

AN IMPROVED UPPER BOUND ON NEIGHBOR EXPANDED SUM DISTINGUISHING INDEX

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Abstract

A total k -weighting f of a graph G is an assignment of integers from the set $\{1, \dots, k\}$ to the vertices and edges of G . We say that f is neighbor expanded sum distinguishing, or NESD for short, if $\sum_{w \in N(v)} (f(vw) + f(w))$ differs from $\sum_{w \in N(u)} (f(uw) + f(w))$ for every two adjacent vertices v and u of G . The neighbor expanded sum distinguishing index of G , denoted by $\text{egndi}_{\Sigma}(G)$, is the minimum positive integer k for which there exists an NESD weighting of G . An NESD weighting was introduced and investigated by Flandrin *et al.* (2017), where they conjectured that $\text{egndi}_{\Sigma}(G) \leq 2$ for any graph G . They examined some special classes of graphs, while proving that $\text{egndi}_{\Sigma}(G) \leq \chi(G) + 1$. We improve this bound and show that $\text{egndi}_{\Sigma}(G) \leq 3$ for any graph G . We also show that the conjecture holds for all bipartite, 3-regular and 4-regular graphs.

Keywords: general edge coloring, total coloring, neighbor sum distinguishing index.

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