# On the Conditional Entropy of Wireless Networks

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***Abstract*—The characterization of topological uncertainty in wireless networks using the formalism of graph entropy has received interest in the spatial networks community. In this paper, we develop lower bounds on the entropy of a wireless network by conditioning on potential network observables. Two approaches are considered: 1) conditioning on subgraphs, and 2) conditioning on node positions. The first approach is shown to yield a relatively tight bound on the network entropy. The second yields a loose bound, in general, but it provides insight into the dependence between node positions (modelled using a homogenous binomial point process in this work) and the network topology.**

***Index Terms*—Graph entropy, random geometric graphs, conditional entropy, network topology.**

## I. Introduction

Uncertainty is pervasive in modern wireless networks. The sources of this uncertainty range from the humans that interact with the networks and the locations of the nodes in space down to the transmission protocols and the underlying scattering processes that affect signal propagation. Over the past decade, tremendous progress has been made towards characterizing how uncertain device locations (transmit/receive pairs as well as interferers) and random propagation conditions affect the distribution of pairwise node connectivity [1], [2]. However, little has been done to develop an understanding of how spatial randomness influences uncertainty in the topological sense.

The topological structure of networks has been studied for many years in various scientific contexts through the lens of graph entropy [3]. This formalism is deeply rooted in statistical physics and information theory, and it allows one to quantitatively characterize the uncertainty or inherent information content of systems that can be described by a graphical model [4]–[9]. Applications of entropy-based methods to the study of networked systems are abundant and include problems related to molecular structure classification [10], social networks [11], [12], data compression [13], and quantum entanglement [14], [15]. With regard to communication networks, graph entropy has been used to quantify node and route stability [16] with the aim of improving link prediction [17] and routing protocols [18], [19]. Topological uncertainty in dynamic mobile ad hoc networks was investigated in [20] from a network layer perspective, and [21] treated self-organization in networks using a basic graph entropy framework. Crucially, those investigations did not facilitate quantitative analysis of wireless systems that experience fading and interference, nor did they account for the spatial embedding of the network.

Very recently, an analytical approach for studying topological uncertainty in wireless networks was proposed in [22]–[24]. This initial work was focused on analyzing network entropy through an upper bound based on the assumption that pairwise connections between devices are statistically independent. In this work, we leverage two different conditioning strategies to obtain lower bounds on network entropy. To this end, we offer the following contributions to the body of knowledge on the subject of wireless network entropy:

- we provide two tight lower bounds on network entropy by conditioning on subgraphs of the network;
- we present an exact analytical framework for studying the network entropy conditioned on the node positions in space;
- we further give a lower bound for the entropy conditioned on positions, which can be useful for estimating network entropy in practice.

The remainder of the paper is organized as follows. In the next section, we give details of the model and assumptions used in this study. In section III, we briefly review recent results on network entropy, before moving on to discuss conditional entropy in section IV. We then discuss the implications of the conditional entropy framework and provide conclusions in section V.

## II. Model and Assumptions

We model a wireless network as a random geometric graph (RGG) with a probabilistic pair connection rule [25], [26]. Consider a set $\mathcal{V}_n = \{1, \ldots, n\}$ of $n$ nodes that are randomly located in a space $\mathcal{K} \subset \mathbb{R}^d$ of finite volume and diameter $D \coloneqq \sup_{u,v\in\mathcal{K}} \|u-v\|$. We assume that the locations $\{Z_i\}_{i\in\mathcal{V}_n}$ of the nodes are independently and uniformly distributed in $\mathcal{K}$. The existence of an (undirected) edge between nodes $i$ and $j$ depends on the Euclidean distance between the two nodes and is indicated by the binary random variable $X_{ij}$ being one. Specifically, given the node locations, the variables $\{X_{ij}\}$ are independent and, each edge $(i,j)$ exists with probability

$$\mathbb{P}(X_{ij} = 1 | z_i, z_j) = p(\|z_i - z_j\|), \tag{1}$$

where $p : [0, \infty) \to [0, 1]$ is the pair connection function. For example, in the hard disk model, $p(\cdot)$ is an indicator function that equals one when its argument is less than $r_0$ and zero otherwise, where $r_0$ denotes the maximum connection range. We define the binary vector $\boldsymbol{X}_n$ to include all the $n(n-1)/2$ edge variables, i.e., $\boldsymbol{X}_n = (X_{ij})_{i<j}$. The RGG $G_n \coloneqq G(\mathcal{V}_n, \mathcal{E}_n)$ with edge set $\mathcal{E}_n = \{(i,j) \mid X_{ij} = 1\}$ is distributed in the set $\mathcal{G}_n$ of all $2^{n(n-1)/2}$ possible graphs.

## III. Network Entropy: A Primer

The topic of wireless network entropy is fairly new. Thus, a brief overview of the fundamental theory is provided in this section for the convenience of the reader and in order to provide context for the main contribution of the paper, which is detailed in section IV. For further details of recent results with application to wireless networks, the interested reader may wish to consult [22]–[24], [27].

Wireless network entropy is typically defined as the Shannon entropy of the RGG $G_n$, which models the network topology. We write the entropy as

$$H(G_n) = -\mathbb{E}[\log_2 \mathbb{P}(G_n)]. \tag{2}$$

Characterized in this way, network entropy can be interpreted as the minimum description length of the network topology (i.e., Kolmogorov complexity) or the logarithm of the number of *typical* networks [8][1]. The distribution of $G_n$ is determined by both the distribution of locations $\{Z_i\}_{i \in \mathcal{V}_n}$ and the probabilistic connection model specified by $p(\cdot)$. The graph $G_n$ is uniquely determined by $\boldsymbol{X}_n$, which has a multivariate Bernoulli distribution. Therefore, we require the pmf $f_{\boldsymbol{X}_n}(\boldsymbol{x}_n) \coloneqq \mathbb{P}(\boldsymbol{X}_n = \boldsymbol{x}_n)$, for each $\boldsymbol{x}_n \in \{0,1\}^{n(n-1)/2}$. The correspondence between $G_n$ and $\boldsymbol{X}_n$ suggests the more explicit formula

$$\begin{aligned} H(G_n) &= H(\boldsymbol{X}_n) \\ &= -\sum_{\boldsymbol{x}_n \in \{0,1\}^{n(n-1)/2}} f_{\boldsymbol{X}_n}(\boldsymbol{x}_n) \log_2 f_{\boldsymbol{X}_n}(\boldsymbol{x}_n). \end{aligned} \tag{3}$$

Since the conditional probability of edge existence depends on distance, it is more convenient to work with inter-node distances instead of node locations. Let $\boldsymbol{R}_n \coloneqq (R_{ij})_{i<j}$ denote the random vector collecting the pair distances $R_{ij} \coloneqq \|Z_i - Z_j\|$, and let $f_{\boldsymbol{R}_n} : [0,D]^{n(n-1)/2} \to [0,\infty)$ be its pdf[2]. We now write

$$\begin{aligned} &f_{\boldsymbol{X}_n}(\boldsymbol{x}_n) \\ &= \int_{\mathcal{R}} f_{\boldsymbol{R}_n}(\boldsymbol{r}_n) \prod_{\substack{i,j=1 \\ i<j}}^{n} p^{x_{ij}}(r_{ij}) \left[1 - p(r_{ij})\right]^{1-x_{ij}} \mathrm{d}r_{ij}. \end{aligned} \tag{4}$$

where the integration domain is $\mathcal{R} = [0,D]^{n(n-1)/2}$. The distribution of $\boldsymbol{X}_n$ is symmetric, since the node locations are identically distributed and the pair connection function is the same for all edges.

[1] In fact, this interpretation is intuitive, but only a conjecture in the case of spatial networks confined within a finite domain, such as wireless networks, where the lack of stationarity must be considered to develop a rigorous result. Such a study has not yet been reported in the literature.

[2] We consider a simple point process to model the node locations in this work. Hence, the pdf $f_{\boldsymbol{R}_n}$ exists.

According to (3) and (4), the calculation of the graph entropy requires the joint pdf of pair distances $f_{\boldsymbol{R}_n}$. Obtaining the joint density is very challenging for $n > 2$, and thus the entropy of $G_n$ cannot be calculated easily in general. To make progress, a simple independence assumption was invoked in [22] to develop a simple bound on entropy

$$H(G_n) \le \binom{n}{2} H(G_2) = \binom{n}{2} H_2(\overline{p}) \tag{5}$$

where

$$H_2(x) = -x \log_2 x - (1-x) \log_2 (1-x) \tag{6}$$

is the binary entropy function, and

$$\overline{p} = \mathbb{E}[p(\|Z_1 - Z_2\|)] \tag{7}$$

is the average probability that two nodes located at the random positions $Z_1$ and $Z_2$ are connected.

The upper bound (5) is obtained by assuming that $\{X_{ij}\}$ (or, equivalently, the inter-node distances $\{R_{ij}\}$) are independent. Clearly, this is not the case, and hence the bound is potentially loose. The recent work [27] obtains a series of tighter upper bounds on graph entropy by aiming to preserve the dependency between inter-node distances. To that end, the graph entropy is related to the entropy of graphs with smaller numbers of nodes, and it is established that the entropy of an RGG normalized by the number of potential edges decreases with the number of nodes, i.e.,

$$\frac{H(G_n)}{\binom{n}{2}} \le \frac{H(G_{n-1})}{\binom{n-1}{2}} \le \ldots \le \frac{H(G_3)}{3} \le H(G_2). \tag{8}$$

In [27], the joint pdf of inter-node distances is obtained for $n = 3$ nodes randomly located in a disk; this enables the evaluation of $H(G_3)$, which can be used as a tighter upper bound, i.e., $H(G_n) \le \binom{n}{2} \frac{1}{3} H(G_3) \le \binom{n}{2} H(G_2)$.

A large number of open problems remain on the topic of wireless network entropy. A discussion of these is included in section V for the reader's interest.

## IV. Conditional Entropy of Wireless Networks

While the fundamental study of RGG entropy, as described in the previous section, is a worthwhile pursuit, it is natural to assume that network designers may, in some cases, have knowledge of particular network properties *a priori*. This leads us to consider the notion of *conditional* RGG entropy. The sources of randomness in the networks under consideration are plentiful. Hence, we can extract different insights by conditioning on different network features. We explore two approaches: 1) conditioning on subgraphs and 2) conditioning on the node positions. The first approach yields useful lower bounds on the entropy of $G_n$, while the second provides information about the mutual information between the node locations and the network topology.

## A. Graph Entropy Conditioned on Subgraphs

The following scenario might be of interest. Let us imagine that information about the local topology (i.e., incident edges) of a subset of the nodes was available. For example, the network designer could interrogate a random subset of the nodes about which nodes they can connect to. In this case, how much uncertainty about the topology of the whole network is still left? The answer is given by the entropy conditioned on the knowledge about whether some of the $n(n-1)/2$ potential edges exist or not.

To be more specific, let us denote a subset of the binary edge variables by $\boldsymbol{X}_{\mathcal{S}} = (X_{ij})_{ij\in\mathcal{S}}$, where $\mathcal{S} \subset \{ij|i,j \in \mathcal{V}_n, i<j\}$; by $\boldsymbol{X}_{\bar{\mathcal{S}}}$ we denote the rest of the edge variables, i.e., those with indices in $\bar{\mathcal{S}}$, the complement of $\mathcal{S}$. Assuming $\boldsymbol{X}_{\bar{\mathcal{S}}}$ is observed, the relevant conditional entropy is given by

$$\begin{aligned} &H(\boldsymbol{X}_{\mathcal{S}}|\boldsymbol{X}_{\bar{\mathcal{S}}}) \\ &\quad = -\sum_{\boldsymbol{x}_n\in\{0,1\}^{n(n-1)/2}} f_{\boldsymbol{X}_n}(\boldsymbol{x}_n)\log_2 f_{\boldsymbol{X}_{\mathcal{S}}|\boldsymbol{X}_{\bar{\mathcal{S}}}}(\boldsymbol{x}_{\mathcal{S}}|\boldsymbol{x}_{\bar{\mathcal{S}}}). \end{aligned} \quad (9)$$

Since partial information reduces uncertainty, conditional entropy gives a lower bound on the graph entropy, i.e., $H(G_n) = H(\boldsymbol{X}_n) \geq H(\boldsymbol{X}_{\mathcal{S}}|\boldsymbol{X}_{\bar{\mathcal{S}}})$ for any choice of $\mathcal{S}$. This bound tends to be rather loose in general. In the following, we provide tighter lower bounds on $H(G_n)$ by applying generic inequalities for the joint entropy of a collection of random variables [28] and exploiting the symmetry of the system.

For illustration, we first consider the case when $n=3$. We consider a collection $\mathcal{C}$ of subsets of $\{ij|i,j\in\mathcal{V}_n, i<j\}$. Choosing $\mathcal{C} = \{\{12\},\{13\},\{23\}\}$ such that each pair index has multiplicity one with respect to $\mathcal{C}$, we have that [28]

$$H(\boldsymbol{X}_3) \geq \sum_{\mathcal{S}\in\mathcal{C}} H(\boldsymbol{X}_{\mathcal{S}}|\boldsymbol{X}_{\bar{\mathcal{S}}}) = 3H(X_{12}|X_{13},X_{23})$$

where the last equality is due to symmetry. The inequality is equivalent to

$$H(G_3) \geq 3H(G_2|\hat{G}_3), \quad (10)$$

where $\hat{G}_3$ stands for the "broken triangle" graph (has only two potential edges), i.e., $\hat{G}_3 := G(\mathcal{V}_3, \hat{\mathcal{E}}_3)$ with $\hat{\mathcal{E}}_3 := \{(i,j)\in\{(1,3),(2,3)\}|X_{ij}=1\}$. Similarly, by considering $\mathcal{C} = \{\{12,13\},\{12,23\},\{13,23\}\}$ (each pair index now has multiplicity two in $\mathcal{C}$), we obtain

$$H(G_3) \geq \frac{1}{2}\sum_{\mathcal{S}\in\mathcal{C}} H(\boldsymbol{X}_{\mathcal{S}}|\boldsymbol{X}_{\bar{\mathcal{S}}}) = \frac{3}{2}H(\hat{G}_3|G_2). \quad (11)$$

Using the joint pdf $f_{\boldsymbol{R}_3}$ calculated in [27], we can evaluate the pmf $f_{\boldsymbol{X}_3}$ given by (4), and then obtain the conditional probabilities required to compute the conditional entropies involved in the lower bounds (10) and (11). Fig. 1 shows the entropy of a three-node graph, the conditional entropies based on (9) and the two lower bounds (10) and (11) as functions of the maximum connection range. The hard disk connection model is assumed, i.e., $p(r)=1$ if $r\leq r_0$ and $p(r)=0$ otherwise.

We now generalize for any $n\geq 3$. We construct the collection $\mathcal{C}$ such that each set of pair indices included in $\mathcal{C}$ corresponds to one of the $\binom{n}{m}$ subgraphs determined by $m$ nodes, where $2\leq m<n$.

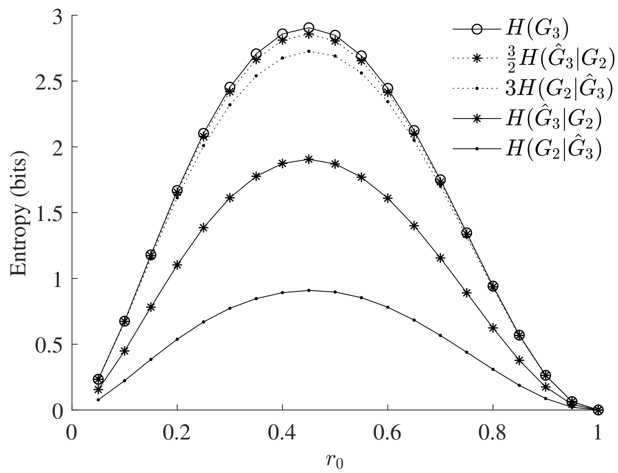


Fig. 1. Entropy of an RGG, conditional entropies, and lower bounds; the $n=3$ nodes are randomly located inside a circle with diameter one; the maximum connection range is $r_0$.

**Proposition 1:** Let $G_n$ be a random graph, where $n\geq 3$. The entropy of $G_n$ satisfies the inequality

$$H(G_n) \geq \frac{n(n-1)}{m(m-1)}H(G_m|G^+_{n-m}) \quad (12)$$

for $n>m\geq 2$, where $G^+_{n-m}$ represents the $(n-m)$-node complement of $G_m$ augmented by the edges that bridge the two subgraphs[3].

We obtain a similar result for the alternative decomposition in which each set of pair indices included in $\mathcal{C}$ corresponds to one of the $\binom{n}{m}$ subgraphs determined by $m$ nodes augmented by the bridging links to the remaining $n-m$ nodes.

**Proposition 2:** Let $G_n$ be a random graph, where $n\geq 3$. The entropy of $G_n$ satisfies the inequality

$$H(G_n) \geq \frac{n(n-1)}{m(2n-m-1)}H(G^+_m|G_{n-m}) \quad (13)$$

for $n>m\geq 1$.

The proof of each proposition follows from a combinatorial argument, application of the generic lower bound developed in [28], and the system's symmetry.

## B. Graph Entropy Conditioned on Node Positions

The upper bound of (5) provides some useful insight into how complexity scales in wireless networks as the number of devices increases or the physical parameters (e.g., transmit power) of the network change [22]–[24]. However, the bound is only relevant to the case where knowledge of the node positions

[3]Note that in (10) and (11), we used the notation $\hat{G}_3$ to denote the single-node complement of $G_2$ augmented by the bridging edges. Here, we choose the notation $G^+_{n-m}$ since it is slightly more general. Re-expressing the broken triangle graph in this more general notation gives $\hat{G}_3 \equiv G^+_1$.

is unavailable. If node positions were static (and known *a priori*), the task of quantifying the network entropy would be made easier, since all edges would be conditionally independent[4]. Indeed, the conditional entropy of general spatial networks was investigated under this assumption in [29]. On the other hand, it is more practical in wireless applications to assume that node locations may be static for a period of time before they change, thus altering the connectivity of the network. Such an assumption gives rise to a different formulation of conditional network entropy to that presented in [29], one that is more familiar to information and communication theorists.

Specifically, let $\Phi_n \subset \mathcal{K}$ denote a homogeneous binomial point process that describes the node positions in $\mathcal{K}$, i.e., there is a one-to-one correspondence between each point in $\Phi_n$ and each node in $\mathcal{V}_n$. We formulate the conditional entropy of the network topology given the node positions as

$$H(G_n|\Phi_n) = H(G_n|Z_1, \ldots, Z_n) \tag{14}$$

where the expectation is taken with respect to the node positions $\{Z_i\}$. Since connectivity in a homogeneous medium depends only on the distance between the nodes, we can express the conditional entropy in the following more convenient form:

$$H(G_n|\Phi_n) = H(G_n|\boldsymbol{R}_n). \tag{15}$$

The edge states in the network are statistically independent given the pairwise distances $\{R_{ij}\}$. Hence, (15) simplifies to

$$H(G_n|\Phi_n) = \sum_{i<j} H(X_{ij}|R_{ij}) = \binom{n}{2}\mathbb{E}[H_2(p(R))]. \tag{16}$$

Eq. (16) is a functional of the pair distance pdf $f_R(\cdot)$ and the pair connection function $p(\cdot)$. Analytic expressions for $f_R(\cdot)$ are known for spherically symmetric geometries [30], regular polygons [31], and various other elementary domains. An appropriate choice for the connection function follows from the application under consideration. Note, however, that choosing $p(\cdot)$ according to a hard connection model, as was done in the previous section, will lead to $H(G_n|\Phi_n) = 0$, since node locations exactly describe connectivity in this scenario. Instead, we might choose the canonical wireless connection function given by

$$p(r) = \mathsf{e}^{-(r/r_0)^\eta} \tag{17}$$

where, in this context, $r_0$ signifies the *typical* connection range (rather than the *maximum*), which encompasses physical system characteristics, such as the transmit power, wavelength, and the noise figure [22], [23], [25]. The parameter $\eta$ is the path loss exponent in this model, and thus it typically takes on values in the range $2 \leq \eta \leq 5$; mathematically, it simply controls the stretch of the decaying exponential, and by letting $\eta \to \infty$, we recover the hard connection model. In Fig. 2, the conditional entropy of a ten-node RGG is plotted for circular and square bounding domains of unit area for the cases where $\eta = 2, 3, 4$. The reduction in entropy for increasing "hardness"

[4]The underlying assumption here is that the fading processes that give rise to uncertainty would be independent for each link.

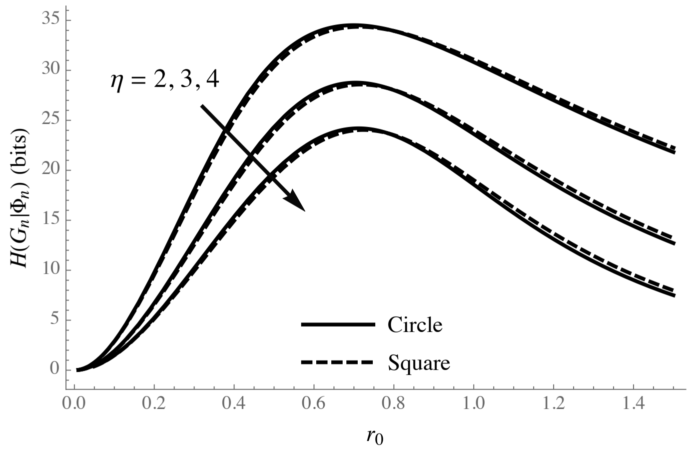


Fig. 2. Conditional entropy of a ten-node RGG with canonical wireless pair connection function in two dimensions; confining geometries: circle of radius $1/\sqrt{\pi}$ and square of unit side length; path loss exponent values are $\eta = 2, 3, 4$; maximum possible entropy is 45 bits.

in the connection function is apparent in the figure. Also, it is observed that the exact geometry of the bounding region does not significantly affect the entropy in this two-dimensional example.

In general, it may be useful to estimate the conditional network entropy using only simple statistics of $p(\cdot)$. To this end, we present the following result that gives a lower bound on $H(G_n|\Phi_n)$.

**Proposition 3:** Define the $q$th moment of the pair connection function as

$$\mu_q = \int_0^D f_R(r)p(r)^q \,\mathrm{d}r, \qquad q \geq 0 \tag{18}$$

where $f_R(r)$ is the pair distance pdf for two points in $\Phi_n$. The entropy of the RGG $G_n$ conditioned on $\Phi_n$ satisfies

$$H(G_n|\Phi_n) \geq 2n(n-1)(\mu_1 - \mu_2) \geq 0. \tag{19}$$

*Proof:* See the appendix. □

Fig. 3 illustrates the bound of Proposition 3 for the canonical wireless pair connection function and $\eta = 2, 3, 4$. The bound is observed to be a relatively good approximation to the actual entropy.

## V. Discussion and Conclusions

The two approaches to characterizing network entropy detailed above yield interesting observations and conclusions. Here, we attempt to summarize some of these and to provide motivation for developing this interesting field of spatial network research further.

Conditioning on subgraphs appears to give tight bounds on the network entropy $H(G_n)$, as illustrated in section IV-B. Unfortunately, it is difficult to calculate these bounds since they rely on knowledge of the joint pdf $f_{\boldsymbol{R}_n}(\cdot)$, about which little is known in general. Nevertheless, Propositions 1 and 2 provide an important first step to uncovering a more tractable and scalable way to estimate the network entropy.

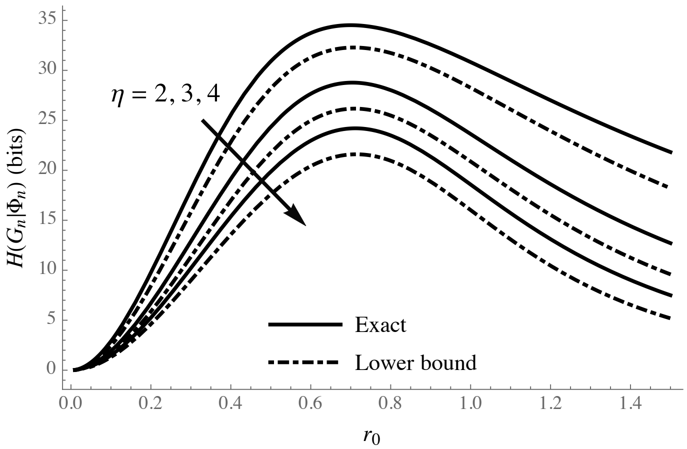


Fig. 3. Conditional entropy of a ten-node RGG with canonical wireless pair connection function in a circle of radius $1/\sqrt{\pi}$; the lower bound of Proposition 3 is shown; path loss exponent values are $\eta = 2, 3, 4$.

In contrast, conditioning on node positions does not necessarily yield a tight bound on the entropy $H(G_n)$ in general, since relatively hard connection rules (e.g., large $\eta$ in (17)) lead to small conditional entropy. Yet, we are able to derive scaling results through this formalism when the connection rule is probabilistic. Conditioning on $\Phi_n$ clearly illustrates the $\mathcal{O}(n^2)$ scaling behavior of network entropy[5], since

$$\binom{n}{2}\mathbb{E}[H_2(p(R))] = H(G_n|\Phi_n) \le H(G_n) \le \binom{n}{2}H_2(\overline{p}). \tag{20}$$

Pair connection functions that are relatively constant over the interval $(0, D)$ lead to fairly tight bounds. Intuitively, one can explain this behavior by observing that a constant connection function implies that connectivity does not depend on the spatial embedding, and hence $H(G_n|\Phi_n) = H(G_n)$.

A number of further open questions exist. For example, (how) can we calculate or simulate entropy efficiently in large networks? Can this theory be generalized to other entropy measures (e.g., Rényi entropy), and what purpose would this generalization serve in a practical context? Can we characterize the dynamics of a temporal network by calculating the entropy rate? Can we use entropy to develop a cost or utility in order to enhance network performance? We hope the contributions given in this paper will motivate some readers to search for answers to some of these questions in their own fields of study.

## Appendix

### Proof of Proposition 3

The proof relies on the following lemma.

**Lemma 1:** The binary entropy function is lower bounded by

$$H_2(x) \ge 4x(1-x), \qquad 0 \le x \le 1. \tag{21}$$

*Proof:* It can be shown that $h(x) = H_2(x)/(x(1-x))$ is a decreasing function of $x$ in the interval $(0, 1/2)$. Hence, $h(x) \ge 4$ on the interval. The result follows from the symmetry of $h(x)$ for $x \in (0, 1)$. □

[5]This scaling can also be conjectured from Proposition 1 by letting $m = 2$.

Now we have

$$\begin{aligned}\mathbb{E}[H_2(p(R))] &= \int_0^D f_R(r)H_2(p(r))\,\mathrm{d}r \\ &\ge 4\int_0^D f_R(r)p(r)(1-p(r))\,\mathrm{d}r \\ &= 4(\mu_1 - \mu_2).\end{aligned} \tag{22}$$

The inequalities stated in the proposition follow from (16) and by recognizing that $\mu_a \ge \mu_b$ for $0 \le a \le b$, with equality occurring when $p(\cdot)$ models the hard connection rule.

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