strator can reduce the risk. In so far as practical it is best that one person set up an experiment alone, especially when reflecting elements must be manipulated. Different output levels, of course, call for different levels of caution, but, in all cases, users should be knowledgeable about applicable regulations and the nature of the type of injury that can result not only from exposure to the coherent beam itself but from associated electrical, chemical, and other dangers.

Sources of appropriate information should be available to all laser users, chosen from the list below or other sources.

186. **Teaching Physics Safely: Some Practical Guidelines in Seven Areas of Common Concern in Physics Classrooms,** A. W. Peterson (AAPT, Stony Brook, NY, 1979). "Specific practical suggestions on working safely with electrical hazards, lasers and light, pressurized and vacuum systems, ionizing radiation, fires, toxic materials, and mechanical dangers." (E)

*187. **"Laser Safety in the Laboratory,"** H. Weichel, W. A. D. Donne, and L. S. Pedrotti, Am. J. Phys. **42,** 1006 (1974). An excellent introduction to the types of hazards associated with the use of lasers together with suggestions for a safe laser laboratory. Written before current regulations were established. (E)

188. **Laser Safety Handbook,** A. Mallow and L. Chabot(Van Nostrand-Reinhold, New York, 1978). A comprehensive volume covering biological effects, laser measurements, protective standards, control of laser radiation and associated hazards, public laws and regulations, and more. Includes a chapter on classroom safety. Certainly not everyone will need all of this material, but just about everything that is needed is there. (I)

189. **Safety with Lasers and other Optical Sources,** D. Sliney and M. L. Wolbarsht (Plenum, New York, 1980). A handbook that "reviews current knowledge of biological hazards from optical radiation and lasers, presents current exposure limits, and provides ... information required for the control of health hazards. Chapters are included on specialized applications of lasers. . . ." (I)

190. **Laser Safety Guide** (Laser Safety Committee, Laser Institute of America, Cincinnati, OH, 1975). Laser hazards, calculations, and measurements are included. (I)

191. **The ANSI Laser Safety Guide** (American National Standards Institute, 1430 Broadway, NY 10018, 1976). Lists maximum permissible exposure levels and safety measures to be employed in using lasers. (I)

192. **HEW (FDA) Publication 76-803: Laser Regulations** (Director, Division of Compliance, Bureau of Radiological Health, 5600 Fishers Lane, Roelsville, MD 20852). These regulations are intended for the manufacturer of lasers and systems incorporating lasers. It is, however, also the defining document for laser classifications and measurements. (A)

XI. LASERS AND ESTHETICS

Fascination with lasers extends far beyond the realms of science and technology, as is obvious to anyone who regularly views contemporary films or visits modern art galleries.

193. **Laser Art and Optical Transforms,** T. Kallard (Optosonic, New York, 1979). Kallard has produced a book that illustrates the range of potential of laser light as an art form. While the section on optical transforms may be useful to teachers of physical optics, its greater value may be in suggesting to students possibilities of special project work that could combine elements of science and art. The organizing intelligence of the artist is seen in a few examples. (E)

We add a cautionary note for those who may be asked to "contribute" a laser display for some purpose beyond the needs of the physics classroom. No one *ever* wants a modest laser display. And a nonmodest, safe laser display may require an expenditure of time and money beyond that which the unwary volunteer may wish to donate.

The laser is indeed a "wonderful" device, in the true sense that it can invoke our capacity to wonder. And it is in wondering about the world that science has its roots. With a small laser and a hologram or two the physics teacher can be a wizard, invoking this sense of wonder, and a scientist, formulating and exploring the questions raised by his or her wizardry.

Interaction-free quantum measurements: A paradox?

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(Received 16 February 1981; accepted 20 May 1981)

In Heisenberg's famous discussion of the measurement of a particle's position using a microscope, the momentum transferred to the particle by the scattered photon makes the particle's momentum uncertain. It is shown that momentum is also transferred when the *lack* of a scattered photon is used to discover that the particle is *absent* from the field of view of the microscope (i.e., located outside the light beam). This apparent paradox, a transfer of momentum and/or energy to a *missing* particle by a light beam (without the scattering of a photon), is discussed and "resolved" using quantum measurement theory.

I. INTRODUCTION

Heisenberg's famous microscope experiment[1,2] is discussed in many elementary textbooks on quantum mechanics. This beautiful "thought experiment" shows clearly that the observation of the position of a particle with an accuracy Δx is accompanied by a momentum transfer to the particle with an uncertainty Δp and that $\Delta x \Delta p \geq \hbar/2$. When a particle is "seen" in the microscope (Fig. 1) the recoil momentum of the photon scattered by the particle is uncertain by the amount $\sim p \sin\theta$ (with $p = h/\lambda$) and the resolving power of the microscope is $\Delta x \sim \lambda/\sin\theta$. Here λ is the photon wavelength. It is presently believed that an essential feature of any determination of a particle's position is some *interaction* with the particle that leaves its momentum uncertain.

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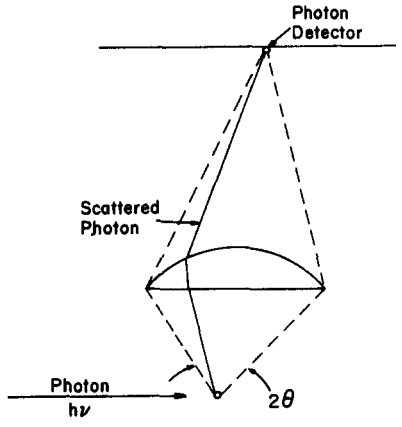

Fig. 1. "Heisenberg microscope." The uncertainty of the recoil direction of the photon scattered into the microscope makes the particle's momentum uncertain.

But what if we fail to see a scattered photon (a null result)? The simple act of *not* detecting a scattered photon provides new knowledge that affects the probability distribution of the particle's position by reducing the probability of the particle being found in the field of view of the microscope. And this seems to imply a change in the particle's wave function and as a result, according to quantum mechanics, possible changes in the particle's momentum and/or energy.

The paradox of such a quantum measurement without a corresponding scattering of a photon raises several questions:

(a) Does the absence of a scattered photon imply a change in the particle's wave function?

(b) Can a properly constructed experiment yield this type of null result?

(c) Does a change in the particle's momentum (and/or energy) without any change in the state of the radiation field imply a violation of conservation laws?

(d) How can the paradox of a change in the particle's momentum or energy without an "interaction" be resolved?

If there really is a paradox we have every reason to believe that the root cause must be some conceptual problem not an inadequacy of quantum mechanics itself.

In addressing these questions our first task is to modify the Heisenberg microscope to make it more suitable for answering the simple query: "Is the particle located in the light beam or not?" This modified microscope is shown in Fig. 2. Using the optics shown, a light beam is brought to a focus of diameter $2a \gg \lambda$. This light beam, moving in the z direction, passes through two transparent parallel walls [(1) and (2)] that confine the particle in the z direction to $z \sim 0$ while permitting it to move freely in the x and y directions. The light beam is then absorbed by being focused on a stop. The photons scattered by the particle generally miss the stop and are detected by the photon counter (ideally a 4π solid angle counter).

If a scattered photon is not detected by the counter an originally motionless particle's wave function is reduced over the beam ($r < a$) and this reduction continues for roughly $\sim Ma^2/h$ seconds, where M is the particle's mass. Thus a meaningful position measurement requires a light pulse of duration $\tau \ll Ma^2/h$.

This light pulse must be sufficiently intense that a particle found in the light beam almost certainly scatters at least one photon. But then, conversely, the absence of a scattered photon implies the absence of the particle from the beam, implying that the particle is "elsewhere."

We shall now address the above four questions:

(a) If the light pulse shows the particle to be absent from the beam, a second pulse (immediately following the first) should almost certainly, find it absent. With the usual meaning of the wave function this implies that after the first light pulse

$$\psi \cong 0 \quad \text{for } r^2 = x^2 + y^2 < a^2.$$

(b) According to the usual interpretation of the wave function the probability of the particle lying outside the focus (hence *not* scattering photons) should be

$$\int |\psi|^2 dx\, dy \neq 0,$$

where the integral is over the domain $r^2 > a^2$.

(c) The uncertainty in the transverse momentum of the radiation pulse is $> h/a$ (because of the focus and also the large number of photons). For the energy the uncertainty is $> h/\tau$. Thus the total momentum and energy of the system are not well enough defined to permit direct tests of the conservation of momentum and energy.

(d) A change in the (well-defined) momentum or energy of a free particle (without some external disturbance) is incompatible with mechanics. Presumably some sort of interaction with the radiation field must be occurring without the scattering of a photon.

A weakness in the above discussion is the implied equivalence of "interaction" with "photon scattering." The distinction between the presence and absence of an interaction can be hardened by letting the interaction destroy the particle. The "particle" could be an atom and the photon energy could be great enough to "destroy" the atom by ionizing it. The criterion for the absence of an interaction between the radiation and the particle now need not be photon scattering. It could be the survival of the particle.

For purposes of discussing this case we introduce another thought experiment. (See Fig. 3.) The particle is an ion in a uniform magnetic field. It is trapped in a quadratic electrostatic potential and oscillates along the magnetic axis with the frequency ω.

In the lowest energy state this oscillation contributes $E_0 = (1/2)\hbar\omega$ to the ion's energy. The corresponding wave function $\psi_0(x)$ is plotted in Fig. 3(c). Now imagine a short ($\tau\omega \ll 1$) intense pulse of energetic photons illuminating the region $-a < x < a$ where

$$\int_{-a}^{a} |\psi_0|^2 dx = \frac{1}{2}.$$

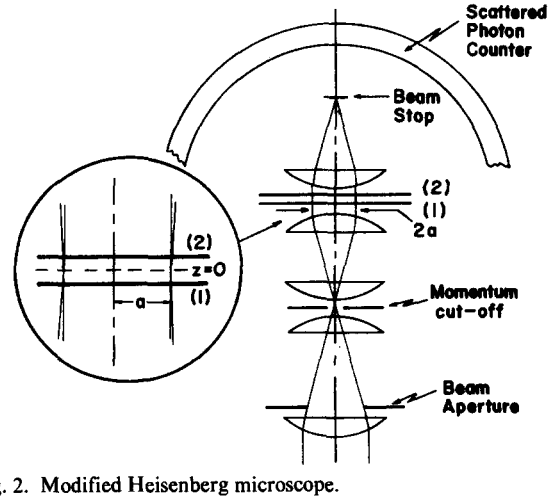

Fig. 2. Modified Heisenberg microscope.

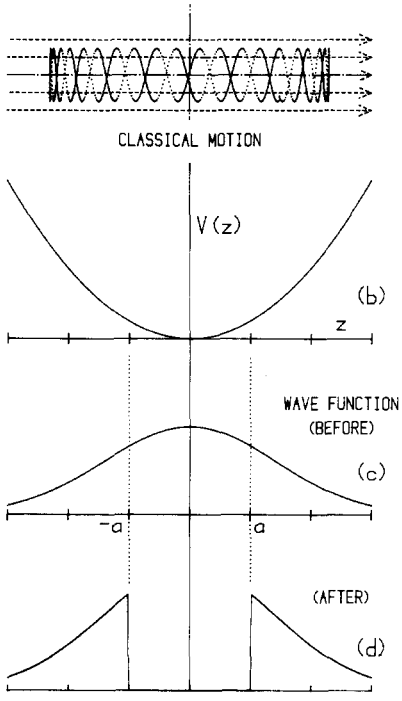

Fig. 3. Particle (ion) trapped in a uniform magnetic field by a parabolic electrostatic potential well (b). Its initial z-component wave function (c) is modified by the intense radiation pulse in the range $-a < z < a$ to (d), which gives the probability distribution of the surviving ion.

It should be emphasized that the frequency ω could be extremely low and the distance a very large compared with a wavelength. Thus there could be a quite sharply defined boundary between the interior and exterior of the radiation beam.

If the ion is found in the radiation beam (probability 0.5) it is multiply ionized (hence destroyed) and the photoelectrons are detected.

If the photoelectrons are not detected the ion survives and it is presumably located outside the radiation beam. Thus the particle's wave function is changed to ψ with

$$\int_{-a}^{a} |\psi|^2 = 0.$$

[See Fig. 3(d).] But then the expectation value of the energy $\langle E \rangle > E_0$. Again an energy change of the particle occurs without any obvious interaction with the radiation beam.

If the wave function after the result "ion intact" were to be left unchanged (i.e., ψ_0) we would face a dilemma, for a quick sequence of n pulses would then leave the ion intact with a probability $(1/2)^n$. But such a series of pulses is equivalent to a single complex pulse for which this probability should be $(1/2)$.

II. QUANTUM MEASUREMENT THEORY AND THE NULL EXPERIMENT

In trying to find a possible conceptual error in the above discussion we now examine the formal measurement theory (due primarily to von Neumann[3]) as applied to a simple example of a null experiment. The quantum-mechanical system will be an atom with a nondegenerate ground state. We shall use resonant fluorescence scattering of a photon by the atom to decide if the atom is in or out of a radiation beam.

The measuring apparatus is that shown in Fig. 2 and the photon wave packet (of wavelength λ) is considered to be part of the apparatus. In making the measurement the electromagnetic field of the wave packet acts on the atom and the presence or absence of scattered photons is used as an indicator for the presence or the absence of the atom from the radiation beam. The interaction between the atom and the field is treated quantum mechanically, but the photon counting is treated nonquantum mechanically as an irreversible process.[4]

Prior to the measurement, the atom and the radiation are independent and the wave function of the combined system is a simple product function of the states of these two components. The interaction leads to a superposition of product states. Each of these product states represents some definite state of the electromagnetic field and the corresponding state of the atom.

The "measurement," an irreversible process, determines the state of the radiation (and the atom) by destroying the superposition and projecting out one of the various product states with the appropriate probability. By observing the presence or absence of scattered photons the state of the radiation is determined and the associated atomic state is also determined (or "measured").

The radiation lifetime τ_r of the excited state of the atom is assumed to be so short that $\tau_r \ll Ma^2/h$. The atom is excited by a radiation pulse of length $\tau \ll \tau_r$. With these inequalities the measurement can be divided into a sequence of well-defined intervals and events: (a) prior to the interaction of the light pulse with the atom, (b) the interaction interval, (c) after the interaction but before an absorbed photon is emitted, (d) after the emission of the absorbed photon but before the recoiling atom can move appreciably, and (e) the detection of an emitted photon, if any.

We assume that the atom is confined to a rectangular box that is large in the x and y directions and very small in the z direction ($\ll \lambda$). It is located at $z \simeq 0$. The center-of-mass wave function of the atom is written $v(x,y,z)$. For simplicity we assume that the atom is initially in its internal ground state and also in the lowest energy state of its center-of-mass motion.

The excited state of the atom is degenerate but for an exciting light pulse of definite polarization a definite spin state of the atom is excited. Thus we can consider the atom to be a two-energy-level system if all nonresonant transitions are ignored. We designate these two internal energy states by $m = 0$ and 1, respectively.

Instead of quantizing the electromagnetic field in the usual set of plane waves we introduce an orthonormal set of "focused waves," focused at $z = 0$ with a discrete frequency spectrum. Waves of different frequency can be superposed to form a second orthonormal set, describing focused wave packets. The initial state of the radiation pulse is a state of n photons created in a particular wave packet. (In the following, units are chosen to set $\hbar$ and $c = 1$.)

As a first step we choose a hypothetical, large, perfectly reflecting enclosure that has the z axis as a symmetry axis and $z = 0$ as a symmetry plane. It is so shaped that the wave amplitude function generated at the plane $z = 0$ by the optical system of Fig. 2 is obtained as one of the normal modes of the enclosure. This hypothetical idealized optical resonator replaces the lens system of Fig. 2. The resonator has a nearly spherical surface of radius $r \sim b$ with $b \gg a > \lambda$. This acts as a perfect reflector to reverse the wave received from the focus at $z = 0$. Owing to the large size of

the enclosure there are many (even) normal modes of different frequency having substantially the same function at the focal plane and they are separated in frequency by the corresponding odd modes (with nodal surfaces at $z = 0$).

To determine the shape of the resonator that generates these focused standing waves as normal modes we start with the (real) amplitude distribution $u(r,\phi,z = 0)$ at the focal plane. This function describes the wave at the focal plane generated by the optical system (Fig. 2) for $\omega = \omega_0$, the resonance frequency of the atom. Using the two-dimensional Fourier transform of u,

$$U(\tilde{k}_x,\tilde{k}_y) = \frac{1}{2\pi}\int u \exp(-i\tilde{\mathbf{k}}\cdot\mathbf{x})d^2x, \tag{1}$$

we obtain

$$u(k_0,\mathbf{x}) = \frac{1}{2\pi}\int U(\tilde{k}_x,\tilde{k}_y)\exp(i\,\tilde{\mathbf{k}}\cdot\mathbf{x})\cos k_z\, z\, d^2\tilde{k} \tag{2}$$

[with $k_x = \sqrt{(k_0^2 - \tilde{k}^2)}$ as the (even) standing wave associated with the amplitude function in the focal plane]. We now choose the shape of the perfectly reflecting resonator to permit its surface to coincide with a nodal surface of u.

The optical system of Fig. 2 employs a circular aperture to define the light beam followed by a circular aperture in Fourier transform space that cuts off waves with large transverse momenta, $\tilde{k} > k_m$. This is followed by another Fourier transform back to x, y space. The resulting wave amplitude over the focal plane ($z = 0$) is

$$u(r,\phi,0) = \frac{1}{\sqrt{\pi}}\int_0^{k_m} J_1(\tilde{k}a)\,J_0(\tilde{k}r)\,d\tilde{k}. \tag{3}$$

(See Fig. 4.)

The normal modes satisfy

$$\nabla^2 u + k^2 u = 0, \tag{4}$$

and are orthogonal for different values of k. Owing to the symmetry axis (z) some modes are degenerate and we use two indices, k and g, to designate the mode $u(k,g,\mathbf{x})$.

We designate the orthonormal modes based on (1) by $g = 0$. For conciseness we designate the set of all orthonormal modes by $u(k,\mathbf{x})$ with k representing both g and k. The incident radiation pulse is assumed to be based on a superposition of the waves given by $g = 0$. For these waves, u is approximately constant over the focus ($r < a$). (See Fig. 4.)

We assume some definite polarization state for the radiation field and an expansion of the electromagnetic vector potential **A**, which is based on the normal modes $u(k,\mathbf{x})$. The quantum state of the electromagnetic radiation is then a superposition of the states $|\ldots,n(k),n(k'),n(k''),\ldots\rangle$. Here $n(k)$ is the number of photons in the mode $u(k,\mathbf{x})$. [If the orthogonal polarization state of the electromagnetic field were also to be excited we would require two photon numbers for each of the radiation modes $u(k,\mathbf{x})$.]

Introducing the annihilation operator $a(k)$ for the kth mode and the corresponding creation operator $a^*(k)$ [the Hermitian adjoint of $a(k)$] gives

$$|\ldots,0,n(k),0,\ldots\rangle = \frac{1}{\sqrt{n!}}[a^*(k)]^n|\ldots,0,0,0,\ldots\rangle. \tag{5}$$

The Hamiltonian of the free field is

$$H_r = \sum_k \omega a^*(k)a(k), \tag{6}$$

where the eigenvalues of $a^*(k)a(k)$ are $n(k)$.

The classical wave packet describing the incident radiation pulse (moving upward) is defined by

$$W(0) = \sum_k C(0,k)u(k,\mathbf{x})\exp(-i\omega t), \tag{7}$$

with $\omega = k$. Here the sum is over all values of g and k but $C(0,k)$ vanishes for $g \neq 0$. $C(0,k)$ is chosen to yield a time dependence of $W(0)$ at the focus ($z = 0$) of the form $f(t)\exp(-i\omega_0 t)$, where $E_0 = \omega_0$ is the excitation energy of the atom and $f(t)$ is the envelope function of the wave packet. Thus in the focal plane ($z = 0$),

$$W(0) = u(k_0,x,y)f(t)\exp(-i\omega_0 t) \tag{8}$$

with $g = 0$.

We adopt (7) as the first member of an orthonormal set of focused wave packets defined by

$$W(j,\mathbf{x},t) = \sum_k C(j,k)u(k,\mathbf{x})\exp(-i\omega t), \tag{9}$$

where the matrix $C(j,k)$ is assumed to be unitary.

The creation operator for creating a photon in the wave packet state j is

$$b^*(j) = \sum_k C(j,k)a^*(k). \tag{10}$$

Hence, since $C(j,k)$ is unitary

$$a^*(k) = \sum \overline{C}(j,k)b^*(j). \tag{11}$$

Instead of expanding the electromagnetic field in the modes $u(k)$ we shall now use the wave packet modes $W(j)$. With this new orthonormal basis the wave function

$$|\ldots,n(j),\ldots\rangle = \frac{1}{\sqrt{n!}}[b^*(j)]^{n*}|\ldots,0,\ldots\rangle, \tag{12}$$

represents a state of $n(j)$ photons in the $W(j)$ wave packet.

The Hamiltonian of the atom is

$$H_a = p^2/2M + \omega_0\sigma^*\sigma, \tag{13}$$

where $p = -i\nabla$ and $\sigma^* = (\sigma_1 + i\sigma_2)/2$ is the Pauli operator that excites the atom ($m = 0 \rightarrow 1$). The eigenvalues of $\sigma^*\sigma$

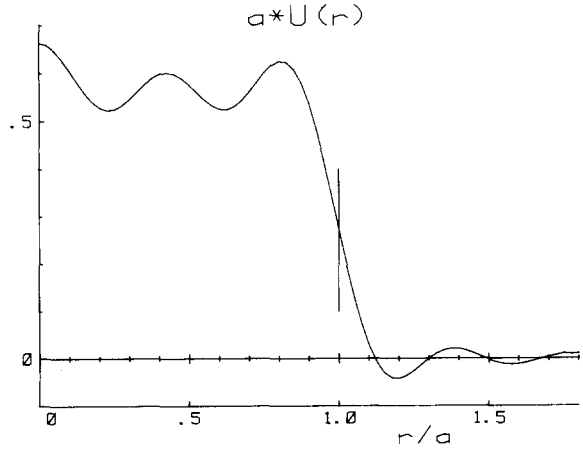

Fig. 4. Wave amplitude $u(r)$ at the focal plane for the incident radiation pulse. Calculated for $k_m = 16/a$.

are $m = 0,1$. The wave function of the free atom is of the form

$$\sum_m v_m(x,y,z)|m\rangle. \tag{14}$$

For the combined system, atom plus radiation, the wave function is in general of the form

$$\sum v(x,y,z)|m,n(0),n(1),\ldots\rangle, \tag{15}$$

where the sum is over all $m,n(0),\ldots$, and the indices $m,n(0),\ldots$ of v have been suppressed.

It is assumed that the initial states of the electromagnetic field and the atom are independent of each other. The wave function is then a simple product function of the atom in its ground state ($m = 0$) and the electromagnetic field in the wave-packet state $n(0) = n$:

$$\psi_0 = v(x,y,z)|0,n,0,0,\ldots\rangle. \tag{16}$$

The interaction-free Hamiltonian is

$$H_0 = p^2/2M + \omega_0\sigma^*\sigma + \sum\omega a^*(k)a(k). \tag{17}$$

The first (kinetic energy) term can be neglected in the interval $0 < t < \tau_r$. The interaction term is

$$H_1 = V[\sigma^* \sum_k a(k)u(k,\mathbf{x}) + \sigma \sum_k a^*(k)\bar{u}(k,\mathbf{x})], \tag{18}$$

where V is a constant and only the resonance terms are included. (The excitation of the atom is accompanied by the annihilation of a photon.)

The time evolution of the wave function ψ can be obtained from Schrödinger's equation written in interaction representation:

$$\tilde{H}_1\tilde{\psi} = i\,\partial\tilde{\psi}/\partial t \tag{19}$$

with

$$\tilde{H}_1 = \exp(iH_0t)H_1\exp(-iH_0t) \tag{20}$$

and

$$\tilde{\psi} = \exp(iH_0t)\psi. \tag{21}$$

The operator $\sigma^*a(k)$ destroys a photon of energy $\omega = k$ and excites the atom with an energy ω_0. Hence

$$\tilde{H}_1 = V\{\sigma^* \sum_k a(k)u(k,\mathbf{x})\exp[i(\omega_0 - \omega)t] + \sigma \sum_k a^*(k)\bar{u}(k,\mathbf{x})\exp[-i(\omega_0 - \omega)t]\}. \tag{22}$$

Substituting (11) in (22) yields

$$\tilde{H}_1 = V[e^{i\omega_0 t}\sigma^* \sum_{kj} C(j,k)u(k,\mathbf{x})e^{-i\omega t}b(j) + \text{Hermitian adjoint}]. \tag{23}$$

The solution to Schrödinger's equation can be written

$$\tilde{\psi} = \exp\left(-i\int_0^t \tilde{H}_1\,dt\right)\tilde{\psi}_0. \tag{24}$$

The principal physically interesting point about (24) can be made with a perturbation carried to second order,

$$\tilde{\psi} = \left(1 - i\int_0^t \tilde{H}_1\,dt - \int_0^t \tilde{H}_1(t')\int_0^{t'} \tilde{H}_1(t'')dt'\,dt''\right)\tilde{\psi}_0. \tag{25}$$

With the assumption that the radiation pulse is very short ($\tau \ll \tau_r$) there are no scattered photons at the time $t = \tau$. The wave function is then a superposition of the original state vector $|0,n,0,0,\ldots\rangle$ with the single state vector $|1,n-1,0,0,\ldots\rangle$ representing the excited atom and $n - 1$ photons in the excited pulse. This latter state vector has its origin in the term that is first degree in $\tilde{H}_1$ and contains the factor σ^* and $b(0)$. All other terms vanish.

By the time $t \gg \tau_r$ the excited state of the atom has decayed generating a photon $[b^*(j)]$ in some state other than $W(0)$, for the wave packet $j = 0$ departs from the surface $z = 0$ at $t = \tau$.

In accordance with the usual interpretation of a quantum measurement as discussed above, if the photon counting measurement ($t \gg \tau_r$) shows no scattered photon, the resulting *new* state of the electromagnetic field is the same as the original state and is obtained from $\tilde{\psi}$ in the form $P\tilde{\psi}$, where P is the projection operator for states without scattered photons

$$P = \prod_{j\neq 0}[1 - b^*(j)b(j)]. \tag{26}$$

The nonvanishing terms in $P\tilde{\psi}$ are found in (25) only in the zeroth and second degree terms in $\tilde{H}_1$. We first evaluate

$$\int_0^t \tilde{H}_1\tilde{\psi}_0\,dt = V\int_0^t \exp(i\omega_0 t)\sum_k C(0,k)u(k,\mathbf{x})\exp(-i\omega t)dt \times \sigma^* b(0)v(\mathbf{x})|0,n,0,0,\ldots\rangle = \sqrt{n}V\int_0^t f(t)dt\,u(k_0,\mathbf{x})v(\mathbf{x})|1,n-1,0,0,\ldots\rangle. \tag{27}$$

[See Eqs. (23), (7), and (8).] In evaluating the second integral the only nonvanishing term in $P\tilde{\psi}$ is the one containing the factor $\sigma b^*(0)$:

$$P\int_0^t \tilde{H}_1\int_0^{t'} \tilde{H}_1\,dt''\,dt'\,\tilde{\psi}_0 = nV^2\int_0^t \bar{f}(t')\int_0^{t'} f(t'')dt''\,dt' \times |u(k_0,\mathbf{x})|^2 v(\mathbf{x})|0,n,0,\ldots\rangle. \tag{28}$$

$f(t)$ vanishes after $t = \tau$, hence for $t = \tau_r$,

$$P\tilde{\psi} = \left(1 - nV^2|u(k_0,\mathbf{x})|^2\int_0^\tau \bar{f}(t)\int_0^t f(t')dt'\,dt\right) \times v(x)|0,n,0,\ldots\rangle. \tag{29}$$

The atom is in its ground state and the expression in brackets is a correction factor to the center of mass wave function. The correction factor for the probability density is

$$[\bar{\ }]^*[\ \ ] = \left(1 - nV^2|u(k_0,\mathbf{x})|^2\left|\int_0^\tau f(t)dt\right|^2\right). \tag{30}$$

The reduction factor for the probability density inside $r = a$, namely,

$$nV^2|u(k_0,\mathbf{x})|^2\left|\int_0^\tau f(t)dt\right|^2|v(\mathbf{x})|^2, \tag{31}$$

is identical with the absolute square of (27) for $t = \tau$,

namely, the probability density for the distribution of the excited atom right after excitation by the radiation pulse.

Consistent with the earlier discussion we find that the observation that a photon is not scattered results in a modification of the center-of-mass wave function of the atom. The resulting reduction in the probability density of finding the atom in the beam ($r < a$) when the photon is not scattered is identical with the probability density of obtaining a scattered photon, hence finding the atom in the beam.

With the assumptions that $\tau \ll \tau_r$ and that $t \leq \tau$, the calculation can be carried out to all orders. The only state vectors appreciably excited are $|0,n,0,\ldots\rangle$ and $|1,n-1,0,\ldots\rangle$. Adopting a real envelope function $f(t)$ and defining a new element of time dt^* such that

$$dt^* = f(t)dt, \tag{32}$$

gives the two equations

$$\begin{aligned} f^{-1}\tilde{H}_1\tilde{\psi}_0 &= i\,\partial\tilde{\psi}_1/\partial t^*, \\ f^{-1}\tilde{H}_1\tilde{\psi}_1 &= i\,\partial\tilde{\psi}_0/\partial t^*, \end{aligned} \tag{33}$$

where $\tilde{\psi} = \tilde{\psi}_0 + \tilde{\psi}_1$ and $P\tilde{\psi} = \tilde{\psi}_0$. Here, $f^{-1}\tilde{H}_1$ is time independent. The equation (33) yields the equation

$$nV^2|u(k_0,\mathbf{x})|^2\tilde{\psi}_{0,1} + \partial^2\psi_{0,1}/\partial t^{*2} = 0, \tag{34}$$

with the solution

$$\begin{aligned} \tilde{\psi}_0 &= \cos(\theta/2)v(\mathbf{x})|0,n,0,\ldots\rangle, \\ \tilde{\psi}_1 &= \sin(\theta/2)v(\mathbf{x})|1,n-1,0,\ldots\rangle, \end{aligned} \tag{35}$$

where $\theta = 2\sqrt{n}V|u(k_0,x)|t^*$. For $t > \tau$ the atom can no longer create or destroy a photon with $j = 0$. Hence $\tilde{\psi}_0$ is constant for $t > \tau$. The measurement projection P carried out at $t \gg \tau_r$ gives this result. To the extent that $|u(k_0,\mathbf{x})|$ is uniform over the focus the pulse intensity n can be adjusted to yield a pulse with $\theta \cong 180°$ for $r < a$. ($\theta \cong 0$ for $r > a$.) For this choice of n the absence of a scattered photon implies the absence of the atom from the radiation beam.

III. SUMMARY AND CONCLUSION

We now seem to have at least a partial resolution of the paradox. The deduction that (after the observation) the atom is *not* located in the radiation beam is based on the *absence* of a scattered photon. This apparently correctly implies that the quantum state of the electromagnetic field has not been affected. Nonetheless, the center-of-mass wave function of the atom has been changed to eliminate the atom from the beam and in the process the expectation value of the atom's center-of-mass motional energy has been increased.

The apparent lack of an interaction between the atom and the electromagnetic field is only illusionary. In lowest order the perturbation calculation shows that the change in the atomic center-of-mass wave function is associated with the absorption of a photon from the incident wave packet and the subsequent return of the photon to the packet. If the absorbed photon could have been assumed to have had a well-defined momentum and energy (as is usually the case) this type of photon absorption and re-emission would not result in the transfer of either momentum or energy to the atom. But, as we have seen, the observation requires the photons to be initially in a state for which neither the momentum nor the energy of the photon is well defined. Consequently, the photon exchange can result in a momentum or energy transfer.

The position measurement of the atom is self consistent in the sense that an immediate repetition of the measurement finds the atom to be missing from the beam if the first one found it missing. But there is an important difference between the two measurements. The second one is not accompanied by any additional transfer of translational energy to the atom. The first one appears to be more than a position measurement. It *generates* a state of definite position and determines that position ($r > a$ or $r < a$). The second measurement only confirms the result.

ACKNOWLEDGMENTS

It is a pleasure to acknowledge a helpful suggestion from Victor Weisskopf about 1940. I also appreciate a helpful conversation in 1974 with my colleague E. P. Wigner. This work was supported in part by the National Science Foundation.

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Albert Einstein in Japan: 1922

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(Received 10 November 1980; accepted 9 March 1981)

TRANSLATOR'S INTRODUCTION

In 1922 Albert Einstein accepted an invitation to visit Japan. Welcomed by the Japanese public with an excitement and enthusiasm reserved in most countries for the idols of stage and screen, he spent most of November and December of that year on lecture tour.

Dr. Einstein was warmly and sincerely admired in Japan, not only for his scientific genius, of which his Japanese hosts had been well aware before his arrival, but also for the extraordinary human qualities he manifested in person, which the Japanese were able to perceive at once, across all barriers of language and culture.

Einstein's trip was extravagantly covered by the press; his itinerary and his speeches were reproduced; and several people who were close to him during those months published their reminiscences. None of these publications, however, surpasses the small masterpiece produced by Ippei Okamoto

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