Network Planning and Routing Problems over Time: Models, Complexity and Algorithms

Lukas Glomb
Department of Data Science, Department Mathematik,
Friedrich-Alexander Universität Erlangen-Nürnberg, Germany

Benno Hoch
Department of Data Science, Department Mathematik,
Friedrich-Alexander Universität Erlangen-Nürnberg, Germany

Frauke Liers
Department of Data Science, Department Mathematik,
Friedrich-Alexander Universität Erlangen-Nürnberg, Germany

Florian Rösel
Department of Data Science, Department Mathematik,
Friedrich-Alexander Universität Erlangen-Nürnberg, Germany

Abstract

In this invited contribution for ESA 2021, we will study the complexity of and algorithms for network optimization tasks with a timing component. They occur, for example, in planning or routing problems that need to be solved repeatedly over time. Typically, already simplified versions of such problems are NP-hard. In addition, the instances typically are too large to be solved straight-forwardly on a time-expanded graph. After an introduction into the area, we state the problem of determining best possible non-stop trajectories in a network that are not allowed to cross at any point in time. For simplified settings, polynomial-time solution approaches are presented whereas already for restricted settings the problems are shown to be NP-hard. When moving to more complex and more realistic settings as they occur, for example, in determining non-stop disjoint trajectories for a set of aircraft, we present heuristic algorithms that adaptively refine coarse disjoint trajectories in the timing dimension. In order to be able to solve the non-stop disjoint trajectories problem over time, the method is integrated in a rolling-horizon algorithm. We present computational results for realistic settings. Motivated by the fact that rolling-horizon approaches are often applied in practice without knowledge on the quality of the obtained solutions, we study this problem from an abstract point of view. In fact, we more abstractly analyze the solution quality of general rolling-horizon algorithms for optimization tasks that show a timing component. We apply it to different planning problems. We end by pointing out some challenges and possibilities for future research.

2012 ACM Subject Classification Theory of computation; Mathematics of computing → Discrete mathematics

Keywords and phrases network problems over time, rolling-horizon, complexity, approximation

Digital Object Identifier 10.4230/LIPIcs.ESA.2021.1

Category Invited Talk

Related Version *Full Version*: `http://www.optimization-online.org/DB_HTML/2020/09/8015.html`
Full Version: `http://www.optimization-online.org/DB_HTML/2020/05/7809.html`

Funding The author acknowleges financial support by BMWi through the projects HOTRUN, FKZ 20E1720B, and OPs-TIMAL, FKZ 20X1711L.

29th Annual European Symposium on Algorithms (ESA 2021).
Editors: Petra Mutzel, Rasmus Pagh, and Grzegorz Herman; Article No. 1; pp. 1:1–1:3
Leibniz International Proceedings in Informatics
Schloss Dagstuhl – Leibniz-Zentrum für Informatik, Dagstuhl Publishing, Germany

1 Summary

In this contribution, we will introduce several network optimization tasks that contain a timing component. Such tasks often occur in planning or routing problems that need to be solved repeatedly over time. Already simplified such network optimization tasks are NP-hard. In addition, realistic instances are typically too large to be solved in an integrated fashion. On the other hand, straight-forward decomposition approaches typically lead to bad solution quality.

We first introduce the class of optimization problems that are considered here. We will then more concretely show complexity results for several settings together with algorithms and computational evaluations. We start by introducing the task of determining best possible non-stop trajectories in a network that are not allowed to cross at any point in time. For simplified settings, polynomial-time solution approaches are presented. Those settings include instances with unit traversal times on the line or grids and meshes, where all commodities have the same destination. For several only slightly more complicated settings the problems are shown to be NP-hard. For example, if time intervals to start the trajectories are given, the problem on the line is hard. On the mesh with arbitrary arc costs the problem is not fixed-parameter tractable in the number of commodities, even if traversal times are unit. More details can be found in [5].

We then move on to more complex and more realistic settings as they occur, for example, in determining non-stop disjoint trajectories for a set of aircraft [4, 7, 8]. We focus on the surroundings of airports where both the planning of conflict-free trajectories as well as the determination of runway schedules are crucial and challenging tasks. In current practice, the resulting continuous and discrete optimization problems are often solved sequentially. In this work, we develop an integrated optimization model for conflict-free multi-aircraft trajectory planning and runway scheduling. We use a space-time discretization and model conflict-free trajectories by an integer linear program that is designed to provide optimal, piecewise-linear reference trajectories and a runway scheduling for multiple aircraft. Even for moderately sized instances, a sufficiently detailed representation of 3D-airspace and time leads to huge models, which cannot be treated by current hard- and software. To overcome this issue, we develop an iterative adaptive-refinement algorithm. Starting from an optimal solution in a coarse discretization, the algorithm re-optimizes trajectories in a neighborhood of the current solution with a higher resolution. The method is integrated into a rolling-horizon approach. The latter repeatedly restricts trajectory determinations within a (sliding) time window. Computational results on realistic instances illustrate the computational efficiency of our approach.

The relevance of the rolling-horizon approach has been demonstrated by applying it to solve a large variety of practical optimization problems, e.g. as in [6] or [1]. Hence it is very interesting to know theoretical properties of the rolling-horizon approach, which have been investigated, e.g. in [2] or [9]. We complement these works in [3]. Hence, we analyze the solution quality of general rolling-horizon algorithms that are applied to multi-period optimization problems with a timing component.

We demonstrate that the solution quality of the standard rolling-horizon procedure can be arbitrarily low considering our general problem setting. We thus adapt the general rolling-horizon procedure such that statements on the quality of solutions obtained from adapted rolling-horizon algorithms can be made.

On the practical side, we present computational results on lot-sizing problems in production planning as well as on tail-assignment problems in aircraft management. The latter assigns available aircraft to specific flights in a best possible way. It can be shown that huge

instances can be solved quickly with an almost negligible loss in solution quality by only a few percent. We end by pointing out some challenges and possibilities for future research.

References

1 Bernardetta Addis, Giuliana Carello, Andrea Grosso, and Elena Tànfani. Operating room scheduling and rescheduling: a rolling horizon approach. *Flexible Services and Manufacturing Journal*, 28(1-2):206–232, 2016.

2 James C Bean and Robert L Smith. Conditions for the existence of planning horizons. *Mathematics of Operations Research*, 9(3):391–401, 1984.

3 Lukas Glomb, Frauke Liers, and Florian Rösel. A rolling-horizon approach for multi-period optimization. *European Journal of Operational Research*, 2021. `doi:10.1016/j.ejor.2021.07.043`.

4 Benedikt Grüter, Matthias Bittner, Matthias Rieck, Johannes Diepolder, and Florian Holzapfel. Optimal sequencing in atm combining genetic algorithms and gradient based methods to a bilevel approach. In *ICAS 30th International Congress of the International Council of the Aeronautical Sciences*, 2016.

5 Benno Hoch, Frauke Liers, Sarah Neumann, and Francisco Javier Zaragoza Martınez. The non-stop disjoint trajectories problem, 2020.

6 Mohammad Mohammadi, SMT Fatemi Ghomi, Behrooz Karimi, and S Ali Torabi. Rolling-horizon and fix-and-relax heuristics for the multi-product multi-level capacitated lotsizing problem with sequence-dependent setups. *Journal of Intelligent Manufacturing*, 21(4):501–510, 2010.

7 Arthur Richards and Jonathan P How. Aircraft trajectory planning with collision avoidance using mixed integer linear programming. In *American Control Conference, 2002. Proceedings of the 2002*, volume 3, pages 1936–1941. IEEE, 2002.

8 Johannes Schmidt and Armin Fügenschuh. A two-time-level model for mission and flight planning of an inhomogeneous fleet of unmanned aerial vehicles. Technical report, BTU Cottbus, 2021.

9 Suresh Sethi and Gerhard Sorger. A theory of rolling horizon decision making. *Annals of Operations Research*, 29(1):387–415, 1991.