

# Detection of inconsistencies in knowledge bases for emission of foreign judgments

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**Abstract.** This paper models the process of emission of judgments considering the personal judgment that an agent has about itself and the impact it causes another agent make a judgment on it, the model is based on the theory of causal learning of Catena and the belief revision process based on learning mechanisms and integration of information.

The knowledge base of the agent is specified by means of 2CNF, considering operations of expansion and contraction of the AGM model, also a criterion of rationality is applied where the agent is presented the list of beliefs (clauses) that can be removed in order to maintain a consistent judgment despite of external stimulus.

**Keywords:** belief revision, 2 CNF, implication digraph, consistency, satisfiability.

## 1. Introduction

The ability to detect causal relationships between environmental events is a major component of adaptive behaviour. Learning that one event is the cause of another is a basic psychological function, given the causal texture of our world. Causal learning allows humans and other organisms to know that two events are connected by some kind of link or mechanism, in such a way that the presence or the absence of the cause is consistently followed by the presence or the absence of the effect.

In the causal learning phenomenon is demonstrated to learn that a "cause produces an effect" requires a preprocessing of the ratio detection covariation between the two events to make causal attribution later in the form of judgment or belief [1].

Alternatively, a recent causal belief revision model [2] defines the manner in which the degree of belief attaching to specific causal hypotheses is updated as a result of observations about a candidate cause and effect. Also, it includes a mechanism that acts each time a judgment is asked of the subject and that serves to update the new evidence obtained from the last judgment. Hence, during contingent training, when subjects are again asked about the relationships between two events after non contingent preexposure, judgment updating will be sensitive to the subjective previous experience, which will especially reduce

the weight of the confirming positive trials. Beliefs are updated primarily as a function of the discrepancy between the ultimate level of belief that the observed evidence should induce and the current level of belief.

So the revision and transformation of knowledge is widely recognized as a key problem in knowledge representation and reasoning. Reasons for the importance of this topic are the facts that intelligent systems are gradually developed and refined, and that often the environment of an intelligent system is not static but changes over time [5].

Belief revision studies reasoning with changing information. Traditionally, belief revision techniques have been expressed using classical logic. Recently, the study of knowledge revision has increased since it can be applied to several areas of knowledge. Belief revision is a system that contains a corpus of beliefs which can be changed to accommodate new knowledge that may be inconsistent with any previous beliefs. Assuming the new belief is correct, as many of the previous ones should be removed as necessary so that the new belief can be incorporated into a consistent corpus. This process of adding beliefs corresponds to a non-monotonic logic [4, 10].

The AGM (Alchourrón, Gärdenfors and Makinson) model addresses the problem of belief revision using the tools of mathematical logic [6]. These works are considered the foundation for studying the problem of knowledge exchange. According to the AGM framework, knowledge  $K$  is represented by propositional logic theories and new information is represented by the same logic formulas.

One way to represent and check the consistency of a knowledge base is modeled by using the 2SAT problem, which has been shown to be solvable in polynomial time [16].

## 2. Basic Concepts

During the 1970's from artificial intelligence and information technology the concept of "default reasoning" was introduced and defined by Raymond Reiter. This kind of logic sustains that in the absence of any contrary information, it is plausible to conclude  $X$ . It is a form of reasoning that takes into account the limitations of the agent and the commonness of things, is pretty close to the way in which everyday reasoning works. Indeed, it is due to this kind of reasoning that we can act in the world.

Well, the notion of plausible or default reasoning led to a vast area now known as non-monotonic logic or common sense, as well as circumscription logic (McCarthy), modal logic (McDermott and Doyle) and autoepistemic logic (Moore and Konolige) [12].

Non-monotonic logic is that form of reasoning under which a conclusion may be recast, retracted or defeated by an increase in information that modifies its premise. For example, the type of inference of everyday life in which people formulate tentative conclusions, reserving the right to withdraw them in light of new information. This logic satisfies the issue considering the defeatable nature of typical inferences of human common sense reasoning. Considering this

type of reasoning, a formal and systematic study of cognitive processes that are present in the manipulation of knowledge structures emerges, by which an intelligent agent can draw conclusions in different ways, without having complete information to do so [13].

In our case, we use the rationality criterion to determine the behavior of changes in beliefs; criterion include the minimum change of preexisting beliefs, the primacy of new information and consistency. Thus for belief revision based on the AGM model using these criteria of rationality, three basic operations are used: expansion, contraction and review [6, 14].

Expansion is the operation that models the process of adding new knowledge to the corpus. This can be thought of as the expression of the learning process and is symbolized by the  $+$  operator, so it is defined as,  $F + p = C(F \cup p)$ , where  $F$  is the knowledge base,  $p$  is the new belief and  $C$  is the function that check new knowledge base.

Contraction is the operation that causes a new belief to remove a piece from the corpus of knowledge, because the agent in question must stop having a certain position on this belief. This becomes complicated when there are other beliefs that would need to be abandoned based on the abandonment of the initial belief, so in the end, only the absolutely necessary beliefs would remain. This is symbolized by the operator  $-$  and is defined as  $F - p = C(F - p)$  where  $F$  is the corpus, the new belief  $p$  and  $C$  is the function that check new knowledge base.

Revision consists of modifying the set of beliefs when a new belief is incorporated into the previous set so that logical consistency is conserved. If the set of beliefs is already consistent with the new information, then the review coincides with expansion, but if new knowledge is inconsistent with any previous beliefs, the operation of review must determine the resulting set of beliefs which keeps only the part of the original which would obtain a consistent result, so the original set of beliefs must be modified by eliminating as many beliefs as necessary to ensure that the resulting set, which includes the new belief, is consistent, and is defined as  $F * p$  where  $F$  is the set of beliefs or knowledge base and  $p$  is the new belief.

To address the problems of belief revision, it is useful to consider the model using propositional logic to verify the consistency of the knowledge base in order to analyze results from adding new beliefs which are considered valid, so it is necessary to define the concepts of propositional logic involved as follows: a formula is said to be in conjunctive normal form (CNF) if it is composed of a conjunction of disjunctive clauses and will be true if all its clauses are [3, 8].

A clause is a disjunction of literals, so that each literal stands for any formula composed of a single proposition symbol  $x$  (positive literal) or its negation  $\neg x$  (negative literal) or a constant  $\perp$  or  $\top$ .

So any formula  $F$  can be translated into an implication digraph (EF), which is a directed graph whose construction is done by taking each of the clauses  $(x_i, x_j)$  of the formula, where vertices of the graph are the  $x_i$  and  $\neg x_i$ . Here, there is a vertex for each variable and another for its negation. For each clause, two edges are generated by applying the following formula:  $(\neg x_i, x_j)$  and  $(\neg x_j, x_i)$ . The

implication digraph is widely used to ensure if a formula is satisfiable or not [17].

The Satisfiability Problem(SAT) is posed as follows: given a set of variables and a constraint in conjunctive normal form, a truth assignment that satisfies the constraint must be found. In our case, we worked on CNF for 2SAT problem, which means the formula consists of clauses consisting of two literals [7].

To solve the 2SAT, the implication digraph is built and the strongly connected components of the digraph are calculated. It is said that the problem is solvable if and only if no variable and its negation belong to the same strongly connected component. There is a theorem that supports this formalism [15]: F is unsatisfiable if and only if a variable  $x$  exists such that there exist trajectories  $x \rightarrow -x$  and  $-x \rightarrow x$  in  $EF$ .

### 3. Inconsistency detection modeling

In [11] a preliminary work was proposed on the use of 2CNF to solve problems in belief revision process, now this methodology is used for modeling an agent's set of personal judgments, where each clause represents a certain judgment and the entire set of clauses represents the knowledge base to be evaluated. In cases where the knowledge base is inconsistent, or unsatisfiable, it is necessary to apply a contraction operation with a rationality criterion which, in this case, is chosen by the agent itself.

#### Evaluation Strategy

Input:

- A set of  $m$  clauses  $(x_i, x_j)$  that make up the personal judgment base over the agent A.
- The external judgment  $C$  over the Agent A

1. Obtain the extended formula  $EF_i$  using equation (1) below:

$$EF = \{(-x_1 \vee x_2) \wedge (-x_2 \vee x_1) \wedge (-x_i \vee x_j) \wedge (-x_j \vee x_i) \dots (-x_m \vee x_n) \wedge (-x_n \vee x_m)\} \quad (1)$$

2. Create the linked list  $L$  to store the implication graph of  $EF_i$ .
3. Calculate the consistency sets  $TX$  for each literal.  
 $TX[x_i] = x_i, L[x_i] \cup L[L[x_i]]$  for each  $L[x_i]$  that does not belong to the set.  $x_i$  is said to be inconsistent if in all of  $TX[x_i]$  there is both a variable  $x_i$  and its negation  $-x_i$ .
4. Verify the consistency of the judgment base  $F$ . If in the calculation of the set  $TX$ , some  $x_i, TX[x_i]$  is inconsistent and  $T[-x_i]$  is also inconsistent, then the base  $F_i$  is unsatisfiable. Otherwise, the base  $F$  is satisfiable.
  - (a) If the base  $F$  is unsatisfiable, we evaluate the new knowledge base  $F * C$ . If the result is unsatisfiable, then we apply the contraction process  $F - C$  on the knowledge base.

In order to test the proposed strategy, a simple application was developed which takes a set of judgments expressed as 2CNF clauses as input. Fig. 1 shows both the application and the rationality criterion, where the right side of the screen indicates the suggested clauses to eliminate to assure base satisfiability, as well as the total number of TX inconsistencies that would be generated if those clauses were eliminated.

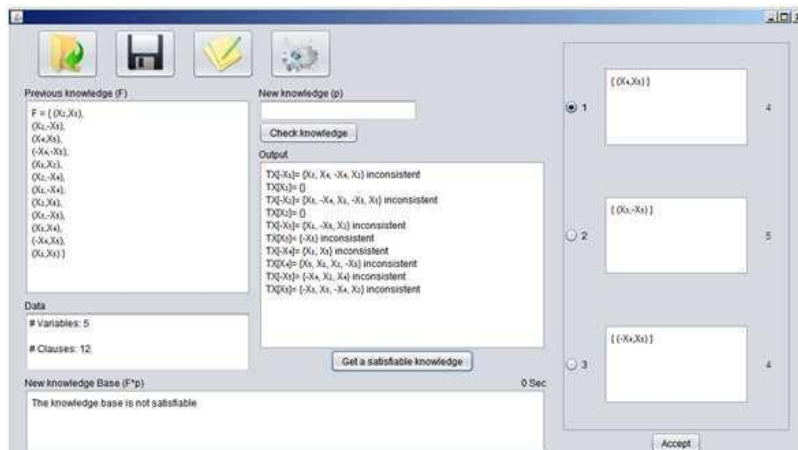


Fig. 1. Application of judgment revision with rationality criterion

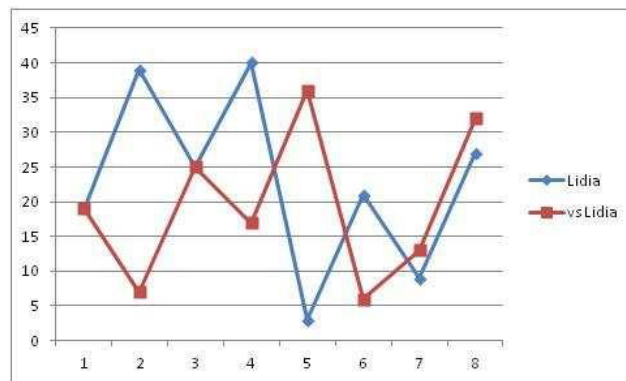


Fig. 2. Lidia's personal judgment against external opinion of her.

### 3.1. Results

This strategy of judgment revision with a rationality criterion was applied to the following problem: A group of students in a certain course were given a survey to determine their personal judgment base. The following list of items from the Honey-Alonso [9] test was used:

1. I always say what I think clearly and to the point.
2. I'm certain of what is right and wrong.
3. I frequently act without thinking about the consequences.
4. I usually try to solve problems methodically, step by step.
5. I prefer to hear other people's opinions before expressing my own.

40 questions (clauses) were formed by combining the above items. Table 1 and 2 show the set of personal judgments of the surveyed students.

**Table 1.** Judgments generated through student survey (first five)

Rugerio	Karen	Chacon	Lidia	Gaby
$(x_2, x_3)$	$(-x_1, x_5)$	$(x_2, x_3)$	$(x_2, x_3)$	$(x_2, x_3)$
$(x_2, x_4)$	$(x_2, x_3)$	$(x_4, -x_5)$	$(x_2, -x_5)$	$(-x_1, x_2)$
$(x_3, -x_4)$	$(-x_1, x_2)$	$(x_2, x_4)$	$(x_4, x_5)$	$(x_2, x_4)$
$(x_2, -x_5)$	$(x_2, x_4)$	$(x_1, -x_3)$	$(-x_4, -x_5)$	$(x_1, -x_3)$
$(x_3, x_4)$	$(-x_3, x_5)$	$(x_3, x_4)$	$(x_1, x_2)$	$(-x_3, x_5)$
$(x_2, -x_3)$	$(x_3, x_4)$	$(x_1, x_2)$	$(x_1, -x_4)$	$(x_1, x_5)$
$(-x_4, -x_5)$	$(x_1, x_5)$	$(x_2, x_5)$	$(x_2, x_5)$	$(x_2, -x_3)$
$(x_1, x_2)$	$(x_1, x_2)$	$(x_1, x_4)$	$(x_3, -x_5)$	$(x_1, x_2)$
$(x_2, -x_4)$	$(-x_1, x_3)$	$(-x_3, x_4)$	$(x_1, x_4)$	$(x_2, x_5)$
$(x_1, -x_4)$	$(x_2, x_5)$	$(x_1, x_3)$	$(-x_4, x_5)$	$(-x_3, x_4)$
$(x_1, -x_5)$	$(x_1, x_4)$		$(x_1, x_3)$	
$(x_3, -x_5)$	$(x_3, x_5)$			
$(x_1, x_3)$	$(x_1, x_3)$			
$(-x_3, -x_4)$				

Three students were randomly chosen and subjected to external judgment by their peers. Table 3 shows the external judgments that affect the personal judgment of each of the three students. In the case of Lidia, for example, it can be seen that in order to maintain a judgment base consistent with the external judgment, she must eliminate some of the following clauses:

- $(x_4, x_5)$ : I usually try to solve problems methodically, step by step, and I prefer to hear other people's opinions before expressing my own.
- $(x_3, -x_5)$ : I frequently act without thinking about the consequences, and I prefer to express my own opinions rather than hearing other people's opinions.
- $(-x_4, x_5)$ : I don't usually solve problems methodically, step by step, and I prefer to hear other people's opinions before expressing my own.

**Table 2.** Judgments generated through student survey (last five)

Jose	Leon	Hugo	Esteban	Anatolio
$(-x_1, x_5)$	$(x_4, -x_5)$	$(-x_1, x_2)$	$(x_2, x_3)$	$(-x_1, x_2)$
$(x_2, x_3)$	$(x_2, x_4)$	$(x_2, x_4)$	$(x_4, -x_5)$	$(x_4, -x_5)$
$(-x_1, x_2)$	$(x_1, -x_3)$	$(x_1, -x_3)$	$(x_2, x_4)$	$(-x_3, x_5)$
$(x_4, -x_5)$	$(-x_3, x_5)$	$(-x_3, x_5)$	$(x_2, -x_5)$	$(x_4, x_5)$
$(x_2, x_4)$	$(x_4, x_5)$	$(x_4, x_5)$	$(x_4, x_5)$	$(x_2, -x_3)$
$(-x_3, x_5)$	$(x_3, x_4)$	$(x_1, x_5)$	$(x_3, x_4)$	$(x_2, x_5)$
$(x_2, -x_5)$	$(x_1, x_5)$	$(x_2, -x_3)$	$(x_1, x_2)$	$(-x_3, x_4)$
$(x_4, x_5)$	$(x_1, x_2)$	$(x_1, x_2)$	$(x_3, -x_5)$	
$(-x_1, -x_3)$	$(x_2, -x_4)$	$(-x_1, x_4)$	$(x_1, x_4)$	
$(x_1, x_5)$	$(x_1, -x_4)$	$(-x_2, -x_3)$	$(-x_3, x_4)$	
$(x_2, -x_3)$	$(x_2, x_5)$	$(x_2, x_5)$	$(x_1, x_3)$	
$(-x_1, x_4)$	$(x_1, x_4)$	$(x_1, x_4)$		
$(-x_3, -x_5)$	$(-x_3, x_4)$	$(-x_3, x_4)$		
$(x_1, -x_5)$	$(x_3, x_5)$	$(-x_2, x_5)$		
$(x_2, x_5)$	$(-x_2, x_5)$			
$(-x_3, x_4)$	$(-x_4, x_5)$			
$(x_1, x_3)$	$(x_1, x_3)$			

**Table 3.** Judgments that affect the students and suggested solutions.

Student	External Judgment	Set of Clauses that must be Eliminated
Rugerio	$(-x_3, x_4)$	$(x_3, -x_4), (x_3, x_4), (-x_3, -x_4)$
Jose	$(x_3, x_5)$	$(-x_1, -x_5)$
Lidia	$(-x_3, -x_5)$	$(x_4, x_5), (x_3, -x_5), (-x_4, x_5)$

In Fig. 1 indicates that by eliminating the first or third clause, only 4 inconsistencies are generated, while eliminating the second clause generates 5. This, along with the meaning of the clause, aids the user in selecting the most convenient clause to eliminate. In Fig. 2 is showed the Lidia's personal judgment against her peer's external judgment of her.

## 4. Conclusions

This paper models the process of judgment emission considering the personal judgment that an agent has of itself and the impact caused when another agent makes a judgment of it.

The judgment base is specified by means of 2CNF, considering operations of expansion and contraction of the AGM model. Furthermore, a rationality criterion is applied where the agent is presented the list of beliefs (clauses) that can be removed in order to maintain a consistent judgment despite external stimuli.

We have a simple method based on the elimination of the clause that generates the fewest inconsistencies by adding new judgment C, this thanks to the calculation of set TX and the implication generated by the implication graph.

The tests to model this strategy were performed in their own application, which receives the set of judgments in the form of 2CNF clauses as input. This program simulates judgment base behavior when new judgments are added. Furthermore, it detects inconsistent based, allowing us to apply a rationality criterion to generate consistent judgment bases. In the future, this strategy could be applied to the diagnosis of learning styles, personality, and user profiles. Furthermore, this proposal could be used in other fields such as lie detection.

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