

NORDHAUS-GADDUM-TYPE RESULTS FOR RESISTANCE DISTANCE-BASED GRAPH INVARIANTS

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Abstract

Two decades ago, resistance distance was introduced to characterize “chemical distance” in (molecular) graphs. In this paper, we consider three resistance distance-based graph invariants, namely, the Kirchhoff index, the additive degree-Kirchhoff index, and the multiplicative degree-Kirchhoff index. Some Nordhaus–Gaddum-type results for these three molecular structure descriptors are obtained. In addition, a relation between these Kirchhoffian indices is established.

Keywords: resistance distance, Kirchhoff index, additive degree-Kirchhoff index, multiplicative degree-Kirchhoff index, Nordhaus–Gaddum-type result.

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REFERENCES

- [1] D. Bonchev, A.T. Balaban, X. Liu and D.J. Klein, *Molecular cyclicity and centrality of polycyclic graphs. I. Cyclicity based on resistance distances or reciprocal distances*, Int. J. Quantum Chem. **50** (1994) 1–20.
doi:10.1002/qua.560500102
- [2] H. Chen and F. Zhang, *Resistance distance and the normalized Laplacian spectrum*, Discrete Appl. Math. **155** (2007) 654–661.
doi:10.1016/j.dam.2006.09.008
- [3] H. Chen and F. Zhang, *Resistance distance local rules*, J. Math. Chem. **44** (2008) 405–417.
doi:10.1007/s10910-007-9317-8
- [4] P. Dankelmann, H.C. Swart and P. van den Berg, *Diameter and inverse degree*, Discrete Math. **308** (2008) 670–673.
doi:10.1016/j.disc.2007.07.053
- [5] K.Ch. Das, I. Gutman and B. Zhou, *New upper bounds on Zagreb indices*, J. Math. Chem. **46** (2009) 514–521.
doi:10.1007/s10910-008-9475-3
- [6] R.M. Foster, *The average impedance of an electrical network*, in: Contributions to Applied Mechanics (Edwards Bros., Michigan, Ann Arbor, 1949) 333–340.
- [7] I. Gutman, L. Feng and G. Yu, *Degree resistance distance of unicyclic graphs*, Trans. Comb. **1** (2012) 27–40.
- [8] I. Gutman and B. Mohar, *The Quasi-Wiener and the Kirchhoff indices coincide*, J. Chem. Inf. Comput. Sci. **36** (1996) 982–985.
doi:10.1021/ci960007t
- [9] D.J. Klein, *Graph geometry, graph metrics, & Wiener*, MATCH Commun. Math. Comput. Chem. **35** (1997) 7–27.
- [10] D.J. Klein, *Centrality measure in graphs*, J. Math. Chem. **47** (2010) 1209–1223.
doi:10.1007/s10910-009-9635-0
- [11] D.J. Klein and O. Ivanciuc, *Graph cyclicity, excess conductance, and resistance deficit*, J. Math. Chem. **30** (2001) 271–287.
doi:10.1023/A:1015119609980
- [12] D.J. Klein and M. Randić, *Resistance distance*, J. Math. Chem. **12** (1993) 81–95.
doi:10.1007/BF01164627
- [13] D.J. Klein and H.-Y. Zhu, *Distances and volumina for graphs*, J. Math. Chem. **23** (1998) 179–195.
doi:10.1023/A:1019108905697
- [14] E.A. Nordhaus and J.W. Gaddum, *On complementary graphs*, Amer. Math. Monthly **63** (1956) 175–177.
doi:10.2307/2306658

- [15] J.L. Palacios and J.M. Renom, *Another look at the degree-Kirchhoff index*, Int. J. Quantum Chem. **111** (2011) 3453–3455.
doi:10.1002/qua.22725
- [16] H. Wiener, *Structural determination of paraffin boiling points*, J. Amer. Chem. Soc. **69** (1947) 17–20.
doi:10.1021/ja01193a005
- [17] W. Xiao and I. Gutman, *Resistance distance and Laplacian spectrum*, Theor. Chem. Acc. **110** (2003) 284–289.
doi:10.1007/s00214-003-0460-4
- [18] Y. Yang, *Relations between resistance distances of a graph and its complement or its contraction*, Croat. Chem. Acta **87** (2014) 61–68.
doi:10.5562/cca2318
- [19] Y. Yang, H. Zhang and D.J. Klein, *New Nordhaus-Gaddum-type results for the Kirchhoff index*, J. Math. Chem. **49** (2011) 1587–1598.
doi:10.1007/s10910-011-9845-0
- [20] B. Zhou and N. Trinajstić, *A note on Kirchhoff index*, Chem. Phys. Lett. **455** (2008) 120–123.
doi:10.1016/j.cplett.2008.02.060
- [21] B. Zhou and N. Trinajstić, *On resistance-distance and Kirchhoff index*, J. Math. Chem. **46** (2009) 283–289.
doi:10.1007/s10910-008-9459-3
- [22] H.-Y. Zhu, D.J. Klein and I. Lukovits, *Extensions of the Wiener number*, J. Chem. Inf. Comput. Sci. **36** (1996) 420–428.
doi:10.1021/ci950116s

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