

MAXIMUM HYPERGRAPHS WITHOUT REGULAR SUBGRAPHS

JAEHOON KIM¹

*Department of Mathematics, University of Illinois
Urbana, IL, 61801, USA*

e-mail: kim805@illinois.edu

AND

ALEXANDR V. KOSTOCHKA²

*University of Illinois at Urbana–Champaign,
Urbana, IL 61801, USA
Sobolev Institute of Mathematics
Novosibirsk 630090, Russia*

e-mail: kostochk@math.uiuc.edu

Abstract

We show that an n -vertex hypergraph with no r -regular subgraphs has at most $2^{n-1} + r - 2$ edges. We conjecture that if $n > r$, then every n -vertex hypergraph with no r -regular subgraphs having the maximum number of edges contains a full star, that is, 2^{n-1} distinct edges containing a given vertex. We prove this conjecture for $n \geq 425$. The condition that $n > r$ cannot be weakened.

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