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Pricing Optimization of Rollover Data Plan

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***Abstract*—Rollover data plans are attractive to mobile users by allowing them to keep their unused data for future use, and hence has been widely implemented by Mobile Network Operators (MNOs) around the world. In this work, we formulate a three-stage Stackelberg game to analyze the interactions between an MNO and its subscribed users under both traditional and rollover data plans. Specifically, in Stage I, the MNO decides which data plan(s) to implement; In Stage II, the MNO decides the price(s) of the data plan(s) to maximize its expected revenue; In Stage III, users make their individual subscription decisions to maximize their expected payoffs. Our analysis shows that in general, high evaluation users are more likely to choose the rollover data plan than medium evaluation users. More precisely, as the network substitutability increases, high evaluation users tend to choose the rollover data plan, while medium evaluation users tend to choose the traditional data plan. We further prove that the MNO can achieve the maximum revenue by only providing the rollover data plan (without bundling with the traditional data plan). Numerical results show that the rollover data plan can increase not only the MNO's revenue but also the users' payoffs (and hence the social welfare) comparing with the traditional data plan. We also compare two rollover data plans that differ in whether the rollover data is consumed prior to monthly data cap, and show that allowing the rollover data to be consumed before the monthly data cap is more beneficial to both users and the MNO.**

I. Introduction

A. Motivations

Due to the increasing competition in the mobile data service market, Mobile Network Operators (MNOs) are under an increasing pressure to increase market shares and improve revenues. A technical approach is to adopt novel wireless technologies (e.g., [1]–[9]) to improve the quality of service (QoS) for users, hence attract more users. However, technology upgrade is often costly and time-consuming. A complementary economical approach is to explore various innovative data plan offerings to meet the requirements of heterogeneous users. The most commonly used data plan is the two-part tariff, where the MNO charges a subscriber a lump-sum montly subscription fee for the data usage up to a fixed monthly data cap, and then charges a linear overage fee for each unit of data usage exceeding the cap [10]. The overage fee can be pretty steep, e.g., \$10/GB to \$15/GB in the US market [11][12]. However, it may be difficult for a user to completely avoid paying for the overage fee due to the stochastic nature of the demand over time. This may discourage the users from subscribing to the data plans, hence reduces the MNOs' revenue. Therefore, the MNOs are motivated to explore various innovative data pricing mechanisms to attract more subscribers by reducing the impact of overage fees on users.

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A common theme of various recently introduced new data pricing mechnisms is to explore the diversity of mobile data demands across several dimentions such as users and time. For example, since 2012 Verizon and AT&T have implemented *shared data plan*, which allows data quota sharing among a small group of family members. Since 2013 China Mobile Hong Kong (CMHK) has implemented the world's first data trading platform (2CM) [13], which allows all 4G users to trade data quota with each other. Since 2015, AT&T, T-Mobile, and Verizon have implemented some versions of *rollover data plan*, which allows a subscriber to rollover the unused portion of his previous monthly data cap to the current month.

Different rollover data plans in practice can be classified according to the consuming priority and the expiring time. In terms of the consuming priority, AT&T specifies that the rollover data will be used after the data cap of the current month is fully used up [14], while China Mobile specifies that the rollover data will be used before consuming the data cap of the current month [15]. As for the expiring time, both AT&T and China Mobile require the rollover data to expire after one month, while T-Mobile allows subscribers to accumulate their rollover data over a time period of several months [16]. In this work, *we will focus on the two rollover schemes offered by AT&T and China Mobile, and analyze the impact of the consuming priority in the rollover data plan implementation and optimization*.

Despite of the increasing popularity of rollover data plan, the related theoretical study just emerged very recently. Zheng *et al.* in [11] compared the rollover data plan with a tradational data plan, and found that moderately price-sensitive users can benefit from subscribing to the rollover data plan. Wei *et al.* in [17] analyzed the rollover data plans with different period lengths through a contract-theoretic approach. In this work, we will focus on a systematical study of the MNOs' optimal price choice of the rollover data plan and the corresponding influences on users' payoffs and the MNO's revenue.

B. Contributions

We focus on two typical rollover data plans adopted by AT&T and China Mobile, and formulate a *three-stage Stackelberg game* to study the interactions between an MNO and users. In Stage I, the MNO decides which data plan(s) to implement, selecting from five possibilities including three individual plans and two combinations of traditional and rollover data plans. In Stage II, the MNO computes the optimal prices for the selected data plan(s). In Stage III, users make

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subscription decisions to maximize their expected payoffs. Different users' preferences to the data plans are characterized by their data evaluations and network substitutabilities. We analyze the equilibrium of the game systematically. In summary, the key contributions of this work are as follows:

- *Comprehensive Model for Rollover Data Plan*: To our best, this is the first work that presents a comprehensive model for the rollover data plan. We propose a three-stage Stackelberg game to analyze users' subscriptions and the MNO's optimal pricing policy under different rollover data mechanisms.
- *Optimal Design of Rollover Data Plan*: We study the MNO's optimal design of the rollover data plan under two different rollover schemes, depending on whether the rollover data is consumed prior to the monthly data cap. Our analysis indicates that both rollover data plans can generate more revenue for the MNO, comparing with the traditional data plan without rollover data.
- *User Subscription Behavior*: We study the subscription behaviors of users with different data evaluations and network substitutabilities. We show that as the network substitutability increases, high evaluation users tend to choose the rollover data plan, while medium evaluation users tend to choose the traditional data plan. In general, high evaluation users are more likely to choose the rollover data plan than medium evaluation users.
- *Rollover Data Plan vs Traditional Data Plan*: We prove that the MNO can achieve the maximum revenue by only providing the rollover data plan, while excluding the traditional data plan. This coincides with the commercial practices of many MNOs in the real world.
- *Performance Evaluation*: Extensive simulation results show that rollover data plan can increase user's expected payoff, bring the MNO more revenue, hence improve the social welfare. In our simulations, the social welfare can be increased by 4.78% (data cap prior) and 8.30% (rollover data prior) on average under the rollover data plan, comparing with the traditional data plan.

The rest of the work is organized as follows. In Section II, we present the system model. In Sections III and IV, we study the users' subscription decisions and the MNO's optimal pricing policy. In Section V, we provide numerical results. Finally, we conclude this work in Section VI.

II. System Model

A. Three-Stage Game

We model the interactions between the MNO and users as a *three-stage Stackelberg game* as illustrated in Fig. 1. Specifically, the MNO decides which data plan(s) to implement in Stage I. We consider five cases including the traditional data plan, two rollover data plans, and two combinations of traditional and rollover data plans. In Stage II, the MNO computes the optimal prices of the chosen data plan(s) in order to maximize its expected revenue from the entire market. In Stage III, users decide which plan to subscribe to maximize their expected payoffs.

Stage I
The MNO decides which data plan(s) to implement.
⇓
Stage II
The MNO decides the price(s) of data plan(s) to maximize its revenue.
⇓
Stage III
Each type-(β,θ) user makes subscription decision to maximize his payoff.

Fig. 1: Three-stage system model.

TABLE I: An Illustrative Example for $\mathcal{T}_0$, $\mathcal{T}_1$, $\mathcal{T}_2$

Month		Jan	Feb	Mar	Apr	May	Jun	Total
Data Usage		2GB	2GB	4GB	4GB	1GB	1GB	14GB
$\mathcal{T}_0$	Payment	50$	50$	65$	65$	50$	50$	**330$**
$\mathcal{T}_1$	r	0	1GB	1GB	0	0	2GB	4GB
	Payment	50$	50$	50$	65$	50$	50$	**315$**
	r'	1GB	1GB	0	0	2GB	2GB	6GB
$\mathcal{T}_2$	r	0	1GB	2GB	1	0	2GB	6GB
	Payment	50$	50$	50$	50$	50$	50$	**300$**
	r'	1GB	2GB	1GB	0	2GB	3GB	9GB

Here r denotes the rollover data from the previous month, and r' denotes the rollover data to the next month. For all three plans, the data cap is 3GB, the fixed lump-sum subscription fee is \$50, and the overage fee is \$15/GB.

B. MNO

In Stage I and Stage II, the MNO will choose and optimize the prices of data plans to maximize its revenue. We consider three different data plans in this work, all of which can be specified by the tuple of $\mathcal{T}_i \triangleq \{Q_i, \Pi_i, \pi\}$, $i = 0, 1, 2$: a subscriber pays a fixed lump-sum subscription fee Π_i for a data usage up to the cap of Q_i data, beyond which the subscriber pays an overage fee π for each unit of additional data consumption. More specifically, $\mathcal{T}_0$ represents the traditional data plan without rollover data, $\mathcal{T}_1$ represents the rollover data plan implemented by AT&T, and $\mathcal{T}_2$ represents the rollover data plan implemented by China Mobile.

In both $\mathcal{T}_1$ and $\mathcal{T}_2$, rollover data will expire after one month, i.e., unused data in the previous month will roll over to the current month, but can no longer be used in the next month. The difference between $\mathcal{T}_1$ and $\mathcal{T}_2$ is the *consuming priority*: in $\mathcal{T}_1$, rollover data from the previous month will be used only after the monthly data cap of the current month is used up, while in $\mathcal{T}_2$, the rollover data from the previous month will be consumed prior to the use of the current monthly data cap. Without loss of generality, we consider an arbitrary month (without specifying the month index). Let r denote the rollover data from the previous month, and let r' represent the rollover data to the next month. Table I shows an illustrative example, where the subscription begins in January.

In this work, we foucs on the MNO's optimal pricing policy under different data plans, i.e., Π_0, Π_1, and Π_2. Therefore, we assume that data caps of all plans are the same, i.e., $Q_0 = Q_1 = Q_2 = Q$. We are going to consider five cases in terms of the MNO's data plan combinations and pricing choices, and the corresponding optimal prices are defined as follows: $\{\mathcal{T}_0\} \Rightarrow \Pi_0^*$, $\{\mathcal{T}_1\} \Rightarrow \Pi_1^*$, $\{\mathcal{T}_2\} \Rightarrow \Pi_2^*$, $\{\mathcal{T}_0, \mathcal{T}_1\} \Rightarrow (\Pi_0', \Pi_1')$, and $\{\mathcal{T}_0, \mathcal{T}_2\} \Rightarrow (\Pi_0'', \Pi_2'')$. We will explain the details in Section IV.

C. User Utility

We model users' satisfaction of data consumption by a utility function, which is often increasing in the data consumption. A widely used utility function is the α-fair utility function [11], [12], [18]. To keep the analysis tractable and obtain clear engineering insights, we set $\alpha = 0$ in the α-fair utility function, which leads to the following linear utility function:

$$U(d) = \theta \cdot d, \tag{1}$$

where d denotes the user data demand and θ denotes the user evaluation for a unit data consumption/demand. We model the user data demand d as a discrete random variable, measured in terms of the minimum data unit ϵ (e.g, 1KB or 1MB according to the MNO's billing practice), to capture its stochastic nature [11], [12]. The probability mass function of d is denoted by $f(d)$ over the feasible discrete set of $\{0, 1, ..., D\}$, where D is the maximum data demand.

Different users have different values of *evaluation* (i.e., θ). For example, students often have low valuations than the businessmen. This will lead to different subscription behaviors even under the same data plan. Another dimension to differentiate users is the *network substitutability*, denoted as β. It represents how much a user will reduce his cellular data usage through other substitutable Wi-Fi networks when exceeding the effective data cap of the current month (taking rollover data into consideration) to avoid high overage fee.[1] The value of β is between 0 and 1, and a higher network substitutability β means more aggressive data demand reduction. Different users have different values of β. For example, it is more difficult for a businessman (with a low β) constantly traveling on the road to reduce overage usage than a student (with a high β) who has frequent access to Wi-Fi at school.

Based on the above, we can characterize a user by (β, θ), which follows a probability density function $h(\beta, \theta)$ over the feasible set of $\mathcal{M} = \{(\beta, \theta) : 0 \le \beta \le 1, 0 \le \theta \le \bar{\theta}\}$. Here $\bar{\theta}$ is the maximum data evaluation among all users. To keep the analysis tractable, we will consdier the uniform distribution, i.e., $h(\beta, \theta) = 1/\bar{\theta}$, in the analysis in Sections III and IV. However, our method applies to a general distribution $h(\beta, \theta)$, and we will illustrate this point through simulations in Section V by using the truncated normal distribution, which includes the uniform distribution as a special case [19]. Specifically, the probability density function of the truncated normal distribution on interval $[a, b]$ is given by [19]

$$f(x; \mu, \sigma, a, b) = \begin{cases} \frac{\frac{1}{\sigma}\phi(\frac{x-\mu}{\sigma})}{\Phi(\frac{b-\mu}{\sigma}) - \Phi(\frac{a-\mu}{\sigma})}, & a \le x \le b, \\ 0, & \text{otherwise}, \end{cases} \tag{2}$$

where $\phi(x)$ is the probability density function of the standard normal distribution, and $\Phi(x)$ is its cumulative distribution function. Note that uniform distribution is a special case of (2) when $\sigma \to \infty$, i.e., $\lim\limits_{\sigma \to +\infty} f(x; \mu, \sigma, a, b) = \frac{1}{b-a}$.

[1] Network substitutability is a user-specific parameter, as a user's mobility pattern can significantly influence the availability of Wi-Fi networks [12].

Furthermore, we use the coefficient of variation (CV), defined as $c_v = \frac{\sigma}{\mu}$, to represent the extent of user diversity based on θ and β. A small c_v represents a market with relatively homogeneous users, while a large c_v represents a market with relatively heterogeneous users.

III. User Subscription Behavior

In this section we first derive a user's expected payoff, and then formulate and solve the user's subscription problem.

A. Tradtional Data Plan – $\mathcal{T}_0$

The payoff of a type-(β, θ) user depends on both the utility by consuming data and the payment to the MNO [11], [12], [18]. We have mentioned earlier that the user will shrink β fraction of his portion of data demand that is subject to the overage fee. This means that if $d > Q_0$, then the realized usage is $d - \beta(d - Q_0)$ after the user's volunteer reduction, the realized overage usage is $(1 - \beta)(d - Q_0)$, and the user's payment owning to exceeding the data cap is $\pi(1-\beta)(d-Q_0)$. Thus, a $\mathcal{T}_0$ subscriber's payoff under a given demand d is

$$S_0(Q_0, \Pi_0, \beta, \theta, d) = \begin{cases} U(d) - \Pi_0, & \text{if } d \le Q_0, \\ U(d - \beta(d - Q_0)) - P_0(\beta, d) - \Pi_0, & \text{if } d > Q_0, \end{cases}$$

where $P_0(\beta, d) = \pi(1-\beta)(d-Q_0)$, π is the unit overage fee, and Π_0 is the lump-sum subscription fee for the data cap.

For a type-(β, θ) user, the monthly data demand d is unknown in advance, so we need compute the user's expected payoff by taking the expectation over data demand d. For the sake of analysis, we assume that d follows the discrete uniform distribution[2] on $\{0, 1, ..., D\}$[3], i.e., $f(d) = \frac{1}{D+1}$. Hence, a type-(β, θ) $\mathcal{T}_0$ subscriber's expected payoff is as follows:

$$\begin{aligned} \bar{S}_0(Q_0, \Pi_0, \beta, \theta) &= \textstyle\sum_{d=0}^{D} S_0(Q_0, \Pi_0, \beta, \theta, d) f(d) \\ &= (\tfrac{D}{2} - A_0\beta)\theta - \pi(1 - \beta)A_0 - \Pi_0, \end{aligned}$$

where $(1-\beta)A_0$ is the expected overage usage of a type-(β, θ) $\mathcal{T}_0$ subscriber, and A_0 is given by

$$A_0 = \textstyle\sum_{d=Q_0}^{D} (d - Q_0) f(d).$$

Note that a higher network substitutability β leads to a reduced overage usage. Therefore, a type-(β, θ) user will subscribe to the traditional data plan $\mathcal{T}_0$ iff $\bar{S}_0(Q_0, \Pi_0, \beta, \theta) \ge 0$.

B. Rollover Data Plan – $\mathcal{T}_1$

Rollover data plan $\mathcal{T}_1$ allows a subscriber to consume the rollover data r from the previous month after the current monthly data cap is used, which means that the subscriber's *effective* data cap in the current month is actually $Q_1 + r$, as showed in Fig. 2. Meanwhile, a subscriber will still incur the

[2] In general, a user's data demand is a complicated function involving the balance between the quota and the time to the end of this month [11]. Here we consider the uniform distribution as a reasonable approximation.

[3] We assume that $D > Q$. If not, there will never be any overage usage, then $\mathcal{T}_0$, $\mathcal{T}_1$ and $\mathcal{T}_2$ are the same for users.

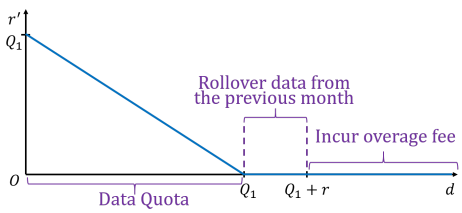


Fig. 2: Transition of rollover data in $\mathcal{T}_1$.

overage fee when $d > Q_1+r$, hence he will shrink the overage usage by β fraction. Thus, a $\mathcal{T}_1$ subscriber's payoff is

$$S_1(Q_1,\Pi_1,\beta,\theta,d,r) = \begin{cases} U(d)-\Pi_1, & \text{if } d \le Q_1+r, \\ U(d-\beta(d-Q_1-r)) - P_1(\beta,d,r) - \Pi_1, & \text{if } d > Q_1+r, \end{cases}$$

where $P_1(\beta,d,r) = \pi(1-\beta)(d-Q_1-r)$ is the payment owning to exceeding the data cap Q_1+r, π is the unit overage fee, and Π_1 is the lump-sum subscription fee.

According to Fig. 2, the amount of rollover data to the next month depends on the data demand d and the data cap Q_1. As the data cap Q_1 will be used before any rollover data r from the previous month, then the rollover data for the next month r' only depends on the relationship between d and Q_1 (and is independent of r). More specifically, if $0 \le d < Q_1$, then $r' = Q_1 - d$ (hence $0 < r' \le Q_1$); if $d \ge Q_1$, then $r' = 0$. As r' computed in the current month will be r in the next month, we can compute the probability mass function of rollover data r in an arbitrary month (except the first month[4]) as

$$p(r) = \begin{cases} f(Q_1-r) = \frac{1}{D+1}, & \text{if } 0 < r \le Q_1, \\ \sum_{d=Q_1}^{D} f(d) = \frac{D-Q_1+1}{D+1}, & \text{if } r = 0. \end{cases}$$

Note that the rollover data r of $\mathcal{T}_1$ subscribers in an arbitrary month (except the first month) is independently and identically distributed. This is a key difference of $\mathcal{T}_1$ and $\mathcal{T}_2$, as will be explained later in Section III.C.

Finally we obtain a type-(β,θ) $\mathcal{T}_1$ subscriber's expected payoff (except the first month) by taking the expectation over data demand d and rollover data r as follows:

$$\begin{aligned} \bar{S}_1(Q_1,\Pi_1,\beta,\theta) &= \sum_{r=0}^{Q_1}\sum_{d=0}^{D} S_1(Q_1,\Pi_1,\beta,\theta,d,r) f(d) p(r) \\ &= (\tfrac{D}{2} - A_1\beta)\theta - \pi(1-\beta)A_1 - \Pi_1, \end{aligned}$$

where $(1-\beta)A_1$ is the expected overage usage of a type-(β,θ) $\mathcal{T}_1$ subscriber, and A_1 is given by

$$A_1 = \begin{cases} \sum_{r=0}^{Q_1}\sum_{d=Q_1+r}^{D}(d-Q_1-r)f(d)p(r), & \text{if } Q_1 \le \frac{D}{2}, \\ \sum_{r=0}^{D-Q_1}\sum_{d=Q_1+r}^{D}(d-Q_1-r)f(d)p(r), & \text{if } Q_1 > \frac{D}{2}. \end{cases}$$

Therefore, a type-(β,θ) user will subscribe to the rollover data plan $\mathcal{T}_1$ iff $\bar{S}_1(Q_1,\Pi_1,\beta,\theta) \ge 0$.

[4]We assume that the contract length is typically many months (e.g., 12 months or 24 months), hence we will ignore the "boundary" effect of the first month here when computing the user's payoff and the MNO's revenue.

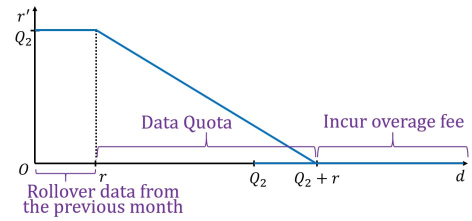


Fig. 3: Transition of rollover data in $\mathcal{T}_2$.

C. Rollover Data Plan – $\mathcal{T}_2$

Rollover data plan $\mathcal{T}_2$ allows a subscriber to consume the unused data from the previous month, therefore a $\mathcal{T}_2$ subscriber's *effective* data cap in the current month is Q_2+r, which is similar to that of $\mathcal{T}_1$ subscribers. Thus, a type-(β,θ) $\mathcal{T}_2$ subscriber's payoff $S_2(Q_2,\Pi_2,\beta,\theta,d,r)$ has the similar expression as $S_1(\cdot)$. However, the rollover data r of a $\mathcal{T}_2$ subscriber will be consumed prior to his monthly data cap Q_2, which leads to a Markov property on rollover data r.

As illustrated in Fig. 3, the distribution of the rollover data to next month, denoted by r', depends on the rollover data r from previous month, the data demand d, and the data cap Q_2. In the following, we will analyze the subscriber's payoff under $\mathcal{T}_2$ in two different cases regarding Q_2 and $\frac{D}{2}$.

Case (i): $Q_2 \le \frac{D}{2}$. In this case, r' is given by

$$r' = \begin{cases} Q_2, & \text{if } 0 \le d \le r, \\ Q_2+r-d, & \text{if } r < d < Q_2+r, \\ 0, & \text{if } Q_2+r \le d \le D. \end{cases}$$

Accordingly, the transition probability from r to r' is

$$p(r,r') = \begin{cases} \sum_{d=0}^{r} f(d) = \frac{r+1}{D+1}, & \text{if } r' = Q_2, \\ f(Q_2+r-d) = \frac{1}{D+1}, & \text{if } 0 < r' < Q_2, \\ \sum_{d=Q_2+r}^{D} f(d) = \frac{D-Q_2-r+1}{D+1}, & \text{if } r' = 0. \end{cases}$$

Thus, we can derive the stationary distribution of r under $Q_2 \le \frac{D}{2}$ through its transition matrix as follows:

$$p(r) = \begin{cases} \frac{2D^2-4DQ_2-5Q_2+4D+Q_2^2+2}{2(D+1)(D-Q_2+1)}, & \text{if } r = 0, \\ \frac{Q_2^2-Q_2+2D+2}{2(D+1)(D-Q_2+1)}, & \text{if } r = Q_2, \\ \frac{1}{D+1}, & \text{if } 0 < r < Q_2. \end{cases}$$

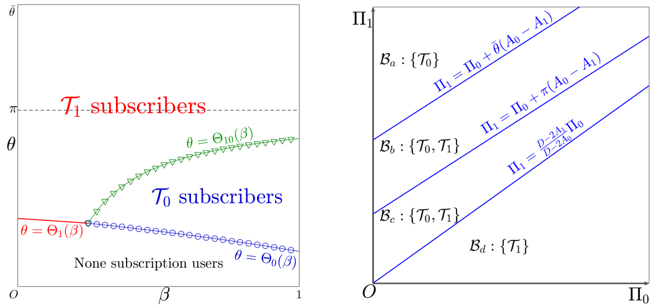
Similarly, we can get the expected payoff of a type-(β,θ) $\mathcal{T}_2$ subscriber by taking the expectation over monthly demand d and rollover data r as follows:

$$\begin{aligned} \bar{S}_2(Q_2,\Pi_2,\beta,\theta) &= \sum_{r=0}^{Q_2}\sum_{d=0}^{D} S_2(Q_2,\Pi_2,\beta,\theta,d,r) f(d) p(r) \\ &= (\tfrac{D}{2} - A_2\beta)\theta - \pi(1-\beta)A_2 - \Pi_2, \end{aligned}$$

where $(1-\beta)A_2$ is the expected overage usage of a type-(β,θ) $\mathcal{T}_2$ subscriber, and A_2 is given by

$$A_2 = \sum_{r=0}^{Q_2}\sum_{d=Q_2+r}^{D}(d-Q_2-r)f(d)p(r)$$

Therefore, a type-(β,θ) user will subscribe to the rollover data plan $\mathcal{T}_2$ iff $\bar{S}_2(Q_2,\Pi_2,\beta,\theta) \ge 0$.



(a) Three groups of users. (b) Four partition modes.

Fig. 4: Illustration of user group and partition mode.

Case (ii): $\frac{D}{2} < Q_2$. In this case, we can derive the subscriber's expected payoff using the similar method. However, there is no closed-form expression of the stationary distribution of rollover data r due to the complex transition matrix. Nevertheless, there exists a unique stationary distribution. For more details, please refer to our technical report [20].

D. Combination of $\{\mathcal{T}_0, \mathcal{T}_1\}$

As a subscriber can reduce his cost through using the rollover data, $\mathcal{T}_1$ is always more beneficial for users than $\mathcal{T}_0$ if the prices are the same[5], i.e., $\Pi_0 = \Pi_1$. Thus, when the MNO offers $\mathcal{T}_0$ and $\mathcal{T}_1$ simultaneously, he can potentially charge a higher price for $\mathcal{T}_1$ to increase his revenue. Accordingly, users will be divided into three groups, i.e., $\mathcal{T}_0$ subscribers, $\mathcal{T}_1$ subscribers, and none subscription users. More specifically, a type-(β, θ) user will choose:

- $\mathcal{T}_0$, if $\bar{S}_0(Q_0, \Pi_0, \beta, \theta) \geq \max\{0,\ \bar{S}_1(Q_1, \Pi_1, \beta, \theta)\}$;
- $\mathcal{T}_1$, if $\bar{S}_1(Q_1, \Pi_1, \beta, \theta) \geq \max\{0,\ \bar{S}_0(Q_0, \Pi_0, \beta, \theta)\}$;
- neither, otherwise.

The boundaries of the three groups of users are given by (3) based on the user parameter (β, θ):

$$\begin{cases} \Theta_0(\beta) \triangleq \pi + \frac{\pi(D-2A_0)-2\Pi_0}{2A_0\beta - D}, \\ \Theta_1(\beta) \triangleq \pi + \frac{\pi(D-2A_1)-2\Pi_1}{2A_1\beta - D}, \\ \Theta_{10}(\beta) \triangleq \pi + \frac{\Pi_1 - \Pi_0 - (A_0 - A_1)\pi}{(A_0 - A_1)\beta}, \end{cases} \tag{3}$$

which are illustrated in Fig. 4(a), where none subscription users are below the blue circle curve and the red solid curve, $\mathcal{T}_0$ subscribers lie between the blue circle curve and the green triangle curve, and the $\mathcal{T}_1$ subscribers are above the green triangle curve and the red solid curve. Proposition 1 summarizes the illustration in Fig. 4(a) more precisely.

Proposition 1: A type-(β, θ) user would choose:

- $\mathcal{T}_0$, if $\Theta_0(\beta) \leq \theta \leq \min\{\Theta_{10}(\beta), \bar{\theta}\}$.
- $\mathcal{T}_1$, iff $\max\{\Theta_1(\beta), \Theta_{10}(\beta)\} < \theta \leq \bar{\theta}$.
- neither, if $0 < \theta < \min\{\Theta_0(\beta), \Theta_1(\beta)\}$.

According to (3), the boundaries of different groups of users depend on Π_0 and Π_1, so the market partition is influenced by (Π_0, Π_1). We illustrate the four market partition modes in Fig. 4(b), where $\mathcal{B}_a, \mathcal{B}_b, \mathcal{B}_c, \mathcal{B}_d$ are given by (4):

[5]Note that we optimize the MNO's revenue over the subscription price and assume data quota is fixed, that is, $Q_0 = Q_1 = Q_2$.

$$\begin{cases} \mathcal{B}_a \triangleq \{\ (\Pi_0, \Pi_1) :\ \ M(\Pi_0) < \Pi_1\ \}, \\ \mathcal{B}_b \triangleq \{\ (\Pi_0, \Pi_1) :\ \ L(\Pi_0) < \Pi_1 \leq M(\Pi_0)\ \}, \\ \mathcal{B}_c \triangleq \{\ (\Pi_0, \Pi_1) :\ \ K(\Pi_0) < \Pi_1 \leq L(\Pi_0)\ \}, \\ \mathcal{B}_d \triangleq \{\ (\Pi_0, \Pi_1) :\ \ 0 < \Pi_1 \leq K(\Pi_0)\ \}, \end{cases} \tag{4}$$

where $K(\Pi_0) = \frac{\Pi_0(D-2A_1)}{D-2A_0}$, $L(\Pi_0) = \Pi_0 + (A_0 - A_1)\pi$, and $M(\Pi_0) = \Pi_0 + (A_0 - A_1)\,\bar{\theta}$.

Next we will discuss each of these four partition modes, with the corresponding graphic illustrations in Fig. 5. For the simplicity of illustration, we fix $\Pi_0 = \$10$ and decrease Π_1 from \$15 to \$10 through Fig. 5(a) to Fig. 5(d).

1) $\mathcal{B}_a$: In this case, there are no $\mathcal{T}_1$ subscribers, as $\mathcal{T}_1$ is relatively expensive comparing with $\mathcal{T}_0$. The user subscriptions correspond to Fig. 5(a), where $\Pi_1 \geq \$12.7$. The blue circle curve $\theta = \Theta_0(\beta)$ corresponds to the boundary between $\mathcal{T}_0$ subscribers and none subscription users.

2) $\mathcal{B}_b$: In this case, both $\mathcal{T}_1$ and $\mathcal{T}_0$ subscribers coexist, as Π_0 and Π_1 are comparable. The user subscriptions correspond to Fig. 5(b), where $\$11.7 < \Pi_1 \leq \$12.7$. The green triangle curves $\theta = \Theta_{10}(\beta)$ are the boundaries between $\mathcal{T}_1$ subscribers and $\mathcal{T}_0$ subscribers under different values of Π_1, where the arrow points to the decreasing direction of Π_1.

From Fig. 5(b), we can see that with a fixed high evaluation θ (larger than π), a user is more likely to choose $\mathcal{T}_1$ as the network substitutability β increases. Mathematically speaking, this is because $\frac{\partial \bar{S}_0}{\partial \beta} < \frac{\partial \bar{S}_1}{\partial \beta} < 0$ for any $\theta > \pi$. Intuitively, a high evaluation user tends to consume more data to achieve a larger payoff, hence the higher network substitutability β he has, the more motivation to maintain his unused data for future use by subscribing to $\mathcal{T}_1$, so that there is less need to shrink the overage data usage.

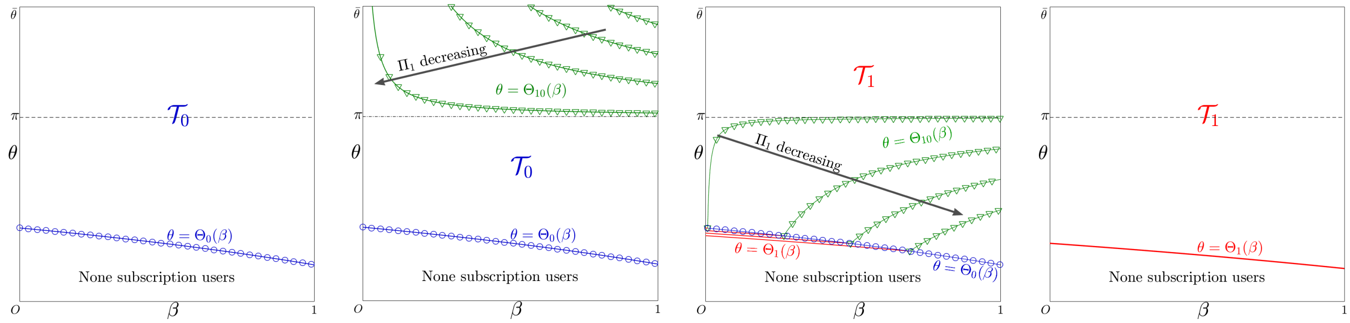
3) $\mathcal{B}_c$: In this case, both $\mathcal{T}_1$ and $\mathcal{T}_0$ subscribers coexist, as Π_0 and Π_1 are comparable. The user subscriptions correspond to Fig. 5(c), where $\$11.1 < \Pi_1 \leq \$11.7$. The meanings of the green triangle curves are the same as in Fig. 5(b). The red solid curves $\theta = \Theta_1(\beta)$ are the boundaries of $\mathcal{T}_1$ subscribers and none subscription users under different values of Π_1.

From Fig. 5(c), we can see that with a fixed medium evaluation θ (smaller than π but not close to 0), a user is more likely to choose $\mathcal{T}_0$ as the network substitutability β increases. Mathematically speaking this is because $\frac{\partial \bar{S}_0}{\partial \beta} > \frac{\partial \bar{S}_1}{\partial \beta} > 0$ for any $\theta < \pi$. Intuitively, a medium evaluation user wants to achieve a larger payoff by reducing the overage cost. Hence the lower network substitutability β he has, the more motivation to reduce his overage usage by subscribing to $\mathcal{T}_1$.

4) $\mathcal{B}_d$: In this case, there are no $\mathcal{T}_0$ subscribers, as Π_1 is close to Π_0. The user subscriptions correspond to Fig. 5(d), where $\Pi_1 < \$11.1$.

E. Combination of $\{\mathcal{T}_0, \mathcal{T}_2\}$

As for the combination of $\{\mathcal{T}_0, \mathcal{T}_2\}$, the formulation and analysis are similar to $\{\mathcal{T}_0, \mathcal{T}_1\}$, and the insights are similar as the ones derived in Section III.D. Please refer to our technical report for details [20].



(a) $\mathcal{B}_a$: $\Pi_1 > \$12.7$ (b) $\mathcal{B}_b$: $\$11.7 < \Pi_1 \leq \$12.7$ (c) $\mathcal{B}_c$: $\$11.1 < \Pi_1 \leq \$11.7$ (d) $\mathcal{B}_d$: $\Pi_1 \leq \$11.1$

Fig. 5: Illustration of four market partition modes ($\Pi_0 = \$10$)

IV. MNO Pricing Policy

In this section we derive the MNO's expected revenue and analyze the MNO's optimal pricing policy.

A. Traditional Data Plan – $\mathcal{T}_0$

For a $\mathcal{T}_0$ subscriber of type (β, θ) with data demand d, the MNO's revenue includes the fixed lump-sum subscription fee and overage payment, which is given by

$$\tilde{R}_0(Q_0, \Pi_0, \theta, \beta, d) = \begin{cases} \Pi_0, & \text{if } 0 \leq d \leq Q_0, \\ \Pi_0 + \pi(1-\beta)(d - Q_0), & \text{if } Q_0 < d \leq D. \end{cases} \quad (5)$$

By taking the expectation over data demand d, the MNO's expected revenue from a $\mathcal{T}_0$ subscriber of type (β, θ) is

$$\begin{aligned} \bar{R}_0(Q_0, \Pi_0, \theta, \beta) &= \textstyle\sum_{d=0}^{D} \tilde{R}_0(Q_0, \Pi_0, \theta, \beta, d) f(d) \\ &= \pi(1-\beta)A_0 + \Pi_0. \end{aligned} \quad (6)$$

By taking the expetation over θ and β, which follows the uniform distribution $h(\beta, \theta) = 1/\bar{\theta}$, the MNO's expected revenue is given by (7), and the corresponding closed-form expression is given in our technical report [20].

$$R_0(Q_0, \Pi_0) = \int_0^1 \int_{\Theta_0(\beta)}^{\bar{\theta}} \bar{R}_0(Q_0, \Pi_0, \theta, \beta) h(\beta, \theta) d\theta d\beta. \quad (7)$$

We further derive the optimal price Π_0^* in Proposition 2.

***Proposition** 2 (Optimal Data Plan $\mathcal{T}_0$):* The revenue-maximizing price of the traditional data plan $\mathcal{T}_0$ is[6]

$$\Pi_0^* = \frac{1}{2}\left[(D - 2A_0)\pi - \frac{A_0(\bar{\theta} - 2\pi)}{\ln(\frac{D-2A_0}{D})}\right]. \quad (8)$$

B. Rollover Data Plan – $\mathcal{T}_1$

For a $\mathcal{T}_1$ subscriber, he neeeds to pay overage fee when his data demand exceeds the effective data cap $Q_1 + r$. Thus, the MNO's revenue from a $\mathcal{T}_1$ subscribers of type (β, θ), with data demand d and rollover data r, is given by

$$\tilde{R}_1(Q_1, \Pi_1, \theta, \beta, d, r) = \begin{cases} \Pi_1, & \text{if } 0 \leq d \leq Q_1 + r, \\ \Pi_1 + \pi(1-\beta)(d - Q_1 - r), & \text{if } Q_1 + r < d \leq D. \end{cases} \quad (9)$$

[6] Since we do not optimize the MNO's revenue over the data cap Q_0, we suppose Q_0 takes practical values (i.e., 50MB would be impractical if the maximum data demand is 2GB), otherwise, Π_0^* may be negative. This applies to later propositions, lemmas, and theorems.

By taking the expectation over data demand d and rollover data r, the MNO's expcted revenue from a $\mathcal{T}_1$ subscriber of type (β, θ) is

$$\begin{aligned} \bar{R}_1(Q_1, \Pi_1, \theta, \beta) &= \sum_{r=0}^{Q_1} \sum_{d=0}^{D} \tilde{R}_1(Q_1, \Pi_1, \theta, \beta, d, r) f(d) p(r) \\ &= \pi(1-\beta)A_1 + \Pi_1. \end{aligned} \quad (10)$$

By taking the expectation over θ and β, the MNO's expected revenue is given by (11). Please refer to our technical report for its closed-form expression [20].

$$R_1(Q_1, \Pi_1) = \int_0^1 \int_{\Theta_1(\beta)}^{\bar{\theta}} \bar{R}_1(Q_1, \Pi_1, \theta, \beta) h(\beta, \theta) d\theta d\beta. \quad (11)$$

And the optimal price Π_1^* is given by Proposition 3.

***Proposition** 3 (Optimal Data Plan $\mathcal{T}_1$):* The revenue-maximizing price of the rollover data plan $\mathcal{T}_1$ is

$$\Pi_1^* = \frac{1}{2}\left[(D - 2A_1)\pi - \frac{A_1(\bar{\theta} - 2\pi)}{\ln(\frac{D-2A_1}{D})}\right]. \quad (12)$$

C. Rollover Data Plan – $\mathcal{T}_2$

As for the rollover data plan $\mathcal{T}_2$, the MNO's expected revenue from a user has a similar expression as in $\mathcal{T}_1$, and its optimal price when $Q_2 \leq \frac{D}{2}$ can be obtained as follows:

***Proposition** 4 (Optimal Data Plan $\mathcal{T}_2$):* When $Q_2 \leq \frac{D}{2}$, the revenue-maximizing price of rollover data plan $\mathcal{T}_2$ is

$$\Pi_2^* = \frac{1}{2}\left[(D - 2A_2)\pi - \frac{A_2(\bar{\theta} - 2\pi)}{\ln(\frac{D-2A_2}{D})}\right]. \quad (13)$$

Moreover, we can compute the unique optimal price for $\frac{D}{2} < Q_2$ numerically using a similar method. Please refer to our technical report for details [20].

D. Comparison Between $\{\mathcal{T}_0\}$, $\{\mathcal{T}_1\}$, $\{\mathcal{T}_2\}$

By summarizing the results from Sections IV.A, IV.B, and IV.C, we have the following result:

***Theorem** 1:* Suppose $Q_0 = Q_1 = Q_2 = Q$. Then, we have:

$$\begin{cases} \Pi_0^* < \Pi_1^* < \Pi_2^*, \\ \bar{S}_0(Q_0, \Pi_0^*, \beta, \theta) < \bar{S}_1(Q_1, \Pi_1^*, \beta, \theta) < \bar{S}_2(Q_2, \Pi_2^*, \beta, \theta), \\ R_0(Q_0, \Pi_0^*) < R_1(Q_1, \Pi_1^*) < R_2(Q_2, \Pi_2^*). \end{cases}$$

Theorem 1 means that the rollover data plan can increase the user payoff and the MNO revenue, and hence improve the social welfare. Additionally, allowing a user to consume the rollover data before his monthly data cap is more beneficial.

E. Combination of $\{\mathcal{T}_0, \mathcal{T}_1\}$

When MNO provides both plans $\{\mathcal{T}_0, \mathcal{T}_1\}$ to users, the MNO's expected revenue depends on the market partition. Specifically, the MNO's expected revenue is (7) when $(\Pi_0, \Pi_1) \in \mathcal{B}_a$, and is (11) when $(\Pi_0, \Pi_1) \in \mathcal{B}_d$. However, when $(\Pi_0, \Pi_1) \in \mathcal{B}_b$, the MNO's expected revenue is

$$\begin{aligned} & R_{10}(Q_0, Q_1, \Pi_0, \Pi_1) \\ =& \int_0^{\hat{\beta}} \int_{\Theta_0(\beta)}^{\bar{\theta}} \bar{R}_0(Q_0, \Pi_0, \theta, \beta) h(\beta, \theta) d\theta d\beta \\ &+ \int_{\hat{\beta}}^1 \int_{\Theta_0(\beta)}^{\Theta_{10}(\beta)} \bar{R}_0(Q_0, \Pi_0, \theta, \beta) h(\beta, \theta) d\theta d\beta \\ &+ \int_{\hat{\beta}}^1 \int_{\Theta_{10}(\beta)}^{\bar{\theta}} \bar{R}_1(Q_1, \Pi_1, \theta, \beta) h(\beta, \theta) d\theta d\beta, \end{aligned} \tag{14}$$

where $(\hat{\beta}, \hat{\theta})$ is the intersection of $\theta = \bar{\theta}$ and $\theta = \Theta_{10}(\beta)$. Please refer to our technical report for details [20].

When market partition mode is $\mathcal{B}_c$, the MNO's expected revenue is given by

$$\begin{aligned} & R_{10}(Q_0, Q_1, \Pi_0, \Pi_1) \\ =& \int_0^{\tilde{\beta}} \int_{\Theta_1(\beta)}^{\pi} \bar{R}_1(Q_1, \Pi_1, \theta, \beta) h(\beta, \theta) d\theta d\beta \\ &+ \int_{\tilde{\beta}}^1 \int_{\Theta_{10}(\beta)}^{\pi} \bar{R}_1(Q_1, \Pi_1, \theta, \beta) h(\beta, \theta) d\theta d\beta \\ &+ \int_{\tilde{\beta}}^1 \int_{\Theta_0(\beta)}^{\Theta_{10}(\beta)} \bar{R}_0(Q_0, \Pi_0, \theta, \beta) h(\beta, \theta) d\theta d\beta, \end{aligned} \tag{15}$$

where $(\tilde{\beta}, \tilde{\theta})$ is the intersection of $\theta = \Theta_1(\beta)$ and $\theta = \Theta_0(\beta)$. Please refer to our technical report for detail [20].

Moreover, we can derive the MNO's optimal pricing policy though the following lemmas.

***Lemma** 1:* Suppose $Q_0 = Q_1 = Q$. For any price pair $(\Pi_0, \Pi_1) \in \mathcal{B}_c$, we can always find a price pair $(\hat{\Pi}_0, \hat{\Pi}_1) \in \mathcal{B}_d$ with $\hat{\Pi}_1 = \Pi_1$ and $\hat{\Pi}_0 \geq \frac{D-2A_0}{D-2A_1}\Pi_1$, such that:

$$R_{10}(Q_0, Q_1, \Pi_0, \Pi_1) \leq R_{10}(Q_0, Q_1, \hat{\Pi}_0, \hat{\Pi}_1). \tag{16}$$

Lemma 1 implies that the MNO's expected revenue obtained from any price pair $(\Pi_0, \Pi_1) \in \mathcal{B}_c$, denoted by the green triangle (or the red star), is no larger than that obtained from any price pair on the purple triangle line (or the blue star line) in $\mathcal{B}_d$.

***Lemma** 2:* Suppose $Q_0 = Q_1 = Q$. Then, for any price pair $(\Pi_0, \Pi_1) \in \mathcal{B}_b$, we have: $\frac{\partial R_{10}}{\partial \Pi_1}|_{(Q_0, Q_1, \Pi_0, \Pi_1)} \leq 0$.

Lemma 2 implies that the MNO's expected revenue obtained from any price pair $(\Pi_0, \Pi_1) \in \mathcal{B}_b$, denoted by the green circle, is no larger than that obtained from the price pair corresponding to the red star. Thus, by combining Theorem 1, Lemma 1, and Lemma 2, we can derive the MNO's optimal pricing policy for $\{\mathcal{T}_0, \mathcal{T}_1\}$ as follows.

***Theorem** 2 (Optimal Prices of $\{\mathcal{T}_0, \mathcal{T}_1\}$):* The optimal prices (Π_0', Π_1') when MNO provides both data plans $\{\mathcal{T}_0, \mathcal{T}_1\}$ satisfy

$$\begin{cases} \Pi_1' = \Pi_1^*, \\ \Pi_0' \geq \frac{D-2A_0}{D-2A_1}\Pi_1^*, \end{cases} \tag{17}$$

where Π_1^* is given by Proposition 2. This means that MNO should charge $\mathcal{T}_1$ the same optimal price as providing $\{\mathcal{T}_1\}$ individually, and charge $\mathcal{T}_0$ a high price that no one will subscribe to $\mathcal{T}_0$; this is equivalent to providing $\{\mathcal{T}_1\}$ alone. This is consistent with the practice, where AT&T, Verizon, and China Mobile have replaced their traditional data plans to rollover data plans instead of offering both plans simultaneously.

TABLE II: Improvement* of rollover data plans

Plan	Price Gain	Revenue Gain	Payoff Gain	Social Welfare
$\mathcal{T}_1$	12.06%	4.77%	4.80%	4.78%
$\mathcal{T}_2$	21.20%	8.36%	8.06%	8.30%

* Compared with the traditional data plan $\mathcal{T}_0$.

F. Combination of $\{\mathcal{T}_0, \mathcal{T}_2\}$

When the MNO provides $\{\mathcal{T}_0, \mathcal{T}_2\}$ to users, the formulation and analysis are similart to Section IV.E. The optimal pricing policy for $\{\mathcal{T}_0, \mathcal{T}_2\}$ is as follows:

***Theorem** 3 (Optimal Prices of $\{\mathcal{T}_0, \mathcal{T}_2\}$):* The optimal prices (Π_0'', Π_2'') when MNO provides $\{\mathcal{T}_0, \mathcal{T}_2\}$ satisfy

$$\begin{cases} \Pi_2'' = \Pi_2^*, \\ \Pi_0'' \geq \frac{D-2A_0}{D-2A_2}\Pi_2^*, \end{cases} \tag{18}$$

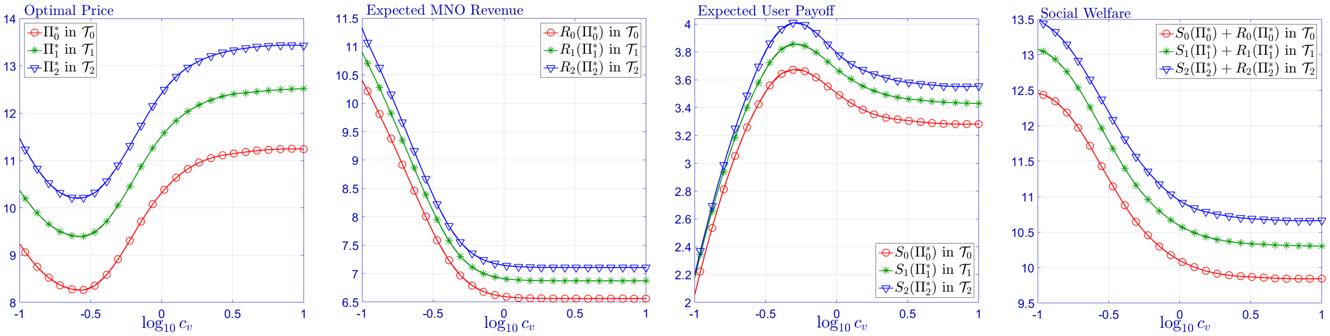
where Π_2^* is given by Proposition 4. This means that MNO should charge $\mathcal{T}_2$ the same optimal price as providing $\{\mathcal{T}_2\}$ individually, and charge $\mathcal{T}_0$ a high price that no one will subscribe to $\mathcal{T}_0$; this is equivalent to providing $\{\mathcal{T}_2\}$ alone.

V. Numerical Results

In this section we focus on providing $\{\mathcal{T}_0\}$, $\{\mathcal{T}_1\}$, and $\{\mathcal{T}_2\}$ separately, since in Theorems 2 and 3 we have shown that the optimal prices of $\{\mathcal{T}_0, \mathcal{T}_1\}$ and $\{\mathcal{T}_0, \mathcal{T}_2\}$ are equivalent to $\{\mathcal{T}_1\}$ and $\{\mathcal{T}_2\}$, respectively. The minimum data unit ϵ is set to 1MB, the data cap is Q_0=Q_1=Q_2=1GB=1000MB, and the overage fee is $15/GB. Users' maximum data demand is 2GB, hence D=2000. Moreover, (β, θ) observes the truncated normal distribution (which includes the uniform distribution used in Sections III and IV as a special case) on $\mathcal{M} = \{(\beta, \theta), 0 \leq \beta \leq 1, 0 \leq \theta \leq 30\}$, where the mean of β is 0.5 and the mean of θ is 15, and its probability density function is given by (2). The numerical results in Fig. 6 capture the influence of use diversity in terms of coefficient of variation $\log_{10} c_v$ ranging in $[-1, 1]$.

A. Performance Improvement

The results in Fig. 6(a) and Fig. 6(b) can validate our analysis in Theorem 1. According to Fig. 6(b), Fig. 6(c), and Fig. 6(d), a rollover data plan can increase users' expected payoff, bring MNO more revenue, and improve the social welfare. According to Table II, $\mathcal{T}_1$ and $\mathcal{T}_2$ can increase the social welfare by 4.78% and 8.30% on average, respectively. Moreover, rollover data plan $\mathcal{T}_2$ is always the most beneficial one to both users and the MNO, which indicates that it is better to allow the rollover data to be consumed prior to the monthly data cap.



(a) Optimal Price. (b) Expected Revenue of the MNO. (c) Expected Payoff of Users. (d) Social Welfare.

Fig. 6: Results in the truncated normal distributed market.

B. Influence of User Diversity

In our simulation, we only consider one data cap. However, it is difficult to satisfy the demand of diverse users with a single data cap. Thus, the MNO's market share will decrease as c_v increases. This is exactly why the MNO's revenue decreases as c_v increases, as showed in Fig. 6(b).

According to Fig. 6(a), in a homogeneous market (i.e., $\log_{10} c_v < -0.6$), when c_v increases, MNO shoud decrease the price to slow down the market share loss. However, in a heterogeneous market (i.e., $\log_{10} c_v > -0.6$), when c_v increases, the optimal pricing policy for MNO is to increase its price and get most revenue from high evaluation users (who are always subscribers).

According to Fig. 6(c), in a homogeneous market (i.e., $\log_{10} c_v < -0.3$), the users' expected payoff will increase with c_v due to the drop of the optimal price. However, in a heterogeneous market (i.e., $\log_{10} c_v > -0.3$), the MNO's optimal price no longer significantly changes but the number of non-subscription users increases with c_v, which reduces the users' expected payoff.

VI. Conclusions

In this work, we studied two rollover data mechanisms as well as the traditional one, and analyzed the interactions between the MNO and users. Our analysis revealed the following insights: (i) comparing with the traditional data plan, a rollover data plan can increase users' expected payoff, bring the MNO more revenue, and improve the social welfare; (ii) allowing a user to consume the rollover data before his monthly data cap is more beneficial than the other way around in terms of users' payoff, the MNO's revenue, and the social welfare; (iii) the MNO can achieve the maximum revenue by providing the rollover data plan without bundling with the traditional data plan; and (iv) high evaluation users are more likely to choose the rollover data plan than medium evaluation users. Specifically, as network substitutability increases, high evaluation users tend to choose the rollover data plan, while medium evaluation users tend to choose the traditional data plan. And low evaluation users usually have no data subscription.

In our future work, we will study the problem under a more general setting by relaxing the uniform distribution of data demand and user market. And we will consider the optimal choice of data caps, and consider the competition among multiple MNOs offering the rollover data plans.

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