Remote Tracking of Distributed Dynamic Sources over A Random Access Channel with One-bit Updates

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Abstract—In this work, we consider a network, where n distributed information sources whose states evolve according to a random process transmit their time-varying states to a remote estimator over a shared wireless channel. Each source generates packets in a decentralized manner and employs a slotted random access mechanism to transmit the packets. In particular, we are interested in networks with a large number of low-complexity devices that share low-capacity random access channels. Accordingly, we investigate update strategies for remote tracking of source states that require each update to constitute as few bits as possible. To that end, we develop update strategies requiring only one-bit of information per update. We first consider a natural benchmark update policy and reveal that the benchmark policy cannot guarantee stability under all conditions. We then introduce an improvement of the benchmark policy that employs a local cancellation strategy, which makes the system always stable. We further analyze and optimize the performance of the cancellation-enabled update policy to bound the estimation error at the receiver. Through simulations, we compare the proposed cancellation-enabled one-bit update policy with zero-wait sampling and threshold-based sampling policies that require more than one-bit of information per update. The comparisons show that the cancellation-enabled update policy at its optimal threshold level outperforms the multi-bit update policies. This suggests that the cancellation-enabled one-bit update policy could be greatly beneficial for applications where transmission power or shared channel capacity are limited.

I. INTRODUCTION

The Internet of Things (IoT) has attracted significant attention resulting in an ever growing number of applications such as traffic monitoring and healthcare monitoring systems [1]. In such systems, where distributed IoT devices/sensors are connected to a remote monitor/controller, the sensors send update packets with time-varying (sensing) information to the monitor so that the monitor can track the state of the monitoring objects. To this end, it is crucial to send timely updates to keep the monitor maintaining fresh information. The timely updates can be challenging in an IoT network where many IoT devices are communicating over a shared channel. This paper tackles this problem by developing strategies that require

each update to constitute as few bits as possible so that a large device population can be served.

Age of Information (AoI) has been introduced and studied to measure the freshness of information [2]–[6], which is defined as the time that has elapsed since the latest packet received at a remote monitor (or a receiver) was generated at a source. In [3], the authors investigate the cases when the zero-wait sampling is not age-optimal with a single source-receiver pair. Networks with multiple sources updating a common receiver over a shared wireless channel are considered in [4]–[6]. Centralized update policies with throughput constraints are studied in [4], and decentralized update policies employing a slotted random access with channel collision feedback are studied in [5]. In [6], a sleep-wake update policy when each source has a limited battery capacity is developed. None of these designs apply to our setting since they measure freshness only via age, whereas in our setting the freshness must be measured via the estimation error.

Remote estimation, where the freshness of information is measured with estimation error (i.e., error between the actual state at a source and the estimate at a receiver), has been studied for networks with a single source-receiver pair [7]–[13] and for networks with multiple sources [14]–[19]. In [7], optimal sampling policies for a Wiener process are developed to minimize the Mean Squared Error (MSE) with the frequency sampling constraints. This problem is also studied when a communication channel has random delay in [8] and it is shown that an optimal policy is a threshold-type. Optimal sampling policies for an Ornstein-Uhlenbeck (OU) process are investigated with a channel having random delay [9] and with average power constraint [10]. In [11]–[13], a source whose state x_t evolves as $x_{t+1} = ax_t + w_t$, where $a \in \mathbb{R}$ and w_t is an independent and identically distributed (i.i.d.) random variable, are considered. In [11], update policies to minimize the MSE subject to a sampling frequency constraint are investigated. In [12], [13], it is assumed that each update pays a communication cost and update policies to minimize estimation error plus communication costs.

In [14]–[17], a network where n sources updating a common receiver is considered and state of each source is modeled as a Linear Time Invariant (LTI) system with an independent zero-mean Gaussian noise. In [14], [15], time-based (centralized) scheduling policies at the receiver are investigated to minimize

the average estimation error covariance when at most one source can update the receiver at a time [14] or when at most m out of n sources can update the receiver at a time and the communication channel has a packet drop probability [15]. In [16], [17], decentralized scheduling policies are investigated, where each source's objective is to minimize its estimation error covariance at the receiver subject to transmission power constraint. This problem is modeled as a multi-player game, and a Nash equilibrium (NE) is found in [16]. In [15], a concept of correlated equilibrium (CE) where the estimation performance can be improved compared with NEs is introduced, and a strategy that achieves the performance at the CE is proposed. In [18], [19], a network with n independent source-receiver pairs communicating over a shared channel is considered. In [18], a centralized scheduling policy is proposed when each transmission incurs a communication cost to minimize the average MSE plus communication costs. In [19], a decentralized scheduling policy is investigated to minimize the transmission power subject to a lower bound constraint on the successful transmission probability.

In this work, we consider a network with n distributed sources updating a common receiver over a shared wireless channel and investigate decentralized update policies to minimize the estimation error. Update policies proposed in this work are different from those proposed in [14], [15], [18] in that our policies are decentralized and different from those proposed in [16], [17], [19] in terms of the objectives. Further, we are interested in networks with a large number of low-complexity devices that share low-capacity random access channels. Such a setting is becoming increasingly more important in massive IoT networks with increasing number of low-complexity devices being connected to the networks such as remote health monitoring or smart architecture. Accordingly, we investigate update policies (i.e., sampling and scheduling policies) that require each update to constitute as few bits as possible. Thus, it is unsuitable for the sampling policies proposed in [7]–[13] to be directly applied in this setting since those sampling policies do not carefully deal with the number of bits per sampling/transmission in the existence of transmission failures.

Our contributions can be summarized as follows.

- We formulate the remote tracking problem to minimize the estimation error with a large number of low-complexity devices updating a common receiver over a low-capacity random access channel when the state of each information source evolves according to a symmetric random walk.
- We develop update strategies that require one-bit of information per update as a case of particular interest. We first consider a natural benchmark update policy and reveal that the benchmark policy will not be able to make the system stable in terms of the estimation error under some conditions.
- We then introduce an improvement on the benchmark policy that employs a local cancellation strategy, which makes the system always stable. We further analyze

and optimize the performance of the cancellation-enabled update policy to bound the estimation error at the receiver.
- We suggest how the proposed one-bit update policy can be applied to more general source models.
- We compare the proposed one-bit update policy with zero-wait sampling and threshold-based sampling policies that require more than one-bit of information per update through simulations. Numerical results show that the proposed one-bit update policy outperforms the multi-bits update policies, which implies that the proposed one-bit update policy is more beneficial when we consider transmission power that is usually increasing as the packet size (i.e., the number of bits per update) increases.

The rest of paper is organized as follows. In Section II, we describe the system model and formulate the problem. In Section III, we develop and analyze update strategies that require only one-bit of information per update. In Section IV, we extend our results to more general source models. In Section V, we compare the proposed one-bit update policy with other update policies through simulations. In Section VI, we conclude our work.

II. SYSTEM MODEL AND PROBLEM FORMULATION

A. Network Model

We consider a fundamental scenario of n distributed information sources (e.g., sensors) whose states evolve according to a random process, and one remote estimator (e.g., sink or collector) that aims to remotely track the time-varying state of the sources over a shared wireless channel, as shown in Fig. 1. In this work, we are interested in developing strategies for remote tracking of source states that require one-bit of information per update as a particular interest, which will be explained in Section II-B.
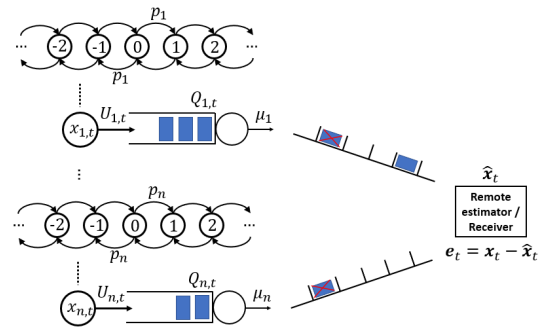

Fig. 1: System model.

Considering a time-slotted system operation, we let $x_{i,t}$ denote the state of source i at the beginning of time t, which evolves over integer values according to a simple random walk. In particular, $x_{i,t}$ evolves as

$$x_{i,t+1} = x_{i,t} + w_{i,t}, \text{ for } t \geq 0, \tag{1}$$

where $w_{i,t}$ is given by

$$
w_{i,t} = \begin{cases} 1, & \text{with probability } p_i, \\ 0, & \text{with probability } 1 - 2p_i, \\ -1, & \text{with probability } p_i, \end{cases} \quad (2)
$$

for some $p_i \in [0, 0.5]$. The transition probability p_i is known to each source. Note that the noise $w_{i,t}$ is independent and identically distributed (i.i.d.) with a zero-mean and finite variance, and that it is symmetric, i.e., $\mathbb{P}(w_{i,t} = 1) = \mathbb{P}(w_{i,t} = -1)$. We note that such a basic evolution lies at the foundation of many important estimation and control mechanisms. By varying the p_i parameter, this process can capture more and less variable source evolution. After developing our results for this model, we will also discuss more general state evolution in Section IV.

Let $U_{i,t} \in \{0,1\}$ denote the packet generation (or sampling) decision of source i at time slot t, where $U_{i,t} = 1$ implies that source i generates a new packet at time slot t. At the end of time slot $t - 1$, the packet generation decision $U_{i,t}$ is made in a decentralized manner by each source based on their own observations up to time slot $t - 1$. Each source maintains a First-Come First-Served (FCFS) queue, and the new generated packet is stored in the queue. The queue length of source i at time slot t is denoted by $Q_{i,t}$.

In view of the low-complexity nature of communication capabilities of these devices, we assume a slotted random access channel for wireless updates whereby if more than one sources transmit packets simultaneously, then all the transmissions fail due to a packet collision. Let $Z_{i,t} \in \{0,1\}$ denote the indicator variable for successful transmission of source i at time slot t. The source i transmits the packet with probability $\mu_i \in (0,1]$ (which is to-be-determined), and idles with probability $1 - \mu_i$. We assume that if queue i is empty (i.e., $Q_{i,t} = 0$) then source i transmits a dummy packet[1]. Then, we have

$$
\mathbb{E}[Z_{i,t}] = \mu_i \prod_{j \neq i} (1 - \mu_j). \quad (3)
$$

If source i is the only source transmitting a packet at time slot t, then the packet is successfully transmitted to the estimator (i.e., $Z_{i,t} = 1$). We assume that the communication channel is error-free and each transmission is done within a time slot.

Let $\hat{x}_{i,t}$ denote the estimated state of source i at the estimator at time slot t, which can be updated using information received by time slot t. Let $e_{i,t}$ denote the information mismatch (or error) between $x_{i,t}$ and $\hat{x}_{i,t}$, i.e., $e_{i,t} = x_{i,t} - \hat{x}_{i,t}$. We assume that $x_{i,0} = \hat{x}_{i,0}$ for all $i \in \{1, ..., n\}$.

B. One-Bit Update Policy at the Sources

In this work, we consider a low-overhead sampling policy, whereby each update constitutes *one-bit* of information so that the shared channel load is minimized for each transmission. This is especially important for wireless channels that serve

[1]This assumption makes the mathematical analysis more tractable. In practical operation, letting source i idle when it has no packet to send can give more transmission opportunity to the other sources and improve the system performance

a large population, as expected in future IoT networks. This motivates us to consider a threshold-type packet generation policy, whereby $\Delta_i \in \mathbb{N}$ denotes the (state) threshold used for sampling. To describe this policy more explicitly, let $\tilde{e}_{i,t}$ denote the virtual error of source i, which is a variable being held by each source i and is updated as

$$
\tilde{e}_{i,t+1} = \begin{cases} 0, & \text{if } U_{i,t} = 1, \\ \tilde{e}_{i,t} + w_{i,t}, & \text{if } U_{i,t} = 0. \end{cases} \quad (4)
$$

Here, the packet generation decision $U_{i,t}$ under the above threshold-base policy at time slot t is given by

$$
U_{i,t} = \begin{cases} 1, & \text{if } |\tilde{e}_{i,t} + w_{i,t}| = \Delta_i, \\ 0, & \text{otherwise.} \end{cases} \quad (5)
$$

That is, when $\tilde{e}_{i,t} + w_{i,t}$ hits the threshold Δ_i or $-\Delta_i$, a packet with one-bit information is generated with the value $+1$ for Δ_i or -1 for $-\Delta_i$, and the value $\tilde{e}_{i,t+1}$ is reset to 0. Fig. 2 shows a trajectory of virtual error $\tilde{e}_{i,t}$, where a new packet with the value $+1$ is generated at time slot τ.
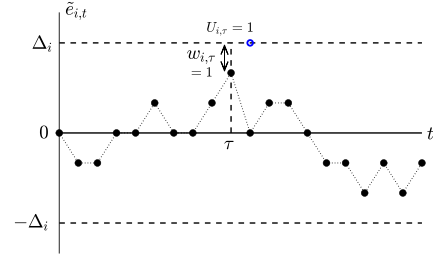

Fig. 2: A trajectory of virtual error $\tilde{e}_{i,t}$ source i.

Next, we provide a few interesting facts about the absolute estimation error performance of such a threshold-based one-bit update rule. These are interesting in explicitly characterizing how the error relate to the threshold level Δ_i and the source dynamics p_i.

Theorem 2.1: *Under the threshold-based one-bit update policy with threshold Δ_i, the long-term expectation of virtual error $\tilde{e}_{i,t}$ of source i is given by*

$$
\mathbb{E}[|\tilde{e}_{i,\infty}|] = \frac{\Delta_i^2 - 1}{3\Delta_i}. \quad (6)
$$

Further, the long-term expectation of update decision $U_{i,t}$ of source i is given by

$$
\mathbb{E}[U_{i,\infty}] = \frac{2p_i}{\Delta_i^2}. \quad (7)
$$

Proof: The virtual error $\tilde{e}_{i,t}$ is a finite-state Markov chain with $2\Delta_i - 1$ states from (4) and (5). Thus by solving global balance equations, we can obtain its stationary distribution $\pi_k^{\Delta_i} = \frac{\Delta_i - |k|}{\Delta_i^2}$ for $k \in \{-\Delta_i + 1, ..., 0, ..., \Delta_i - 1\}$, from which we can obtain the long-term expected virtual error $\mathbb{E}[|\tilde{e}_{i,\infty}|]$:

$$
\mathbb{E}[|\tilde{e}_{i,\infty}|] = \frac{2}{\Delta_i^2} \sum_{k=1}^{\Delta_i - 1} k(\Delta_i - k) = \frac{\Delta_i^2 - 1}{3\Delta_i}. \quad (8)
$$

Further, since each source independently generates a packet, we can consider $\tilde{e}_{i,t}$ as an independent renewal process, which is reset to 0 upon every packet generation. In [20], it is shown that

$$\mathbb{E}[U_{i,\infty}] = \lim_{t \to \infty} \mathbb{P}(U_{i,t} = 1) = \frac{2p_i}{\Delta_i^2} \qquad (9)$$

using Blackwell's renewal theorem (Theorem 4.6.2 in [21]). ∎

C. Estimation at the Receiver

Now that we described the policy at the sources, we turn to the corresponding estimation process at the receiver. We denote $V_{i,t}^k \in \{-1, 1\}$ for $k \in \{1, ..., Q_{i,t}\}$ as the value of k-th packet in queue i at time slot t, where 1 is the index for the head of the queue. If $Z_{i,t} = 1$, then the packet with value $V_{i,t}^1$ is successfully sent to the receiver and $V_{i,t+1}^k = V_{i,t}^{k+1}$. Then, at the receiver, the estimate $\hat{x}_{i,t}$ is updated as

$$\hat{x}_{i,t+1} = \hat{x}_{i,t} + V_{i,t}^1 Z_{i,t} \Delta_i. \qquad (10)$$

That is, when a new packet is received from source i, the estimated $\hat{x}_{i,t}$ is either increased by Δ_i if the received information is 1, or decreased by Δ_i if -1 is received. Then, we have, with $e_{i,0} = \tilde{e}_{i,0} = 0$, that

$$e_{i,t} = \tilde{e}_{i,t} + \Delta_i \sum_{k=1}^{Q_{i,t}} V_{i,t}^k. \qquad (11)$$

D. Distributed Remote-Estimation Problem

Given the one-bit update policy at the sources and the estimation policy at the receiver, the goal of the remote tracking problem is to optimize the choices of thresholds $\boldsymbol{\Delta} \triangleq \{\Delta_1, ..., \Delta_n\}$, and the probabilities $\boldsymbol{\mu} \triangleq \{\mu_1, ..., \mu_n\}$ for random access transmissions that minimize the mean absolute estimation error. Mathematically, our objective is to design $(\boldsymbol{\Delta}, \boldsymbol{\mu})$ given the source dynamics $\mathbf{p} \triangleq (p_1, ..., p_n)$ to minimize the expected average absolute-error over infinite time horizon:

$$\min_{\boldsymbol{\Delta}, \boldsymbol{\mu}} \ J(\boldsymbol{\Delta}, \boldsymbol{\mu}) = \lim_{t \to \infty} \frac{1}{tn} \sum_{s=1}^{t} \sum_{i=1}^{n} \mathbb{E}_\pi \left[|e_{i,s}| \right]. \qquad (12)$$

III. DESIGN AND ANALYSIS OF ONE-BIT UPDATE POLICIES FOR REMOTE ESTIMATION

In this section, we attack the problem formulated in the previous section by designing one-bit update policies for distributed remote tracking. At the outset, it is even unclear whether there exists a policy that can guarantee a bounded absolute estimation error. In fact, in Section III-A, we investigate a class of First-Come-First-Serve (FCFS) policies to find a condition on the (source-dynamics, threshold-level) pairs, $(\mathbf{p}, \boldsymbol{\Delta})$, that can be stabilized by such policies. The negative result from this design motivates us in Section III-B to propose an improved class of policies that employ a *cancellation* strategy within the transmission queues in order to guarantee stability for all possible source dynamics $\mathbf{p}$. Then, in Section III-D, we undertake the optimization of our cancellation-enabled design with performance guarantees.

A. Benchmark Analysis for First-Come First-Serve Updates

To develop a basic understanding of the system operation, let us consider the operation of the one-bit update and random-access service policy from the perspective of a given source i. Omitting the subscript i for notational simplicity, suppose that the source uses a threshold level of Δ and achieves a transmission success probability of μ in each transmission. The next theorem establishes a condition between Δ, p, and μ that would make the FCFS update policy unstable.

Theorem 3.1: *Under the threshold-based one-bit sampling and the First-Come First-Serve update policy, if $\Delta \leq \sqrt{\frac{2p}{\mu}}$, then the system is unstable, i.e.,*

$$\lim_{t \to \infty} \mathbb{E}[|e_\infty|] = \infty. \qquad (13)$$

This follows from the fact that, to make the system stable, the source has to make the queue stable and the condition for queue stability is that, in the long-term, the arrival rate must be less than the service rate, i.e., $\frac{2p}{\Delta^2} < \mu$. The detailed proof is in [22]. In the next section, we shall show that this deficiency can be eliminated through a cancellation mechanism within the transmission queue of each source.

B. One-Bit Update Policies with Packet Cancellation

The performance of FCFS update policy revealed that the estimation error will be unbounded if $\frac{2p}{\Delta} > \mu$. In this subsection, we introduce an improvement on these benchmark policies with substantial improvement. To that end, we first note that the dynamics of $x_{i,t}$ in (2) is symmetric, i.e., $\mathbb{P}(x_{i,t_0+t} = x \mid x_{i,t_0} = 0) = \mathbb{P}(x_{i,t_0+t} = -x \mid x_{i,t_0} = 0)$, due to symmetry of noise $w_{i,t}$. Using this symmetry of the dynamics, we can manipulate the FCFS queue if the information of packets in the queue can be accessed. If the values of the newly generated packet and the packet at the tail of the queue are the opposite, then those two packets *cancel* each other and are discarded from the queue before transmission. Let $D_{i,t} \in \{0, 1\}$ be the indicator variable for this event, where $D_{i,t} = 1$ indicates the packet cancellation occurs. Note that $\mathbb{E}[D_{i,t}] = \frac{1}{2}\mathbb{E}[U_{i,t}]\mathbb{P}\{Q_{i,t} > 0\}$ since $\mathbb{P}(x_{i,t} = \Delta_i \mid U_{i,t} = 1) = \mathbb{P}(x_{i,t} = -\Delta_i \mid U_{i,t} = 1) = \frac{1}{2}$ from symmetry of the dynamics of $x_{i,t}$.

Under this cancellation-enabled policy, the values of all the packets at the queue i must be the same at all times, i.e., $V_{i,t}^1 = \cdots = V_{i,t}^{Q_{i,t}}$. We assume that departure happens after arrival. Under this queueing discipline, the queue length $Q_{i,t}$ evolves as

$$Q_{i,t+1} = Q_{i,t} + U_{i,t} - 2D_{i,t} \qquad (14)$$
$$- Z_{i,t}[(1 - U_{i,t})\mathbb{I}\{Q_{i,t} > 0\} + U_{i,t}(1 - D_{i,t}\mathbb{I}\{Q_{i,t} = 1\})]$$

or we also can write

$$Q_{i,t+1} = [Q_{i,t} + U_{i,t} - Z_{i,t} - 2U_{i,t}D_{i,t}]^+, \qquad (15)$$

where $[\cdot]^+ = \max\{\cdot, 0\}$. Let $\lambda_{i,t} = \mathbb{E}[U_{i,t}] = \mathbb{P}(U_{i,t} = 1)$ denote the packet generation probability of source i at time t, which converges to $\lambda_i = \frac{2p_i}{\Delta_i^2}$ as $t \to \infty$ from Theorem 2.1.

C. Analysis of One-Bit Updates with Cancellation

In this subsection, we present fundamental results on the error performance of cancellation-enabled one-bit update policies that is introduced in the previous subsection. We start with the next lemma that establishes the strongly ergodic (non-stationary) nature of the transmission queue-length $\{Q_{i,t}\}_t$.

Lemma 3.1: *For each source i, the queue length process $\{Q_{i,t}\}_{t\geq 0}$ under the cancellation-enabled one-bit update policy described in (14) forms a strongly ergodic Markov Chain for any $\Delta_i > 0$, $\mu_i > 0$, and $p_i \in [0,1/2]$.*

The detailed proof is in [22]. In contrast to the FCFS policy performance (see Theorem 3.1), Lemma 3.1 proves that cancellation-enabled policy can stabilize the error level for any $\Delta_i > 0, \mu_i > 0$ and any feasible p_i. Specifically, it proves that there exists a unique steady-state distribution and the queue length process $\{Q_{i,t}\}_{t\geq 0}$ to the unique steady-state distribution.

Next, we note that, under the cancellation-enabled policy, we can write the error $e_{i,t}$ between the estimator and source i as

$$e_{i,t} = \tilde{e}_{i,t} + \Delta_i V^1_{i,t} Q_{i,t}. \tag{16}$$

Then, we can obtain bounds on the expected long-term absolute error $\mathbb{E}[|e_{i,\infty}|]$ using $\mathbb{E}[|\tilde{e}_{i,\infty}|]$ and $\mathbb{E}[Q_{i,\infty}]$ as follows.

Lemma 3.2: *Under the cancellation-enabled one-bit update policy with parameter $(\boldsymbol{\Delta}, \boldsymbol{\mu})$, we have*

$$\frac{\Delta_i}{2}\mathbb{E}[Q_{i,t}] + \mathbb{E}[|\tilde{e}_{i,t}|] \leq \mathbb{E}[|e_{i,t}|] \leq \Delta_i\mathbb{E}[Q_{i,t}] + \mathbb{E}[|\tilde{e}_{i,t}|] \tag{17}$$

for all $t \in \mathbb{N}$.

We obtain (17) from (16), with details provided in [22]. Building on this lemma, the next theorem bounds the long-term expected queue length $\mathbb{E}[Q_{i,\infty}]$ and absolute-error $\mathbb{E}[|e_{i,\infty}|]$ of source i under the cancellation-enabled one-bit update policy described in Section II-B with parameter $(\boldsymbol{\Delta}, \boldsymbol{\mu})$.

Theorem 3.2: *Under the cancellation-enabled one-bit update policy with parameter $(\boldsymbol{\Delta}, \boldsymbol{\mu})$, the long-term expected queue length of source $i \in \{1,...,n\}$ is bounded by*

$$\frac{p_i}{\Delta_i^2 M_i} \leq \mathbb{E}[Q_{i,\infty}] \leq \frac{p_i}{\Delta_i^2 M_i} + \frac{1}{2}, \tag{18}$$

where $M_i = \mu_i \prod_{j\neq i}(1-\mu_j)$. Further, the long-term expected absolute-error of source i is bounded by

$$\frac{p_i}{2\Delta_i M_i} + \frac{\Delta_i^2 - 1}{3\Delta_i} \leq \mathbb{E}[|e_{i,\infty}|] \leq \frac{p_i}{\Delta_i M_i} + \frac{5\Delta_i}{6} \tag{19}$$

for $i \in \{1,...,n\}$.

The lower and upper bounds for the long-term expected queue length $\mathbb{E}[Q_{i,\infty}]$ of source i can be obtained from Lyapunov analysis using (14) and (15), respectively. Further, we can obtain (19) from (17). The detailed proof is in [22].

From Theorem 3.2, we can observe that a remote tracking system is always stable under the cancellation-enabled one-bit update policy for any $(\boldsymbol{\Delta}, \boldsymbol{\mu})$ with strictly positive entries, i.e.,

$$\lim_{t\to\infty} \frac{1}{tn}\sum_{s=1}^{t}\sum_{i=1}^{n}\mathbb{E}[|e_{i,s}|] < \infty. \tag{20}$$

D. Optimum Design for One-Bit Updates with Cancellation

Now that we have shown that the system is always stable with packet cancellation, we next aim to select the design parameters $(\boldsymbol{\Delta}, \boldsymbol{\mu})$ to minimize the upper bound on (12):

$$\min_{\boldsymbol{\Delta},\boldsymbol{\mu}} \lim_{t\to\infty} \frac{1}{tn}\sum_{s=1}^{t}\sum_{i=1}^{n}\mathbb{E}[|e_{i,s}|] \leq \min_{\boldsymbol{\Delta},\boldsymbol{\mu}} f_u(\boldsymbol{\Delta}, \boldsymbol{\mu}), \tag{21}$$

where

$$f_u(\boldsymbol{\Delta}, \boldsymbol{\mu}) := \frac{1}{n}\sum_{i=1}^{n}\frac{p_i}{\Delta_i\mu_i\prod_{i\neq j}(1-\mu_j)} + \frac{5\Delta_i}{6}. \tag{22}$$

We undertake this optimization for homogeneous and heterogeneous sources separately.

1) Homogeneous sources: When the sources are homogeneous, i.e., $p_i = p$ for $i \in \{1,...,n\}$, we find the optimal values $\Delta_i = \Delta \in \mathbb{N}$ and $\mu_i = \mu \in (0,1]$. In this homogeneous source case, we can rewrite (22) as

$$f_u(\Delta, \mu) := \frac{p}{\Delta\mu(1-\mu)^{n-1}} + \frac{5\Delta}{6}. \tag{23}$$

The next theorem, provides the optimal (Δ^u, μ^u) that minimizes (23).

Theorem 3.3: *When the sources are homogeneous, the optimal threshold Δ^u and activation probability μ^u of $f_u(\Delta, \mu)$ are given by*

$$\mu^u = \frac{1}{n} \text{ and}$$
$$\Delta^u = \left\lfloor \sqrt{\frac{1.2p}{\mu^u(1-\mu^u)^{n-1}}} \right\rfloor \text{ or } \left\lceil \sqrt{\frac{1.2p}{\mu^u(1-\mu^u)^{n-1}}} \right\rceil. \tag{24}$$

Proof: We first ignore that Δ is integer-valued. Then, since $f(\Delta, \mu)$ is convex in (Δ, μ) for $\Delta > 0$ and $\mu \in (0,1)^2$, by solving $\frac{\partial f(\bar{\Delta},\mu^u)}{\partial \Delta} = 0$ and $\frac{\partial f(\bar{\Delta},\mu^u)}{\partial \mu} = 0$, we can obtained $\bar{\Delta}$ and μ^u as

$$\mu^u = \frac{1}{n} \text{ and } \bar{\Delta} = \sqrt{\frac{1.2p}{\mu^u(1-\mu^u)^{n-1}}}. \tag{25}$$

Now, since Δ is integer-valued, the optimal threshold Δ^u is given by (24) by comparing (23). ∎

2) Heterogeneous sources: Now, we consider remote tracking of heterogeneous sources. The next theorem provides the optimal $(\boldsymbol{\Delta}^u, \boldsymbol{\mu}^u)$ that minimizes (22).

Theorem 3.4: *When the sources are heterogeneous, the optimal thresholds Δ_i^u $f_u(\Delta, \mu)$ are given by*

$$\Delta_i^u = \left\lfloor \sqrt{\frac{1.2p_i}{\mu_i^u\prod_{j\neq i}(1-\mu_j^u)}} \right\rfloor \text{ or } \left\lceil \sqrt{\frac{1.2p_i}{\mu_i^u\prod_{j\neq i}(1-\mu_j^u)}} \right\rceil, \tag{26}$$

for all i, and the optimal activation probabilities $\boldsymbol{\mu}^u$ of $f_u(\Delta, \mu)$ are obtained by solving the convex problem:

$$\min_{\boldsymbol{\mu}\in\mathbb{R}^n} \sqrt{\frac{10p_i}{3\mu_i\prod_{j\neq i}(1-\mu_j)}}$$
$$\text{subject to } \sum_{i=1}^{n}\mu_i = 1; \tag{27}$$
$$\mu_i \in (0,1), \forall i \in \{1,...,n\}.$$

Proof: We first ignore that Δ_i is integer-valued for all i. Then, $f(\boldsymbol{\Delta}, \boldsymbol{\mu})$ is convex in $(\boldsymbol{\Delta}, \boldsymbol{\mu})$ since $\Delta_i > 0$ and $\mu_i \in$

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[2] $1/\prod_{i=1}^{n}x_i$ is convex on $\{(x_1,...,x_n) \in \mathbb{R}^n : x_i > 0, \forall i\}$.

$(0, 1)$. By solving $\frac{\partial f(\bar{\boldsymbol{\Delta}}, \boldsymbol{\mu}^u)}{\partial \Delta_i} = 0$ and $\frac{\partial f(\bar{\boldsymbol{\Delta}}, \boldsymbol{\mu}^u)}{\partial \mu_i} = 0$ for all i, we can obtain

$$\bar{\Delta}_i = \sqrt{\frac{1.2 p_i}{\mu_i^u \prod_{j \neq i}(1 - \mu_j^u)}}, \qquad (28)$$

$$\frac{1 - \mu_i^u}{\mu_i^u} \sqrt{\frac{1 - \mu_i^u}{\mu_i^u}} = \sum_{j \neq i} \sqrt{\frac{p_j}{p_i}} \sqrt{\frac{1 - \mu_j^u}{\mu_j^u}}. \qquad (29)$$

for all i and $\sum_{i=1}^n \mu_i^u = 1$. Although solving (29) is intractable, we have

$$f(\bar{\boldsymbol{\Delta}}, \boldsymbol{\mu}^u) = \sqrt{\frac{10 p_i}{3 \mu_i^u \prod_{j \neq i}(1 - \mu_j^u)}}, \qquad (30)$$

which is convex in $\boldsymbol{\mu}^u$ and thus we can numerically find $\boldsymbol{\mu}^u$ solving the convex optimization problem (27). Also, since Δ_i is integer-valued, the optimal thresholds $\boldsymbol{\Delta}^u$ are given by (26) by comparing (22). ∎

Combining (18) and (28), we have $\mathbb{E}[Q_{i,\infty}] \leq \frac{4}{3}$ for all i. That is, given some $\boldsymbol{\mu}$ (not necessarily optimal service rates), the behavior of the optimal sampling policy is to make the queue length close to 1 by increasing its threshold level Δ_i. As such, when source i makes a successful transmission, it can make the error $e_{i,t}$ close to $\tilde{e}_{i,t}$. Suppose there are two sources: one with 10 packets in its queue having values Δ and another with 1 packet in its queue having value 10Δ. After one successful transmission, the first source decreases its error by Δ while the second source decreases its error by 10Δ. However, if the threshold is too large, the source will lose their chance to update the receiver and have high $\tilde{e}_{i,t}$.

Remark 3.1: *Let $\boldsymbol{\mu}^u(\mathbf{p})$ denote the optimal service rates that minimize (22) given parameters $\mathbf{p} = \{p_i\}_i$. Let $p_i = \gamma_i \alpha$ and $q_i = \gamma_1 \beta$ for all i, where $0 < \alpha, \beta < 1/2 \max_i \gamma_i$. Then, we can notice that $\boldsymbol{\mu}^u(\mathbf{p}) = \boldsymbol{\mu}(\mathbf{q})$ from (29).*

The next theorem provides the asymptotic performance guarantee of the parameters $(\boldsymbol{\Delta}^u, \boldsymbol{\mu}^u)$ obtained in Theorem 3.4.

Theorem 3.5: *Under the cancellation-enabled one-bit update policy with parameters $(\boldsymbol{\Delta}^u, \boldsymbol{\mu}^u)$, we have*

$$\lim_{n \to \infty} \frac{J(\boldsymbol{\Delta}^u, \boldsymbol{\mu}^u)}{\min_{\boldsymbol{\Delta}, \boldsymbol{\mu}} J(\boldsymbol{\Delta}, \boldsymbol{\mu})} \leq \sqrt{5}. \qquad (31)$$

The detailed proof is in [22]. This theorem states that the parameters $(\boldsymbol{\Delta}^u, \boldsymbol{\mu}^u)$ obtained by optimizing the upper bound of the objective function will not be worse than $\sqrt{5}$ of optimal performance of the cancellation-enabled one-bit update policy. However, it will be shown that their performance gap is insignificant through simulations in Section V.

IV. EXTENSION TO MORE GENERAL SOURCE DYNAMICS

In this section, we provide extensions of our results to sources with more general dynamics. In particular, suppose that the state x_t of a source changes as

$$x_{t+1} = x_t + w_t, \text{ for } t \geq 0, \qquad (32)$$

where w_t is a noise with zero mean and finite variance σ^2, and a symmetric pdf. A new packet is generated (i.e., $U_t = 1$)

if $|\tilde{e}_t + w_t| \geq \Delta$ for $\Delta \in (0, \infty)$, and the virtual error $\tilde{e}_t$ is updated as

$$\tilde{e}_{t+1} = \tilde{e}_t + w_t - \Delta\mathbb{I}\{\tilde{e}_t \geq \Delta\} + \Delta\mathbb{I}\{\tilde{e}_t \leq -\Delta\}. \qquad (33)$$

Also, the source randomly accesses the channel with the successful transmission probability of $\mu \in (0, 1]$. Then, the next theorem provides the long-term expected absolute error performance under the cancellation-enabled one-bit update policy.

Theorem 4.1: *Under the cancellation-enabled one-bit update policy with parameter (Δ, μ) when a noise is a Gaussian random variable with zero mean and finite variance σ^2, we have*

$$\mathbb{E}[|e_\infty|] \leq \frac{\sigma^2 + \mathbb{P}(|\tilde{e}_\infty| \geq \Delta)\Delta^2}{2\Delta\mathbb{P}(|\tilde{e}_\infty| \geq \Delta)} + \frac{\Delta\mathbb{E}[U_\infty]}{2\mu} + \frac{\Delta}{2}. \qquad (34)$$

To prove this, we first show that the virtual error process $\tilde{e}_t$ with a Gaussian noise with zero mean and finite variance σ^2 forms a positive Harris recurrent Markov chain with a unique invariant distribution. If one can show that the virtual error process $\tilde{e}_t$ with an arbitrary symmetric noise with zero mean and finite variance σ^2 forms a positive Harris recurrent Markov chain with a unique invariant distribution, then Theorem 3.4 holds for the particular symmetric noise. The detailed proof is in [22].

Note, for $\Delta \in (0, \infty)$, that $\mathbb{P}(|\tilde{e}_\infty|) \geq \Delta$; otherwise, i.e., $\mathbb{P}(|\tilde{e}_\infty| < \Delta)$ with probability 1, the system is naturally stable with $\mathbb{E}[|e_\infty|] < \Delta$. Further, from Theorem 4.6.2 in [21], we have $\mathbb{E}[U_\infty] = \lim_{t \to \infty} \mathbb{P}(U_t = 1) = \frac{1}{\mathbb{E}[T]}$, where T is the packet generation period. Since $\mathbb{P}(|\tilde{e}| \geq \Delta) > 0$, we have $\mathbb{E}[T] \in [1, \infty)$. Thus, the upper bound in (34) is finite, which implies that the system is always stable for any $\sigma \in (0, \infty)$. Further, if we can analytically obtain the long-term probability $\mathbb{P}(|\tilde{e}_\infty| \geq \Delta)$ of packet generation and the long-term expected packet generation period $\mathbb{E}[U_\infty]$, then we can optimize the upper bound in (34) and have a sub-optimal update policy.

V. NUMERICAL RESULTS

In this section, we verify the performance of our threshold-based one-bit update policies. We first compare three different one-bit update policies: updates without packet cancellation proposed in Section III-A (denoted by No-pck-cancel), cancellation-enabled updates (denoted by Pck-cancel) proposed in Section III-B and threshold-based updates keeping queue length at most one (denoted by Q-at-most-one (1 bit)). Given (Δ, μ), the Q-at-most-one (1 bit) policy tries to generates a new packet after a successful transmission thus the queue being empty. If $\tilde{e}_t \geq \Delta$ (or $\leq -\Delta$), then a packet having Δ (or $-\Delta$) is generated and the virtual error $\tilde{e}_t$ decreases (or increases) by Δ, i.e., $\tilde{e}_{t+1} = \tilde{e}_t - \Delta$ (or $\tilde{e}_{t+1} = \tilde{e}_t + \Delta$). If $|\tilde{e}_t| < \Delta$, then it waits until $\tilde{e}_t$ hits the thresholds Δ or $-\Delta$.

We first consider remote tracking of a single source. The source has transition probability $p = 0.4$ and activation probability $\mu = 0.04$, and the simulations run for $T = 10^5$ time slots and are averaged over 20 repetitions. Fig. 3 shows the average absolute error of three different one-bit update
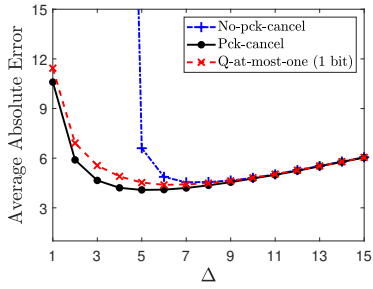
Fig. 3: Average absolute error of three different 1-bit update policies for a single source with different thresholds Δ given $p = 0.4$ and $\mu = 0.04$.
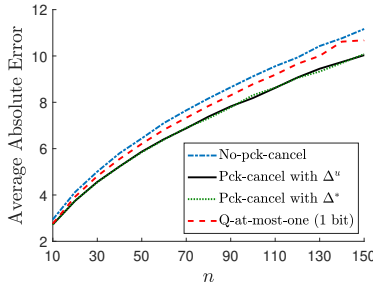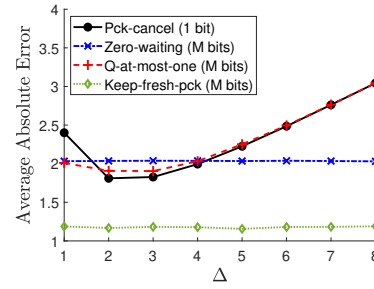

Fig. 5: Average absolute error of the 1-bit Pck-cancel policy and three different M bits update policies for a single source with different thresholds Δ given $p = 0.4$ and $\mu = 0.2$.

Fig. 4: Average absolute error of four different 1-bit update policies for homogeneous sources with different number n of sources given $p = 0.4$.
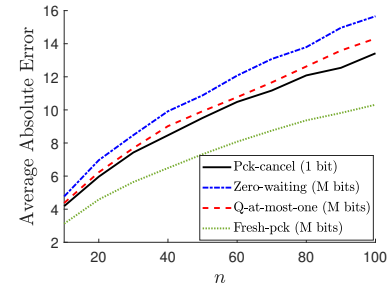

Fig. 6: Average absolute error of four different update policies for homogeneous sources with respect to the number n of sources given $p = 0.4$.

policies with respect to threshold Δ. For No-pck-cancel policy, the thresholds $\Delta > \sqrt{\frac{2p}{\mu}} \approx 4.4741$ is the stability condition as stated in Theorem 3.1, while the other two policies (Pck-cancel. and Q-at-most-one) make the system always stable. Further, Pck-cancel policy outperforms the other one-bit update policies for all Δ.

Next, consider remote tracking of multiple homogeneous sources with $p_i = p = 0.4$ for all i. Since the sources have the same dynamics, it is reasonable to set the activation probabilities $\mu = \frac{1}{n}$ for all the sources given n number of sources in the system. For the cancellation-enabled one-bit updates, we use two different thresholds: one is the threshold $\Delta^u = \left\lfloor \sqrt{\frac{1.2p}{\frac{1}{n}(1-\frac{1}{n})^{n-1}}} \right\rfloor$ or $\left\lceil \sqrt{\frac{1.2p}{\frac{1}{n}(1-\frac{1}{n})^{n-1}}} \right\rceil$, which is the optimal threshold of the upper bound f_u of the objective obtained in Theorem 3.3, and another one is the optimal threshold Δ^*, which is numerically found through exhaustive search. For No-pck-cancel and Q-at-most-one policies, the optimal thresholds Δ are also numerically found. The simulations run for $T = 10^5$ time slots and are averaged over 20 repetitions. Fig. 4 shows the average absolute error of four different one-bit update policies with respect to the number n of sources. We have showed that the optimality ratio of the Pck-cancel with threshold Δ^u is $\sqrt{5}$ for large n in Theorem 3.5. However, the Fig. 4 shows that the gap between the cancellation-enabled one-bit updates with thresholds Δ^u and Δ^* is unnoticeable, and Pck-cancel policies with Δ^* and Δ^U outperform the other

two update policies.

Next, we compare the cancellation-enabled one-bit update policy with three different multi-bits update policies: updates keeping the queue with the freshest packet (denoted by Keep-fresh-pck), zero-waiting updates (denoted by Zero-waiting) and threshold-based updates keeping queue length at most one (denoted by Q-at-most-one (M bits)). The Keep-fresh-pck policy generates a new packet at every time slot and replaces the packet in the queue with the new packet. Thus, when a source has a transmission opportunity, it always sends the freshest information to the receiver. The Zero-waiting policy generates a new packet with the actual state value after a successful transmission. The Q-at-most-one (M bits) policy is similar with the Q-at-most-one (1 bit) policy except that, if $\tilde{e}_t \geq \Delta$ (or $\leq -\Delta$), a packet having the actual value $\tilde{e}_t$ is generated and the virtual error $\tilde{e}_t$ becomes 0. If $|\tilde{e}_t| < \Delta$, then it waits until $\tilde{e}_t$ hits the thresholds Δ or $-\Delta$. Note that the Pck-cancel and Keep-fresh-pck policies are preemptive in the sense that it interrupts the packet in service by eliminating it from the queue or replacing it with a new packet while the Zero-waiting and Q-at-most-one policies are non-preemptive. Among non-preemptive policies, it is known that Zero-waiting is age-optimal and Q-at-most-one is error-optimal for a Wiener process [8] with a channel with random delay.

Fig. 5 shows the average absolute error of the four different update policies with respect to threshold Δ with a single source having transition probability $p = 0.4$ and activation

probability $\mu = 0.2$, and Fig. 6 shows the average absolute error with respect to the number n of homogeneous sources with $p = 0.4$. The simulations run for $T = 10^5$ time slots and are averaged over 20 repetitions. As can be seen in Figs. 5 and 6, the Pck-cancel policy at its optimal threshold level outperforms the multi-bits Zero-waiting and Q-at-most-one policies. In general, transmission time and power are increasing as the packet size (i.e., the number of bits for the state information) is increasing. This suggests that the cancellation-enabled one-bit update policy could be greatly beneficial for applications where transmission power or shared channel capacity are limited.

VI. CONCLUSION

Motivated by massive IoT network applications, we considered the scenario of a large number of low-complexity devices update their evolving state to a receiver over low-capacity random access channels. In particular, we developed decentralized update policies that require one-bit of information per update for minimizing the expected absolute (estimation) error when states of sources evolve according to symmetric random walks. We first studied a benchmark first-come first-serve (one-bit) update policy and showed that this policy will fail to stabilize the system under some conditions. Then, we introduced a cancellation-enabled one-bit update policy that improves the performance of the benchmark policy and makes the system always stable with appropriate threshold parameter selection. We analyzed and optimized the performance of the cancellation-enabled one-bit update policy to bound the estimation error, and obtained a closed-form expression of sub-optimal parameters of cancellation-enabled update policies for homogeneous sources. We showed that the cancellation-enabled policy with sub-optimal parameters has optimality ratio $\sqrt{5}$ to the optimal performance of the cancellation-enabled policy. Through simulations, we identified that the sub-optimal parameters are robust to errors compared with the optimal parameters obtained through exhaustive search, and compared the cancellation-enabled one-bit update policy with zero-wait sampling and threshold-based sampling policies that require more than one-bit of information per update. The comparison showed that the cancellation-enabled update policy at its optimal threshold level outperforms the multi-bits update policies. This suggests that the cancellation-enabled one-bit update policy could be greatly beneficial for applications where transmission power or shared channel capacity are limited.

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