Impact of Hostile Interference on Information Freshness: A Game Approach

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Abstract. For time critical updates, it is desirable to maintain the freshness of the received information. We address the impact of hostile interference on information freshness by formulating a non-zero-sum two-player game, in which one player is the transmitter aiming to maintain the freshness of the information updates it sends to its receiver, and the other player is the interferer aiming to prevent this. The strategy of a player is the power level transmitted by that player. We then derive the equilibria for both Nash and Stackelberg strategies. We show that both players have the same power cost at Nash equilibrium. In addition, the Stackelberg strategy dominates the Nash strategy, i.e., the Stackelberg utility function exceeds the Nash utility function.

Index Terms. Age of information, information freshness, interference game, SINR, Nash equilibrium, Stackelberg equilibrium

1 Introduction

We consider a wireless communication problem in which a transmitter desires to maintain the freshness of the information updates it sends to a receiver, in the presence of a hostile interferer who desires to degrade the freshness of the updates (Fig. 1). We can model the communication-interference problem as a two-player game. One player is the transmitter aiming to maintain the freshness of the updates to its receiver, and the other player is the interferer aiming to prevent this.

Each player chooses a transmission power level to optimize its utility function, which incorporates both the reward and operational cost for that player. The players' rewards are related to the age of the information update as observed at the receiver, while the cost incurred by a player is proportional to the power or energy used by that player. The age of an update process at time t refers to the length of the interval from the timestamp of the most recently received update to t [4, 8, 12, 13, 17]. Our communication-interference model forms a continuous game, i.e., each player chooses a power level as a strategy from a continuous set.

Game theory has many applications to computer and communication systems [5, 7]. Several versions of the interference game can be studied. In particular, if the players act simultaneously, the solution is the Nash equilibrium (NE) [15], i.e., the point at which neither player could improve its performance by changing its strategy while the other player's strategy remains unchanged. Alternatively, if one player (the leader) chooses its strategy and the other player (the follower) reacts by choosing its own strategy, the solution is then the Stackelberg equilibrium (SE) [15]. For example, there are situations in which the interferer would like to choose its strategy to maximize its utility in response to already established communications, or conversely the transmitter would like to choose its strategy to maximize its utility in response to already established interference. In this paper we consider both the Nash and Stackelberg strategies.

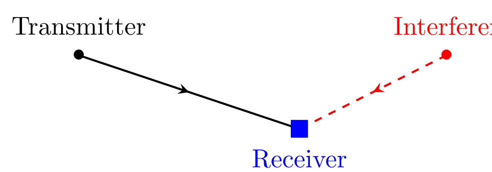

Fig. 1 Information updates under hostile interference.

Applications of game theory, in which NE and SE are the main analysis tools, to systems affected by hostile interference are well known. However, the game reward functions in existing papers typically emphasize throughput performance (for example, see [1, 2, 9, 16, 19]). In contrast, our game reward function is based on the age measure that emphasizes information freshness.

In Section 2, we review the idea of age for a random process, which is used to quantify the freshness of the received information. In Section 3, we formulate the interaction between information updates and hostile interference as non-zero-sum two-player games. In Section 4, we compute the best responses from the players (see Theorem 2), which are needed later for deriving the NE and SE. In Sections 5 and 6, we derive the NE and SE for the case of zero receiver noise (see Theorems 3-5). The results show that both players have the same power cost at NE, whereas the interferer's power cost is half of transmitter's power cost at SE (when the interferer is the

978-3-901882-90-6/17 ©2017 IFIP

leader). Furthermore, the Stackelberg strategy dominates the Nash strategy, i.e., the SE utility function exceeds the NE utility function. In Section 7, we summarize our main results.

2 Age of a Random Process

A *renewal process* is an arrival process in which the interarrival intervals are independent and identically distributed (i.i.d.) random variables [4]. Let $(S_i, i \geq 1)$ be a renewal process, i.e., the interarrival intervals $X_i = S_i - S_{i-1}$ are i.i.d. random variables, where S_i denotes the arrival instant of arrival i (with $S_0 = 0$). Let $N(t)$ be the number of arrivals in the interval $(0, t]$. Thus, $N(t) = \max\{n \mid S_n \leq t\}$. In particular, if t is the current time, then $N(t)$ is also the most recent arrival. Thus, $S_{N(t)}$ is the arrival time of the most recent arrival. The *age* of the renewal process at time t is defined by $Z(t) = t - S_{N(t)}$, which is the interval from the arrival time of the most recent arrival to t. A sample function of the age process $Z(t)$ is shown in Fig. 2, which forms a sawtooth curve consisting of isosceles right triangles [4]. The time average of $Z(t)$ is defined by $A_{\text{ave}} = \lim_{t\to\infty} \frac{1}{t}\int_0^t Z(u)du$. For the renewal process, it can be shown that [4]

$$A_{\text{ave}} = \frac{E[X_1^2]}{2E[X_1]} \tag{1}$$

An alternative to the *average age* A_{ave} in (1) is the *average peak age* A_{p} [3], which is the average of the peak values in the sawtooth curve in Fig. 2. For the renewal process (see Fig. 2), we have

$$A_{\text{p}} = E[X_1] \tag{2}$$

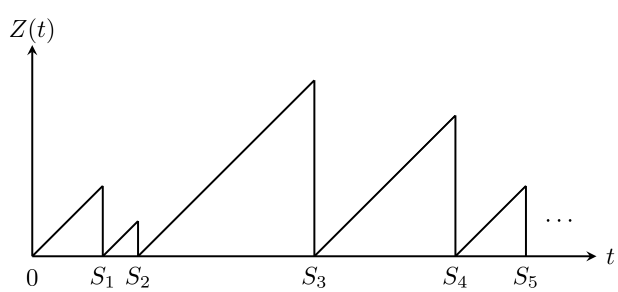

Fig. 2 Age of a renewal process.

The age measures (1) and (2) above can be applied and extended to the case of communication between a source-destination pair in a network [8, 12, 13, 17]. Suppose that packets arrive (or are generated) at the source node, and these packets are timestamped and are transmitted through the network to its destination. In general, these packets are subject to queueing management before transmission. They are also affected by delay and/or packet loss during transmission. When the packets arrive at the destination, the destination can read the timestamps contained in these packets. The age (of the transmitted packets) at time t, as observed by the destination, is defined by $Z(t) = t - S_{D(t)}$, where $D(t)$ is the most recently received packet up to time t. A sample function of the age process $Z(t)$ is shown in Fig. 3, which forms a sawtooth curve consisting of right trapezoids [8, 12, 13, 17]. Furthermore, it can be shown that [3]

$$A_{\text{ave}} = \lambda_e \left(\frac{E[(Y_i^d)^2]}{2} + E[Y_{i-1}^s Y_i^d] \right)$$

$$A_{\text{p}} = E[Y_{i-1}^s] + E[Y_i^d]$$

where λ_e is the (effective) rate of transmitted packets, Y_i^s is the system time (the sum of the waiting time and service time) for packet i, and Y_i^d is the interdeparture time between packets $(i-1)$ and i.

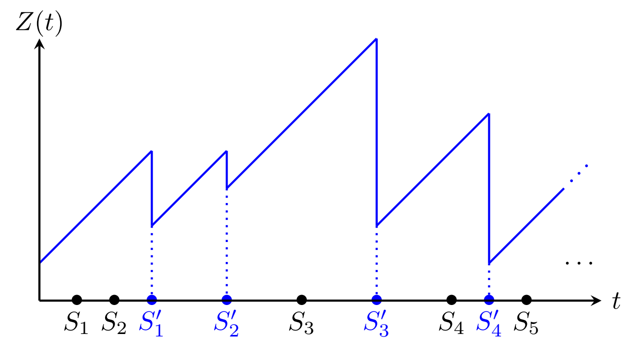

Fig. 3 Age of a random process affected by queueing and service delay (S_i = timestamp of packet i, S_i' = received time of packet i).

The communication channel between a source-destination pair in a network can often be modeled as a general A/S/n/K queue, where A specifies the arrival process, S specifies the service time distribution (M: exponential, G: general), n is number of servers, and K is the capacity of the system (i.e., the maximum number of jobs allowed in the system including those in service). Early results on age for the cases of M/M/n/K queues appear in [8, 12, 13, 17], while more recent developments on age appear in [3, 6, 10, 11, 18].

3 System Model

We consider a wireless link on which transmitter T sends time critical updates that it generates to a receiver (Fig. 1). This receiver is affected by hostile interference from interferer I, as well as by noise of power N. Let p and q be the power levels transmitted by T and I, respectively. Let g and h be the channel power gain from T and I to the receiver, respectively. Thus, the signal-to-interference-plus-noise ratio (SINR) at the receiver is

$$\text{SINR}(p, q) = \frac{gp}{N + hq} \tag{3}$$

3.1 Assumptions

To make the model more specific and to simplify the analysis, we assume the following.

(A1) The update packets are generated at the transmitter according to a Poisson process with rate λ.

(A2) The interfering signal does not carry information that can be exploited to enhance the transmitter's performance. The receiver's decoder considers the interfering signal as additive white Gaussian noise (AWGN). Propagation delay is assumed to be negligible.

(A3) The transmitter can transmit with bit rate r that is proportional to the SINR at the receiver, i.e., $r = c\ \mathrm{SINR} = c\frac{gp}{N+hq}$, where $c > 0$ is a constant. For the case of binary phase-shift keying (PSK) transmission, it is shown in Remark 1 that $c = \frac{2W}{[Q^{-1}(P_e)]^2}$, where $W =$ system bandwidth, $P_e =$ required bit error rate (BER), and Q^{-1} is the inverse Q-function.

(A4) All transmitted update packets are received successfully. This assumption is approximately true, for example, when a good error-correcting code is used to protect the packets. □

Let L_i is the length (in bits) of update packet i. Assume that $(L_i, i \geq 1)$ are i.i.d. random variables. Note that the packet length L_1 can have arbitrary distribution. Let T_i be the random variable representing the duration of update packet i, i.e., $T_i = L_i/r$, where r is the bit rate, $i \geq 1$. Note that $(T_i, i \geq 1)$ are i.i.d. random variables with $T_1 = L_1/r$ and $E[T_1] = E[L_1]/r$. Thus, the update packets are transmitted with rate $\mu = r/E[L_1]$. Using (3) and assumption (A3), the packet transmission rate associated with power profile (p, q) is then

$$\mu(p,q) = z\ \mathrm{SINR}(p,q) = z\frac{gp}{N+hq} \tag{4}$$

where $z = c/E[L_1]$ is a constant.

Recall from assumption (A1) that update packets are generated at the transmitter according to a Poisson process with rate λ. Thus, the transmission from the transmitter to the receiver can be modeled as an M/G/1/K queue (for some $1 \leq K \leq \infty$) in which the server is the communication channel, and the service time is the packet transmission duration.

In this paper we are interested in the average peak age metric, which reflects the need for fresh updates [3]. Furthermore, to simplify the packet management, we assume that there is no queue (i.e., $K = 1$) for the update packets. In particular, any update packet that arrives while another update packet is being transmitted is discarded. Thus, the transmission of the update packets can be modeled as an M/G/1/1 queue. The average peak age A_{p}, which is now denoted by A, is then given by [3, 6]

$$A = \frac{1}{\lambda} + \frac{2}{\mu} \tag{5}$$

where λ is the packet arrival rate and μ is the packet service rate. The average peak age is now denoted by $A(p, q)$ to show its dependence on power profile (p, q). Substituting (4) into (5), we have

$$A(p,q) = \frac{1}{\lambda} + \frac{2\,(hq+N)}{gpz} \tag{6}$$

In our non-cooperative game, the transmitter wants to reduce the average peak age $A(p, q)$ [equivalently, to increase $-A(p, q)$], whereas the interferer wants to increase $A(p, q)$.

Let $c_T(p, q)$ and $c_I(p, q)$ be the costs of transmitter T and interferer I for playing the game. We now formulate a two-player game in which one player is transmitter T wanting to maximize its utility function u_T, whereas the other player is interferer I wanting to maximize its utility function u_I, where

$$u_T(p,q) = -A(p,q) - c_T(p,q) \tag{7}$$

$$u_I(p,q) = \quad A(p,q) - c_I(p,q) \tag{8}$$

We require that $p > 0$ and $q \geq 0$. In particular, $q = 0$ when interferer I does not transmit. We exclude the case of $p = 0$ (i.e., when transmitter T does not transmit), because the age in (6) is infinity for this case. In the following we present two versions of the game, depending on whether the players' costs are based on energy or power.

Remark 1. Consider binary PSK transmission on a link without fading. From assumption (A2), the receiver's decoder considers the interfering signal as noise. Assume further that the sum of the receiver noise and the received interfering signal approximates AWGN of power spectral density N_0. We have $N + hq = WN_0$, where W is the system bandwidth. The BER is then given by $P_e = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$, where $Q(x) = \frac{1}{\sqrt{2\pi}}\int_x^\infty e^{-x^2/2}dx$ [14]. Let r be the bit rate. We then have $E_b = gp/r$. Thus, $\frac{E_b}{N_0} = \frac{Wgp}{r(N+hq)}$. Using (3), we have

$$P_e = Q\left(\sqrt{\frac{2W\ \mathrm{SINR}}{r}}\right)$$

which implies that $r = \frac{2W}{[Q^{-1}(P_e)]^2}\ \mathrm{SINR}$.

3.2 Two-Player Game: Energy-Based Costs

In this version of the game, we assume that the players' costs are proportional to the average energy used to transmit or to interfere with an update packet. Here we assume that interferer I can detect whether transmitter T transmits or is idle. For each update packet transmitted by transmitter T, interferer I emits an interfering signal that lasts for the duration of that packet. Note that the average packet duration is $1/\mu(p, q)$, which implies that the average transmission energy for transmitter T is $p/\mu(p, q)$. Similarly, the average transmission energy for interferer I is $q/\mu(p,q)$. Thus, the players' energy costs are $c_T(p,q) = v_T p/\mu(p,q)$ and $c_I(p,q) = v_I q/\mu(p,q)$, where v_T and v_I are the unit energy costs of transmitter T and interferer I. From (4), (7), and (8), it can be shown that

$$u_T(p,q) = -\frac{1}{\lambda} - \frac{2\,(hq+N)}{gpz} - v_T\frac{N+hq}{gz} \tag{9}$$

$$u_I(p,q) = \frac{1}{\lambda} + \frac{2(hq+N)}{gpz} - v_I\frac{q(N+hq)}{gpz} \tag{10}$$

Recall that $p > 0$ and $q \geq 0$. Note that a strategy profile (p_n, q_n) is said to be an NE if $u_T(p_n, q_n) \geq u_T(p, q_n)$ for all $p > 0$, and $u_I(p_n, q_n) \geq u_I(p_n, q)$ for all $q \geq 0$. The following result follows directly from the definition of NE.

Theorem 1. For the game version with the utility functions (9) and (10), the set of all NE is given by $\{(\infty, q) : q \in [0, \infty)\}$.

Proof. Assume that (p^*, q^*) is an NE. Suppose that $0 < p^* < \infty$. We have $u_T(p^*, q^*) < -\frac{1}{\lambda} - v_T\frac{N+hq^*}{gz} = u_T(\infty, q^*)$. Thus, (p^*, q^*) is not an NE, which contradicts the assumption. Thus, $p^* = \infty$. It follows that if an NE exists, it has the form (∞, q) for $q \geq 0$.

Consider any $q \in [0,\infty)$. We have $u_T(\infty, q) = -\frac{1}{\lambda} - v_T\frac{N+hq}{gz} \geq -\frac{1}{\lambda} - \frac{2\,(hq+N)}{gpz} - v_T\frac{N+hq}{gz} = u_T(p,q)$ for all $p > 0$, and $u_I(\infty, q) = \frac{1}{\lambda} \geq \frac{1}{\lambda} = u_I(\infty, q_1)$ for all $q_1 \geq 0$. Thus, (∞, q) is an NE for any $q \in [0, \infty)$. Note that $u_T(\infty,\infty) = -\infty$, but $u_I(\infty, \infty) = \lim_{p\to\infty, q\to\infty} u_I(p,q)$ does not exist. □

Theorem 1 shows that any NE has the form $(p_n, q_n) = (\infty, q)$, where $q \geq 0$. Thus, the NE solution is degenerate. It can be shown that the SE solution is also degenerate. Thus, we will not discuss any further the energy-cost-based version of the game. Next, we present another version of the game, which is based on the power costs and will be our main focus of this paper. As shown later, the power-cost-based version of the game yields NE and SE that are nondegenerate.

3.3 Two-Player Game: Power-Based Costs

For this version of the game, we assume that the cost incurred by a player is the power cost of that player. Here we assume that transmitter T transmits two types of traffic: regular traffic and update (high-priority) traffic, using the same power level. Transmitter T transmits continuously, i.e., we assume that it always has data to transmit. Update traffic is generated at random times, and has preemptive priority over regular traffic, i.e., update packets are transmitted immediately upon their generation (thus interrupting the transmission of the regular packet currently in progress). Our focus in this paper is on the information freshness of the update traffic, which is not affected by the regular traffic. We assume that the interferer transmits (at the same power level) continuously, because it cannot distinguish between regular traffic and update traffic. Let w_T and w_I be the unit power costs of transmitter T and interferer I. For a given power profile (p,q), the power costs for T and I are then $w_T p$ and $w_I q$. Thus, $c_T(p,q) = w_T p$ and $c_I(p,q) = w_I q$.

From (4), (7), and (8), it can be shown that

$$u_T(p,q) = -\frac{1}{\lambda} - \frac{2\,(hq+N)}{gpz} - pw_T \tag{11}$$

$$u_I(p,q) = \frac{1}{\lambda} + \frac{2\,(hq+N)}{gpz} - qw_I \tag{12}$$

where p is the power transmitted by T and q is the power transmitted by I. Recall that $p > 0$ and $q \geq 0$.

In summary, for the rest of this paper we focus on the non-zero sum game for which the utility functions of the two players are given by (11) and (12). The set of strategies, from which the players choose their power levels, is $\{(p,q) : p > 0, q \geq 0\}$. Our goal is to find the equilibria for the game, in two forms of pure strategies: NE (p_n, q_n) and SE (p_s, q_s).

4 Best Responses of Players

We now determine the best responses from the players, which are needed later for deriving the NE and SE. The best response set of transmitter T to an interfering power level $q \geq 0$ is defined by $B_T(q) = \arg\max_{p>0} u_T(p,q)$. Similarly, the best response set of interferer I to a transmitting power level $p > 0$ is defined by $B_I(p) = \arg\max_{q\geq 0} u_I(p,q)$.

Theorem 2. The players' best responses for the game with the utility functions (11) and (12) are given by

$$B_T(q) = \begin{cases} \text{not exist} & \text{if } q = 0 \text{ and } N = 0 \\ \sqrt{\frac{2(hq+N)}{gw_T z}} & \text{if } q > 0 \text{ and } N \geq 0 \end{cases} \tag{13}$$

$$B_I(p) = \begin{cases} \infty & \text{if } 0 < p < \frac{2h}{gw_I z} \\ 0 & \text{if } p > \frac{2h}{gw_I z} \\ [0,\infty) & \text{if } p = \frac{2h}{gw_I z} \end{cases} \tag{14}$$

where N is the receiver noise power.

Proof.

a Best response from transmitter T

(1) Case: $N = 0$ and $q = 0$. For $p > 0$, we have from (11) that

$$u_T(p,0) = -\frac{1}{\lambda} - pw_T$$

Thus, there does not exist $p^* > 0$ such that $u_T(p^*, 0) \leq u_T(p, 0)$ for all $p > 0$, which implies that $B_T(0)$ does not exist.

(2) Case: $N \geq 0$ and $q > 0$. From (11), we have

$$\frac{\partial}{\partial p} u_T(p,q) = -w_T + \frac{2\,(hq+N)}{gp^2 z}$$

It can be shown that the unique solution of $\frac{\partial}{\partial p} u_T(p,q) = 0$ is $p^* = \sqrt{\frac{2(hq+N)}{g w_T z}}$. Note that

$$\frac{\partial^2}{\partial p^2} u_T(p,q) = -\frac{4\,(hq+N)}{g p^3 z} < 0$$

which implies that $u_T(.,q)$ is concave for all $q > 0$. Thus, p^* maximizes $u_T(p,q)$, i.e.,

$$B_T(q) = \sqrt{\frac{2(hq+N)}{g w_T z}}$$

(3) Case: $N > 0$ and $q = 0$. For $p > 0$, from (11) we have

$$u_T(p,0) = -\frac{1}{\lambda} - \frac{2N}{gpz} - p w_T$$

Similarly to Case (2), it can be shown that

$$B_T(0) = \sqrt{\frac{2N}{g w_T z}}$$

b Best response from interferer I

We can rewrite (12) as

$$u_I(p,q) = q\left(\frac{2h}{gpz} - w_I\right) + \frac{1}{\lambda} + \frac{2N}{gpz}$$

Thus, for each $p > 0$, $u_I(p,.)$ represents a straight line.
(1) Case: $p < \frac{2h}{g w_I z}$. Then $\frac{2h}{gpz} - w_I > 0$. Thus, $B_I(p) = \infty$.
(2) Case: $p > \frac{2h}{g w_I z}$. Then $\frac{2h}{gpz} - w_I < 0$. Thus, $B_I(p) = 0$.
(3) Case: $p = \frac{2h}{g w_I z}$. Then $\frac{2h}{gpz} - w_I = 0$. We have $u_I(p,q) = \frac{1}{\lambda} + \frac{2N}{gpz} = \frac{1}{\lambda} + \frac{N w_I}{h}$, for all $q \in [0,\infty)$. Thus, $B_I(p) = [0,\infty)$. □

In the following, we focus on the case of zero receiver noise power (i.e., $N = 0$), which approximates the case of negligible receiver noise. From (11), (12), and (13), we have

$$u_T(p,q) = -\frac{1}{\lambda} - \frac{2hq}{gpz} - p w_T \tag{15}$$

$$u_I(p,q) = \frac{1}{\lambda} + \frac{2hq}{gpz} - q w_I \tag{16}$$

$$B_T(q) = \begin{cases} \text{not exist} & \text{if } q = 0 \\ \sqrt{\frac{2hq}{g w_T z}} & \text{if } q > 0 \end{cases} \tag{17}$$

5 Nash Strategies

A strategy profile (p_n, q_n) is said to be an NE if $u_T(p_n, q_n) \geq u_T(p, q_n)$ for all $p > 0$, and $u_I(p_n, q_n) \geq u_I(p_n, q)$ for all $q \geq 0$.

Theorem 3 (NE). Suppose that the receiver noise power is zero, i.e., $N = 0$. Then the NE is unique and is given by $(p_n, q_n) = \left(\frac{2h}{g w_I z}, \frac{2h w_T}{g w_I^2 z}\right)$.

Proof. Suppose that (p^*, q^*) is an NE, i.e., $q^* \in B_I[B_T(q^*)]$ and $p^* = B_T(q^*)$. From (14) and (17), we have $B_I[B_T(\infty)] = B_I(\infty) = \{0\}$, which does not contain ∞. Thus, $q^* \neq \infty$. Then $p^* \neq B_T(\infty) = \infty$.

Suppose that $q^* = 0$. From (15), we have

$$u_T(p^*, 0) = -\frac{1}{\lambda} - p^* w_T$$

Let $0 < p_0 < p^*$. Then $u_T(p_0, 0) > u_T(p^*, 0)$. Thus, $(p^*, 0) = (p^*, q^*)$ is not an NE, which contradicts the assumption.

In summary, if (p^*, q^*) is an NE then $p^*, q^* \in (0,\infty)$.

Let $q_1 = \frac{2h w_T}{g w_I^2 z}$. It can then be shown that $\frac{2h}{g w_I z} = \sqrt{\frac{2\,h q_1}{g w_T z}}$. Note that $B_T(q_1) = \sqrt{\frac{2\,h q_1}{g w_T z}}$. Thus, $B_T(q_1) = \frac{2h}{g w_I z}$. Let $p_1 = \frac{2h}{g w_I z}$. We then have $q_1 \in [0,\infty) = B_I\left(\frac{2h}{g w_I z}\right) = B_I(p_1)$ and $p_1 = B_T(q_1)$. Thus, $(p_1, q_1) = \left(\frac{2h}{g w_I z}, \frac{2h w_T}{g w_I^2 z}\right)$ is an NE.

Next, we show that the NE is unique. Thus, suppose that there is another NE $(p_2, q_2) \neq (p_1, q_1)$. Then $p_2 = B_T(q_2)$ and $q_2 \in B_I(p_2)$. As discussed above, we have $p_2, q_2 \in (0,\infty)$.

(1) Case: $p_2 < p_1 = \frac{2h}{g w_I z}$. Then $B_I(p_2) = \{\infty\}$. Because $q_2 \in B_I(p_2)$, we have $q_2 = \infty$. Thus, $p_2 = B_T(q_2) = B_T(\infty) = \infty$, which contradicts the assumption $p_2 < \frac{2h}{g w_I z}$.

(2) Case: $p_2 > p_1 = \frac{2h}{g w_I z}$. Then $B_I(p_2) = \{0\}$, which implies that $q_2 = 0$, which contradicts the fact that $q_2 \in (0,\infty)$.

(3) Case: $0 < q_2 < q_1$. Then $B_T(q_2) < B_T(q_1)$. Using $p_2 = B_T(q_2)$ and $p_1 = B_T(q_1)$, we have $p_2 < p_1$, which leads to the same contradiction mentioned in Case (1).

(4) Case: $q_2 > q_1$. Then $B_T(q_2) > B_T(q_1)$. Using $p_2 = B_T(q_2)$ and $p_1 = B_T(q_1)$, we have $p_2 > p_1$, which leads to the same contradiction mentioned in Case (2). □

Theorem 3 implies that the NE (p_n, q_n) does not depend on the arrival rate λ. Thus, the players do not need to know λ to compute the NE. Furthermore, $w_T p_n = w_I q_n = \frac{2h w_T}{g w_I z}$. Thus, the players incur the same power costs at NE. Note also that $u_T(p_n, q_n) = -\frac{4h w_T}{g w_I z} - \frac{1}{\lambda}$, $u_I(p_n, q_n) = \frac{1}{\lambda}$, and

$$A(p_n, q_n) = \frac{1}{\lambda} + \frac{2h w_T}{g w_I z} \tag{18}$$

which implies that $A(p_n, q_n) = \frac{1}{\lambda} + w_T p_n = \frac{1}{\lambda} + w_I q_n$.

6 Stackelberg Strategies

In the following we show that, when transmitter T is the leader, an SE does not exist. In contrast, when interferer I is the leader, the SE exists and dominates the NE.

6.1 Transmitter is the Leader

Suppose that transmitter T is the leader. The set $(p_s, B_I(p_s)) = \{(p_s, q) : q \in B_I(p_s)\}$ is said to be an SE (under the leadership of transmitter T) if, for all $q_s \in B_I(p_s)$, we have $u_T(p_s, q_s) \geq u_T(p, q)$ for all $p > 0$ and all $q \in B_I(p)$.

Theorem 4 (SE). Suppose that the receiver noise power is zero, i.e., $N = 0$, and transmitter T is the leader. Then there is no SE.

Proof. We provide a proof by contradiction, by assuming that an SE $(p^*, B_I(p^*))$ exists. Recall that

$$u_T(p, q) = -pw_T - \frac{1}{\lambda} - \frac{2hq}{gpz}$$

$$B_I(p) = \begin{cases} \infty & \text{if } 0 < p < \frac{2h}{gw_I z} \\ 0 & \text{if } p > \frac{2h}{gw_I z} \\ [0, \infty) & \text{if } p = \frac{2h}{gw_I z} \end{cases}$$

Let $p_1 = \frac{2h}{gw_I z}$ and $q_1 = 0$. Then $q_1 \in [0, \infty) = B_I(p_1)$ and $u_T(p_1, q_1) = -p_1 w_T - \frac{1}{\lambda} = -\frac{2hw_T}{gw_I z} - \frac{1}{\lambda} > -\infty$.

(1) Case: $0 < p^* < \frac{2h}{gw_I z}$. Then $B_I(p^*) = \infty$. Thus, $u_T(p^*, B_I(p^*)) = -\infty$. We have $u_T(p^*, B_I(p^*)) < u_T(p_1, q_1)$, which implies that $(p^*, B_I(p^*))$ is not an SE.

(2) Case: $p^* > \frac{2h}{gw_I z}$. Then $B_I(p^*) = 0$. We have $u_T(p^*, B_I(p^*)) = u_T(p^*, 0) = -p^* w_T - \frac{1}{\lambda} < -\frac{2hw_T}{gw_I z} - \frac{1}{\lambda}$. Recall that $u_T(p_1, q_1) = -\frac{2hw_T}{gw_I z} - \frac{1}{\lambda}$. Thus, $u_T(p^*, B_I(p^*)) < u_T(p_1, q_1)$, which implies that $(p^*, B_I(p^*))$ is not an SE.

(3) Case: $p^* = \frac{2h}{gw_I z}$. Then $B_I(p^*) = [0, \infty)$. Let $q = 1$. Then $q \in B_I(p^*)$. We have $u_T(p^*, q) = u_T(p^*, 1) = -p^* w_T - \frac{1}{\lambda} - \frac{2h}{gp^* z} = -\frac{2hw_T}{gw_I z} - \frac{1}{\lambda} - w_I$.

Let $0 < e < w_I / w_T$ and $p_2 = \frac{2h}{gw_I z} + e$. Then $B_I(p_2) = 0$. Thus, $u_T(p_2, B_I(p_2)) = u_T(p_2, 0) = -p_2 w_T - \frac{1}{\lambda} = -\left(\frac{2h}{gw_I z} + e\right) w_T - \frac{1}{\lambda} = -\frac{2hw_T}{gw_I z} - \frac{1}{\lambda} - ew_T$.

From $0 < e < w_I / w_T$, we have $-w_I < -ew_T$. Thus, $u_T(p^*, q) < u_T(p_2, B_I(q_2))$, which implies that the set $(p^*, B_I(p^*))$ is not an SE. □

6.2 Interferer is the Leader

Suppose that interferer I is the leader. The set $(B_T(q_s), q_s) = \{(p, q_s) : p \in B_T(q_s)\}$ is said to be an SE (under the leadership of interferer I) if, for all $p_s \in B_T(q_s)$, we have $u_I(p_s, q_s) \geq u_I(p, q)$ for all $q \geq 0$ and all $p \in B_T(q)$.

Theorem 5 (SE). Suppose that the receiver noise power is zero, i.e., $N = 0$, and interferer I is the leader. Then the SE is $(p_s, q_s) = \left(\frac{h}{gw_I z}, \frac{hw_T}{2gw_I^2 z}\right)$. Furthermore, the Stackelberg strategy dominates the Nash strategy, i.e., $u_T(p_s, q_s) > u_T(p_n, q_n)$ and $u_I(p_s, q_s) > u_I(p_n, q_n)$.

Proof. Suppose that $q > 0$. From (17), $B_T(q)$ exists and $B_T(q) = \sqrt{\frac{2hq}{gw_T z}}$. Using (16), we have

$$u_I[B_T(q), q] = -qw_I + \frac{1}{\lambda} + \frac{2hq}{gz\sqrt{\frac{2hq}{gw_T z}}}$$

It can be shown that

$$\frac{\partial}{\partial q} u_I[B_T(q), q] = -w_I + \left(\frac{hw_T}{2gqz}\right)^{\frac{1}{2}}$$

Then the unique solution of $\frac{\partial}{\partial q} u_I[B_T(q), q] = 0$ is $q^* = \frac{hw_T}{2gw_I^2 z}$. Note that

$$\frac{\partial^2}{\partial q^2} u_I[B_T(q), q] = -\left(\frac{hw_T}{8gz}\right)^{\frac{1}{2}} q^{-\frac{3}{2}} < 0$$

Thus, $u_I[B_T(q), q]$ is concave down, which implies that q^* maximizes $u_I[B_T(q), q]$. Thus, $q_s = q^*$ and $p_s = B_T(q^*) = \frac{h}{gw_I z}$.

For the case of $N = 0$, we have at NE

$$u_T(p_n, q_n) = -\frac{4hw_T}{gw_I z} - \frac{1}{\lambda}$$

$$u_I(p_n, q_n) = \frac{1}{\lambda}$$

When I is the leader, we have at SE

$$u_T(p_s, q_s) = -\frac{2hw_T}{gw_I z} - \frac{1}{\lambda}$$

$$u_I(p_s, q_s) = \frac{hw_T}{2gw_I z} + \frac{1}{\lambda}$$

Thus, $u_T(p_s, q_s) > u_T(p_n, q_n)$ and $u_I(p_s, q_s) > u_I(p_n, q_n)$. □

From Theorems 3 and 5, when interferer I is the leader, we have $(p_s, q_s) = (p_n/2, q_n/4)$ and $q_s w_I = p_s w_T / 2 = \frac{hw_T}{2gw_I z}$. Thus, the Stackelberg strategy uses less power than the Nash strategy, and interferer I's power cost is half of transmitter T's power cost. Note also that

$$A(p_s, q_s) = \frac{1}{\lambda} + \frac{hw_T}{gw_I z} \tag{19}$$

which implies that $A(p_s, q_s) = \frac{1}{\lambda} + p_s w_T = \frac{1}{\lambda} + 2q_s w_I$. Furthermore, it follows from (18) and (19) that $A(p_n, q_n) = A(p_s, q_s) + \frac{hw_T}{gw_I z}$. Similar to the case of NE, the players do not need to know the arrival rate λ to compute the SE.

7 Summary

This paper studies the interaction between communication and hostile interference in a time-critical wireless system, which is formulated as a non-zero-sum two-player game. The transmitter's strategy is a transmission power level, while the interferer's strategy is an interfering power level. As shown in the paper, when the payoff is the information freshness quantified by the age metric, both players have the same power cost at NE. In addition, the Stackelberg strategy, when led by the interferer, dominates the Nash strategy, i.e., the SE utility function exceeds the NE utility function.

Acknowledgment

This work was supported by the Office of Naval Research.

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