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ON THE n-PARTITE TOURNAMENTS WITH EXACTLY $n-m+1$ CYCLES OF LENGTH m

Qiaoping Guo and Wei Meng

School of Mathematical Sciences
Shanxi University, Taiyuan, 030006, China

e-mail: guoqp@sxu.edu.cn
mengwei@sxu.edu.cn

Abstract

Gutin and Rafiey [*Multipartite tournaments with small number of cycles*, Australas J. Combin. 34 (2006) 17–21] raised the following two problems: (1) Let $m \in \{3, 4, \ldots, n\}$. Find a characterization of strong n-partite tournaments having exactly $n-m+1$ cycles of length m; (2) Let $3 \leq m \leq n$ and $n \geq 4$. Are there strong n-partite tournaments, which are not themselves tournaments, with exactly $n-m+1$ cycles of length m for two values of m? In this paper, we discuss the strong n-partite tournaments D containing exactly $n-m+1$ cycles of length m for $4 \leq m \leq n-1$. We describe the substructure of such D satisfying a given condition and we also show that, under this condition, the second problem has a negative answer.

Keywords: multipartite tournaments, tournaments, cycles.

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