Reshaping Design Search Spaces for Efficient Computational Design Optimization in Architecture

Likai WANG[1], Patrick JANSSEN[2], Guohua JI[1]
1. School of Architecture and Urban Planning, Nanjing University, China
2. School of Design and Environment, National University of Singapore, Singapore
dg1436002@smail.nju.edu.cn, patrick@janseen.name, jgh@nju.edu.cn

Abstract

This paper focuses on the use of using appropriate parametric modelling approaches for computational design optimization in architecture. In many cases, architects do not apply appropriate parametric modelling approaches to describe their design concepts, and as a result, the design search space defined by the parametric model can be problematic. This can further make it difficult for the computational optimization process to produce optimized designs. As a result, the design search space needs to be reshaped in order to allow the computational design optimization process to fully exploit the potential of the design concept on improving the design quality. In this paper, we identify two common types of inappropriate modelling approaches. The first one is related to the design search space that lacks proper constraints, and the second is related to the design search space fixed by the conventional design knowledge. Two case studies are presented to exemplify these two types of inappropriate parametric modelling approaches and demonstrate how these approaches can undermine the utility of computational design optimization.

Introduction

In recent years, computational design optimization has been gaining popularity in architecture because it provides an efficient method for helping architects solve many performance-based building design problems related to material or energy use. By defining a parametric model for the building design and an evaluative model for the building performance, architects or engineers are able to use computational optimization algorithms such as genetic algorithms or direct search algorithms to establish an automated design optimization system. Such systems enable architects to use computers for tedious “number-crunching” where the computer explores the design search space defined by the parametric model. Once the population has been evolved, architects can then identify optimal design variants that can achieve a set of performance requirements. As such, the process of design variants can be driven by building performance. Such design optimization processes can be referred to as *performance-based* (Oxman, 2009) or *performance-driven design* (Shi & Yang, 2013).

In the research literature, there a numerous successful examples of computational design optimization on improving the building performance. However, in practice, the performance improvements that can be achieved can be limited if only relying on computational design optimization itself.

When applying computational design optimization in architecture, a key task for the architect is to encode their design concept as a parametric model. The parametric model delineates a specific design search space with a family of design variants (candidate solutions) sharing specific characteristics. This model is composed of a set of rules and constraints defining the associative relationships among various parts and components. Ideally, the parametric model should capture the design concept which the architects believe is capable of solving the design challenges.

However, the relationship between the design concept and the parametric model is complex, and for architects, creating such a model in the midst of their design exploration process is often difficult. This is due to that fact that, with different parametric modelling approaches, the particular constraints and rules that the architects define will result in a search space that only includes certain design variants and will also impose biases favour some design variants over others within that search space. As a result, an inappropriate parametric modelling approach may inadvertently end up including too many low-performance designs or excluding the most interesting high-performance design variants from the design search space (Figure 1). In this respect, creating an appropriate parametric model can be more decisive than performing the computational design optimization process itself. This issue is highlighted by Rittel & Webber as follows: “*setting up and constraining the solution space and constructing the measure of performance is ... likely ... more essential than the remaining steps of searching for a solution ...*” (Rittel & Webber, 1973).

Taking this as the point of departure, this paper first identifies two common inappropriate parametric modeling approaches and describes the weakness of the design search space if the space is defined by these two modelling approaches. In order to overcome these weaknesses, the design search spaces need to be iteratively modified in order to produce an improved design search space for the computational optimization, which can be referred to as *design search space reshaping*. Next, two case studies are presented to crystallise the idea of design search space reshaping and demonstrate how computational design optimization can be undermined by an inappropriate design search space as well as how it can benefit from reshaping.

Design Search Space Reshaping

In the context of performance-based architectural design, exploring design variants is the primary task after the design concept has been defined. With design processes that do not use optimization systems, architects might iteratively generate and evaluate design variants reflecting a particular design concept. The evaluations might use building performance simulations such as computational fluid dynamic (CFD) simulations and energy simulations. By iteratively producing and evaluating new design variants, architects were able to gradually improve design performance (Gero, 1990; Liu, Bligh, & Chakrabarti, 2003). However, such design processes are typically inadequate and inefficient since only a small number of designs could be explored. As a result, the chances of discovering unexpected high-performance designs would be very low.

The emergence of computational design optimization helped to resolve this challenge. With such algorithms, architects were able to define a design search space by encoding their design concepts and then let the computer search the design search space for the optimal design variants. Such automated design optimization methods rapidly become popular in research over the past decade.

While some researchers believe that performance-based design can be fully automated, while others have become aware of the limitation of computational design optimization. Many researchers argue in favour of a human-in-the-loop optimization process, with architect playing a critical role in guiding and filtering the computational design optimization process (Bradner, Iorio, & Davis, 2014; Negendahl, 2015; Stouffs & Rafiq, 2015; Wortmann & Nannicini, 2017). In this respect, Bradner et al. point out that "*the computed optimum was often used as the starting point for design exploration*" (Bradner et al., 2014), and similarly, Wortmann takes computational design optimization as a "*medium of reflection*" (Wortmann, 2018). In reality, however, for those architects who are interested in using computational design optimization either for design exploration or reflection, a challenge they may first encounter is encoding their design concepts with an appropriate parametric modelling approach.

In practice, the problem of inappropriate parametric modelling approaches is not uncommon. Nonetheless, it is often overlooked by architects due to the fact that the parametric models defined by such inappropriate modelling approaches will still result in a computational design optimization process that seems to progressively discover better design variants. This will often give architects a false sense of confidence with regards to the actual quality (fitness) of the design variants.

On our observation, there are two key reasons for the misapplication of parametric modelling approaches. The first reason relates to the lack of knowledge on optimization (Wang, Janssen, & Ji, 2018). Many architects who use computational design optimization have little knowledge of the complexity and limitations of such optimization algorithms. The second reason relates to architects' design fixation, stemming from conventional design knowledge based on past experiences (Wang, Janssen, & Ji, 2019b). Such knowledge can result in the architect overlooking alternative parametric modelling approaches that could result in design variants with significant performance improvements.

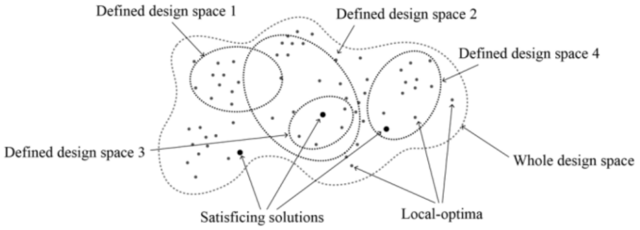

Figure 1. Relationship of design search spaces

In order to overcome these two problems, it requires architects to be more critical on learning from the feedback offered by the computational design optimization process rather than directly using the result obtained by the process. In this regard, the outcomes of the computational design optimization processes can encourage architects to reflect on and improve their parametric modelling approaches in order to reshape the design search space.

In the next two sections, two case studies are presented to illustrate how the above mentioned parametric modelling approaches, as well as the associated problematic design search space, can degrade the result of computational design optimization processes, and how these problematic design search spaces can be modified to allow computational optimization to achieve better results.

Case Study 1

The first case study serves to exemplify the design search space without proper constraints. The design describes a 40-storey high-rise building centred with an atrium. A series of vertical garden voids connecting to the atrium are inserted into the building. The combination of an atrium and vertical gardens are widely used to enhance natural ventilation and moderate temperatures. However, vertical gardens can also occupy a large amount of rental space of the building and increase the overall cost. Therefore, for this case study, the design objective is to search for design variants that can optimize the economic performance taking into account various factors including potential rental profit, façade cost, and construction cost. Thus, high-performance design variants have a rental profit that can significantly outweigh the accumulated cost if the building facades and structures.

In this design, the building was first voxelised in order to insert vertical gardens into the building mass. The floors are divided into multiple fixed-size voxels (Figure 2). Except for the voxels representing the structural cores and the atrium, all other perimeter voxels can be switched between an indoor floor and outdoor void, thereby allowing for complex patterns of interlocking indoor and outdoor space to be created.

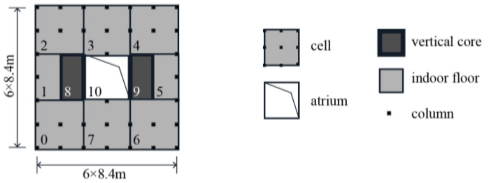

Figure 2. The subdivision of the floor plan into voxels.

Three alternative parametric models were created: without constraints (the first one) and with constraints (the second and the third one). Each parametric model was used to evolve a population of designs with evolutionary algorithms. The models were created in the Rhino-Grasshopper environment, one of the most popular parametric modelling platforms among architects. Design optimization processes were run using Galapagos, an inbuilt optimizer in Grasshopper, which provides a simple genetic algorithm. In order to achieve higher statistical significance, the computational design optimization process was repeated five times for each model.

Design Search Space without Constraints

When encoding this design concept into a parametric model, many designers would prefer a simple and unconstrained parametric modelling approach which independently assigns void-solid conditions for each of the voxels from external parameters. With this approach, the parametric model delineates a design search space with a rich diversity of design variants. At the same time, such parametric models are easy to implement, thereby, making such approaches attractive to architects who often have limited programming skills. However, the downside of using this modelling approach is the extremely large design search space. Moreover, the design search space may also include a great many chaotic design variants which may be too expensive to build. The model is referred to as the Naïve Model.

The drawbacks of this parametric modelling approach and the corresponding design search space can be discovered by running the optimization processes. The first line in Figure 3 presents the result of each of the five design optimization processes. All resultant design solutions have distinct geometric characteristics, and some of these design variants have unexpected geometric features. For instance, the middle one has the merge of voids from consecutive floors allowing for an impressive spatial flowing form. However, from the architectural point of view, most of these can be regarded as infeasible in terms of building geometry.

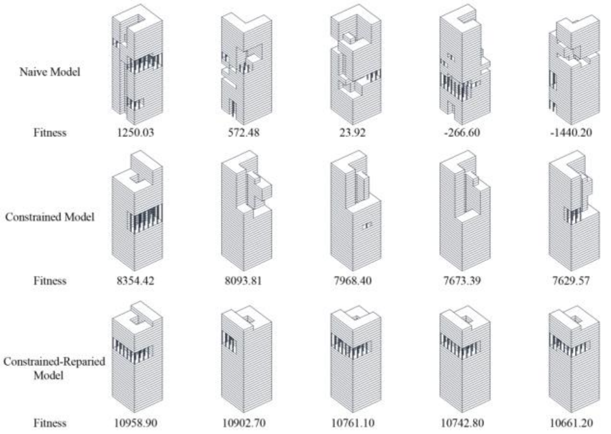

Figure 3. Results of the design optimization processes

In addition to the infeasible building geometry, the design variants can also not meet the required economic performance criteria. Two out of the five resultant designs have a fitness value below zero, which means that these solutions fail to make a rental profit covering the cost, while the other three merely have limited profitability. On the whole, even though these design variants are deemed "optimal", they fail the basic requirements for feasibility. Meanwhile, even in the sense of supporting reflection, these designs cannot offer much information for architects to extrapolate these to discover some hidden trends or trade-offs of the design problem.

With regards to weaknesses that these design variants have, the unconstrained design search space is the major issue hampering the computational design optimization process to find feasible design solutions. As Rasheed (1998) argued, many parametric models can result in a large proportion of infeasible or invalid design variants in the design search space. This can make it extremely difficult for computational design optimization processes to identify even one single feasible design variant if the design search space is huge. Considering the weaknesses inherited from the unconstrained design search space, the parametric modelling approach needs reformulation by introducing constraints, to reshape the design search space.

Design Search Space with Constraints

Excluding those infeasible design variants within the design search space is necessary to achieve meaningful results from design optimization processes. Hence, direct constraint handling strategies such as *special representations* and *repair functions* (Eiben & Smith, 2004) can be used to achieve such exclusions by avoiding the creation of infeasible design variants. Therefore, the parametric modelling approach was reformulated by applying these constraint handling strategies into the parametric model.

Firstly, with regards to special representations, a set of predefined floor layout patterns were applied to the design (Figure 4), which ensure that the insertion of vertical garden voids only occupies a reasonable size of indoor floors and all vertical garden can connect to the atrium to facilitate natural ventilation. The parametric model applying this constraint handling design strategy is referred to as the *Constrained Model*. Likewise, based on the same simple genetic algorithm for optimizing the Naïve Model, five design optimization processes were carried out to investigate the effect of the constraints applied in this Model

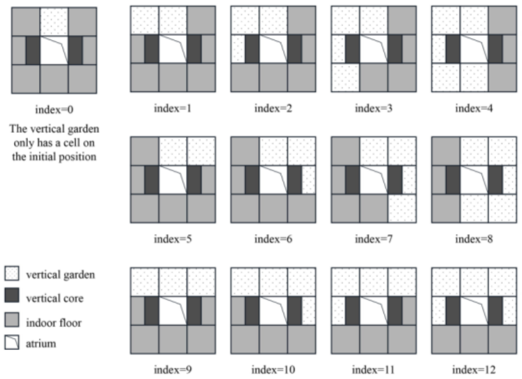

Figure 4. Predefined floor layout patterns

The second line of Figure 3 shows the optimal design variants found across the five design optimization processes. Compared with those obtained with the Naïve Model, these optimal designs have better economic performance in terms of the fitness. In addition, the constraints also make the design optimization processes find design variants with similar geometric characteristics, which help architects extract reliable information from the result.

However, also shown in Figure 3, the building geometries still lack feasibility, which implies that the design search space defined by the Constrained Model still contains a large number of infeasible design variants which prevent feasible designs from being identified. These infeasible design variants have a common undesirable feature: a large hole at the top of the building. It is mostly due to the stacking of floors with the same floor layout pattern. Such floor stacking can result in oversized voids in cases where many voids overlap one another and become merged. In this regard, the constraint embedded in this model is unable to exclude all unwanted infeasible design variants, and the design search space of the Constrained Model needs to be further reshaped (shrunken).

Considering the infeasible feature uncovered by the optimization result, a repair function was applied into Constrained Model, which is able to correct the identified problematic features in the design. The parametric model with the repair function is referred to as the *Constrained-Repaired Model*. The repair function is primarily aimed to control the vertical size of façade voids by preventing voids from stacking.

The repair function is not explicitly enforced under all circumstances, and rather, it is triggered implicitly when violations of the vertical size limit are detected. For this case study, if the void vertically exceeds six stories, the repair function will be activated, and the over-sized void will be iteratively shrunken from the top and the bottom until a suitable height is reached (*a-a'* in Figure 5). In addition, the repair function also fixes another problematic feature, where two voids meet at a point on the diagonal resulting in a pair of cross-diagonal voids (*b-b'* in Figure 5). Likewise, Constrained-Repaired Model was used to optimize the design.

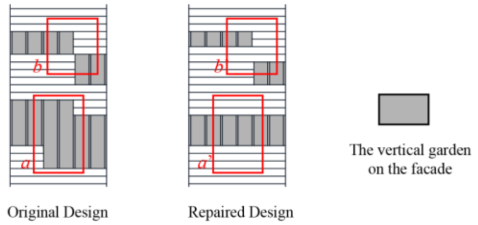

Figure 5. An example of design being fixed by the repair function

The third line of Figure 3 presents the optimal designs found by the design optimization processes with Constrained-Repaired Model. One can find that all these selected design variants have similar geometric features and fitness. On the one hand, the building geometries are more desirable from architectural perspectives compared with those obtained with the Constrained Model. On the other hand, the fitness is also further improved, which implies that these design variants are more profitable. In this respect, the combination of the special representation and the repair function makes the design search space well-constrained. Only with that design search space, can the computational design optimization play a meaningful role in helping architects either understand the design problem or facilitate better decision-making.

With progressively applied constraints into the parametric model, the design search space is iteratively reshaped (shrunken) with the exclusion of a great many infeasible design variants. The first case study not only shows how poorly-constrained design search space can obstacle computational design optimization processes to improve the design but also demonstrates how constraints can be applied to overcome the weaknesses inherited from the unconstrained design search space. However, we should point out the incorporating these constraints will inevitably reduce the variety of design variants in the design search space, which further make the optimization result less unexpected. It is the trade-off the architect should carefully consider.

Case Study 2

The second case study serves to exemplify the parametric modelling approach framed by conventional design knowledge. The design describes a fixed four-story low-rise building centered with a quadrilateral courtyard space. Courtyards have been widely applied in architectural design to improve indoor lighting quality, as it can allow much light to reach the inner part of the building. However, the form and the size of the courtyard volume can result in great differences in its ability to catch sufficient natural lighting for the indoor space. In general, larger courtyards allow for larger natural-lit indoor space, while these also undermine the profitability of the building since the courtyard occupies much indoor space. Thus, when designing buildings with a courtyard, it is crucial to restrict the size of the courtyard volume while searching for the form of the courtyard can catch as much daylight as possible.

In this case study, three alternative parametric models were created, and these model either stem from paper-based design knowledge or spatial design knowledge. In order to investigate the capability of these models in facilitating computational design optimization to optimize the natural lighting performance of the building, design optimization processes combined lighting simulations were conducted. The simulation was performed by DIVA (Jakubiec & Reinhart, 2011), and Spatial Daylight Autonomy (sDA) was taken as the performance indicator. sDA calculates the percentage of floor area that receives at least 300 Lux for at least 50% of the annual occupied hours (Sterner, 2014).

At the same time, in order to restrict the size of the courtyard, the gross floor area of the building was also taken into account. The gross area of the building is normalized to an area ratio by dividing the actual gross area with a target gross area. The area ratio is exponentially decreased along with the decrease in the gross floor area. The area ratio is applied to penalize natural lighting performance. Thus, the value of sDA value is multiplied with the area ratio which is always equal or less than 1. As a result, only when the courtyard with a reasonable size, is the natural lighting performance achieved by the courtyard valid.

The lighting simulation is time-consuming, where each simulation lasts around 1-to-2 minutes. Thus, running the computational design optimization process multiple times in impractical. Hence, for this case study, the optimization process based on each parametric model only ran once. At the same time, an island-model-based evolutionary algorithm (Wang, Janssen, & Ji, 2019a) was applied to optimize the design because this algorithm can yield several diverse design variants, which can facilitate a better understanding of the design search space. In this case study, five parallel search processes were set, so that an equal number of design variants were obtained by the design optimization process.

Design Search Space of Paper-based Design Knowledge

In architecture, the paper-based design method and the rationalist ideology have a great impact on the way how architects describe their design concept, even in digital and computational design environments where higher spatial freedom is allowed by 3D modelling software. One example is 3D parametric modelling approaches that are inadvertently based on 2D conceptual thinking. Such thinking has been deeply rooted in the architectural design knowledge, where building geometry is habitually defined by floor plans and floor height. Therefore, when implementing parametric modelling for courtyard design, the courtyard is often defined by the shape in plan and its height. In this case study, we call the parametric model using this parametric modelling approach the *Plan-based Model*. For this model, the courtyard shape in plan is defined by the four corner points, and then the shape is extruded to create the courtyard volume.

The first line of Figure 6 presents the five selected design variants found across the optimization process based on the Plan-based Model. In terms of the courtyard shape, these design variants have different shapes but with identifiable shared characters. From all presented design variants, one can find that the courtyards tend to have a triangle or near-triangle shape in plan. This is most likely due to the fact that such triangle shapes can result in a long distance among the courtyard façades so that the mutual shading becomes reduced, thereby, allowing much more natural light to go into the building.

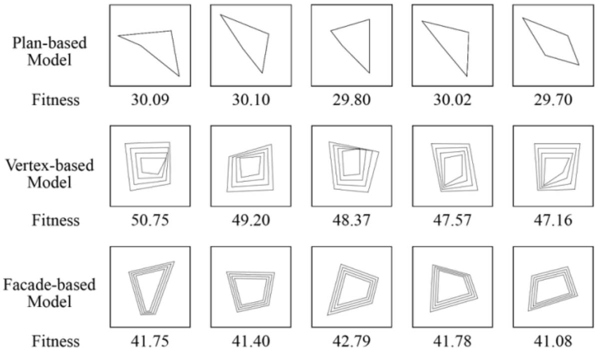

Figure 6. Results of the design optimization processes

However, even the triangle-shape courtyard can reduce the mutual shading, the natural-lit area in the building remain limited. The left column in Figure 7 visualizes the sDA value on each analysis grid based on the building with the highest fitness. Apparently, only the area close to the courtyard can be fully-lit, and the width of the area reduces sharply on lower levels and leave a large area on the lower floors under-lit. It is mostly due to the vertical facades cannot catch daylight when the incident angle of the light is either too high or too low. Hence, in order to achieve bigger improvement by the courtyard, the computational design optimization process should be allowed to search design space outside the one only with the courtyards characterized by vertical facades.

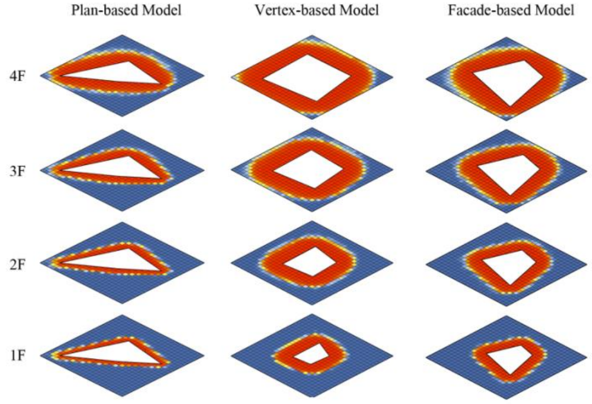

Figure 7. Visualization of sDA analysis

Design Search Space of Spatial Design Knowledge

In order to overcome the weakness stemming from vertical facades, a direct solution is to liberate the inclination angle of the façades as well as the cross-sectional profile of the courtyard. A simple way to free the courtyard from orthogonal volumes is to treat the courtyard volume as a spatial entity. Spatially, the volume of the courtyard is a hexahedron. Thus it can be defined by the eight vertices. Therefore, we reformulated the parametric modelling approach, which defines the courtyard with these vertices, and the corresponding parametric model is called the *Vertex-based Model*. In this model, each vertex is allowed to move independently. By moving these vertices, rich diverse courtyard volumes with inclined and twisted facades can be generated. Such façades are more effective in catching natural daylight from various incident angles and can also reduce mutual shading.

With the Vertex-base Model, the computational design optimization process was performed, which was also based on the same evolutionary algorithm used for Plan-based Model. The second line of Figure 6 shows the optimal design variants found across the design optimization process. The positive effect of using the Vertex-based Model can be immediately noticed in the sharp increase in fitness, where there is approximately 80% improvement on average. It implies that by allowing for higher geometrical freedom, a great many promising design variants are included in the design search space. Even if the inclusion of these design variants enlarges the design search space significantly and thereby may theoretically make the design search space difficult to search, however, the computational design optimization process can quickly identify several design variants with excellent lighting and economic (in terms of the gross floor area) performance.

The middle column of Figure 7 illustrates how the inclined and twisted facades actually improve the natural lighting performance of the building. Compared with the design obtained with the Plan-based Model, the natural-lit area of design obtained with Vertex-based Model is dramatically enlarged. Not only does the top floor can be nearly fully-lit, but also does the bottom floor have a much widened natural-lit area. At the same time, such a significant improvement is achieved without compromising on additional floor area losses.

However, even the natural lighting performance is substantially improved, the use of the Vertex-based Model is not without problems. A major downside comes along with the liberation of the courtyard volume is that most facades are twisted, and such twisted facades are difficult and expensive to construct. As a result, stakeholders could be not interested in such designs with twisted facades, and, if the economic feasibility is the priority for the project, the design variants created by the Vertex-based Model might be problematic. Therefore, economic feasibility and constructability should be considered in order to make the evolutionary result more practical.

With regards to economic feasibility and constructability, we came to the third parametric modelling approach, which can largely satisfy the economic requirement while allowing a significant improvement on natural lighting performance to be obtained. The third parametric modelling approach defines the courtyard volume by its façades, which we call the *Façade-based Model*. Compared with the Vertex-based Model, this model does not allow the façade surface to be twisted, and all façades are a planar surface. Thus, the courtyard volume is changed by rotating the façade vertically and horizontally and moving it along x/y axis. With the Façade-based Model, the façades can still allow being inclined while no twisted façade surfaces will be created.

The third line of Figure 6 shows the optimal design variants found across the computational design optimization process. The fitness indicates that the use of Façade-based Model results in a marked decrease in natural lighting performance compared with those found with the Vertex-based Model due to the exclusion of design variants with twisted facades. However, the planar façade surface makes these design variants more practical in real-world projects. Despite the decrease in performance, the optimal design variants obtained with Façade-based Model can still significantly outperform those obtained with the Plan-based Model due to the inclined façades.

The right column of Figure 7 illustrates the actual natural-lit area of the design variant with the highest fitness value among those found by the optimization process. One can find that the depth of the natural-lit area of this design variant is smaller than that of the design variant found with the Vertex Model. It is mostly because the planar surface is inflexible in controlling the trade-off between catching natural daylight and minimizing the loss of indoor space. In contrast, the twisted façade surface can incline in different angles on each side of the façade. Therefore, such façade surfaces can make the disadvantageous façade section that is blocked by other facades steeper to minimize occupied indoor space, while making the section that can receive much natural light more inclined to maximize the advantage.

For the second case study, the reshaping process of the design search space is in the relatively opposite direction compared with that in the first case study, where the design search space is first significantly enlarged and then slightly shrunken. The reshaping operations used in this case study respectively consider the natural lighting performance and the requirement of economic feasibility. Finally, the Façade-based Model finds a desirable balance between improving natural lighting performance and maintaining economic feasibility.

Moreover, using the models like the Vertex-based or the Façade-based Model can drive the computational design optimization process to maximize the potential of a design concept on improving the concerned performance. Only with this limit known by the architects, can they extrapolate the trends or trade-offs revealed by these optimal design variants to conceive better design concepts. In this regards, better representation of the design concept lay a more reliable foundation for the reflection on design concepts.

Discussion and Conclusion

This research focuses on the use of appropriate parametric modelling approaches for computational design optimization in architecture. Inappropriate parametric modelling approaches can produce poor design search spaces which cannot well represent architects' design concepts. Such search spaces make the design problem unsolvable either for the reason that the size of the design search space is too large or for the reason that the design search space does not include high-performance design variants. As a result, the potential of a design concept on improving the building performance cannot be efficiently explored by computational optimization.

With two presented case studies, this research identifies and exemplifies two sources that can cause the application of inappropriate parametric modelling approaches. Two case studies also show how these approaches can degrade the usefulness of computational design optimization. The first one stems from the ignorance of the search complexity of the design problem characterized by huge unconstrained design search spaces. In the first case study, even the design search space of the Naïve Model fully covers the design search spaces of the Constrained and the Constrained-repaired Models. However, the enormous infeasible design variants conceal the design sub-spaces characterized by the Constrained and the Constrained-repaired Model for the computational design optimization process to explore.

The second one stems from the design fixation inherited from the conventional design knowledge. In architecture, even 3D modelling techniques have been widely spread, the design knowledge characterized by orthogonal geometries persist. It is also pointed out by Menges as follows: "*In the history of architecture and construction ground breaking technologies have often been initially employed to facilitate projects that were conceived through, and indeed embraced, well established design concepts and construction logics. There is ample evidence of this inertia in design thinking in the context of technological progress*" (Menges, 2007). As a result, fixed by such conventional design knowledge, the design search space is also restricted, and as mentioned earlier, such design search space could not include the high-performance design variants.

In order to address the weakness inherited from the problematic design search space, the design search space needs to be reshaped, which requires the architects to be more critical on their parametric modelling approaches. Moreover, computational design optimization is also indispensable in such a reshaping process. Computational design optimization can help exhibit the design search spaces because the weakness in the design search space is often not explicit, and reaching to the limit of the design search space with computational design optimization can make the problems more "visible".

To conclude, using computational design optimization methods do not guarantee the architectural design can be truly optimized. One of the crucial promises of performing valid computational optimization is applying appropriate parametric modelling approaches, as inappropriate parametric modelling approaches can end up with "garbage-in-garbage-out". By reflecting the parametric modelling approaches, the iterative reshaping process towards the design search space not only helps the architects to obtain a better result with computational design optimization but also offer them insight into the performance implication of their design concepts, with which building performance can become a catalyst in architects' design synthesis.

Acknowledgments

This paper is supported by the National Natural Science Foundation of China (51378248) and the China Scholarship Council (201706190203).

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