Optimization at the Second Level

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──── **Abstract** ────

This report documents the program and the outcomes of Dagstuhl Seminar 22441 "Optimization at the Second Level". The seminar was held on October 30 – November 4, 2022 in Schloss Dagstuhl – Leibniz-Zentrum für Informatik. Participants gave overview talks and presented recent results in bilevel, robust, and stochastic optimization. These three areas have in common that they typically deal with optimization problems which are contained in the second level of the polynomial hierarchy. The goal of the seminar was to bring together experts of bilevel, robust, and stochastic optimization in order to connect and work towards new insights and approaches for such problems. During the seminar, the relationships between these different areas of optimization were intensively discussed and interesting connections were identified.

1 Executive Summary

Luce Brotcorne (INRIA Lille, FR)
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Topic of the Seminar

The second level of the polynomial hierarchy contains a variety of problems that allow natural simple formulations with one existential and one universal quantifier. For instance, a typical problem in robust optimization asks whether there EXISTS some production plan that performs reasonably well under ALL possible price scenarios for electricity in the coming two years. A typical problem in bilevel optimization asks whether there EXISTS a way of setting taxes so that ALL possible behaviors of the citizens generate a reasonable tax revenue. A typical problem in Stackelberg games asks whether there EXISTS a starting move for the first player that wins the game against ALL possible counter-moves of the second player.

Problems of this type are usually complete for the class Σ_2^p and hence are most likely not contained in the class NP. For that reason, the methodologies that have been developed for NP-complete problems over the last 50 years do not directly apply to robust and/or bilevel optimization problems. Up to the current moment, most of the work on such problems is purely computational and without any deeper theoretical understanding. Most approaches simply try to carry over the well-developed machinery from integer programming to concrete robust problems and bilevel problems. We will need to develop new techniques, new tricks, new insights, new algorithms, and new theorems to get a grip on this area.

The goal of this Dagstuhl Seminar was to bring together experts in theoretical computer science and experts in combinatorial optimization, and to work towards the following goals:

- summarize the status quo of robust optimization and bilevel optimization,
- identify central research lines on the computational and implementational front,
- identify central research lines in theoretical computer science, as for instance in parameterized complexity and approximability.

The list of participants perfectly reflected these goals, it included experts in complexity theory as well as researchers interested in the development of effective algorithms and the practical solution of real-world bilevel or robust optimization problems.

Implementation and Conclusions

In this seminar we brought together, for the first time, leading researchers from three different communities (robust optimization, stochastic optimization, and bilevel optimization) in order to bridge the gap between these fields from a theoretical and practical point of view.

Considering the different backgrounds of participants, we scheduled several talks with an introductory character in the first half of the week: Marc Goerigk and Frauke Liers gave overview talks on robust optimization, from a combinatorial and continuous perspective, respectively. Martine Labbé presented the state of the art of bilevel optimization, while Martin Schmidt combined both topics in his talk on bilevel optimization under uncertainty. Bernardo Pagnoncelli gave an introduction into the related topic of stochastic optimization. These presentations laid a common foundation for all further presentations and discussions and were thus a crucial prerequisite for the success of the seminar.

The contributed talks covered a wide range of topics, including complexity theoretic results for (certain or uncertain) bilevel optimization, new models and new methods for bilevel or robust optimization, and new approaches for solving bilevel or robust optimization problems arising in practice. Apart from the exciting contents of these talks, a particularly positive aspect was the large representation of young researchers. In fact, seven out of 18 contributed talks were given by PhD students.

The whole seminar was marked by a very open and constructive atmosphere and by an extraordinarily interactive approach: many presentations quickly turned into lively discussions involving many different participants, often making the original schedule obsolete, but with the benefit of a better common understanding and often new insights. One of the recurrent topics arising in many of these discussions was the connection between bilevel and robust optimization, the two main subjects of the seminar. The fact that both problem classes lead to potentially Σ_2^p-hard problems, as mentioned above, yields a connection of rather theoretical nature. From a more concrete point of view, the discussion was about whether one of the two problem classes can be seen (or modeled) as a special case of the other. Additionally, interesting links to game theory and stochastic optimization were identified.

Even though a conclusive answer to these questions could not be given (and probably does not exist), one of the main insights of the seminar was that bilevel and robust optimization, though being investigated in separate communities, share many structural and algorithmic

properties. It is worth studying these connections and sharing the knowledge of both communities in order to profit from one another. The Dagstuhl Seminar on "Optimization at the Second Level" was a first significant step into this direction, which is hopefully followed by further progress.

The seminar was a big success, it stimulated new and very fruitful collaborations. We got laudatory feedback from many participants who were already thinking of organizing another seminar on a the same topic in the future.

In memoriam

At this point, our thoughts go out to our late colleague and friend Gerhard J. Woeginger. It was his idea to organize a seminar on "Optimization at the Second Level", and without him the seminar would never have become real. Unfortunately, he could no longer witness how his idea was put into practice and how fruitful it turned out to be.

References

1   Gerhard J. Woeginger: "The trouble with the second quantifier", 4OR – A Quarterly Journal of Operations Research, Vol. 19(2), pp. 157–181, 2021.

2   Table of Contents

3   Overview of Talks

3.1   On a Computationally Ill-Behaved Bilevel Problem with a Continuous and Nonconvex Lower Level

Yasmine Beck (Universität Trier, DE)

It is well known that bilevel optimization problems are hard to solve both in theory and practice. In this talk, we highlight a further computational difficulty when it comes to solving bilevel problems with continuous but nonconvex lower levels. Even if the lower-level problem is solved to ε-feasibility regarding its nonlinear constraints for an arbitrarily small but positive ε, the obtained bilevel solution as well as its objective value may be arbitrarily far away from the actual bilevel solution and its actual objective value. This result even holds for bilevel problems for which the nonconvex lower level is uniquely solvable, for which the strict complementarity condition holds, for which the feasible set is convex, and for which Slater's constraint qualification is satisfied for all feasible upper-level decisions. Since the consideration of ε-feasibility cannot be avoided when solving nonconvex problems to global optimality, our result shows that computational bilevel optimization with continuous and nonconvex lower levels needs to be done with great care. Finally, we show that the nonlinearities in the lower level are the key reason for the observed bad behavior by proving that this behavior cannot appear for linear bilevel problems.

3.2   A pricing and routing problem for last-mile delivery

Martina Cerulli (ESSEC Business School – Cergy Pontoise, FR)

The Profitable Tour Problem (PTP) belongs to the class of Vehicle Routing Problems with profits. In PTP, a vehicle, starting from a central depot, can visit a subset of the available customers, collecting a specific revenue whenever a customer is visited. The objective of the problem is the maximization of the net profit, i.e., the total collected revenue minus the total route cost. Most of the literature in this field considers only one decision maker. However, in several real-world routing applications, and in particular in the last-mile delivery, there are different involved agents with conflicting goals. If the decisions are made in a hierarchical order, this problem can be modeled with bilevel programming, with the PTP at the lower level. In this talk, we consider a company, which acts as a leader and offers disjoint subsets of a given set of items to a set of independent drivers. Each driver solves a PTP communicating to the company the items she accepts to serve. Both the company and the drivers aim at maximizing their net profit, which is calculated differently in the two levels. We propose a bilevel formulation that models this interaction allowing the leader not only to anticipate the best followers' response, but also to find the optimal pricing scheme for each carrier. The value function reformulation of this bilevel program is considered, and further

reformulated by projecting out some of the lower-level variables. We find exact solutions to these models using a branch and cut approach, leveraging on an alternative reformulation of the lower-level problem, which benefits from a particular monotonicity property.

3.3 Reformulation-Perspectification Technique for nonconvex (robust) optimization

Danique de Moor (University of Amsterdam, NL)

In this talk, a new technique is introduced, called the Reformulation-Perspectification Technique (RPT), to obtain convex approximations of nonconvex continuous (robust) optimization problems. RPT consists of two steps, those are, a reformulation step and a perspectification step. The reformulation step generates redundant nonconvex constraints from pairwise multiplication of the existing constraints. The perspectification step then convexifies the nonconvex components by using perspective functions. The proposed RPT extends the existing Reformulation-Linearization Technique (RLT) in two ways. First, it can multiply constraints that are not linear or not quadratic, and thereby obtain tighter approximations than RLT. Second, it can also handle more types of nonconvexity than RLT. A numerical experiment on convex maximization problems demonstrates the effectiveness of the proposed approach.

3.4 Two-stage robust optimization with objective uncertainty

Boris Detienne (University of Bordeaux, FR)

In this talk, we study optimization problems where some cost parameters are not known at decision time and the decision flow is modeled as a two-stage process within a robust optimization setting. We address general problems in which all constraints (including those linking the first and the second stages) are defined by convex functions and involve mixed-integer variables. We first give a short proof that the special case of the problem with linear constraints is NP-complete. We then show how the general problem can be reformulated using Fenchel duality, allowing to derive an enumerative exact algorithm, for which we prove asymptotic convergence in the general case, and finite convergence for cases where the first-stage variables are all integer. An implementation of the resulting algorithm, embedding a column generation scheme, is then computationally evaluated on a variant of the Capacitated Facility Location Problem with unknown transportation costs, using instances that are derived from the existing literature.

3.5   Integer Programming Games (and Robust optimization)

Gabriele Dragotto (Princeton University, US)

Joint work of Gabriele Dragotto, Rosario Scatamacchia
Main reference Gabriele Dragotto, Rosario Scatamacchia: "The ZERO Regrets Algorithm: Optimizing over Pure
          Nash Equilibria via Integer Programming", arXiv, 2021.
     URL https://doi.org/10.48550/ARXIV.2111.06382

In this talk, we briefly survey Integer Programming Games (IPGs), i.e., simultaneous one-shot non-cooperative games where each player solves a parametrized integer program. After discussing a few motivating examples, we introduce the concept of Nash equilibrium for IPGs, namely a stable solution where no single agent has the incentive to defect from it profitably. Although the concept of Nash equilibrium has a natural and simple interpretation, determining if an IPG instance has a Nash equilibrium is generally a Σ_2^p-complete problem. We present ZERO Regrets, an algorithm to compute, enumerate and optimize over the Nash equilibria of IPGs. We propose some ideas for its extension to generalized IPGs, i.e., IPGs where each player's feasible set depends on their opponent's variables. Finally, we sketch the connection between Nash equilibria and robust optimization. In particular, we argue that robust combinatorial optimization and IPGs are equivalent problems, and the robust solution is indeed a Nash equilibrium.

3.6   Results and Challenges in Robust Combinatorial Optimization

Marc Goerigk (Universität Siegen, DE)

Robust optimization over combinatorial problems often behave quite differently to their more general counterparts. In many settings, we only consider uncertainty in the objective and assume that the combinatorial structure is known (of course, other settings are considered as well). In this talk I try to give a general introduction to this topic, covering min-max, min-max regret, two-stage and multi-stage problems with different types of uncertainty (discrete, budgeted, intervals), as well as pointing out some of the more recent results and challenges. In particular, many complexity questions remain open (e.g, the approximability of min-max regret problems with interval uncertainty, the complexity of two-stage assignment with continuous budgeted uncertainty, and many multi-stage or explorable problems). Furthermore, for experiments, often random data is generated, but the algorithm performance on random instances is poorly understood. To help a systematic approach towards experimental work, we are currently building a benchmark website for datasets, see `https://robust-optimization.com/`

3.7 The Robust Bilevel Selection Problem

Dorothee Henke (TU Dortmund, DE)

In bilevel optimization problems, there are two players, the leader and the follower, who make their decisions in a hierarchy, and both decisions influence each other. Usually one assumes that both players have full knowledge also of the other player's data and objective. Although bilevel problems are often already NP- or even Σ_2^p-hard to solve under this assumption, a more realistic model might sometimes be needed. Uncertainty can be quantified, for example, using the robust optimization approach: Assume that the leader does not know the follower's objective function precisely, but only knows an uncertainty set of potential follower's objectives, and the leader's aim is to optimize the worst case of the corresponding scenarios. Now the question arises how the computational complexity of bilevel optimization problems changes under the additional complications of this type of uncertainty.

We make a step towards answering this question by investigating a simple bilevel problem that can be solved in polynomial time without uncertainty. In the bilevel selection problem, the leader and the follower each select a number of items, while a common number of items to select in total is given, and each of the two players maximizes the total value of the selected items, according to different sets of item values. We compare several variants of this problem and show that all of them can be solved in polynomial time. We then investigate the complexity of the robust version of the problem. If the item sets controlled by the leader and by the follower are disjoint, it can still be solved in polynomial time in case of a finite uncertainty set or interval uncertainty. If the two item sets are not disjoint, the robust problem version becomes NP-hard, even for a finite uncertainty set, which shows that uncertainty can indeed add additional complexity to a bilevel optimization problem.

3.8 Quadratic Regularization of Unit-Demand Envy-Free Pricing Problems and Application to Electricity Markets

Quentin Jacquet (EDF – Paris, FR)

We consider a profit-maximizing model for pricing contracts as an extension of the unit-demand envy-free pricing problem: customers aim to choose a contract maximizing their utility based on a reservation bill and multiple price coefficients (attributes). A classical approach supposes that the customers have deterministic utilities; then, the response of each customer is highly sensitive to price since it concentrates on the best offer. A second approach is to consider logit model to add a probabilistic behavior in the customers' choices. To circumvent the intrinsic instability of the former and the resolution difficulties of the latter, we introduce a quadratically regularized model of customer's response, which leads to a quadratic program under complementarity constraints (QPCC). This allows to robustify the deterministic model, while keeping a strong geometrical structure. In particular, we

show that the customer's response is governed by a polyhedral complex, in which every polyhedral cell determines a set of contracts which is effectively chosen. Moreover, the deterministic model is recovered as a limit case of the regularized one. We exploit these geometrical properties to develop an efficient pivoting heuristic, which we compare with implicit or non-linear methods from bilevel programming. These results are illustrated by an application to the optimal pricing of electricity contracts on the French market.

3.9 A comparison of k-adaptability and min-max-min robustness: new results and insights

Jannis Kurtz (University of Amsterdam, NL)

Robust optimization is one of the most successful concepts to tackle optimization problems under uncertainty. In this talk we compare two popular robust optimization models, namely *k-adaptability* [1] and *min-max-min robust optimization* [2], both under objective uncertainty. While k-adaptability was introduced to approximate two-stage robust problems by calculating k second-stage solutions already in the first stage, the concept of min-max-min robustness was introduced as a less conservative version of the classical robust optimization problem where the user can calculate k instead of a single solution. This provides more flexibility, better performance for future uncertain scenarios and a small set of solutions which, in contrast to the two-stage robust problem, do not change in the future. This is especially useful for applications from disaster management, where a team has to be trained on an ideally small set of solutions.

While at a first glance both problems are very similar, it turns out they significantly differ in terms of computational complexity. However recent results derived for min-max-min robust optimization can partly be extended to k-adaptability problems and give new insights into the latter problem class.

We analyze the three cases where k, the number of calculated (second-stage) solutions, is a) larger than the dimension of the problem, b) a small value, c) smaller but close to the dimension of the problem. It turns out that, if the underlying deterministic problem is tractable, then the min-max-min robust problem is tractable in case a), often tractable in case c), and NP-hard in case b). On the other hand the k-adaptability problem can be NP-hard for all cases and provides the exact optimal value of the two-stage robust problem in case a). We provide an efficient heuristic algorithm which calculates good solutions for both problems for every k and give theoretical and computational proof that the optimality gap of these heuristic solutions decreases if k increases. We incorporate this heuristic into a branch & bound framework which finds optimal solutions for both problems for any k. Our computations show that the larger k the more instances of both problems can be solved during the timelimit which is mainly due to the decreasing optimality gap of our heuristic. This gives rise to the conclusion that solving the exact two-stage robust optimization problem is computationally more efficient than approximating it by the k-adaptability approach in case of objective uncertainty.

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3.10 Bilevel Optimization with a focus on the linear case

Martine Labbé (UL – Brussels, BE)

A bilevel optimization problem consists in an optimization problem in which some of the constraints specify that a subset of variables must be an optimal solution to another optimization problem. This paradigm is particularly appropriate to model competition between agents, a leader and a follower, acting sequentially.

In this talk I focus on the simplest bilevel problems, those that are linear. I present the main characteristics, properties and algorithms for these problems. Then, I discuss some recent results showing that these problems are already extremely challenging. Finally, I show how linear bilevel optimisation may help to improve the resolution of stochastic optimisation problems whose objective involve the Value at Risk (VaR) or quantile.

3.11 Incentivizing Truthfulness in Core-Stable Intermediated Combinatorial Exchanges with Budget Constraints

Christina Liepold (TU München, DE)

Joint work of Christina Liepold, Maximilian Schiffer

Servitization markets that center their business around operationally connecting buyers and suppliers of goods, services, or capacities can be modeled as combinatorial exchanges. Usually, the markets employ a profit-oriented intermediary to coordinate the process. Particularities of the servitization market are that (I) potential buyers face budget constraints, which provide an incentive to overstate valuations, resulting in exchanges that are not welfare-maximizing and (II) core-stability of the market is essential to foster participation and, thus, the attractiveness of the heterogeneous exchange. However, it has been proven that no truthful mechanism exists under private budget constraints in combinatorial exchanges. We propose the introduction of a profit-oriented intermediary, which allows for the incentivization of truthfulness in these markets. We utilize the properties of Mixed Integer Bilevel Linear Problems to formulate the allocation and pricing problem for core-stable intermediated combinatorial exchanges under private budget constraints. We find that intermediated combinatorial exchanges with a profit-oriented intermediary lead to higher social welfare than truthful non-intermediated benchmarks while restricting the matching of untruthful buyers.

3.12   Robust Optimization, including Data, Nonlinearity, and Current Research Questions

Frauke Liers (Universität Erlangen-Nürnberg, DE)

In this talk, I first reviewed robust convex optimization, focussing on duality-based reformulations of robust counterparts to finite and algorithmically tractable problems as well as cutting plane approaches. For constraints that are convex in the decisions and concave in the uncertainty, Fenchel duality can be used. For combinatorial optimization problems with nonlinear objective, I showed new reformulations that combine Gamma-robust counterparts with Fenchel duality (This part is joint work with Dennis Adelhütte, FAU). A novel approach (joint with Martina Kuchlbauer and Michael Stingl, both FAU) for mixed-integer nonlinear robust optimization was summarized. Here, an adaptive bundle method uses nonconvex adversarial problems that are solved up to any given error, e.g., via piecewise linear relaxations. For problems that are convex in the uncertainty and potentially nonconvex in the uncertainty, the bundle method is embedded in an outer approximation scheme. I then continued with reviewing distributional robustness, focussing on reformulations and the construction of ambiguity sets based on historical data. I concluded by pointing out current research questions.

3.13   SOCP-Based Disjunctive Cuts for a Class of Integer Nonlinear Bilevel Programs

Ivana Ljubić (ESSEC Business School – Cergy Pontoise, FR)

We study a class of integer bilevel programs with second-order cone constraints at the upper level and a convex-quadratic objective function and linear constraints at the lower level. We develop disjunctive cuts (DCs) to separate bilevel-infeasible solutions using a second-order-cone-based cut-generating procedure. We propose DC separation strategies and consider several approaches for removing redundant disjunctions and normalization. Using these DCs, we propose a branch-and-cut algorithm for the problem class we study, and a cutting-plane method for the problem variant with only binary variables. We present an extensive computational study on a diverse set of instances, including instances with binary and with integer variables, and instances with a single and with multiple linking constraints. Our computational study demonstrates that the proposed enhancements of our solution approaches are effective for improving the performance. Moreover, both of our approaches outperform a state-of-the-art generic solver for mixed-integer bilevel linear programs that is able to solve a linearized version of our binary instances.

3.14 Oracle-base methods to solve multi-stage adjustable robust optimization with right-hand-side uncertainty

Ahmadreza Marandi (TU Eindhoven, NL)

In this talk, we address multi-stage adjustable robust optimization with right-hand-side uncertainty. We consider both continuous cases and mixed integer cases. For continuous cases, we show that the problem can be solved by enumerating the vertices of the uncertainty set. So, for a polyhedral uncertainty set, we develop a simplex-type method that starts from a vertex and moves to an adjacent vertex with a worse objective value. For the mixed integer case, we developed an iterative approach that converges to the optimal solution. The algorithm is based on partitioning the uncertainty set into smaller sets and obtaining a lower and upper bound for each subset. Since the algorithm itself is an exhaustive search, we also developed a heuristic method based on the limited discrepancy search, where we develop a branching tree and construct waves to explore the tree. We show that both algorithms work in practical problems, as it is based on an oracle that solves the deterministic problem in each iteration.

3.15 On the complexity of bilevel bottleneck assignment problem

Komal Muluk (RWTH Aachen, DE)

Bilevel assignment problem is an example of bilevel optimization problems. In an instance of bilevel assignment problem, we are given a bipartite graph $G = (V, E)$, and the edge set E is partitioned into two sets E_ℓ and E_f. The set E_ℓ is controlled by the leader and the set E_f is controlled by the follower. Additionally, leader and follower have their own weight functions w_ℓ and w_f defined on the edge set E. Both decision makers have their own objective functions c, d for the leader and the follower respectively. We consider the settings in which the objective functions of the leader and the follower can be of sum or bottleneck type. Moreover, the follower can behave according to the optimistic or pessimistic rule. The goal of the leader is to find $L \subseteq E_\ell$ which minimizes her objective function subject to an optimal reaction of the follower $F \subseteq E_f$ such that $L \cup F$ forms a perfect matching in the graph G. We give a unified NP-hardness proof for the eight variants of bilevel assignment problem; the eight variants arise from the combinations of sum or bottleneck type of objective functions c and d, and from the pessimistic or optimistic behavior of the follower. Consequently, we show non-existence of an approximation algorithm for this problem.

3.16 Stochastic optimization: a tutorial to establish connections

Bernardo Pagnoncelli (SKEMA Business School – Lille, FR)

In this presentation I will give a broad introduction to stochastic optimization, focusing on static problems. I will start with the classic paradigm of minimizing expected costs, and then discuss why the inclusion of risk measures is desirable in many practical situations. I will exemplify the effect of risk aversion in the newsvendor problem, using the CVaR as the risk measure.

I will briefly introduce the dynamic setting, and the different algorithms to solve the problems in sequential decision-making. I will conclude with the current challenges in end-to-end learning, which consist of problems that integrate prediction and prescription. Bilevel optimization is an important tool in this integration, and I will define precisely how it can build the bridge between machine learning and optimization.

3.17 Projections are the new binaries

Jean Pauphilet (London Business School, GB)

Product recommendation, matrix factorization, or Euclidean embedding can be formulated as optimization problems with a constraint on the rank of the decision variables (a matrix). These optimization problems constitute hard non-convex optimization problems that cannot be modeled via convex mixed-integer optimization. Alternatively, we propose to introduce projection matrices (i.e., matrices that satisfy $P^2 = P$) to encode for the span of the decision variables and linearize the rank constraint. Projection matrices are not only a generalization of binary variables (i.e., scalar roots of the equation $z^2 = 2$) but we show that the resulting optimization framework, which we coin mixed-projection optimization, can effectively be used to solve low-rank optimization problems to provable optimality. In particular, some of the most successful ideas from mixed-integer optimization such as big-M formulations, perspective cuts, relax-then-round strategies generalize to low-rank problems by using projection matrices.

3.18 K-adaptability with few recourse solutions

Michael Poss (University of Montpellier & CNRS, FR)

We present a variant of adaptive robust combinatorial optimization problems where the decision maker can prepare K solutions and choose the best among them upon knowledge of the true data realizations. We suppose that the uncertainty may affect the objective and the constraints through functions that are not necessarily linear.

In the first part of the talk, we propose a new exact algorithm for solving these problems when the feasible set of the nominal optimization problem does not contain too many good solutions. Our algorithm enumerates these good solutions, generates dynamically a set of scenarios from the uncertainty set, and assigns the solutions to the generated scenarios using a vertex p-center formulation, solved by a binary search algorithm. Our numerical results on adaptive shortest path and knapsack with conflicts problems show that our algorithm compares favorably with the methods proposed in the literature. We additionally propose a heuristic extension of our method to handle problems where it is prohibitive to enumerate all good solutions. This heuristic is shown to provide good solutions within a reasonable solution time limit on the adaptive knapsack with conflicts problem. Finally, we illustrate how our approach handles non-linear functions on an all-or-nothing subset problem taken from the literature.

In the second part of the talk, we discuss extensions of these algorithms to K-adaptability, where the decision maker must set first-stage optimization variables in addition to the preparation of the K second-stage solutions. We reformulate again the problem along the lines of p-center location problems, this time including additional coupling constraints. We discuss how subtle pricing and adversarial problems may lead to efficient branch-and-cut-price algorithms for solving the problem optimally.

3.19 80–20 optimization under uncertainty

Krzysztof Postek (TU Delft, NL)

In this talk, we consider the design choices one has to make when considering optimization under uncertainty. Expanding the model form useful for solving the deterministic problem into an uncertainty-including form is often hard. This calls, in line with the seminar description, for completely new ways of modelling Σ_2^p problems. We illustrate one heuristic idea (evaluating incumbent solutions of a deterministic MILP for a given problem with stochastic simulations and picking the best one among those) on the fleet assignment problem in airline optimization, which performs similarly to a massive, exact MILP reformulation of the stochastic problem.

3.20   Duality and Value Functions

Ted Ralphs (Lehigh University – Bethlehem, US)

Duality is a central concept in optimization from which optimality conditions arise, among other important implications. It is duality-based optimality conditions that enable the reformulations that are the basis for many algorithms for addressing multistage optimization problems in which the problem to be solved at each stage is parametric in the decisions and information revealed at previous stages. Examples include multilevel optimization, robust optimization, and multistage stochastic programming with recourse. By replacing later stage problems with their optimality conditions, it is possible to reformulate these multistage problems as traditional mathematical optimization problems, albeit ones involving complex value function. While it is often stated that there is no strong duality for nonconvex optimization problems, such as mixed integer linear optimization problems (MILPs), we discuss in this talk that this is indeed not the case. We describe a duality theory that generalizes the classical LP/convex duality theory to the MILP setting and show that it can be derived from first principles beginning from each of three seemingly disparate starting points. The first is the traditional notion based on bounding of the value function, the second is based on a generalized Farkas lemma that leads to a complexity-theoretic interpretation, and the third is based on reformulation via projection, which leads to a generalized Benders decomposition. The relationship between these three different derivations and the generalized notions that arise shed light on the nature of the duality relations at the core of optimization theory and enable duality principles to be applied to a much wider range of problems.

3.21   Some recent results and thoughts on bilevel optimization under uncertainty

Martin Schmidt (Universität Trier, DE)

Bilevel optimization is a very active field of applied mathematics. The main reason is that bilevel optimization problems can serve as a powerful tool for modeling hierarchical decision making processes. This ability, however, also makes the resulting problems challenging to solve – both in theory and practice. In this talk, we focus on bilevel optimization problems under uncertainty. First, we give an overview about this rather young field. We particularly show that the sources of uncertainty are much richer in bilevel optimization when compared to "usual", i.e., single-level, optimization. The reason is that, besides data uncertainty, one can also face uncertainties w.r.t. the decisions of the other player. Second, we briefly present two recent papers on uncertain bilevel optimization. One in which we develop a branch-and-cut framework for bilevel knapsack interdiction, where the lower-level is a Gamma-robustified model, and another one, in which we try to model the situation of a follower that is uncertain w.r.t. the leader's decision. Third and finally, we sketch some open problems in the field of bilevel optimization under uncertainty to propel some further discussions during the seminar and beyond.

3.22   A Reverse Stackelberg Game Model for Grid Usage Pricing with Local Energy Markets

Juan Sepúlveda (INRIA Lille, FR)

In the context of the massive penetration of renewables and the shift of the energy system operation towards more consumer-centric approaches, local energy markets are seen as a promising solution for prosumers' empowerment. Various local market designs have been proposed that often ignore the laws of physics ruling the underlying distribution network's power flows. This may compromise the power system's security or lead to operational inefficiencies. Therefore, including the distribution network in clearing the local market arises as a challenge. We propose using grid usage prices (GUPs) as an incentive mechanism to drive the system towards an economically and operationally efficient market equilibrium, subject to security constraints. Our approach requires expressing the incentive policies as affine functions of the prosumers' active and reactive power outputs. This setting falls into the category of reverse Stackelberg games, where we look for the optimal policy in the space of affine functions [2]. This approach takes advantage of controllability guarantees for the problem's unconstrained setting, which hopefully will enable the DSO to influence the output of the market towards an optimally determined target point. Market-related properties of the policy, such as economic efficiency, individual rationality, incentive compatibility, and fairness, will be rigorously studied. Two alternative solution approaches are proposed: The first is based on reformulating the resulting bilevel problem into a single-level problem employing the KKT conditions of the underlying lower-level problem. Then, the resulting quadratically constrained quadratic problem is solved applying a trust-region method. The second solution approach is based on first obtaining a team-problem solution and solving a feasibility problem to induce it [1]. Finally, extensive computational experiments will be carried out on different IEEE test feeders to assess the performance of the proposed approach statistically.

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3.23   Dynamic pricing for Public Cloud Computing

Nathalia Wolf (INRIA Lille, FR)

In this work, we present a cloud sharing system that increases the utilization rate of public cloud resources. The system is modeled as a mixed integer bilevel problem with two independent follower lever problems. The leader interacts with the followers by (i) offering rewards to long-term consumers to lease their resources and (ii) setting prices to short-term consumers to access available resources. We solve the model using an optimal value function algorithm.

Participants

- Yasmine Beck
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