

NOTE

A NOTE ON FACE COLORING ENTIRE WEIGHTINGS OF PLANE GRAPHS¹

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Abstract

Given a weighting of all elements of a 2-connected plane graph $G = (V, E, F)$, let $f(\alpha)$ denote the sum of the weights of the edges and vertices incident with the face α and also the weight of α . Such an entire weighting is a proper face colouring provided that $f(\alpha) \neq f(\beta)$ for every two faces α and β sharing an edge. We show that for every 2-connected plane graph there is a proper face-colouring entire weighting with weights 1 through 4. For some families we improved 4 to 3.

Keywords: entire weighting, plane graph, face colouring.

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