Trade Information, Not Spectrum: A Novel TV White Space Information Market Model

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***Abstract*—In this paper, we propose a novel *information market* for TV white space networks, where the spectrum database operator sells the information regarding TV white space to secondary users. Different from the traditional spectrum market, the information market processes the unique property of *positive externality*, as more users purchasing the information service will increase the value of the service to each buyer. We systematically characterize the market equilibrium and the database operator's optimal information pricing strategy. Specifically, we first study how the market share dynamically evolves over time and eventually converge to a market equilibrium. We show that the market equilibrium increases with the initial market share, and there exist several *tipping points* of the initial market share, around which a slight change will lead to a significant change on the emerging market equilibrium. Based on the market equilibrium analysis, we further study the impact of the database operator's information pricing strategy on the market equilibrium, and derive the optimal information price that maximizes the database operator's revenue. Theoretical analysis and numerical result indicate that this is a promising business model for creating incentives for the database operator in TV white space networks.**

I. Introduction

A. Background and Motivation

TV white space networks [1]–[3] can effectively improve the TV spectrum efficiency and alleviate the spectrum scarcity, and thus is a promising approach to solve the spectrum shortage problem. In a TV white space network, unlicensed wireless devices (called white space devices, WSDs) opportunistically exploit the unused or under-utilized channels (called TV white space, TVWS[1]) in the broadcast television spectrum band. The successful deployment of a TV white space network requires many technical innovations, among which an important one is to reliably detect the available channel and accurately estimate the channel quality at different times and locations.

Most early studies on the channel detection and quality estimation focused on the spectrum *sensing* technique [4]. However, recent studies [5] pointed out that spectrum sensing alone is often inefficient, due to the high operational cost as well as the low detection performance. As an alternative, spectrum regulatories (such as FCC in the US and Ofcom in the UK) advocate the use of a *geo-location* white space database [3]. In this database approach, unlicensed WSDs obtain the channel information via querying a geo-location database, rather than sensing the wireless environment. Accordingly, the database is required to house an up-to-date repository of TV licensees, and periodically update the channel occupation by TV licensees. Such a database-assistant TV white space network architecture has also been widely supported by standards bodies and industrial organizations [6]–[12].

While most prior studies focused on the technical issues such as the design and deployment of the white space database, there lacks a proper *business model* that offers necessary economic incentives to the database operators.[2] Prior studies related to the business modeling of TV white space network mainly focused on the *spectrum market* [13]–[17], where the database operators, acting as brokers or agents, purchase channels from the TV licensees, and then sell the purchased channels to unlicensed WSDs at a relatively higher price. However, the TV spectrum market model may *not* be suitable due to some regulatory considerations. For example, TV white spaces sometimes are treated as the *public* spectrum resource by regulators, whose goal is to make more spectrum available for public and shared usage. Because of this, the TV spectrum may not always be traded in the spectrum markets like other licensed spectrum bands.

To this end, a new business model without involving the trading of spectrum is desired. Spectrum Bridge, the world's first white space database certified by the FCC, proposed an alternative business model called "White Space Plus" [12]. The basic idea is to sell certain advanced information regarding TV channel to WSDs, such that they can choose and operate on the most desirable channel. An example of such information is the degree of interference on every TV channel, which may come from either the nearby TV stations or the unlicensed WSDs operating on that channel. This essentially leads to an *information market*, where WSDs purchase the information regarding channel quality, instead of purchasing the channel itself. Clearly, the successful deployment of such an information market requires (i) an accurate model to evaluate the value of information for WSDs (buyers), and (ii) a carefully designed pricing plan for the database operator. However, none of these two issues has been considered in the current White Space Plus.[3] This motivates us to study the information market model for white space databases in this paper.

B. Contributions

In this paper, we model and study an information market for TV white space network, where the database operator (seller) sells the following information to WSD users (buyers): *the interference levels on TV channels*. We focus on designing the optimal information pricing plan that maximizes the database

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[1]For convenience, we will also refer to TV white space as "TV channel" or just "channel" in this paper.

[2]Such incentives are necessary, for example, for covering the database operators' capital expenditure (CapEX) and operational expenditure (OpEX).

[3]Currently Spectrum Bridge just offers a one year free trial to use this White Space Plus service [12].

operator's revenue. To achieve this, we need to accurately evaluate the value of information to users, and the users' reactions under any information price.

Information Value. We propose a general framework for evaluating the value of information to users. Notice that the interference on a channel may come from the nearby TV stations operating on that channel or the nearby WSDs using that channel. The database can (relatively) precisely predict the interference from TV stations, as it maintains a repository of TV licensees. However, it may not be able to predict the precise interference from WSDs, either because some users may not want to inform the database their choices of channels, or simply because some users may interact with another database in the same area. Therefore, the overall interference information that the database provides may *not* be accurate. This will affect the value of information for WSD users, which in turn will affect the database operator's optimal pricing plan.

Market Equilibrium. After characterizing the value of information to users, we are able to derive the stable market share (i.e., the percentage of users who purchase information from the database operator), called the *market equilibrium*. In contrast to traditional spectrum markets which are usually congestion-oriented (i.e., the more users purchasing and using spectrum, the less value of spectrum for users due to the potential co-interference among users), we show that the information market has the nice property of *positive externality* [18]–[20]. That is, the more users purchasing the information from the database, the higher value of the information for each buyer. This is because when more users purchase the information and reveal their channel selections to the database implicitly, the database can predict the interference information more accurately.

Due to the positive network externality, the market equilibrium increases with the initial market share. Interestingly, there exist several critical points (called *tipping points*) of the initial market share, around which a slight change will result in a significant change on the emerging market equilibrium. Such a "Small Changes, Big Impact " [21] phenomenon implies that the database needs to initialize the market with a large enough market share, so that the market can eventually evolve to a desirable market equilibrium. We propose a *refund* mechanism to motivate users to purchase the information at the initial stage, so as to successfully pass the largest tipping point. This has a similar spirit of the Spectrum Bridge's current marketing strategy, which offers one year free trial to use the White Space Plus service. Finally, based on the market equilibrium analysis, we derive the optimal information pricing plan that maximizes the database revenue.

In summary, our main contributions are as follows.

- To the best of our knowledge, this is the first paper proposing and studying an information market for TV white space networks. Compared with the spectrum market model, this model better satisfies the requirements from the regulators and the practice of the industry.
- We propose a general framework to evaluate the value of information to WSD users. This framework considers both the potential error of the information and the heterogeneity of users.
- We characterize the positive network externality of the information market, and study the market equilibrium systematically. Based on this, we further derive the optimal information pricing plan that maximizes the database operator's revenue.
- Theoretical analysis and numerical result indicate that the database operator can make a significant profit from such an information market. Thus, this is a promising business model to give the commercial entities necessary incentives to operate and maintain white space databases.

The rest of the paper is organized as follows. In Section II we present the system model. In Sections III and IV, we analyze the users' best behaviors and the database operator's optimal information pricing plan, respectively. We present the simulations in Section V, and finally conclude in Section VI.

II. System Model

We consider a TV white space network, where a set $\mathcal{N} = \{1, ..., N\}$ unlicensed white space device users (*end-users*) operate on idle TV channels requested from a white space database. Let $\mathcal{K} \triangleq \{1, \ldots, K\}$ denote the set of idle TV channels in the area of the network. Each end-user can only transmit on one of these channels at any given time. We consider a time-slotted system (consistent with many TV white space trial systems [11]), where end-users interact with the database periodically to obtain the available channel information, usually with a period ranging from several minutes to several hours (e.g., 2 hours suggested by Ofcom [3]).

For each end-user $n \in \mathcal{N}$, each channel k is associated with an *interference level*, denoted by $Z_{n,k}$, which reflects the aggregate interference from all other nearby devices (including TV stations and other end-users) operating on this channel. Due to the fast varying nature of wireless channels and the uncertainty of end-users' activities, the interference $Z_{n,k}$ is a random variable. We assume that $Z_{n,k}$ is *temporal-independence* and *frequency-independence*. That is, (i) the interference $Z_{n,k}$ on channel k is independent identically distributed (i.i.d.) across time periods, and (ii) the interferences on different channels, $Z_{n,k}, k \in \mathcal{K}$, are also i.i.d. in the same time period. [4] As we are talking about a general WSD n, **we will omit the WSD index n in the notations (e.g., write $Z_{n,k}$ as Z_k), whenever there is no confusion caused.** Let $F_Z(\cdot)$ and $f_Z(\cdot)$ denote the cumulative distribution function (CDF) and probability distribution function (PDF) of Z_k, $\forall k \in \mathcal{K}$.

White Space Database. According to the regulator's ruling (e.g., FCC [1]), a white space database needs to provide the following information to end-users: (i) the list of all available TV channels, (ii) the maximum transmission power on every channel, and (iii) some other optional requirements. This is the *basic service* that every database is required to provide to any interested user free of charge.

Beyond the basic service, the database can also provide an *advanced service* to make profit, under the constraint that it does not conflict with the basic service. Motivated by the practice of Spectrum Bridge [12], we consider such a scenario

[4] Note that the i.i.d. assumption is a reasonable approximation of the practical scenario, where all channel quality distributions are the same but the realized instant qualities of different channels are different (e.g., [22]).

where the database provides the interference level Z_k of every available channel k to those end-users who subscribe to this advanced service[5]. With this advanced information, end-users are able to pick and operate on the best available channel. Accordingly, the database will charge a *subscription fee* (denoted by π) for such an advanced service. This constitutes an *information market*.

White Space Device Users (End-Users). Basically, after obtaining the available channel list through the free basic service, each end-user has 3 choices (denoted by l) in terms of channel selection: (i) $l = \mathsf{a}$: subscribing to the advanced service and pick the channel with the minimum interference, (ii) $l = \mathsf{s}$: sensing the available channels to figure out the best one, or (iii) $l = \mathsf{b}$: randomly choosing a channel from the list of available channels. Different choices may bring different benefits and incur different costs for/on end-users.

We assume that each end-user is rational, and will choose the strategy that maximizes its payoff. The *payoff* of an end-user is defined as the difference between (i) the benefit (*utility*) achieved from transmitting data on the selected channel, and (ii) the *subscription fee* (if choosing to subscribe to the advanced service) or the *sensing cost* (if choosing to sense the channels). We consider heterogeneous end-users, where different end-users value the same data transmission rate or utility differently (due to different wireless applications). Let θ denote an end-user's evaluation for its achieved utility. For the analytical convenience, we assume θ is uniformly distributed in $[0, 1]$. The payoff of a type-θ end-user is defined as:

$$\Pi^{\text{EU}} = \begin{cases} \theta \cdot g\left(R_{[\mathsf{b}]}\right), & \text{if } l = \mathsf{b}, \\ \theta \cdot g\left(R_{[\mathsf{s}]}\right) - c, & \text{if } l = \mathsf{s}, \\ \theta \cdot g\left(R_{[\mathsf{a}]}\right) - \pi, & \text{if } l = \mathsf{a}, \end{cases} \tag{1}$$

where $R_{[l]}$ denotes the expected data rate when the end-user chooses a strategy $l \in \{\mathsf{b}, \mathsf{s}, \mathsf{a}\}$, and $g(\cdot)$ is the utility function of end-user, which is a concavely increasing function of $R_{[l]}$. Here we assume that all end-users have the same sensing cost c when $l = \mathsf{s}$, and are charged by the same price π when $l = \mathsf{a}$. In other words, the database is not allowed to engage in either QoS discrimination or price discrimination for the simplicity of practical implementation. Let us further assume that there is no sensing error.[6] Then, the end-user's expected data rates under the strategies $l = \mathsf{b}$ (random selection) and $l = \mathsf{s}$ (sensing) can be computed by:

$$\begin{aligned} R_{[\mathsf{b}]} &= \mathrm{E}_Z\left[r(Z)\right] = \textstyle\int_z r(z) \mathrm{d}F_Z(z), \\ R_{[\mathsf{s}]} &= \mathrm{E}_{Z_{(1)}}\left[r\left(Z_{(1)}\right)\right] = \textstyle\int_z r(z) \mathrm{d}F_{Z_{(1)}}(z), \end{aligned} \tag{2}$$

where $Z_{(1)} \triangleq \min\{Z_1, ..., Z_K\}$ denotes the minimum interference on all channels, $F_{Z_{(1)}}(z) = [1 - F_Z(z)]^K$ is the CDF of $Z_{(1)}$, and $r(\cdot)$ is the transmission rate function (e.g., the Shannon capacity). It is important to note that the end-user's expected data rate $R_{[\mathsf{a}]}$ under the strategy $l = \mathsf{a}$ (subscribing to the advanced service) depends on the accuracy of the advanced information the database offers. Intuitively, we have: (i) $R_{[\mathsf{a}]} = R_{[\mathsf{s}]}$ in the extreme case that the database's information is fully accurate, and (ii) $R_{[\mathsf{a}]} = R_{[\mathsf{b}]}$ in another extreme case that the database does not have any accurate information regarding the interferences. However, in the general case where the database's information is partially accurate, $R_{[\mathsf{a}]}$ is generally different from $R_{[\mathsf{s}]}$ and $R_{[\mathsf{b}]}$. We will provide the detailed characterization of $R_{[\mathsf{a}]}$ in (7) after we define the accuracy of the database's information.

[5]"Subscribe to the advanced service" is used throughout this paper to mean an end-user's behavior of purchasing the advanced information.

[6]Our analysis can be directly applied to the case with sensing error [23].

Interference Level (Information). For a particular end-user, its experienced interference Z_k on a channel k is the aggregate interference from all other (nearby) devices operating on channel k, and usually consists of three components:

1) U_k: the interference from licensed TV stations;
2) $W_{k,m}$: the interference from another end-user m operating on the same channel k;
3) V_k: any other interference from outside systems.

The total interference on channel k is $Z_k = U_k + W_k + V_k$, where $W_k \triangleq \sum_{m \in \mathcal{N}_k} W_{k,m}$ is the total interference from all other end-users operating on channel k (denoted by $\mathcal{N}_k$). Similar to Z_k, we assume that $U_k, W_k, W_{k,m}$, and V_k are random variables with temporal-independence (i.e., i.i.d. across time periods) and frequency-independence (i.e., i.i.d. across channels). We further assume that $W_{k,m}$ is user-independence, i.e., $W_{k,m}, m \in \mathcal{N}_k$, are i.i.d. Let $F_U(\cdot)$, $F_W(\cdot)$, and $F_V(\cdot)$ denote the CDFs of $U_k, W_{k,m}$, and V_k, respectively. It is important to note that **different end-users may experience different interferences U_k (from TV stations), $W_{k,m}$ (from another end-user), and V_k (from outside systems) on a channel k, as we have omitted the end-user index n for all these notations for clarity.**

Next let us discuss the above interference components more detailedly. First, based on the knowledge about the location and channel occupancy of TV stations, the database is able to compute the interference U_k from TV stations to a particular end-user (on channel k). Second, due to the lack of outside interference source information, the database cannot compute the interference V_k from outside systems accurately. Thus, the information about V_k will *not* be included in the database's advanced information sold to end-users, which reduces the accuracy of the database's advanced information. Third, the computation of the interference $W_{k,m}$ from another end-user m (operating on channel k) is more complicated. Notice that the database knows precisely the location information of every end-user who requests TV channels (as end-users are mandatorily required to report their location information [1], [2]). Thus, the database can precisely compute the interference of one end-user to another one, if it is able to get the operational channels of end-users. However, the database may or may not know the exact channel selection of an end-user, depending on whether the end-user subscribes to the advanced service or not. Specifically, if an end-user subscribes to the advanced service, the database can predict the end-user's channel selection, since the end-user is fully rational and will always choose the channel with the minimum interference level indicated by the database (in the advanced service). However, if an end-user does not subscribe, the database cannot predict its channel selection, since the end-user's sensing result may not be the same as that provided from the database's advanced information (due to the missing of V_k in the database's information as we explained above), or the end-user may even choose a channel randomly.

Let $\mathcal{N}_{k[\mathsf{a}]}$ denotes the set of end-users (operating on channel k) subscribing to the advance service, and $\mathcal{N}_{k[\mathsf{x}]}$ denotes those *not* subscribing to the advance service. That is, $\mathcal{N}_{k[\mathsf{a}]} \bigcap \mathcal{N}_{k[\mathsf{x}]} = \emptyset$ and $\mathcal{N}_{k[\mathsf{a}]} \bigcup \mathcal{N}_{k[\mathsf{x}]} = \mathcal{N}_k$. Then, for a particular channel k, the interference known by the database (and thus will be included in its advanced information) is

$$X_k \triangleq U_k + \textstyle\sum_{m \in \mathcal{N}_{k[\mathsf{a}]}} W_{k,m}. \quad (3)$$

The interference *not* known by the database (and thus will not be included in its advanced information) is

$$Y_k \triangleq V_k + \textstyle\sum_{m \in \mathcal{N}_{k[\mathsf{x}]}} W_{k,m}. \quad (4)$$

Thus, the total interference level on channel k is

$$Z_k \triangleq X_k + Y_k = U_k + V_k + \textstyle\sum_{m \in \mathcal{N}_k} W_{k,m}. \quad (5)$$

Obviously, Y_k and X_k are also random variables with temporal-independence and frequency-independence. Since the database knows only X_k, it will provide this information as the *advanced service* to end-users. It is easy to see that the more end-users subscribing to the advanced service, the more information the database can provide (and thus the more accurate the database's information will be).

Based on the above, we can characterize the accuracy of the database's information under different number of end-users subscribing to the advanced service. Let η denote the percentage of end-users subscribing to the advanced service, called the *market share* of the database. By the assumption of the frequency independence of Z_k (i.e., the total interferences on different channels $Z_k, k \in \mathcal{K}$ are i.i.d.)[7], each end-user will be "assigned" to each channel with an equal probability.[8] Therefore, there are, *on average*, $\frac{N}{K}$ end-users operating on each channel k, with $\frac{N}{K} \cdot \eta$ end-users subscribing to the advanced service and $\frac{N}{K} \cdot (1-\eta)$ end-users not subscribing to the advanced service. That is, $|\mathcal{N}_k| = \frac{N}{K}$, $|\mathcal{N}_{k[\mathsf{a}]}| = \frac{N}{K} \cdot \eta$, and $|\mathcal{N}_{k[\mathsf{x}]}| = \frac{N}{K} \cdot (1-\eta)$.[9] Then, by (3) and (4), we can immediately obtain the distributions of X_k and Y_k under any given market share η.

Information Value. Now we evaluate the value of database's information $\{X_k\}_{k\in\mathcal{K}}$ to end-users, which is reflected by the end-user's benefit (i.e., the utility $g(\cdot)$ in Eq. (1)) that can be achieved from utilizing this information.

We first consider an end-user's utility without this information (i.e., when not subscribing to the advanced service). In this case, end-users can decide either to randomly select a channel ($l = \mathsf{b}$), or to sense all channels for the best one ($l = \mathsf{s}$). The end-user's expected utilities under strategies $l = \mathsf{b}$ and $l = \mathsf{s}$ are, respectively,

$$B \triangleq g\left(R_{[\mathsf{b}]}\right) \quad \text{and} \quad S \triangleq g\left(R_{[\mathsf{s}]}\right), \quad (6)$$

where $R_{[\mathsf{b}]}$ and $R_{[\mathsf{s}]}$ are the respective data rates defined in (2). Obviously, B and S depend only on the distribution of the total interference Z_k, while not on the specific distributions of X_k and Y_k. This implies that the accuracy of the database's information does not affect the utilities of those end-users not subscribing to the advanced service.

Then we consider an end-user's expected utility with this information (i.e., when subscribing to the advance service, $l = \mathsf{a}$). In this advanced service, the database returns the interference $\{X_k\}_{k\in\mathcal{K}}$ to end-users, together with the basic information such as the available channel list. For a rational end-user, it will always choose the channel with the minimum X_k (since $\{Y_k\}_{k\in\mathcal{K}}$ are i.i.d.). Let $X_{(1)} = \min\{X_1, \ldots, X_k\}$ denote the minimum interference provided by the database. Then, the actual interference experienced by an end-user can be formulated as a random variable $Z_{[\mathsf{a}]} = X_{(1)} + Y$. Accordingly, the end-user's expected data rate and utility are

$$\begin{aligned} R_{[\mathsf{a}]} &= \mathrm{E}_{Z_{[\mathsf{a}]}}\left[r\left(Z_{[\mathsf{a}]}\right)\right] = \textstyle\int_z r(z) \mathrm{d}F_{Z_{[\mathsf{a}]}}(z), \\ A &\triangleq g\left(R_{[\mathsf{a}]}\right), \end{aligned} \quad (7)$$

where $F_{Z_{[\mathsf{a}]}}(z)$ is the CDF of $Z_{[\mathsf{a}]}$. It is easy to see that both $R_{[\mathsf{a}]}$ and A in (7) depend on the distributions of both X_k and Y_k, and thus depend on the market share η. Therefore, we will also write A as $A(\eta)$. We can further check that $A(\eta)$ increases with η, which shows that the information market has the property of *positive network externality*. This is because the more end-users subscribing to the advanced service, the more accurate the database's information is, and further the more benefit for the end-users subscribing to the advanced service. We further assume that $A(\eta)$ is concave in η, which is verified by simulations.

By (6)–(7), we can find the following useful properties.

Proposition 1.

- *B and S in (6) do not depend on η;*
- *$A(\eta)$ in (7) monotonously increases with η;*
- *$B \le A(\eta) \le S$, $\forall \eta \in [0,1]$;*
- *$A(1) = S$, if there is no outside interference $\{V_k\}_{k\in\mathcal{K}}$;*
- *$A(0) = B$, if there is no licensee interference $\{U_k\}_{k\in\mathcal{K}}$.*

Problem Formulation. We will study the optimal information price π that maximizes the database revenue Π^{DB}, where

$$\Pi^{\text{DB}} = \pi \cdot \eta \cdot N.$$

Note that the market share η of the database is a function of the information price π, and thus can be written as $\eta(\pi)$. Moreover, the change of η will affect the interference X_k (known by the database) and Y_k (not known by the database), which in turn will affect the end-user's utility $A(\eta)$ achieved from the advanced service, and the end-user's decision.

III. End-user Subscription Dynamics and Market Equilibrium

In this section, we will study the end-user subscription dynamics and the market equilibrium. Specifically, we will first study the end-user's best choice under a given information

[7]This assumption is used for analysis convenience. Note that the actual distribution depends on the users' subscription behaviors. The more detailed analysis and simulation verifications will be left for our future work.

[8]Each end-user with the strategy $l = \mathsf{s}$ (sensing) will be "assigned" to each channel with an equal probability (as such end-users will choose the best channel with the lowest $Z_k, k \in \mathcal{K}$). Similarly, by the assumption of the frequency independence of X_k, each end-user with the strategy $l = \mathsf{a}$ (subscribing) will be "assigned" to each channel with an equal probability (as such end-users will choose the best channel with the lowest $X_k, k \in \mathcal{K}$). Furthermore, each end-user with the strategy $l = \mathsf{b}$ (randomly selecting) will straightforwardly be "assigned" to each channel with an equal probability.

[9]Note that the above discussion is from the aspect of expectation, and in a particular time period, the realized numbers of end-users in different channels may be different.

price and initial market share. Then we will study how the end-user subscription dynamically evolves over time, and what is the eventual stable market share (*market equilibrium*).

A. End-user's Best Choice

We first consider an end-user's best choice under a particular information price π and initial market share $\eta^0 \in [0,1]$. As mentioned earlier, each end-user has three choices: (i) subscribing to the advanced service, i.e., $l = \mathsf{a}$; (ii) sensing channels to figure out the best one, i.e., $l = \mathsf{s}$; and (iii) randomly choosing a channel from the available channel set, i.e., $l = \mathsf{b}$. The respective payoffs under different choices are given in (1), where $g\left(R_{[\mathsf{s}]}\right) = S$, $g\left(R_{[\mathsf{b}]}\right) = B$, $g\left(R_{[\mathsf{a}]}\right) = A(\eta^0)$; moreover, $B < A(\eta^0) < S$. For convenience, we will write $A(\eta^0)$ as A in this subsection.

First, let us compare the end-user's choices b and s (under the basic service). By (1), we can find that a type-θ end-user prefers the choice b to s, if and only if

$$\theta \cdot B \geq \theta \cdot S - c.$$

Thus, we immediately have the following proposition.

Proposition 2. *There exists a user-type threshold*

$$\theta_{\mathrm{BS}} = \tfrac{c}{S-B}, \tag{8}$$

such that (i) the end-users with type $\theta < \theta_{\mathrm{BS}}$ prefer the choice $l = \mathsf{b}$ (randomly choosing a channel) to $l = \mathsf{s}$ (sensing channels), and (ii) the end-users with type $\theta > \theta_{\mathrm{BS}}$ prefer the choice $l = \mathsf{s}$ to $l = \mathsf{b}$.

Figure 1 illustrates the threshold θ_{BS}, which is denoted by the intersection of the blue curve (the end-user's payoff when $l = \mathsf{b}$) and the red curve (the end-user's payoff when $l = \mathsf{b}$. Notice that if $c > S-B$, then $\theta_{\mathrm{BS}} = \frac{c}{S-B} > 1$, which implies that none of end-users will choose s (as the end-user type θ is defined in $[0,1]$) due to the high sensing cost. Therefore, in the following analysis we will focus on the scenario of $c \leq S-B$.

Next, let us compare all of the three choices of end-users. By (1), we can easily find that a type-θ end-user will choose to subscribe to the advanced service ($l = \mathsf{a}$), if and only if

$$\theta \cdot A - \pi \geq \theta \cdot B, \quad \text{and} \quad \theta \cdot A - \pi \geq \theta \cdot S - c.$$

Then, we further have the following proposition.

Proposition 3. *There exist two user-type thresholds*

$$\theta_{\mathrm{BA}} = \tfrac{\pi}{A-B}, \quad \textit{and} \quad \theta_{\mathrm{AS}} = \tfrac{c-\pi}{S-A}, \tag{9}$$

such that (i) the best choice of end-users with type $\theta \in [\theta_{\mathrm{BA}}, \theta_{\mathrm{AS}}]$ is $l = \mathsf{a}$, (ii) the best choice of end-users with type $\theta \in [0, \min\{\theta_{\mathrm{BA}}, \theta_{\mathrm{BS}}\}]$ is $l = \mathsf{b}$, and (ii) the best of choice of end-users with type $\theta \in [\max\{\theta_{\mathrm{BA}}, \theta_{\mathrm{BS}}\}, 1]$ is $l = \mathsf{s}$.

Figure 1 illustrates the thresholds θ_{BA} and θ_{AS}, which are denoted by the intersection of the green curve (the end-user's payoff when $l = \mathsf{a}$) and the blue curve, and the intersection of the green curve and the red curve, respectively. It is easy to see that the end-users with type $\theta < \theta_{\mathrm{BA}}$ (Region I) prefer the choice b, the end-users with type $\theta > \theta_{\mathrm{BS}}$ (Region III) prefer the choice s, and the percentage of end-users subscribing to the advanced service is $\eta = \theta_{\mathrm{AS}} - \theta_{\mathrm{BA}}$, called the *derived* market share. Notice that if $\pi > c$, then $\theta_{\mathrm{AS}} < 0$, which implies that none of end-users will subscribe to the advanced service due to the high subscription fee.

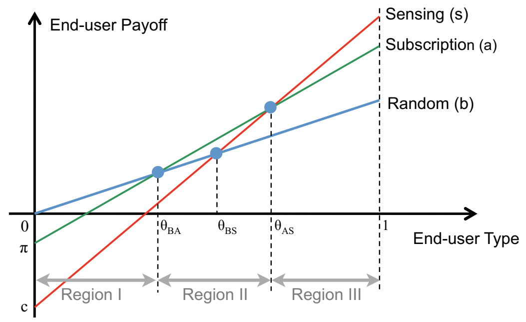

Fig. 1. End-user payoff vs End-user type θ. In Region I, the best choice of end-user is $l = \mathsf{b}$ (random choice), in Region II, the best choice of end-user is $l = \mathsf{a}$ (subscribing to advanced service), and in Region III, the best choice of end-user is $l = \mathsf{s}$ (sensing).

To facilitate the characterization of the derived market share $\eta(\pi)$, we introduce the following two critical prices:[10]

$$\pi_{\mathrm{BS}} = \tfrac{c\cdot(A-B)}{S-B}, \qquad \pi_{\mathrm{AS}} = c - (S - A). \tag{10}$$

Then, under a particular information price π and an initial market share, the derived market share $\eta(\pi)$ is given by

- If $\pi > \pi_{\mathrm{BS}}$, then $\eta(\pi) = 0$ (as $\theta_{\mathrm{AS}} < \theta_{\mathrm{BA}}$);
- If $\pi_{\mathrm{AS}} \leq \pi \leq \pi_{\mathrm{BS}}$, then $\eta(\pi) = \theta_{\mathrm{AS}} - \theta_{\mathrm{BA}} = \frac{c-\pi}{S-A} - \frac{\pi}{A-B}$;
- If $0 \leq \pi \leq \pi_{\mathrm{AS}}$, then $\eta(\pi) = 1 - \theta_{\mathrm{BA}} = 1 - \frac{\pi}{A-B}$.

Formally, we have the following derived market share.

Proposition 4. *The derived market share under price π is*

$$\eta(\pi) = \max\left\{\min\{\theta_{\mathrm{AS}}, 1\} - \theta_{\mathrm{BA}},\ 0\right\}. \tag{11}$$

B. End-user Subscription Dynamics

Eq. (11) shows that the derived market share η depends not only on the information price, but also on the initial market share η^0 (as A is a function of η^0). Notice that the changing of market share will affect the end-users' evaluation for the database's information (i.e., A), and thus affect the end-users' future subscribing decisions. Thus, the market share will dynamically evolve, until it reaches a *stable* market share (called *market equilibrium*). Now we study such an end-user subscription dynamics, and characterize the market equilibrium.

To characterize such a dynamics of end-user's subscription, we construct a virtual time-discrete system with slots $t = 1, 2, ..., T$ (each with a sufficently small time period), and allow end-users change their decisions in every time slot based on the new addressed market share.[11] At each time slot t, each end-user will form a belief, or expectation, on the current market share η, and thereby on the $A(\eta)$, before it makes a subscription decision. If the end-users' belief is higher than the real market share η, some end-users subscribing to the advanced service will cancel their subscription in the next time slot. If the end-users' belief is lower than the real market share η, some end-users not subscribing to the advanced service will start their subscription in the next time slot.

Let us denote η^t as the market share at time slot t. Based on the market share η^{t-1} in the previous slot $t-1$, every

[10] Intuitively, the critical price π_{BS} corresponds to the case that $\theta_{\mathrm{BA}} = \theta_{\mathrm{AS}}$ (which must be same as θ_{BS}), i.e., three curves in Figure 1 intersect at the same point. The critical price π_{AS} corresponds to the case that $\theta_{\mathrm{AS}} = 1$, i.e., the green and red curves in Figure 1 intersect at $\theta = 1$.

[11] Notice that the addressed market share at each slot in the virtual system corresponds to the end-user's belief of market share in the real system.

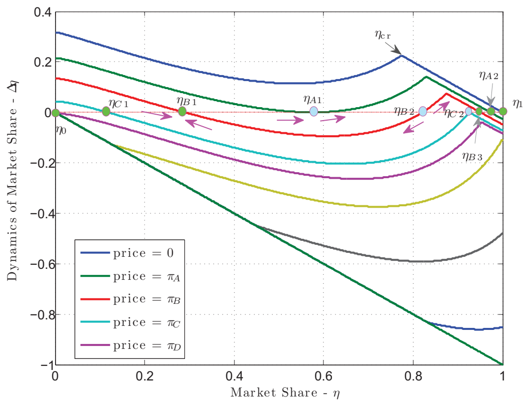

Fig. 2. Dynamics of η under different prices (low sensing cost: $\alpha = 0.2$)

end-user makes the subscription decision in the current slot t in a myopic way, that is, aiming at maximizing its expected payoff in the current slot. Section III-A explains how an end-user makes such a decision. Specifically, by introducing the result in (11), we have the following derived market share at the beginning of the time slot $t = 1, ..., T$:

$$\eta^t = \max\left\{ \min\left\{\tfrac{c-\pi}{S-A(\eta^{t-1})}, 1\right\} - \tfrac{\pi}{A(\eta^{t-1})-B},\ 0\right\}, \quad (12)$$

where η^0 is the initial market share at the beginning.

Let $\triangle\eta$ denote the change (dynamics) of the market share η between two successive time slots t and $t-1$, i.e.,

$$\triangle\eta = \eta^t - \eta^{t-1}, \quad (13)$$

where η^t is a function of η^{t-1} given in (12). Thus, $\triangle\eta$ is also a function of η^{t-1} (and hence is a function of t). Note that a positive (or negative) $\triangle\eta$ implies that the market share η will increase (or decrease) along the dynamics.

An *equilibrium* is defined as such a market share where no end-user has an incentive to change its action. Formally,

Definition 1. *A market share η^* is a market equilibrium if and only if $\triangle\eta(\eta^*) = 0$.*

In the following analysis, we will study the equilibrium market share systematically. Specifically, we will show that under a given price, there may be one or multiple *tipping points* of the initial market share, around which a slight change will leads to a significant change on the market equilibrium. We will also show that under a given price, there may be *multiple* equilibria, and which will eventually emerge depends on the end-user's initial belief on the market share. Besides, some equilibria are *stable* in the sense that a small fluctuation around these equilibria will not drive the market share away from the equilibria, while some equilibria are *un-stable* in the sense that a tiny fluctuation on these equilibria will drive the market share to a different equilibrium.

A key system parameter that affects the characterization of the market equilibria is the sensing cost c. Next we will consider both low and high sensing cost. For convenience, we denote the the magnitude of the end-user's sensing cost as $\alpha \triangleq \frac{c}{S-B}$, where $\alpha \in [0, 1]$.

*1) **Low Sensing Cost:*** We first consider the scenario with a low sensing cost c (i.e., a small α). We illustrate the dynamics of the market share η (i.e., $\triangle\eta$) in Figure 2, where each curve denotes the dynamics $\triangle\eta$ under different prices $0 \le \pi_A \le \pi_B \le \pi_C \le \pi_D$, which will be discussed one by one.

(A) $\pi = 0$. The corresponding dynamics $\triangle\eta$ (or $\triangle\eta|_{\pi=0}$) is denoted by the top blue curve. We notice that $\triangle\eta|_{\pi=0}$ is always larger than zero except the last point $\triangle\eta|_{\pi=0}(1) = 0$. Thus, there is a unique equilibrium $\eta_1 = 1$. That is, if the database offers the advanced service for free, then all end-users will subscribe to the advanced service eventually. Moreover, this equilibrium is stable.

We further notice that this blue curve is indifferentiable at a point $\eta = \eta_{\rm cr}$, which is a critical point satisfies $\frac{c-\pi}{S-A(\eta)} = 1$. Specifically, (i) before the critical point (i.e., $\eta \le \eta_{\rm cr}$), we have $\frac{c-\pi}{S-A(\eta)} < 1$, and thus the blue curve is characterized by $\triangle\eta = \frac{c-\pi}{S-A(\eta)} - \frac{\pi}{A(\eta)-B} - \eta$; (ii) after the critical point (i.e., $\eta \ge \eta_{\rm cr}$), we have $\frac{c-\pi}{S-A(\eta)} \ge 1$, and thus the blue curve is characterized by $\triangle\eta = 1 - \frac{\pi}{A(\eta)-B} - \eta$. Intuitively, if $\frac{c-\pi}{S-A(\eta)} \ge 1$, all end-users can achieve a higher payoff by subscribing to advanced service than sensing (i.e., the green curve is always higher than the red curve in Figure 1), and thus no end-user will choose sensing. If $\frac{c-\pi}{S-A(\eta)} < 1$, some end-users can achieve a higher payoff by sensing than by subscribing to the advanced service (i.e., there is an intersection of the green and red curves in Figure 1). This leads to the different characterizations of $\triangle\eta$. When $\eta \le \eta_{\rm cr}$, we have the following first-order derivative:

$$\frac{\mathrm{d}\triangle\eta}{\mathrm{d}\eta} = \left(\tfrac{c-\pi}{[S-A(\eta)]^2} + \tfrac{\pi}{[A(\eta)-B]^2}\right) \cdot \frac{\mathrm{d}A(\eta)}{\mathrm{d}\eta} - 1, \quad (14)$$

which is negative initially, and then becomes positive with the increase of η. This explains the shape of the blue curve before $\eta_{\rm cr}$. When $\eta \ge \eta_{\rm cr}$, we have $\frac{\mathrm{d}\triangle\eta}{\mathrm{d}\eta} \le 0$, and thus the blue curve decreases with η in this range. Notice that for every price described below, there exists a similar critical point $\eta_{\rm cr}$ (but with a different value).

(B) $\pi = \pi_{\rm A}$. The corresponding dynamics $\triangle\eta$ (or $\triangle\eta|_{\pi=\pi_{\rm A}}$) is denoted by the second (green) curve. This curve is below the blue curve (when $\pi = 0$), since $\triangle\eta$ decreases with π. For better illustration, we intentially choose a price $\pi_{\rm A}$ such that the smallest point before the critical point meets zero. In this case, there are two equilibria: η_{A1} and η_{A2}. We further notice that the equilibrium η_{A2} (illustrated by the green dot) is stable, since any fluctuation of market share around η_{A2} will come back to η_{A2} eventually, whereas η_{A1} (illustrated by the gray dot) is *not* stable, as a tiny increase on η_{A2} will drive the market share to the larger equilibrium η_{A2}. In this sense, η_{A2} is the *tipping point.*

(C) $\pi = \pi_{\rm B}$. The corresponding dynamics $\triangle\eta$ (or $\triangle\eta|_{\pi=\pi_{\rm B}}$) is denoted by the third (red) curve. As the price increases to $\pi_{\rm B}$, there are three equilibria η_{B1}, η_{B2}, and η_{B3}, where η_{B1} and η_{B3} are stable, and η_{B2} is not. Note that η_{B2} is the *tipping point*, since a tiny increase on η_{B2} will drive the market share to the larger equilibrium η_{B3}, while a tiny decrease on η_{B2} will drive the market share to the smaller equilibrium η_{B1}.

(D) $\pi = \pi_{\rm C}$. The corresponding dynamics $\triangle\eta$ (or $\triangle\eta|_{\pi=\pi_{\rm C}}$) is denoted by the forth curve. For better illustration, we intensionally choose a price $\pi_{\rm C}$ such that the critical point meets zero. There are two equilibria η_{C1} and η_{C2}, where η_{C1} is stable, and η_{C2} is not. η_{C2} is the *tipping point.*

(E) $\pi = \pi_{\rm D}$. The corresponding dynamics $\triangle\eta$ (or $\triangle\eta|_{\pi=\pi_{\rm D}}$) is denoted by the fifth curve. For better illustration, we intensionally choose a price $\pi_{\rm D}$ such that the initial point meets zero $\triangle\eta(0) = 0$. In this case, $\triangle\eta$ is always smaller than zero (except the initial point). Thus, there is an unique equilibria $\eta_0 = 0$ which is stable.

Lemma 1 summarizes the above discussions regarding the stable equilibrium, where $\pi_{\text{A}}, \pi_{\text{B}}, \pi_{\text{C}}, \pi_{\text{D}}$ are the prices illustrated in Figure 2.

Lemma 1 (Stable Equilibrium)**.** *The stable market equilibrium under the low sensing cost scenario is given by*

- *if $\pi \geq \pi_{\text{D}}$, there is a unique stable equilibrium: $\eta^{\text{EQ}} = 0$;*
- *if $\pi_{\text{C}} \leq \pi < \pi_{\text{D}}$, there is a unique stable equilibrium: $\eta^{\text{EQ}} = \eta_{C1}$, where η_{C1} is given by*
$$\tfrac{c-\pi}{S-A(\eta)} - \tfrac{\pi}{A(\eta)-B} - \eta = 0;$$
- *if $\pi_{\text{A}} < \pi < \pi_{\text{C}}$, there exist two stable equilibria: η_{B1} and η_{B3}, which are respectively given by*
$$\tfrac{c-\pi}{S-A(\eta)} - \tfrac{\pi}{A(\eta)-B} - \eta = 0, \quad 1 - \tfrac{\pi}{A(\eta)-B} - \eta = 0;$$
- *if $\pi \leq \pi_{\text{A}}$, there exists a unique stable equilibrum $\eta^{\text{EQ}} = \eta_{A2}$, which is given by $1 - \frac{\pi}{A(\eta)-B} - \eta = 0$.*

Lemma 1 illustrates that there may be multiple stable equilibria under a particular price. Next, we show which stable equilibrium will eventually emerge depends on the initial market share (or the initial belief of the market share). Let us take the case $\pi = \pi_{\text{B}}$ in Figure 2 as an illustration, where there are two stable equilibria η_{B1} and η_{B3}. If the initial market state $\eta^0 < \eta_{B1}$, then the market share will gradually increase to η_{B1} as $\triangle\eta > 0$. Similarly, if $\eta_{B1} < \eta^0 < \eta_{B3}$, then the market share will gradually decrease to η_{B1} as $\triangle\eta < 0$. Only if $\eta^0 > \eta_{B2}$, the highest stable equilibrium η_{B2} will emerge. Notice that given the price, the database always prefers the highest stable equilibrium if multiple equilibria exist. Thus, some incentive mechanism is necessary to motivate more end-users subscribing to the advanced service earlier, so as to construct a higher initial market share and achieve a higher stable equilibrium. We will study this in Section IV-B.

*2) **High Sensing Cost**:* The analysis for the high sensing cost case is similar to that for the low sensing cost case, but the detailed results are different due to the difference between the shapes of the dynamics $\triangle\eta$. Due to space limit, we will leave the detailed analysis in our technical report [23].

IV. Database Optimal Information Pricing

In this section, we will study the optimal information pricing strategy for the database operator to maximize its revenue, based on the market equilibrium analysis in the previous section. In the following analysis, we first suppose that there is an effective mechanism such that the highest stable equilibrium will emerge if multiple equilibria exist, and derive the optimal pricing strategy accordingly. Then we propose a *refund* mechanism for the database to achieve this goal.

A. Best Pricing Decision

By Lemma 1, we can easily find that (i) if $\pi \geq \pi_{\text{D}}$, then $\eta^{\text{EQ}} = 0$, (ii) if $\pi_{\text{C}} \leq \pi < \pi_{\text{D}}$, then $\eta^{\text{EQ}} = \eta_{C1}$ which is given by $\frac{c-\pi}{S-A(\eta)} - \frac{\pi}{A(\eta)-B} - \eta = 0$; and (iii) if $\pi < \pi_{\text{C}}$, then the *highest* stable equilibrium $\eta^{\text{EQ}} = \eta_{B3}$ or η_{A2}, both given by $1 - \frac{\pi}{A(\eta)-B} - \eta = 0$. We illustrate this stable market share and the database's revenue under different prices in Figure 3, where $\eta^C(\pi)$ and $\eta^D(\pi)$ are respectively given by
$$1 - \tfrac{\pi}{A(\eta)-B} - \eta = 0, \quad \tfrac{c-\pi}{S-A(\eta)} - \tfrac{\pi}{A(\eta)-B} - \eta = 0.$$

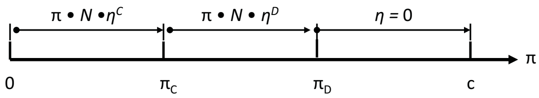

Fig. 3. Database revenue under different prices

1) Low price region: $\pi \leq \pi_{\text{C}}$. The database 's revenue is: $\Pi^{\text{DB}}(\pi) = \pi \cdot \eta^{\text{C}}(\pi) \cdot N$, which is concave in the database's price π. Thus, by the KKT analysis, we have:
$$\pi^* = \pi^\dagger \triangleq (1 - \eta^\dagger) \cdot \big[A(\eta^\dagger) - B\big] \tag{15}$$
where $\eta^\dagger$ is the solution of $A(\eta) - B + \frac{(1-\eta)\cdot\eta}{1-2\eta} \cdot \frac{\mathrm{d}A(\eta)}{\mathrm{d}\eta} = 0$. For more details, please refer to [23].

2) High price region: $\pi_{\text{C}} < \pi \leq \pi_{\text{D}}$. The database's revenue is $\Pi^{\text{DB}}(\pi) = \pi \cdot \eta^{\text{D}}(\pi) \cdot N$. The optimal price in this case is
$$\pi^* = \pi^\ddagger \triangleq \tfrac{A(\eta^\ddagger)-B}{S-B} \cdot \big[c - (S - A(\eta^\ddagger)) \cdot \eta^\ddagger\big] \tag{16}$$
where $\eta^\ddagger$ is solved by $\frac{\mathrm{d}}{\mathrm{d}\eta}\big(\frac{A(\eta)-B}{S-B} \cdot [c\cdot\eta - (S-A(\eta))\cdot\eta^2]\big) = 0$.

By comparing the optimal pricing and the corresponding maximum revenue in different price regions, we can obtain the database's optimal pricing decision.

Lemma 2 (Optimal Information Pricing)**.** *The database's optimal pricing decision is given by (15) or (16), depending on which of $\pi^\dagger \cdot \eta^{\text{C}}(\pi^\dagger)$ and $\pi^\ddagger \cdot \eta^{\text{D}}(\pi^\ddagger)$ is the larger one.*

B. Refund Policy

Now we propose a mechanism to ensure the highest stable equilibrium. As mentioned previously, the emerging equilibrium depends on the initial market share η^0. Moreover, the larger the initial market share, the higher possibility the emerging of the highest stable equilibrium. Therefore, the main purpose of the mechanism is to motivate more end-users subscribing to the service in the early stage, so as to construct a high enough initial market share.

First, the end-user subscription dynamics in Section III shows that if the initial market share $\eta^0 = 1$, then the market will always converge to the highest stable equilibrium. This implies that we only need to find a mechanism such that the initial market share is $\eta^0 = 1$. A natural approach is to provide the advanced service for free for a certain time (as Spectrum Bridge did) to achieve a larger initial market share. Although this approach can increase the probability of the highest equilibrium, it will incur considerable revenue loss on the database, and moreover, it still cannot guarantee the highest equilibrium (see the example in [23]).

We propose a *refund* policy. The basic idea is as follows. The database first announces a high enough *hypothetic* market share (e.g., $\eta^0 = 1$) to end-users, and then end-users decide whether to subscribe to the service. Notice that end-users may not believe the market share announced by the database, since the database may announce an inflated market share (to enlarge its revenue potentially). To avoid this, the database adopts the following **refund policy**: *Refund the subscription fee to an end-user who is not satisfied with the information obtained.* Meanwhile, to avoid end-users frequently ask for refund (even when it is satisfied with the information), the database will adopt the following **stopping-service policy**: *stop to serve an end-user who ask for refund for a certain long time*. Obviously, by this refund policy, end-users will subscribe to the advanced service without hesitation, as they are freely to ask for refund.

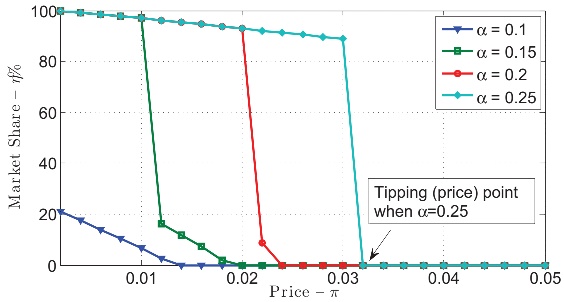

Fig. 4. Market equilibrium under different information price.

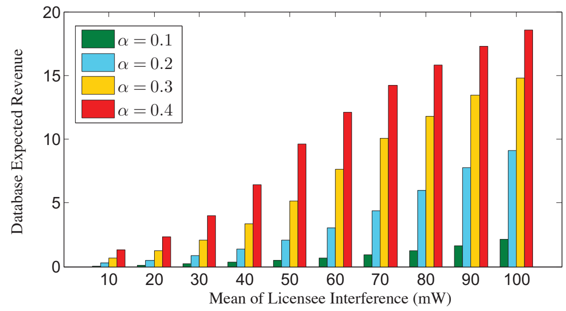

Fig. 5. Database's revenue under different licensee interferences.

Moreover, by adopting the stopping-service policy for a long enough time, end-users who are satisfied with the service will not ask for refund, since this will discontinue in a long time and incur a large loss.[12] This implies that the database does not loss any revenue, in contrast to the previous free-serving policy.

V. Simulation Result

In this section, we use numerical results to evaluate the performance of the proposed information pricing scheme. The following settings are used in our simulations: $N = 80$, $K = 20$, and U, V, and W follow the truncated normal distributions. The transmission data rate is defined by the Shannon capacity: $r(Z) = \log(1 + \frac{P}{Z})$. The end-user utility is simply defined as the expected data rate: $g(R) = R$.

Market Equilibrium. Figure 4 illustrate the market equilibrium (i.e., the stable percentage of end-users subscribing to the advanced service) vs the information price π. We can see that the market share decreases with the price. This means that less end-users are willing to purchase information under a higher price. We can also see that the market share increases with the end-user's sensing cost (recall that $\alpha = \frac{c}{S-B}$). This means that more end-users are willing to purchase information if the sensing is expensive. More interestingly, we can see that there exists the *tipping price point*, at which a slight change will lead to a dramatic decrease on the market equilibrium.

Database Revenue. Figure 5 illustrates the database's revenue under different licensee interferences U (with a mean changing from 10mw to 100mw). The mean value of V and W are fixed at 40mw and 10mw, respectively. We can see that the database's revenue increases with the mean of the licensee interference. This is because the licensee interference is known by the database, and a larger licensee interference makes the database's information more valuable for the end-users. We can further see from each bar group that the database's revenue increases with the end-user's sensing cost.

VI. Conclusion

In this paper, we study a novel information market for the spectrum database in TV white space networks, which allows the database to sell information to end-users for revenue. We show that the information market has the property of positive network externality, and study the subscription dynamics and market equilibrium systematically. Based on this equilibrium analysis, we propose a refund policy to guarantee the desirable market equilibrium, and further derive the database's optimal pricing strategy. Our theoretical analysis and numerical result show that the information market can bring significant revenue for the database. There are some possible directions to extend the results in this paper. A natural extension is to consider the oligopoly market with multiple database, where different databases sell their respective information to end-users. On one hand databases compete with each other for end-users (e.g., the duopoly competition studied in [24]); on the other hand, databases have strong incentives to share information with each other, as such a cooperation will increase the accuracy of their information.

[12]This is because the interference is temporal-independence (i.e., randomly changing over different subscription periods), and thus end-users need to subscribe to the advanced service periodically in order to get the updated information. Note that the interference keeps unchanged within each subscription period, otherwise the interference information is meaningless.

References

[1] FCC 10-174, *Second Memorandum Opinion and Order*, 2010.
[2] OFCOM, *TV White Spaces - A Consultation on White Space Device Requirements*, Nov. 2012.
[3] Ofcom, *Implementing Geolocation*, Nov. 2010.
[4] R. Saifan, A. E. Kamal, and Y. Guan, "Efficient Spectrum Searching and Monitoring in Cognitive Radio Network", *IEEE MASS*, 2011.
[5] V. Goncalves and S. Pollin, "The Value of Sensing for TV White Spaces," *IEEE DySpan*, 2011.
[6] Google Spectrum Database, http://www.google.org/spectrum/whitespace/
[7] Spectrum Bridge TV White Space, [Online] http://www.spectrumbridge.com/ProductsServices/WhiteSpacesSolutions/WhiteSpaceOverview.aspx
[8] IEEE 802.22 WRAN, [Online] http://www.ieee802.org/22/
[9] Whitespace Alliance, [Online] www.whitespacealliance.org
[10] Microsoft Reserach WhiteFiServiice, http://whitespaces.msresearch.us/
[11] Microsoft Report, *Cambridge TV White Space Trial: A Summary of the Technical Findings*, April. 2012.
[12] Spectrum Bridge White Space Plus, http://www.spectrumbridge.com/ProductsServices/WhiteSpacesSolutions/OperatorsUsers.aspx
[13] L. Gao, X. Wang, Y. Xu, and Q. Zhang, "Spectrum Trading in Cognitive Radio Networks: A Contract-Theoretic Modeling Approach", *IEEE JSAC*, vol. 29, no. 4, pp. 843-855, 2011.
[14] H. Jin, G. Sun, X. Wang, and Q. Zhang, "Spectrum Trading with Insurance in Cognitive Radio Networks", *IEEE INFOCOM*, 2012.
[15] Y. Luo, L. Gao, and J. Huang, "Spectrum Broker by Geo-location Database", *IEEE GLOBECOM*, 2012.
[16] X. Feng, J. Zhang, and J. Zhang, "Hybrid Pricing for TV White Space Database," *IEEE INFOCOM*, 2013.
[17] Y. Luo, L. Gao, and J. Huang, "White Space Ecosystem: A Secondary Network Operator's Perspective", *IEEE GLOBECOM*, 2013.
[18] J. J. Laffont, *et al*, "Internet Interconncetion and the Off-Net-Cost Pricing Principle", *The RAND Journal of Economics*, 32(2), 2003.
[19] O. Candogan, K. Bimplikis, and K. Ozadaglar, "Optimal Pricing in Networks with Externalities", *Operation Research*, 60(4), 2012.
[20] Y. Jin, *et al*, "Dynamics of Competition Between Incumbent and Emerging Network Technologies", *IEEE NetEcon*, 2008.
[21] M. Gladwell, *The Tipping Point: How Little Things Can Make a Big Difference*, Back Bay Books, Jan. 2002.
[22] N. B. Chang, and M. Liu, "Optimal Channel Probing and Transmission Scheduling for Opportunistic Spectrum Access", *IEEE/ACM Transactions on Networking*, vol. 17, no. 6, pp. 1805-1818, Dec. 2009.
[23] Y. Luo, L. Gao, and J. Huang, Tech. Report, [Online] http://jianwei.ie.cuhk.edu.hk/publication/AppendixDatabaseInformation.pdf
[24] Y. Luo, L. Gao, and J. Huang, "Information Market for TV Wthie Space," in *Workshop of IEEE INFOCOM on Smart Data Pricing (SDP'14)*, 2014.