Comment on "Avalanches and Non-Gaussian Fluctuations of the Global Velocity of Imbibition Fronts"

The theoretical study of interfaces in disordered media has led to many interesting results, many of which have never been experimentally verified. The work [1] by Planet, Santucci, and Ortín is therefore of great importance. However, as shown here, their data analysis used in [1] predetermines the universal exponents to unity.

The avalanche size distribution in interfacial growth [1] is expected to display simple scaling in the form

$$P_s(s) = a_s s^{-\alpha} \mathcal{G}_s(s/s_c), \qquad (1)$$

where $P_s(s)$ describes the probability density of finding an avalanche of size s. A corresponding scaling ansatz with exponent τ is made for the avalanche duration T.

In [1], to dedimensionalize the data, measurements of individual avalanche sizes were divided by the average; i.e., the histogram $H_u(u)$ was analyzed where $u = s/\langle s\rangle$, with the aim to extract α as defined in Eq. (1). The same technique had been employed for the avalanche duration distribution $P_T(T)$, using the histogram $H_w(w)$ with $w = T/\langle T\rangle$.

As shown in the following, the scaling exponents α and τ are actually bound to be unity once the histograms $H_u(u)$ and $H_w(w)$ display a collapse, as found in [1].

The dimensionless histogram $H_u(u)$ is related to $P_s(s)$ via $H_u(u)du = P_s(s)ds$ where $du/ds = 1/\langle s\rangle$. According to Eq. (1), $\langle s\rangle = A_s s_c^{2-\alpha}$ for $\alpha < 2$, with constant amplitude A_s, so that

$$H_u(u) = \tilde{A}_s u^{-\alpha} s_c^{(1-\alpha)(2-\alpha)} \mathcal{G}_s(u A_s s_c^{1-\alpha}). \qquad (2)$$

The *only* way for this function to collapse, i.e., to be independent from s_c, as found by Planet *et al*. is that $\alpha = 1$ and by the same argument $\tau = 1$. To prevent artefacts, the *only* scale allowed to dedimensionalize the observable (as often required in experimental data analysis) is its cutoff or any measure thereof, e.g., $u = s/s_c$. To produce a collapse, its suitably rescaled histogram, for example $H(u)u^\alpha s_c^{\alpha-1}$, is to be plotted versus the dimensionless quantity u.

A few remarks are in order. First, the slope of $H_u(u)$ in a double logarithmic plot measures the scaling of the product of the power law prefactor and the scaling function in Eq. (2), which says little about the actual scaling exponents α and τ as defined through Eq. (1). This form of an "apparent exponent" [2] is what was actually measured in [1] in an intermediate region of the incomplete collapse. Second, a proper data collapse is achieved for $P(s) \times \langle s\rangle^{\alpha/(2-\alpha)}$ or $P(s)s^\alpha$ plotted versus $s/\langle s\rangle^{1/(2-\alpha)}$. Figure 1 shows attempts to collapse a (mock) data set known analytically to have $\alpha = 4/3$ along the lines above.

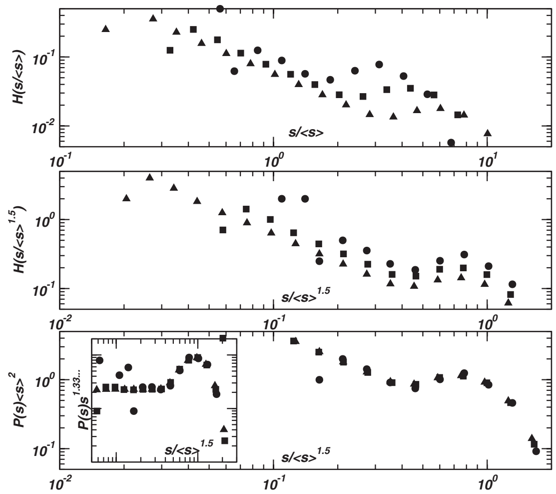

FIG. 1. Attempted collapse of noisy data, known analytically to follow Eq. (1) with $\alpha = 4/3$ [6], using $H_u(u) = P_s(s)ds/du$ versus $u = s/\langle s\rangle$ in the top graph, $H_u(u) = P_s(s)ds/du$ versus $u = s/\langle s\rangle^{1/(2-\alpha)}$ in the middle graph, and $P_s(s)\langle s\rangle^{\alpha/(2-\alpha)}$ versus $s/\langle s\rangle^{1/(2-\alpha)}$ in the bottom graph. The inset shows the widely used collapse $P_s(s)s^\alpha$ versus $s/\langle s\rangle^{1/(2-\alpha)}$, where s_c is expressed as $\langle s\rangle^{1/(2-\alpha)}$.

The other scaling feature Planet *et al.* determine is the joint histogram $H(u, w)$, suggesting that its center of mass follows roughly $u \propto w^{1.31}$. This behavior, again, is normally expected for the dimensionful observables s and T [3]. If the conditional average $\langle s|T\rangle$ follows asymptotically T^γ, then $u = \frac{s}{\langle s\rangle} \propto \frac{T^\gamma}{s_c^{2-\alpha}} \propto (\frac{T}{T_c^{2-\alpha}})^\gamma$, where $s_c \propto T_c^\gamma$ was used. The scaling $u \propto w^x$ found by Planet *et al.* follows only if $\alpha = \tau$.

One might speculate that the process considered by Planet *et al.* belongs to the C-DP universality with $\alpha = 1.11(5)$, $\tau = 1.17(5)$ [4] and $x = 1.5(2)$ [5], which is possibly revealed by an appropriate data collapse.

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[1] R. Planet, S. Santucci, and J. Ortín, Phys. Rev. Lett. **102**, 094502 (2009).
[2] K. Christensen *et al.*, Eur. Phys. J. B **62**, 331 (2008).
[3] K. Christensen *et al.*, J. Stat. Phys. **63**, 653 (1991).
[4] J. A. Bonachela, Ph.D. thesis, Granada, Spain, 2008.
[5] H. Nakanishi and K. Sneppen, Phys. Rev. E **55**, 4012 (1997).
[6] G. Pruessner, J. Phys. A **37**, 7455 (2004).

