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ON AN EXTREMAL PROBLEM IN THE CLASS OF BIPARTITE 1-PLANAR GRAPHS

Július Czap

Department of Applied Mathematics and Business Informatics
Faculty of Economics, Technical University of Košice
Němcovej 32, 040 01 Košice, Slovakia

e-mail: julius.czap@tuke.sk

Jakub Przybyło[1]

AGH University of Science and Technology
Faculty of Applied Mathematics
al. A. Mickiewicza 30, 30–059 Kraków, Poland

e-mail: jakubprz@agh.edu.pl

and

Erika Škrabuľáková[2]

Institute of Control and Informatization of Production Processes
Faculty of Mining, Ecology, Process Control and Geotechnology
Technical University of Košice, Košice, Slovakia

e-mail: erika.skrabulakova@tuke.sk

Abstract

A graph $G = (V, E)$ is called 1-planar if it admits a drawing in the plane such that each edge is crossed at most once. In this paper, we study bipartite 1-planar graphs with prescribed numbers of vertices in partite sets. Bipartite 1-planar graphs are known to have at most $3n - 8$ edges, where n denotes the order of a graph. We show that maximal-size bipartite 1-planar graphs which are almost balanced have not significantly fewer edges than indicated by this upper bound, while the same is not true for unbalanced

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ones. We prove that the maximal possible size of bipartite 1-planar graphs whose one partite set is much smaller than the other one tends towards $2n$ rather than $3n$. In particular, we prove that if the size of the smaller partite set is sublinear in n, then $|E| = (2 + o(1))n$, while the same is not true otherwise.

Keywords: 1-planar graph, bipartite graph, graph size.

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