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THE NICHE GRAPHS OF INTERVAL ORDERS

Jeongmi Park

Department of Mathematics
Pusan National University
Busan 609-735, Korea

e-mail: jm1015@pusan.ac.kr

and

Yoshio Sano

Division of Information Engineering
Faculty of Engineering, Information and Systems
University of Tsukuba
Ibaraki 305-8573, Japan

e-mail: sano@cs.tsukuba.ac.jp

Abstract

The *niche graph* of a digraph D is the (simple undirected) graph which has the same vertex set as D and has an edge between two distinct vertices x and y if and only if $N_D^+(x) \cap N_D^+(y) \neq \emptyset$ or $N_D^-(x) \cap N_D^-(y) \neq \emptyset$, where $N_D^+(x)$ (resp. $N_D^-(x)$) is the set of out-neighbors (resp. in-neighbors) of x in D. A digraph $D = (V, A)$ is called a *semiorder* (or a *unit interval order*) if there exist a real-valued function $f : V \to \mathbb{R}$ on the set V and a positive real number $\delta \in \mathbb{R}$ such that $(x, y) \in A$ if and only if $f(x) > f(y) + \delta$. A digraph $D = (V, A)$ is called an *interval order* if there exists an assignment J of a closed real interval $J(x) \subset \mathbb{R}$ to each vertex $x \in V$ such that $(x, y) \in A$ if and only if $\min J(x) > \max J(y)$.

Kim and Roberts characterized the competition graphs of semiorders and interval orders in 2002, and Sano characterized the competition-common enemy graphs of semiorders and interval orders in 2010. In this note, we give characterizations of the niche graphs of semiorders and interval orders.

Keywords: competition graph, niche graph, semiorder, interval order.

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