technology of the city
$mt_i(t)$	Military technology of the city
$cd_i(t)$	Defence of the city

Table 1
List and description of state variables for a city-state C_i

the military field and the city's defences, only undergo positive increments whenever the city decides to embark on a development phase compatible with the available resources. Figure 3 exemplifies the economic dynamics of a city-state agent, highlighting the dependencies that the different state variables have on each other.

City-states are decision-making entities. In this sense, they are undertaking a decision at each time-step, regarding in which kind of activity to invest the resource, or if to create villages or battalions, or how to use the battalions. The economic phase of a city-state C_i decision-making

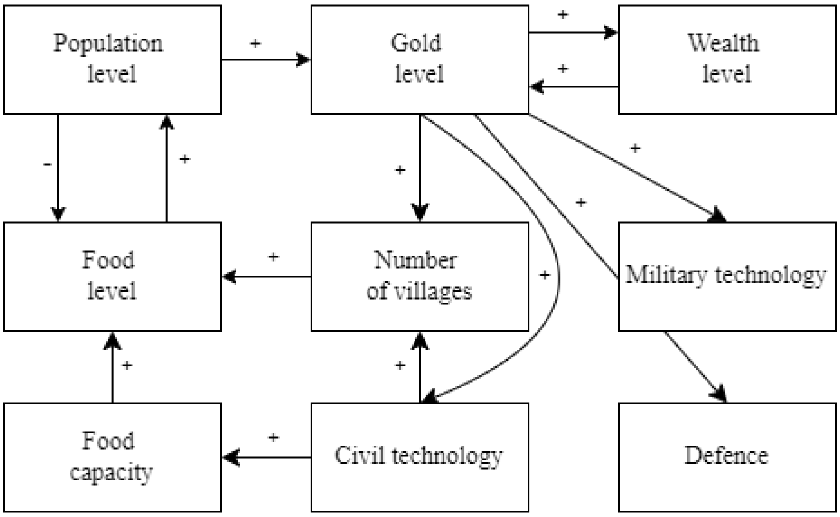


Figure 3: Graph of functional dependencies depicting the economical dynamics of a city-state C_i

Name	Description	Allowed Values
pv_i	Preference to found a village	$pv_i \in [0,1]$
pct_i	Preference to invest in civil technology	$pct_i \in [0,1]$
pmt_i	Preference to invest in military technology	$pmt_i \in [0,1]$
pw_i	Preference to invest in wealth	$pw_i \in [0,1]$
pd_i	Preference to invest in defences	$pd_i \in [0,1]$
pb_i	Preference to recruit a battalion	$pb_i \in [0,1]$
pp_i	Preference to send protecting troops	$pp_i \in [0,1]$
pm_i	Preference to organize a mission	$pm_i \in [-0,1]$
pva_i	Preference to attack a village	$pva_i \in [0,1]$
pca_i	Preference to attack a city	$pca_i \in [0,1]$

Table 2
List and description of strategic parameters for a city-state C_i

divides into two phases. First, a city-state collect gold and food based on their gold-rate and population values and the village production. Then, a city-state decides if to improve wealth, technology, or defense, to build a battalion, or to found new villages. The military phase instead consists in the decision of what to do with the battalion: the city-state can organize missions to directly attack enemy cities or their villages, or alternatively, it can send battalions to defend a village and protect it from possible enemy attacks. Each decision can be trigger by two elements: a specific internal or external condition, and a set of behavioural parameters. Behavioural parameters can hence be divided into two categories: strategic parameters (2) and the tactical parameters (3).

The strategic parameters can assume a value $x_i \in R : x_i \in (0,1) \wedge \sum x_i = 1$, with the exception of pva_i and pca_i, which can value $y_i \in R : y_i \in (0,1) \wedge \sum y_i = 1$. This difference is due to the

Name	Description	Allowed Values
α_1	Coefficient of target decision regarding enemy's defence	$\alpha_1 \in$ [-1,1]
α_2	Coefficient of target decision regarding enemy's number of battalions	$\alpha_2 \in$ [-1,1]
α_3	Coefficient of target decision regarding enemy's distance	$\alpha_3 \in$ [-1,1]
α_4	Coefficient of target decision regarding enemy's military technology level	$\alpha_4 \in$ [-1,1]
α_5	Coefficient of target decision regarding enemy's gold	$\alpha_5 \in$ [-1,1]
α_6	Coefficient of target decision regarding enemy's food	$\alpha_6 \in$ [-1,1]
α_7	Coefficient of target decision regarding enemy's population	$\alpha_7 \in$ [-1,1]

Table 3
List and description of tactical preference parameters for a city-state C_i

Name	Description	Allowed Values
N	Number of starting city-states	N $\in$ [5, 20)
bsc	Base battalion recruitment cost	$bsc \in$ [20000, 2000000]
pgp	Person gold production	$pgp \in$ [1, 1000]
bsp	Base village food production	$bvp \in$ [1 , 1000]

Table 4
List and description of environmental parameters

fact that pva_i and pca_i pertain to the city's preference to directly attack enemy cities or their villages. These values are subordinate to the value of pm_i, which represents the city-state's preference for organizing offensive missions. Once the mission has been organized, the city must choose which type of target to direct its attack towards. These parameters determine the strategy each city decides to undertake on the resource management. For instance, if $pv_i = 0.2$, it means that the probability for a city-state C_i to build a new village during the economic phase of the decision-making process, and only when the option is available, is $P(v) \propto 0.2$.
The tactical parameters can all assumes the value $z_i \in R \, : \, z_i \in (-1, 1)$, and are used to decide which enemy to attack in the moment where the decision to attack has already been taken. Each of these parameters acts as a multiplier on specific characteristics of the enemy C_i with which the C_i interacts. The sum of these values determines a final score, where the C_i will choose to attack the C_i with the highest score. Each value is compared with the total amount present on the map. For example, α_2 multiplies, for each C_i, the number of B_i it possesses divided by the total number of battalions present on the map. This helps to return a value of the target's "danger level." The choice to vary these values between -1 and 1 was driven by the desire to better explore which characteristics of the target cities were taken most into account. Each C_i will have its own unique set of preferences, assigning different positive or negative importance to various aspects. These parameters play a pivotal role within the model: given that attacking is the only way of interaction in the model, and that each set of parameters is unique for each city-state C_i, they are regulating the decision of the target, and so it makes the way in which the economics output of two city-state agents are tested.
Finally, there are some environmental parameters of interest (see 4), such as the initial number of city-states N, the rate of production of the two resources (respectively pgp for the gold and bsp for the food), and the cost of production of a battalion bsc.

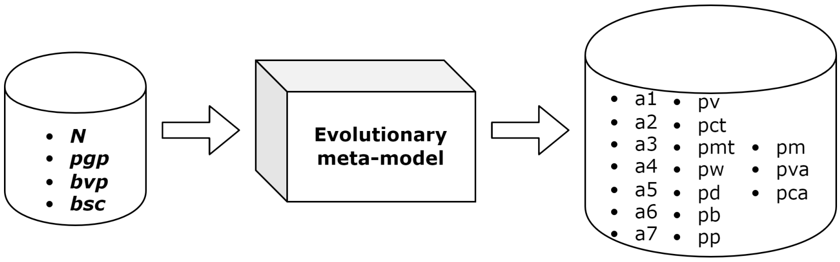


Figure 4: Black box diagram of the experimental setting

2.2. Experimental design

To implement the model described in the previous paragraph, we used NetLogo 6.3.0. This software was chosen for its simplicity and because the number of agents in the model was limited, eliminating the need for high performance computing. The experiments were conducted using NetLogo's BehaviorSpace module, which facilitates grid sampling. Through 1250000 simulations, a wide range of scenarios was analyzed. This number of repetitions was sufficient to ensure statistically robust results and allowed us to explore the effects of various input variables on the interactions between cities, villages, and battalions through simulation data analysis.

The grid sampling exploration was performed by sampling four key inputs, with each input variable varied across a specified range to cover both extreme and moderate values. Each variable was collected from a uniform random distribution. The decision to adopt a random grid sampling system was guided by the fact that, not knowing what result to expect a priori, it was considered the best way to examine as many combinations as possible and discover interesting patterns within the model. In future developments of the model, one could explore using, for example, a genetic algorithm to find the best possible strategy within the pool of numerous combinations available. For each simulation run, data was collected on key outcome variables for the surviving cities, enabling the generation of various statistical analyses that could provide insights into how different environmental parameters influenced the overall dynamics of the system. The data was analyzed and processed using Python 3.11.3 in a Jupyter Notebook. These experiments allowed us to observe how different scenarios impact cities' preferences, resource management, and overall economic and military dynamics.

3. Results and discussion

Figure 5 illustrates the share of C_i that survived at the end of the simulation relative to the starting number N. The graph suggests that often only a low share of C_i survives, with this share gradually decreasing in frequency as the survival rate increases. However, there is a noticeable increase in survival values close to 1, indicating that specific parameter combinations exist where all the city-states could survive. It is interesting to observe how the behavioral parameters of the city-states change with environmental inputs, which depict an elementary form of fitness to the environment and suggest the best behavior under certain conditions.

The following analysis, depicted in Figure 6a, Figure 6b, Figure 6c, and Figure **??**, involves

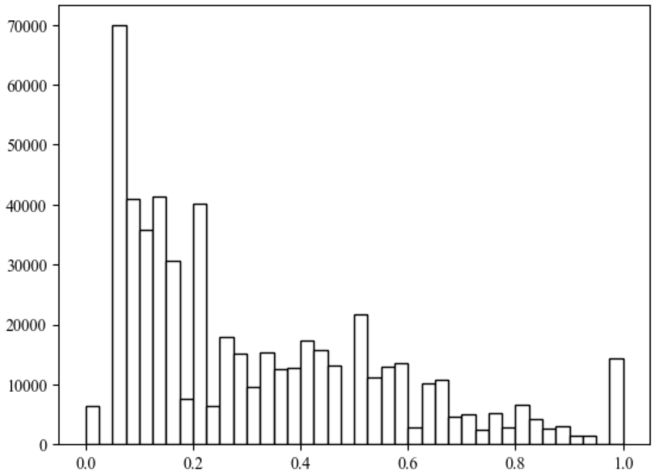


Figure 5: Histogram of frequency of percentage number of survived C_i respect to N

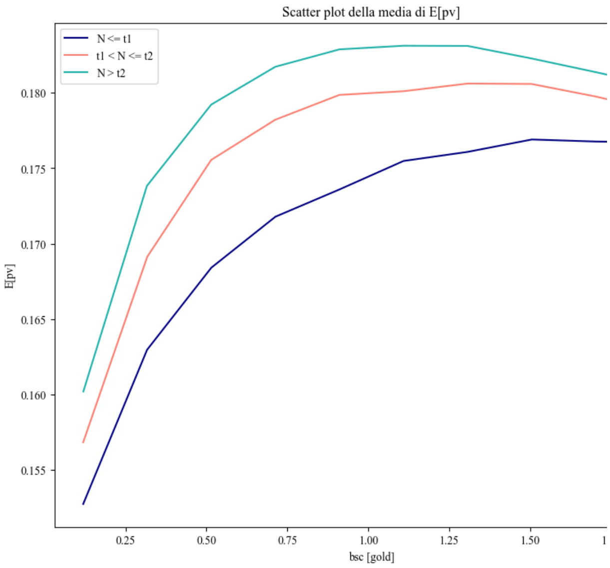
plotting a behavioral output on the y-axis, while observing the co-effect of two different inputs: one on the x-axis and the other used to divide the data into three clusters by tertiles, which boundaries are respectively called t_1 and t_2 for each variable. These graphs represent on the y-axis the average values of pv and pm, indicated respectively as $E[pv]$ and $E[pm]$.

Figure 6a depicts the relationship between bsc and pv, clustered by N. For all values of N, pv initially increases with bsc, exhibiting different peaking points followed by a subsequent decrease. This non-monotonic behavior varies with N: the higher the number of city-states, the greater the preference for founding villages.

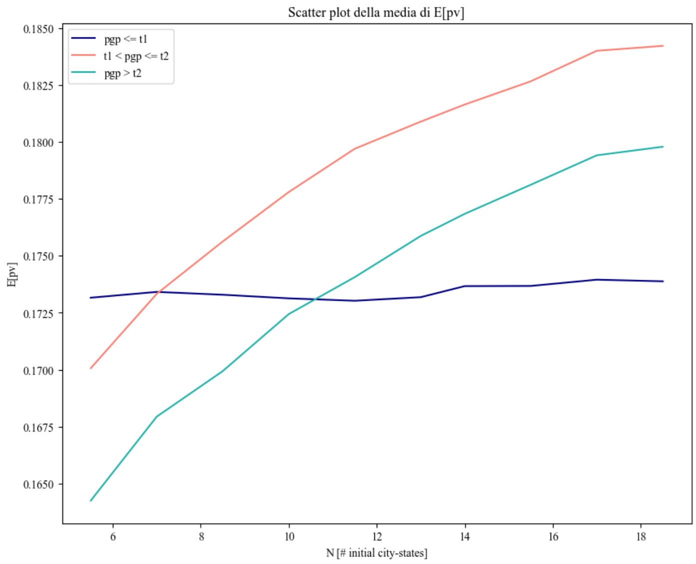
Larger C_i seem to sustain a higher preference for a higher cost longer than smaller C_i. This can be connected to the varying success of different risk-related attitudes. Notably, as bsc increases, a more expansive and risk-prone strategy emerges, aiming to seize as much territory as possible by founding villages until the area is saturated. Additionally, since each C_i can only perform one action per turn, it exposes itself to the risk of enemy offensives targeting its villages. This occurs because the city-state would be less protected due to its lower pb_i in favor of pv_i.

Figure 6b shows the relationship between N and pv, clustered by pgp. It is observable that for $pgp > t_1$, there is an equal increase in pv with N, although with a different intercept. On the other hand, when $pgp < t_1$, there is almost a null trend. Similarly to what was mentioned for the previous graph, the focus is indirectly on the cost of the external environment. A high number of N on the map leads to greater resource scarcity, making these resources more valuable. Consequently, the propensity to expand increases as the number of N grows. However, this reasoning does not seem to apply when the C_i's ability to generate resources remains excessively low.

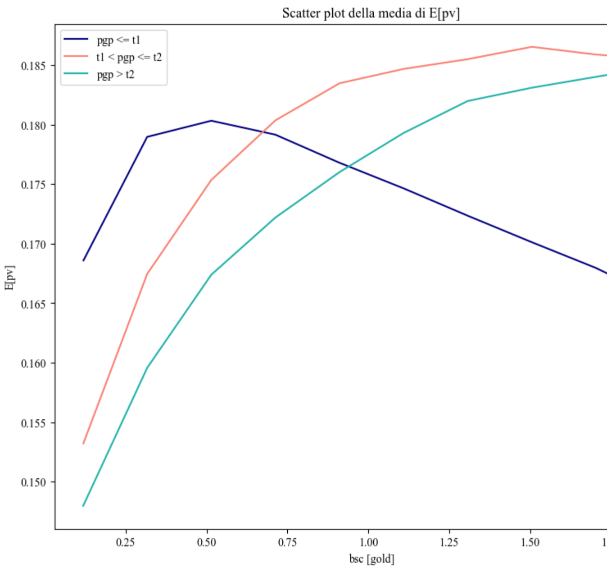
The link between bsc and pv, clustered by pgp, is depicted in Figure 6c. It is possible to observe that when $pgp > t_1$, there is a non-linear growing relationship between bsc and $E[pv]$, which saturates after a certain level. For $pgp < t_1$, this saturation occurs much earlier, and the values of $E[pv]$ start decreasing notably even for low values of bsc. In this sense, economic



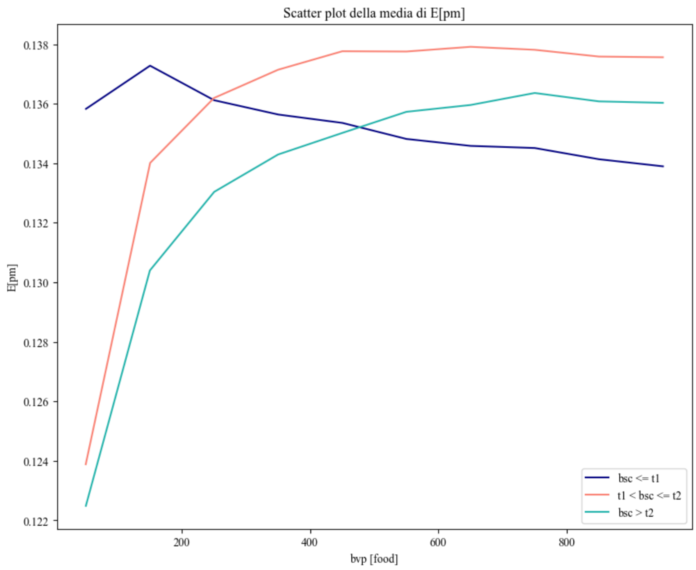
(a) Line plot of $E[pv]$ of C_i related to bsc clustered by number of N



(b) Line plot of $E[pv]$ of C_i related to N clustered by pgp



(c) Line plot of $E[pv]$ of C_i related to bsc clustered by pgp

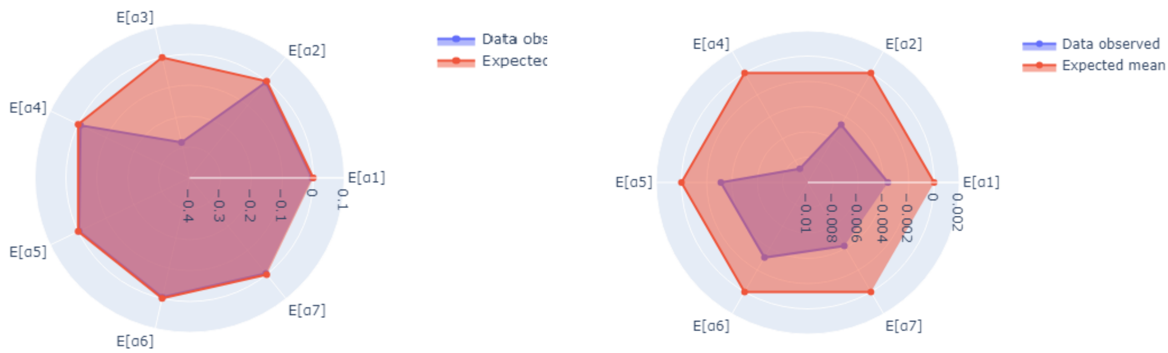


(d) Line plot of $E[pm]$ of C_i related to bvp clustered by bsc

Figure 6: Line plot of $E[pv]$ and $E[pm]$ related to different clusters and x-axis

strength seems to buffer the impact of *bsc*. This chart effectively illustrates the relationship between environmental cost and internal production. As *bsc* increases, so does the propensity for expansion by the C_i. However, as expected, when productivity is excessively low, this propensity drops drastically since the C_i is not able to sustain such high costs.

Figure **??** depicts the relationship between bvp and V_i, clustered by pm. For all values of bsc, pm initially increases with bvp, then peaks and either plateaus or decreases. Interestingly, for low values of bsc, this pattern differs significantly. We notice a particular balance that aligns with the previous statements. As we have learned, the expansive effect increases with bvp. In this case, we observe the pm_i values. Filtering by bsc, we see how this phenomenon is accentuated. However, in the case of low bsc, despite the increase in bvp, the pm_i tends to decrease slightly. When the pgp is higher, C_i tend to have higher pv_i and can tolerate higher costs, whether for soldier recruitment or otherwise. Larger C_i tend to sustain higher pv_i for longer and can support higher costs better than smaller C_i. This indicates economies of scale and possibly better resource distribution and management in larger C_i. There is a noticeable cost tolerance threshold in both pv_i and pm_i. Beyond certain bsc, preferences decline, indicating a balance point in economic and operational planning. Higher production, both in villages and gold, positively correlates with higher preferences up to a point. However, after certain production levels, the incremental benefits reduce, suggesting optimal production ranges for maximizing preferences.



(a) Radar plot of the mean values of tactical parameters for all the C_i lasting in a simulation (in red), compared with the related to expected values (in blue)

(b) Radar plot of the mean values of tactical parameters without $E[a3]$ for all the C_i lasting in a simulation (in red), compared with the related to expected values (in blue)

Figure 7: Radar plots of tactical parameters

In these proposed charts, it was decided to compare the expected values of some parameters with the actual average values observed in the C_i that survived at the end of the simulations. The expected values of the parameters were calculated based on the range of admissible values by definition.

The tactical parameters can take any value in the interval $[-1, 1]$ with equal probability distribution. Therefore, their expected value is 0.

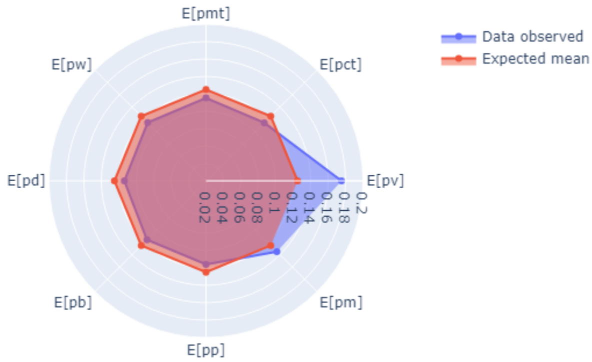


Figure 8: Radar plot of the mean values of strategical parameters for all the C_i lasting in a simulation (in red), compared with the related to expected values (in blue)

A similar reasoning was applied to the strategic parameters. The strategic parameters can take any value in the interval $[0, 1]$ with equal probability, but the sum of these parameters must equal 1. For this reason, since there are 8 primary strategic parameters, the expected value for each of them is $1/8$, or 0.125. Figure 7a illustrate the differences between the expected values of tactical parameters if the environment had no effect on the simulation, and the actual average values obtained from simulations, for strategic and tactical parameters. It is shown that the C_i have maintained values close to those expected for all α_i except for α_3. This markedly negative value indicates an aversion on the part of the C_i to selecting targets located farther away.
Figure 7b was created by removing $E[\alpha_4]$ from the visualization to better appreciate the differences of the other tactical parameters relative to their expected values. In the graph, all values are slightly negative and thus below the expected value of 0. However, the α_4 parameter, although only slightly, stands out as the most negative, highlighting it as the second major contributing factor in determining the target for missions by the cities.
Figure 8 shows the average preferences of actions that each C_i can take, comparing them with their expected values. It is possible to appreciate how C_i have significantly prioritized the $E[pv]$ at the expense of almost all other preferences. Only the $E[pm]$ is slightly above the expected value. This graph offers one final insight. Previously, we discussed more conservative or expansive strategies. We notice how the two main expansive preferences, pv_i and pm_i, stand out compared to the others in terms of the C_i preferences. This tends to indicate a greater preference among C_i for an expansive strategy, which evidently tends to perform better in different scenarios.

4. Conclusions

The ABM of a city-states system presented in this paper aims to analyze the different mixes of preferences and, consequently, the possible strategies that the primary agents of the model, the city-states, might choose to exploit. The objective of each city-state is survival, which can be achieved through absolute conquest or partial coexistence with other city-states. The study of these dynamics has revealed a particular pattern. As previously stated, the expansive attitude and resulting risk propensity of city-states emerge counter-intuitively in response to external environmental and internal resource characteristics.

An increase in productive capacity leads city-states to adopt a more aggressive stance toward their neighbors. One might expect similar behavior when the cost of external goods is particularly low. However, this phenomenon does not occur; instead, city-states in this circumstance tend to adopt a conservative attitude. In the event of conflicting internal and external pressures, the external environment exerts a dominant influence on the strategic direction of the city-state, despite the mitigating effect of internal pressures.

Future developments include a broader and deeper analysis of the model's behavior. Additionally, agents could be enhanced with memory regarding past events, allowing them to learn which other city-states attack them more often and adjust their behavior accordingly; a retaliation behavioral parameter could also be included. Finally, it could be relevant to include the possibility of trading for agents, thereby incorporating cooperative behavior.

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