Temporal Correlation of Interference Under Spatially Correlated Shadowing

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***Abstract*—In this paper, we study the temporal correlation of interference in mobile ad-hoc networks under a correlated shadowing environment. By modeling the node locations as a 1-D Poisson point process with an i.i.d. mobility model and considering spatially correlated shadowing that depends on the distance between nodes, we derive a simple asymptotic expression of the temporal correlation coefficient of interference as the variance of log-normal shadowing increases. This shows a readable relationship between the correlation distance of log-normal shadowing and the temporal correlation of interference and thus can be useful for modeling general wireless systems with spatially correlated shadowing.**

***Index Terms*—Interference, correlated shadowing, temporal correlation, Poisson point process, stochastic geometry**

I. Introduction

Interference is an important factor in the analysis of various wireless systems. More specifically, correlation of interference may cause significant degradation of the performance of wireless communications in both spatial and temporal means, e.g., end-to-end throughput, retransmissions [1], cooperative relaying [2], broadcast communications, multiple antennas [3], and handovers [4]. For example, if interference at a certain receiver is temporally correlated, quick retransmission after a transmission failure is likely to fail again. Moreover, in broadcast communications of vehicular networks under strong spatial correlation of interference, outage of vehicles in a dense cluster can be highly correlated, which leads to performance degradation in information dissemination. Therefore, correlation of interference needs to be analyzed to efficiently design various wireless network systems or protocols.

Due to the importance of correlation of interference, many researchers have studied them in various settings in the past decade [4]–[10]. Basically, several major factors can cause correlation of interference: node locations, fading, and channel access. Indeed, Ganti and Haenggi [5] showed that interference can be temporally correlated due to the temporal correlation of node locations among successive times slots even when ALOHA is used as the media access control (MAC) protocol. In addition, the temporal correlation of traffic or channel status and low mobility of nodes can increase the correlation of interference [6], [7].

Although the effects of the correlation of the node locations (including mobility) or traffic have been well studied, few researchers have considered the impact of *spatially correlated shadowing* on the correlation of interference. Most previous work assumed spatially i.i.d. fading; however, shadowing (i.e., slow fading) is spatially correlated on a scale from 50 to 200 m [11] due to the effects of blockage and reflection in transmission channels. Shadowing is commonly modeled by a log-normal distribution, and a widely accepted model for the correlation of shadowing was proposed by Gudmundson [12]. Typically, the distance at which the correlation of shadowing remains depends on environments and is called a *correlation distance*. However, the relationship between the correlation distance and the correlation of interference has not been studied. Furthermore, the impact of node mobility in the correlated shadowing environment has not been reported.

In this paper, we study the temporal correlation of interference in mobile ad-hoc networks under a correlated shadowing environment. By modeling the node locations as a 1-D Poisson point process (PPP) with an i.i.d. mobility model and the correlated shadowing as Gudmundson's model [12], we analyze the temporal correlation coefficient of interference in a correlated shadowing environment. Since the exact expression of the temporal correlation coefficient is not tractable, we derive its simple asymptotic expansion when the *variance* of the log-normal shadowing increases by using Watson's lemma (see e.g., [13]). Our results show a readable relationship between the correlation distance of the shadowing and the correlation of interference and indicate that the temporal correlation of the interference mainly depends on the probability that a node stays in the same position. Furthermore, we found through numerical examples that the obtained asymptotic expansion can be used as a tight approximate formula and so is useful for estimating the impacts of other various system parameters on the temporal correlation of interference.

Although we consider a 1-D model, we believe that it gives us a critical insight into the spatial correlation of shadowing and can be applied to various network models such as vehicular networks.

II. Related Work

The spatial and temporal correlation coefficients of interference in various settings have been studied [4]–[10]. Those in ad-hoc networks with ALOHA were first studied by Ganti and

Haenggi [5], who modeled the node locations as a PPP under an i.i.d. fading assumption. Schilcher et al. [6] extended this work by considering Rayleigh block fading and temporally correlated traffic. The impact of the node mobility on the correlation of interference was studied by Gong and Haenggi [7]. For extensions of a model of node locations, Wen et al. considered K-tier heterogeneous networks [8] and also studied the cases when nodes are distributed with a clustered point process and a repulsive one [9]. The correlation of interference in cellular networks was recently studied by Krishnan and Dhillon [4]. Many researchers also studied the performance of wireless systems under correlated interference. Crismani et al. [14] considered a decode-and-forward relaying system in which consecutive transmission attempts are temporally correlated. Afify et al. [15] presented a unified mathematical framework for cellular networks with multiple-input-multiple-output (MIMO) and studied the temporal correlation in retransmissions. The effect of the spatial correlation of interference on opportunistic secure information transfer was analyzed in [16]. However, the above studies considered i.i.d. fading and did not take into account spatially correlated shadowing.

Since the shadowing is spatially correlated [11], several analytical models have been proposed [12], [17], [18]. Gudmudonson [12] proposed a widely-accepted model in which the shadowing variable of the channel between a fixed base station (BS) and a moving user is modeled by an autoregressive process with exponentially decaying autocorrelation subject to the moving distance. Several extensions of this model have also been proposed: the case of multiple BSs [17] and multi-hop networks [18]. The correlation distance or other parameters of this model have also been experimentally studied [12], [19], [20]. Baek et al. [20] recently reported that the correlation distance for the mm-wave frequency band is similar to that for other lower frequency bands.

A stochastic geometry based approach often gives us tractable results in the interference analysis. However, models with (even uncorrelated) log-normal shadowing lead to intractable results. For this problem, Baccelli and Zhang [21] proposed a correlated shadowing model, in which correlated log-normal shadowing is approximated by a random variable depending on the number of buildings that a transmission channel penetrates. In addition, Koufos et al. [10] studied the temporal correlation of interference in MANETs with blockage by modeling obstacles and their penetration loss. In contrast with the above studies, we formally incorporate the correlated log-normal shadowing into our model on the basis of Gudmundson's model [12]. By doing this, we obtain a simple connection between the correlation distance of shadowing and the temporal correlation of interference.

III. Model Description

In this section, we explain our model. We consider a line on $\mathbb{R}$ and a target receiver fixed at the origin. We assume that nodes (i.e., potential transmitters) are randomly distributed on the line and independently move in each time-slot whereas the target receiver does not move. More precisely, we model the node locations by a 1-D homogeneous PPP Φ with intensity λ. Each point $x_i(t) \in \Phi(t)$ ($i \in \mathbb{Z}_+ \triangleq \{0, 1, 2, \ldots, \}$) on $\mathbb{R}$ represents the position of the i-th transmitter (vehicle, mobile user, BS) at a time slot $t \in \mathbb{Z}_+$. We omit t of $x_i(t)$ and $\Phi_i(t)$, such as x_i and Φ, when considering a certain fixed time-slot. For the node mobility, we assume that the moving distance of the node i at time t, $x_i(t+1) - x_i(t) =: v$, is independently distributed with a p.d.f. $\psi(v)$ ($v \in \mathbb{R}$), which does not depend on $x_i(t)$ and satisfies $\psi(0) > 0$. Due to the displacement theorem of PPPs (see e.g., Theorem 2.33 in [22]), the realization of $\Phi(t)$ at fixed time slot t remains a homogeneous PPP if $\Phi(0)$ is homogeneous.

We next explain our channel model. We assume that all nodes have the unit transmission power. The path loss model is assumed as $\ell(r) = (1+r)^\alpha$ ($r \in \mathbb{R}_+$) where $\alpha > 1$ is a path loss exponent. Furthermore, we assume that each transmission channel has the effect of shadowing and h denotes the shadowing variable. According to a widely-accepted assumption for the shadowing effect, for any fixed channel, h is assumed to be log-normally distributed, i.e.,

$$h = \exp\left(-\sigma_{\mathrm{dB}}^2/2 + \sigma_{\mathrm{dB}} Z\right), \tag{1}$$

where $Z \sim \mathcal{N}(0, 1)$. Note that $\mathbb{E}[h] = 1$ and $\mathbb{E}[h^2] = e^{\sigma_{\mathrm{dB}}^2}$. Thus, we can consider a stationary marked point process $\hat{\Phi} = \{(x_i, h_i); i \in \mathbb{Z}_+\}$ with the mark distribution of a typical point as (1). We also assume that the shadowing variables are correlated depending on the distance between nodes. More precisely, if we consider two nodes i and j, the corresponding shadowing variables h_i and h_j have the following correlation coefficient in the logarithmic sense:

$$\rho_{\mathrm{dB}} \triangleq \frac{\mathbb{E}[\ln h_i \ln h_j]}{\sigma_{\mathrm{dB}}^2} = e^{-\frac{|x_i - x_j|}{d_{\mathrm{cor}}} \ln 2}, \tag{2}$$

where d_{cor}, called a correlation (decorrelation) distance, depends on environments and represents the distance at which the correlation coefficient ρ_{dB} is equal to 0.5. For simplicity, we write $d_0 \triangleq d_{\mathrm{cor}}/\ln 2$ hereafter. This is the widely-accepted model proposed by Gudmundson [12] for the spatial correlation of shadowing. In this paper, we only consider the shadowing effect and do not take into account the multi-path fading, such as Rayleigh fading. In addition, the shadowing variable is assumed to be time-invariant, i.e., the value of the shadowing variable at a fixed node location does not change over time.

By definition, the received power from the node i at time slot t can be represented as $h_i(t)/\ell(x_i(t))$. Thus, the total interference power received at the origin equals

$$I(t) = \sum_{x_i(t) \in \Phi(t)} \frac{h_i(t) \mathcal{S}_i(t)}{\ell(x_i(t))}, \tag{3}$$

where $\mathcal{S}_i(t)$ denotes an indicator that equals 1 when the node i is transmitting radio waves at the beginning of time slot t and 0 otherwise.

We assume that all nodes use slotted-ALOHA as the MAC. Thus, each node transmits radio waves or not with probability

p in each transmission time-slot. Note that the transmission time slot is different from the mobility time-slot. We assume that the transmission time-slot is fixed and smaller than the mobility time-slot. In other words, the transmission of each vehicle at time t does not continue until time $t+1$, and thus $\mathcal{S}_i(t)$ and $\mathcal{S}_i(t+1)$ are independent for all i.

IV. Main Results

In this section, we present our main results, the temporal correlation of interference, i.e., the correlation between interferences at different two time slots. Due to the spatially correlated shadowing, the interference received at the same receiver is expected to be more correlated in a lower mobility environment. In what follows, we first derive the first and second moments of interference, which will be used for the derivation of the temporal correlation coefficient. Although the mean interference can be easily obtained, the second moment does not have an explicit form due to the cross correlation of shadowing variables. Therefore, by using Watson's lemma (see e.g., [13]), we derive a simple but useful asymptotic expansion of the second moment of interference for sufficiently large σ_{dB}. We then derive an asymptotic expansion of the temporal correlation coefficient of interference that is valid for general i.i.d. mobility models. Finally, we give several mobility models as examples and derive the corresponding temporal correlation coefficient for each model.

A. First and second moments of interference

Since the marked point process $\hat{\Phi}$ is stationary, the mean interference can be easily obtained by applying Campbell's theorem (see e.g., [22]) to (3) as follows:

$$\mathbb{E}[I] = \mathbb{E}\left[\sum_{x_i\in\Phi} \frac{h_i\mathcal{S}_i}{\ell(x_i)}\right] = \int_{\mathbb{R}} \frac{\lambda p \mathbb{E}_0[h]}{(1+|x|)^{\alpha}}\mathrm{d}x = \frac{2\lambda p}{\alpha-1}, \tag{4}$$

where $\mathbb{E}_0$ denotes the expectation with the distribution of a typical point. We next consider the second moment of interference, $\mathbb{E}[I^2]$. By definition, this can be rewritten as

$$\mathbb{E}[I^2] = \mathbb{E}\left[\sum_{x_i,x_j\in\Phi^{(2)}_{\neq}} \frac{h_ih_j\mathcal{S}_i\mathcal{S}_j}{\ell(x_i)\ell(x_j)}\right] + \mathbb{E}\left[\sum_{x_i\in\Phi}\frac{h_i^2\mathcal{S}_i}{(\ell(x_i))^2}\right],$$

where $\Phi^{(2)}_{\neq}$ denotes all the set of the distinct pairs in Φ. By applying Campbell's theorem (e.g., [22]) to the above and using (2), we obtain

$$\begin{aligned}\mathbb{E}[I^2] &= \iint_{(\mathbb{R})^2} \frac{\lambda^2p^2\mathbb{E}_{ij}[h_ih_j]}{\ell(x_i)\ell(x_j)}\mathrm{d}x_i\mathrm{d}x_j + \int_{\mathbb{R}}\frac{\lambda p\mathbb{E}_0[h^2]}{(1+|x|)^{2\alpha}}\mathrm{d}x \\ &= \iint_{(\mathbb{R})^2}\frac{\lambda^2p^2e^{\sigma_{\mathrm{dB}}^2 e^{\frac{-|x_i-x_j|}{d_0}}}}{\ell(x_i)\ell(x_j)}\mathrm{d}x_i\mathrm{d}x_j + \frac{2\lambda p e^{\sigma_{\mathrm{dB}}^2}}{2\alpha-1}. \end{aligned}\tag{5}$$

The first term in the above equation can be numerically computed, but it does not have a closed-form. However, when the variance of shadowing (i.e., σ_{dB}) is large, we have the following simple asymptotic expansion. The proof of the lemma is given in Appendix B.

Lemma 1 *The second moment of interference has the following asymptotic expansion as $\sigma_{\mathrm{dB}}\to\infty$:*

$$\mathbb{E}[I^2] \sim \frac{2\lambda p e^{\sigma_{\mathrm{dB}}^2}}{2\alpha-1}\left[1+\frac{2\lambda p d_0}{\sigma_{\mathrm{dB}}^2}+\frac{2\lambda p d_0}{\sigma_{\mathrm{dB}}^4}+O\left(\frac{1}{\sigma_{\mathrm{dB}}^6}\right)\right] + \frac{2\lambda^2p^2d_0^2e^{\sigma_{\mathrm{dB}}^2}}{\sigma_{\mathrm{dB}}^4}. \tag{6}$$

B. Temporal correlation coefficient

On the basis of the previous results, we next aim to derive the temporal correlation coefficient of interference, which for time interval τ slots ($\tau\in\mathbb{Z}_+$) is defined as,

$$\rho_\tau \triangleq \frac{\mathrm{Cov}[I(t),I(t+\tau)]}{\sqrt{\mathrm{Var}[I(t)]\mathrm{Var}[I(t+\tau)]}} = \frac{\mathbb{E}[I(t)I(t+\tau)]-(\mathbb{E}[I])^2}{\mathbb{E}[I^2]-(\mathbb{E}[I])^2}.$$

Here, we call $\mathrm{Cov}[I(t),I(t+\tau)]$ the temporal covariance of interference. Recall that the moving distance of the node i during one time-slot is independently distributed in accordance with the p.d.f. $\psi(v)$. Therefore, the moving distance of the node i in τ time-slots, i.e., $\overline{v}_i(\tau)\triangleq x_i(t+\tau)-x_i(t)$, follows the p.d.f. $\psi_\tau(v)$, which is the τ-th convolution of $\psi(v)$ and independent of $x_i(t)$. The cross-correlation of $I(t)$ and $I(t+\tau)$ can be represented as

$$\begin{aligned}&\mathbb{E}[I(t)I(t+\tau)] \\ &= \mathbb{E}\left[\sum_{x_i(t),x_j(t)\in\Phi^{(2)}_{\neq}(t)}\frac{h_i(t)h_j(t+\tau)\mathcal{S}_i(t)\mathcal{S}_j(t+\tau)}{\ell(x_i(t))\ell(x_j(t)+\overline{v}_j(\tau))}\right] \\ &\quad+\mathbb{E}\left[\sum_{x_i(t)\in\Phi(t)}\frac{h_i(t)h_i(t+\tau)\mathcal{S}_i(t)\mathcal{S}_i(t+\tau)}{\ell(x_i(t))\ell(x_i(t)+\overline{v}_i(\tau))}\right].\end{aligned}\tag{7}$$

The following is an asymptotic expression of the temporal covariance of interference.

Lemma 2 *Suppose that $\psi_\tau(0)>0$ and $\psi'_\tau(0)$ exists. The temporal covariance of interference for time interval τ has the following asymptotic expansion as $\sigma_{\mathrm{dB}}\to\infty$:*

$$\begin{aligned}\mathrm{Cov}[I(t),I(t+\tau)] \sim 2\lambda p^2 d_0 e^{\sigma_{\mathrm{dB}}^2}\Bigg[&\frac{2}{2\alpha-1}\left\{\frac{(\lambda+\psi_\tau(0))}{\sigma_{\mathrm{dB}}^2}\right. \\ &+\left.\frac{\lambda+\psi_\tau(0)+d_0\psi'_\tau(0)}{\sigma_{\mathrm{dB}}^4}\right\}+\frac{d_0(\lambda+\psi_\tau(0))}{\sigma_{\mathrm{dB}}^4}+O\left(\frac{1}{\sigma_{\mathrm{dB}}^6}\right)\Bigg].\end{aligned}$$

Proof. The proof of this lemma is given in Appendix C. □

By applying Lemmas 1 and 2 to the definition of ρ_τ, we can immediately obtain our main result below.

Theorem 1 *Suppose that $\psi_\tau(0)>0$ and $\psi'_\tau(0)$ exists. The temporal correlation coefficient of interference of time interval τ is asymptotically equivalent to, when $\sigma_{\mathrm{dB}}\to\infty$,*

$$\begin{aligned}\rho_\tau \sim\ &\frac{1}{\frac{1}{2\alpha-1}\left(\frac{1}{d_0p}+\frac{2\lambda}{\sigma_{\mathrm{dB}}^2}+\frac{2\lambda}{\sigma_{\mathrm{dB}}^4}\right)+\frac{d_0\lambda}{\sigma_{\mathrm{dB}}^4}} \\ &\times\left[\frac{2}{2\alpha-1}\left\{\frac{\lambda+\psi_\tau(0)}{\sigma_{\mathrm{dB}}^2}+\frac{\lambda+\psi_\tau(0)+d_0\psi'_\tau(0)}{\sigma_{\mathrm{dB}}^4}\right\}\right. \\ &\left.+\frac{d_0(\lambda+\psi_\tau(0))}{\sigma_{\mathrm{dB}}^4}\right].\end{aligned}\tag{8}$$

Theorem 1 indicates that ρ_τ depends only on $\psi_\tau(0)$ and $\psi'_\tau(0)$ when $\sigma_{\rm dB} \to \infty$. In other words, the probability that the nodes do not move has the dominant impact on the asymptotic behavior of the temporal correlation coefficient of interference.

Remark 1 By removing the terms of $O(1/\sigma_{\rm dB}^4)$, we can obtain a simpler asymptotic expression of ρ_τ, as $\sigma_{\rm dB} \to \infty$:

$$\rho_\tau \sim \frac{2d_0 p(\lambda + \psi_\tau(0))}{2\lambda d_0 p + \sigma_{\rm dB}^2}. \tag{9}$$

C. Mobility models

We next consider several commonly used mobility models as examples and show the temporal correlation coefficients corresponding to each model. In this paper, we choose three models: (i) *constrained i.i.d. mobility (CIM) model*; (ii) *random walk (RW) model*; and (iii) *discrete-time Brownian motion (BM) model*. The same models are considered by Gong and Haenggi [7]. We describe the details of each model below.

(i) *constrained i.i.d. mobility (CIM) model*: In the CIM model, the location $x_i(t+1) \in \Phi(t+1)$ of the node i at the time slot $(t+1)$ is determined independently of $x_i(t)$ such that

$$x_i(t+1) := x_i(0) + v_i(t), \qquad t \in \mathbb{Z}_+.$$

Here, $v_i(t)$'s are drawn from a uniform distribution with range $[-V_{\max}, V_{\max}]$. Clearly, $\psi_\tau(0) = 1/2V_{\max}$ and $\psi'_\tau(0) = 0$.

(ii) *random walk (RW) model*: In the RW model, the location of the node i at time $(t+1)$ is determined as follows:

$$x_i(t+1) := x_i(t) + v_i(t), \qquad t \in \mathbb{Z}_+, \tag{10}$$

where $v_i(t)$'s are drawn from a uniform distribution with range $[-V_{\max}, V_{\max}]$. Note that the RW model differs from the CIM model because the location of the node i at time $(t+1)$ depends on that at time t. Under the RW model, $\psi_\tau(0)$ becomes (see e.g., [23])

$$\psi_\tau(0) = \frac{1}{2V_{\max}} \sum_{i=0}^{\lfloor \frac{\tau}{2} \rfloor} (-1)^i \frac{\tau}{\tau!(\tau-i)!} \left(\frac{\tau}{2} - i\right)^{\tau-1}.$$

Note also that $\psi_\tau(v)$ is not differentiable at $v = 0$ when τ is an even number. Thus, to utilize (8), τ must be an odd number.

(iii) *discrete-time Brownian motion model (BM)*: In the random walk model, the location of the node i at time $(t+1)$ is determined by (10) with $v_i(t) \sim \mathcal{N}(0, \sigma_V^2)$. In this model, the p.d.f. $\psi_\tau(v)$ is simply equal to $\mathcal{N}(0, \tau\sigma_V^2)$ due to a property of a normal distribution; and thus $\psi_\tau(0) = 1/\sqrt{2\pi\tau\sigma_V^2}$ and $\psi'_\tau(0) = 0$.

V. Numerical Examples

In this section, we present several numerical examples. We compared the exact values of the temporal correlation coefficient computed by numerical integration with those approximated by using the asymptotic expansions provided in Section IV. By doing this, we show how the correlated shadowing affects the correlation of the interference under various parameter settings and also that the temporal correlation can be well approximated by our simple asymptotic formulas. In our numerical examples, we set $d_{\rm cor} = 100$ [m], i.e., $d_0 = 100/\ln 2$ according to [11] and $\alpha = 4$ unless otherwise noted. Furthermore, we chose the best parameters for all numerical integrations after several numerical calculations.

A. Impacts of various parameters

We first investigate the impacts of various parameters on the temporal correlation coefficient. Fig. 1 shows ρ_τ from numerical computation (plotted as 'exact') and approximation based on (8) (plotted as 'approx') with different $\sigma_{\rm dB}$ and τ in the CIM, RW, and BM models. Since the positions of nodes do not depend on τ under the CIM model, we set $V_{\max}$ to the x-axis in the left graph. In addition, we set $\lambda = 10$ [1/km]. We can see from the graphs that the temporal correlation coefficient decreases as the elapsed time slots increase or the mobility increases, but it does not converge to zero because the mobility effect is averaged out in the calculation of an integral term in the temporal covariance (see (16), (17)). This indicates that the temporal correlation of interference may remain for a long time. We can also see that if $\sigma_{\rm dB}$ increases, the temporal correlation coefficient decreases. Furthermore, the approximate values based on (8) well fit to the exact values in all cases.

Figs. 2 and 3 compare results of the temporal correlation coefficients under the RW model with different $\sigma_{\rm dB}$ when varying $d_{\rm cor}$ and λ, respectively. Clearly, when the density of nodes increases, the correlation also increases. In addition, the temporal correlation becomes higher when correlation distance becomes longer.

B. Accuracy as approximate formulas

We next evaluate accuracy as an approximate formula for the temporal correlation coefficient. We compare the approximated values when using (8) and its simpler version (9). Fig. 4 compares ρ_τ from numerical computation and approximate formulas based on (8) (corresponding to app(A) in the graph) and (9) (corresponding to app(B)). We can see from the figure that in all cases, both asymptotic formulas well fitted to the results from numerical computation although the simpler version (app(B)) has larger errors than the original one (app(A)) when $\sigma_{\rm dB}$ is small. However, since $\sigma_{\rm dB}$ typically takes the values in 4–13 dB in a realistic situation [11], we can say that the simpler version of the asymptotic formula is also sufficiently accurate to roughly estimate how the interference is temporally correlated under a correlated shadowing environment.

VI. Conclusion

In this paper, we studied the temporal correlation of interference in a correlated shadowing environment. On the basis of Gudmundson's model [12], we derived a simple asymptotic expansion of the temporal correlation coefficients of interference. We also showed in numerical results that the asymptotic expansion can be used as tight approximate formulas.

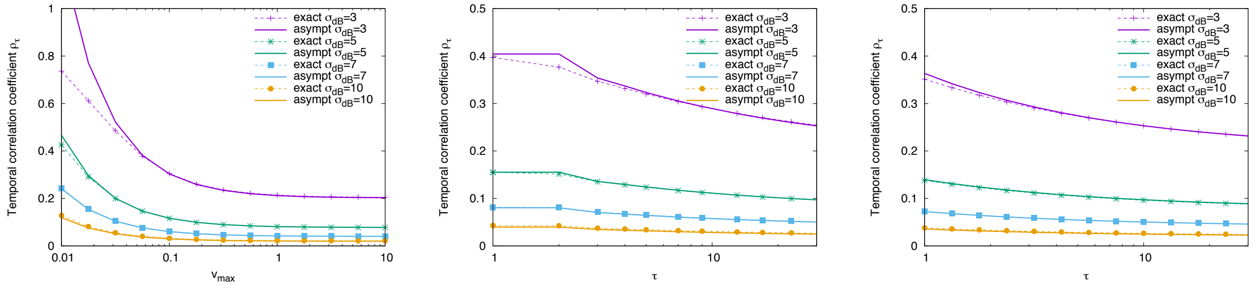

Fig. 1. Comparison of values of ρ_τ from numerical computation and asymptotic expansions under CIM (left), RW (middle), and BM (right) models with different σ_{dB}. Horizontal axis in CIM is $V_{\max}$ and those in RW and BM are τ. In RW and BM, $V_{\max} = 0.05$, $\sigma_V = 0.05$.

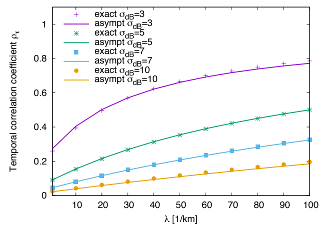

Fig. 2. Comparison of ρ_τ ($\tau = 1$) in RW model from numerical computation and approximation with different σ_{dB} when varying λ.

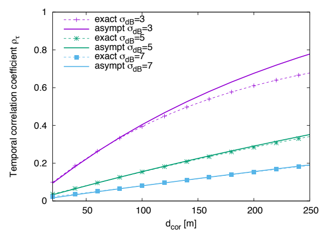

Fig. 3. Comparison of ρ_τ ($\tau = 1$) in RW model from numerical computation and approximation with different σ_{dB} when varying d_{cor}.

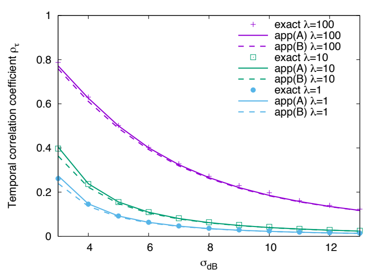

Fig. 4. Comparison of ρ_τ ($\tau = 1$) in RW model from numerical computation and approximation with different λ when varying σ_{dB}.

Our future work includes an extension to two-dimensional space. More general mobility models are also our future work. In addition, the correlation of outage in a correlated log-normal shadowing environment needs to be considered for the performance evaluation of wireless communications.

Appendix A: Watson's lemma

In this appendix, we explain a useful mathematical tool, known as Watson's lemma, which gives an asymptotic expression of the following form of an exponential integral:

$$\widetilde{F}(\lambda) = \int_a^{a+\delta} e^{\lambda R(t)} g(t)\mathrm{d}t = \int_0^{\delta} e^{\lambda R(a+\tau)} g(a+\tau)\mathrm{d}\tau,$$

where $R(t)$ has its maximum at $t = a$.

Watson's lemma (Miller [13] (Section 3.3)): Suppose that $R(t)$ and $g(t)$ have an infinite number of continuous derivatives for $a \le t < a + \delta$ and $R'(a) < 0$. We then have the following asymptotic expansion as $\lambda \to \infty$[1]:

[1]In the original version of Watson's lemma, asymptotic expansions with arbitrary terms $O(1/\lambda^n)$ ($n \in \mathbb{N}$) can be obtained (for details, see [13]).

$$\widetilde{F}(\lambda) = e^{\lambda R(a)} \left(-\frac{g(a)}{R'(a)} \frac{1}{\lambda} + \frac{g'(a)R'(a) - g(a)R''(a)}{(R'(a))^3} \frac{1}{\lambda^2} + O\left(\frac{1}{\lambda^3}\right) \right). \quad (11)$$

Appendix B: Proof of Lemma 1

The first term in (5) can be rewritten as

$$\iint_{(\mathbb{R})^2} \frac{e^{\sigma_{\text{dB}}^2 e^{\frac{-|x_i - x_j|}{d_0}}}}{\ell(x_i)\ell(x_j)} \mathrm{d}x_i \mathrm{d}x_j = \int_{\mathbb{R}} \frac{1}{\ell(x)} \int_{\mathbb{R}} \frac{e^{\sigma_{\text{dB}}^2 e^{\frac{-|s|}{d_0}}}}{\ell(x+s)} \mathrm{d}s\mathrm{d}x. \quad (12)$$

The above integral does not have an explicit form and requires numerical integration for its calculation. However, as proven in Lemma 3 below, we can obtain a simple asymptotic expansion for this integral as $\sigma_{\text{dB}} \to \infty$.

Lemma 3 *Let $f(s)$ denote an arbitrary function on $\mathbb{R}$ such that $f(0) \neq 0$ and $f'(0)$ exits. We then have, as $\sigma \to \infty$,*

$$\iint_{(\mathbb{R})^2} \frac{e^{\sigma^2 e^{\frac{-|s|}{d_0}}} f(s)}{\ell(x)\ell(x+s)} \mathrm{d}s\mathrm{d}x \sim 2d_0 e^{\sigma^2} \left[\frac{2}{2\alpha - 1} \times \left(\frac{f(0)}{\sigma^2} + \frac{f(0) + d_0 f'(0)}{\sigma^4} \right) + \frac{d_0 f(0)}{\sigma^4} + O\left(\frac{1}{\sigma^6}\right) \right], \quad (13)$$

Proof of Lemma 3. By definition, we have, for any $x \in \mathbb{R}_+$,

$$\int_{\mathbb{R}} \frac{e^{\sigma^2 e^{-\frac{|s|}{d_0}}} f(s)}{\ell(x+s)} \mathrm{d}s = \underbrace{\int_0^{\infty} \frac{e^{\sigma^2 e^{-\frac{s}{d_0}}} f(s)}{(1+x+s)^{\alpha}} \mathrm{d}s}_{A_1} + \underbrace{\int_0^{x} \frac{e^{\sigma^2 e^{-\frac{s}{d_0}}} f(s)}{(1+x-s)^{\alpha}} \mathrm{d}s}_{A_2} + \underbrace{\int_x^{\infty} \frac{e^{\sigma^2 e^{-\frac{s}{d_0}}} f(s)}{(1+s-x)^{\alpha}} \mathrm{d}s}_{A_3}. \quad (14)$$

By applying (11), the first and second integrals in (14) lead to the following asymptotic expansions as $\sigma^2 \to \infty$:

$$A_1 \sim d_0 e^{\sigma^2} \left[\frac{f(0)}{(1+x)^{\alpha}} \frac{1}{\sigma^2} + \left(\frac{f(0) + d_0 f'(0)}{(1+x)^{\alpha}} - \frac{\alpha d_0 f(0)}{(1+x)^{\alpha+1}} \right) \frac{1}{\sigma^4} + O\left(\frac{1}{(1+x)^{\alpha}\sigma^6} \right) \right],$$

$$A_2 \sim d_0 e^{\sigma^2} \left[\frac{f(0)}{(1+x)^{\alpha}} \frac{1}{\sigma^2} + \left(\frac{f(0) + d_0 f'(0)}{(1+x)^{\alpha}} + \frac{\alpha d_0 f(0)}{(1+x)^{\alpha+1}} \right) \frac{1}{\sigma^4} + O\left(\frac{1}{(1+x)^{\alpha}\sigma^6} \right) \right].$$

Thus, combining the above yields

$$\int_{\mathbb{R}_+} \frac{A_1+A_2}{\ell(x)}\mathrm{d}x \sim 2d_0 e^{\sigma^2}\int_{\mathbb{R}_+}\Bigg[\frac{f(0)}{(1+x)^{2\alpha}\sigma^2} + \frac{f(0)+d_0 f'(0)}{(1+x)^{2\alpha}\sigma^4} + O\left(\frac{1}{(1+x)^{2\alpha}\sigma^6}\right)\Bigg]\mathrm{d}x$$
$$= \frac{2d_0 e^{\sigma^2}}{2\alpha-1}\left[\frac{f(0)}{\sigma^2} + \frac{f(0)+d_0 f'(0)}{\sigma^4} + O\left(\frac{1}{\sigma^6}\right)\right]. \quad (15)$$

By changing the variables, the last integral in (14) can be rewritten as

$$\int_{\mathbb{R}_+}\frac{A_3}{\ell(x)}\mathrm{d}x = \int_{\mathbb{R}_+}\int_{\mathbb{R}_+}\frac{f(s+x)e^{\sigma^2}e^{-\frac{s+x}{d_0}}}{(1+x)^\alpha(1+s)^\alpha}\mathrm{d}s\mathrm{d}x$$
$$\overset{(a)}{=} \int_{\mathbb{R}_+}\int_{-1}^{1}\frac{yf(y)e^{\sigma^2}e^{-\frac{y}{d_0}}}{2(1+\frac{y+yz}{2})^\alpha(1+\frac{y-yz}{2})^\alpha}\mathrm{d}z\mathrm{d}y,$$

where we use $y=x+s$ and $z=\frac{x-s}{x+s}$ in (a). Furthermore, it follows from (11) that

$$\int_{\mathbb{R}_+}\frac{A_3}{\ell(x)}\mathrm{d}x \sim \int_{-1}^{1} e^{\sigma^2}\left[\frac{d_0^2 f(0)}{2\sigma^4} + O\left(\frac{1}{\sigma^6}\right)\right]\mathrm{d}z$$
$$= e^{\sigma^2}\left[\frac{d_0^2 f(0)}{\sigma^4} + O\left(\frac{1}{\sigma^6}\right)\right].$$

As a result, combining this and (15) with (14) leads to (13). $\square$

Substituting $f(s)\equiv 1$ into (13) and using (12) and (5), we obtain (6).

Appendix C: Proof of Lemma 2

Since the shadowing variable is assumed to be time-invariant, by using Campbell's theorem, the first term in (7) can be rewritten as

$$\mathbb{E}\left[\sum_{x_i(t),x_j(t)\in\Phi^{(2)}_{\neq}(t)}\frac{h_i(t)h_j(t+\tau)\mathcal{S}_i(t)\mathcal{S}_j(t+\tau)}{\ell(x_i(t))\ell(x_j(t+\tau))}\right]$$
$$= \lambda^2 p^2\int_{\mathbb{R}}\int_{\mathbb{R}}\mathbb{E}_v\left[\frac{e^{\sigma_{\mathrm{dB}}^2}e^{-\frac{|x-(y+v)|}{d_0}}}{\ell(x)\ell(y+v)}\right]\mathrm{d}x\mathrm{d}y$$
$$= \lambda^2 p^2\int_{\mathbb{R}}\int_{\mathbb{R}}\frac{e^{\sigma_{\mathrm{dB}}^2}e^{-\frac{|s|}{d_0}}}{\ell(x)\ell(x+s)}\mathrm{d}x\mathrm{d}s, \quad (16)$$

which can be calculated by (13). The second term in (7) can be also rewritten as

$$\mathbb{E}\left[\sum_{x_i(t)\in\Phi(t)}\frac{h_i(t)h_i(t+\tau)\mathcal{S}_i(t)\mathcal{S}_i(t+\tau))}{\ell(x_i(t))\ell(x_i(t+\tau)}\right]$$
$$= \lambda p^2\int_{\mathbb{R}}\mathbb{E}_v\left[\frac{e^{\sigma_{\mathrm{dB}}^2}e^{-\frac{|v|}{d_0}}}{\ell(x)\ell(x+v)}\right]\mathrm{d}x$$
$$= \lambda p^2\iint_{(\mathbb{R})^2}\frac{e^{\sigma_{\mathrm{dB}}^2}e^{-\frac{|v|}{d_0}}\psi_\tau(v)}{\ell(x)\ell(x+v)}\mathrm{d}v\mathrm{d}x, \quad (17)$$

which can be also evaluated by using (13). Therefore, by applying Lemma 3 to (16) and (17) and combining them with (7), the proof is completed.

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