Dagstuhl Seminar

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Algorithmic Combinatorial Game Theory

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Contents

1 Summary

Games are as old as humanity. The combinatorial game theory community has studied games extensively, resulting in powerful tools for their analysis, like the notion of game-theoretic value. This theory provides a high-level understanding of how to play combinatorial games, but to completely solve specific games requires algorithmic techniques. So far, algorithmic results are rare and mainly negative, e.g., the proofs that Chess and Go are EXPTIME-complete. There are also some positive results on endgames of Go and on various classes of impartial games. But most games lie in "Wonderland", i.e., we are wondering about their complexity/efficiency. (We are not normally interested in exhaustive approaches like the recent world-class computer players for Checkers and Chess.)

The two large communities of combinatorial game theory and algorithmics rarely interact. This is unfortunate. Game theory could benefit from applying algorithmic techniques to games with known outcomes but no known efficient strategies, e.g., Hex and poset games such as Chomp. On the other hand, better knowledge of the game-theoretic tools could help researchers in algorithmics to develop more efficient or more general algorithms for games whose complexity is barely known, e.g., Hex and Chomp and epidemiography games such as Nimania. Maybe the game-theoretic framework can even be extended to noncombinatorial games, like geometric games.

There has been a recent surge of interest in algorithmic combinatorial game theory from both communities. The goal of this workshop was to bring these two communities together, to advance the area of algorithmic combinatorial game theory from infancy to maturity.

In all, 46 researchers with affiliations in Austria (1), Canada (5), the Czech Republic (4), Germany (16), Hong Kong (1), Israel (2), the Netherlands (4), Poland (1), Sweden (1), Switzerland (1), and the USA (10) participated in the meeting (some of them EU citizens working abroad). Nineteen participants were graduate students or postdocs. Four invited keynote speakers, Elwyn Berlekamp, Aviezri Fraenkel, Joel Spencer, and Jürg Nievergelt, gave one-hour position talks. The remaining 31 presentations given by participants of the meeting covered a wide range of topics, ranging from complexity theoretic results up to experimental studies. Game-theoretic analysis of popular board games like Go and Amazons never ceases to be interesting. The algorithmicians on the other hand provided NP-hardness proofs of games like Clickomania and variants of Pushing Block games, or efficient strategies for game playing. And all younger participants were eager to learn the differences between the US and European tenure game. The abstracts of these presentations are contained in this seminar report. A special issue of TCS-A (Theoretical Computer Science, series A), edited by R. Fleischer and R. Nowakowski, containing selected papers presented at this Workshop is in preparation.

The evening sessions were devoted to the discussion of open problems and a

Clobber tournament (played on a 5×6 board). The winners of this tournament were Tomáš Tichý and Jiří Sgall (runner-up). The computer Clobber tournament (all programs were written on the first day of the Workshop) was won by R. Hearn. Clobber is a new two-player game, recently invented by Albert, Grossmann, and Nowakowski, and not much is known about it (inspired by our tournament, the upcoming Third International Conference on Computers and Games in Edmonton will have a Clobber Problem Composition Contest). During the workshop, two papers on Clobber were written that will also be submitted to the TCS special volume. Actually, we expect many more papers to originate from this very successful workshop, as several of the proposed open problems were already solved during the week (and solutions presented in a special session at the end of the Workshop), and other problems at least partially solved.

As usual, Schloß Dagstuhl proved to be an excellent place to hold a great meeting, so we would not only like to thank the participants of the seminar for making this a very successful event but also the Dagstuhl staff for providing a friendly and stimulating working environment.

2 Program

Monday, February 18

9:00 Rudolf Fleischer: Introduction

9:15 Elwyn Berlekamp: Idempotents among combinatorial games

10:30 **Coffee Break**

11:00 Richard Nowakowski: Periodicity and arithmetico-periodicity in hexadecimal games

11:25 David Wolfe: On the order of games

12:15 **Lunch**

14:30 Ingo Althöfer: Gameplaying with multiple choice systems in combinatorial games

14:55 Rainer Feldmann: Parallel evaluations of quantified Boolean formulas

15:20 **Coffee Break**

16:00 Walter Kern: Solution concepts for cooperative games

16:25 Stefan Schwarz: Playing FreeCell

16:50 Tomasz Luczak: Random graphs and combinatorial games

18:00 **Dinner**

20:00 **Special Session**: Open problems

Tuesday, February 19

9:00 Aviezri Fraenkel: Complexity, appeal and challenges of combinatorial games

10:00 J. P. Grossman: One-dimensional Peg Duotaire and Clobber

10:25 **Coffee Break**

11:00 Robert A. Hearn: The complexity of sliding-block puzzles

11:25 Michael Hoffmann: Block pushing puzzles

12:15 **Lunch**

14:30 Sándor Fekete: Traveling Salesmen in the age of competition

14:55 Markus Holzer: Assembling molecules in Atomix is hard

15:20 **Coffee Break**

16:00 Wolfgang Slany: On the lengths of symmetry breaking-preserving games on graphs

16:25 Uri Zwick: Jenga

16:50 Cyril Banderier: Average time for trivial pursuit-like games

17:15 Jiří Sgall: A solution of David Gale's man and lion problem

18:00 **Dinner**

20:00 **Special Session**: Clobber tournament, first two rounds

Wednesday, February 20

9:00 Joel Spencer: Living up to your potential function

10:00 Stefan Pickl: CSoMTFCtOSotEP—TGCaaAS (full title available on demand)

10:25 **Coffee Break**

11:00 Ulf Lorenz: Error analysis in game trees

11:25 Alejandro Lopez-Ortiz: Processor scheduling for BFS/DFS in search game spaces

12:15 **Lunch**

13:45 Trip to Völklinger Hütte, UNESCO World Heritage Museum

18:00 (**Dinner**)

20:00 **Special Session**: Clobber tournament, rounds 3 and 4

Thursday, February 21

9:00 Jürg Nievergelt: Man vs. machine in Chess and Go

10:00 Rudolf Fleischer: Xiangqi and combinatorial game theory

10:25 **Coffee Break**

11:00 Benjamin Doerr: Vector balancing and tenure games

11:25 Ioana Dumitriu: Half-lies and the Ulam searching problem

12:15 **Lunch**

14:30 Georg Snatzke: Exhaustive search for combinatorial games in Amazons

14:55 Theodore Tegos: Combinatorial endgame databases in Amazons

15:20 **Coffee Break**

16:00 Martin Müller: Capturing races in Go and their combinatorial game values

16:25 Jos Uiterwijk: Application of proof number-search variants to Lines of Action

16:50 Solved problems session: Rudolf Fleischer (competitive TSP), Stefan Schwarz (reals representation), Erik Demaine (PushPush)

18:00 **Dinner**

20:00 **Special Session**: Clobber tournament, final round(s)

Friday, February 22

9:00 Ian Munro: Clickomania

9:25 Ulrich Faigle: Graph coloring games

9:50 Alejandro Lopez-Ortiz: Battleship and other games on interval graphs

10:20 **Coffee Break**

11:00 J. P. Grossman *et al.*: Clobber tournament wrap-up and Clobber programs

11:25 J. P. Grossman: One-dimensional Phutball

12:00 Concluding remarks

12:15 **Lunch**

18:00 **Dinner**

3 Abstracts

Idempotents Among Combinatorial Games

Elwyn Berlekamp

Combinatorial Game Theory offers powerful mathematical methods for attacking a variety of Two-Person games with Perfect Information, by Dividing and Conquering. The theory is especially useful for dealing with games whose endgame positions naturally split up into sums of smaller pieces. A thorough local analysis of any particular region yields a simplified data structure, and then appropriate techniques facilitate the rapid combination of these extracted summaries to determine whether or not there exists any overall winning move, and if so, where. The earliest and most basic axiomatization of the theory, due to John Conway, determines canonical forms for positions in loop-free games in which the last move looses. Such games form an Abelian group under addition. More general games, which may contain loopy positions, or special restrictions such as the compulsory capture rule in checkers, the complimentary extra moves in Dots-and-Boxes, or the ko ban rule in Go, have been successfully treated by introducing into the basic theory additional values which do not have negatives. They are idempotents in the algebraic sense that when such a value is added to itself, the result is itself. As in the applied analysts use of terms such as o(t), combinatorial game theorists have found other idempotents such as ish which similarly simplify many calculations by suppressing less significant terms. One idempotent which has proved especially valuable in studies of Go and Domineering is the thick stack of coupons.

The full version of this paper will appear in "More Games of No Chance", a volume now in press at Cambridge University Press, edited by Richard Nowakowski.

Periodicity and Arithmetico-Periodicity in Hexadecimal Games

Richard Nowakowski

We give a theorem for deciding when an hexadecimal game is periodic. We also extend Autin's arithmetic periodicity theorem to cover all saltuses. We then examine all 1- and 2-digit Hexadecimal games and some 3-digit games. We also note that there are hexadecimal games which are neither periodic nor arithmetic periodic.

On the Lattice Structure of Finite Games

David Wolfe

(This work was conducted with William Fraser, Dan Calistrate and Marc Paulhus)

Under Conway's simple but powerful game theory axioms, games form a group with a partial order. While a great deal has been known about the group structure of large subsets of games, surprisingly little was known about the overall partial order. We prove that games lasting a fixed number of turns form a distributive lattice, but that the collection of all finite games does not form a lattice.

We will also present theorems about the structure of this lattice. A direct corollary of these theorems is that all maximal chains in the day n lattice are of the same length, that length being exactly one plus double the number of games born on day $n-1$.

This talk will be introductory and I'll explain the relevant game theory and lattice theory as we go.

Multiple Choice System for Decision Support

Ingo Althöfer

Programs compute a clear handful of candidate solutions. Then a human has the final choice amongst these candidates.

Often we shortly call such systems „Multiple Choice Systems". In the field of electronic commerce Multiple Choice Systems are/were sometimes advertised under the names „Recommender System" and „Recommendation System".

We describe three different Multiple Choice Systems which were designed for playing chess.

„3-Hirn" / „Triple Brain": Two independent programs make one proposal each.

„Double-Fritz with Boss": Since 1995 the chess program „Fritz" has a 2-best-mode where it computes not only its best but its two best candidate moves. The human gets the choice amongst these top-2 candidates.

„Listen-3-Hirn" / „List Triple Brain": Two independent chess programs are used which both have k-best modes. Program 1 proposes its k best candidate moves, whereas program 2 gives its top m candidates. (The numbers k and m are chosen appropriately, for instance k=m=3.)

In all three examples the human has the final choice amongst the computer proposals. However, he is not allowed to outvote the programs.

I am an amateur chess player and experimented with these Multiple Choice Systems in performance-oriented chess: with 3-Hirn from 1985 to 1996, Double-Fritz with Boss in 1996, List Triple Brain in 1997. The systems almost always performed 200 rating points above the performances of the single programs in it. The „final" success was a 5-3 match win of List Triple Brain over Germany's no. 1 player Arthur Yusupov in 1997. After this event top human players were no longer willing to play against these Multiple Choice Systems.

Theses:

1. Human (singular!) and computers (plural!) together may be much stronger than human alone or computers alone.
2. In Multiple Choice Systems the human is not allowed to outvote the program. This often helps to improve the performance, especially in realtime applications.
3. Combinatorial games are nice testbeds for decision support systems in general.

Natural building blocks of a Multiple Choice System are programs that compute more than one candidate solution. In this context k-best algorithms are often not suited because they tend to give only **micro mutations instead of true alternatives** (master example: the shortest path problem). The task to generate true alternatives of „good" quality is typically non-trivial.

A Distributed Algorithm for QBF

Rainer Feldmann

We start our talk by introducing QBF, the problem to decide whether a given quantified boolean formula is satisfiable or not. We show that QBF is a two-person zero-sum game with complete information. Using game tree search for the QBF problem results in an algorithm which is known as Davis-Putnam algorithm.

We then investigate the most important characteristics of the game trees resulting from this approach.

In the last part of the talk we show how to adapt a distributed game tree search algorithm to QBF and present experimental results indicating that the algorithm gains considerable efficiency on random inputs as well as on inputs from real world problems.

Solution Concepts of Cooperative Games

Walter Kern

There are many classical so-called solution concepts for cooperative games that have been proposed in the literature, e.g. the core, the kernel, the nucleolus and the Shapley value. During the last years, people started to study the computational complexity of these solution concepts. We outline this kind of research.

Playing Freecell Faster

Stefan Schwarz

In this joint work with André Große[1] we present the program *BigBlackCell* (`http://www.uni-jena.de/~BigBlackCell`), which evaluates moves in the solitaire card game *Freecell*. The aim of *BigBlackCell* is to help humans, playing *Freecell* much more faster. We consider this solitaire game as a test case for decision making processes, where fast (but not perfect!) computer programs can support and speed up the human decisions.

Random Graphs and Combinatorial Games

Malgorzata Bednarska and Tomasz Luczak

Let $\Gamma(n; p, q; \mathcal{F})$ be a perfect information game played by two players, Maker and Breaker, who alternately claim edges of the complete graph on n vertices. In each round Maker selects up to p edges; then Breaker answers with at most q edges, chosen among all pairs which have not been claimed so far. The aim of Maker is to build a graph from a given family $\mathcal{F}$; if he fails to do that he loses. In the talk we study $\Gamma(n; p, q, \mathcal{F})$ for some families $\mathcal{F}$. In particular, we explore somewhat surprising connections between $\Gamma(n; 1, q; \mathcal{F})$ and properties of the random graph $\mathbf{G}(n, 1/(1+q))$.

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Complexity, Appeal and Challenges of Combinatorial Games

Aviezri S. Fraenkel

Studying the precise nature of the complexity of games enables *gamesters* to attain a deeper understanding of the difficulties involved in certain new and old open game problems, which is a key to their solution. For *algorithmicians*, such studies provide new interesting algorithmic challenges. Substantiations of these claims are illustrated on hand of many sample games, leading to a definition of the tractability, polynomiality and efficiency of subsets of games. In particular, there are tractable games that are not polynomial, polynomial games that are not efficient. The nature of subclasses of *PlayGames* and *MathGames* are also explored.

One Dimensional Peg Solitaire and Clobber

J. P. Grossmann

We consider the single-hop version of one dimensional peg duotaire, a two player version of peg solitaire in which players alternate moves and the last player to move wins. We solve for the nim-value of two sets of consecutive pegs separated by a hole.

We conjecture that two classes of positions produce periodic sequences of nim-values. The shorthand representation for a closed set of duotaire positions motivates a new partizan game called clobber. Clobber is played on an arbitrary graph with white and black stones. A player moves by picking up one of their own stones and "clobbering" an adjacent stone of the opponent's colour; the clobbered stone is removed. The last player to move wins. We present some preliminary results and conjectures for this game.

The Complexity of Sliding-Block Puzzles

Robert A. Hearn

We present a model of computation (*Nondeterministic Constraint Logic*) based on reversing edge directions in weighted directed graphs with minimum in-flow constraints on vertices. Deciding whether this simple graph model can be manipulated in order to reverse the direction of a particular edge is shown to be PSPACE-complete by a reduction from Quantified Boolean Formulas. We prove this result in a variety of special cases including planar graphs and highly restricted vertex configurations, some of which correspond to a kind of passive

constraint logic. We illustrate the importance of this model of computation by giving simple reductions to show that multiple motion-planning problems are PSPACE-hard. Our main result along these lines is that classic unrestricted sliding-block puzzles are PSPACE-hard, even if the pieces are restricted to be all dominoes (1×2 blocks) and the goal is simply to move a particular piece. No prior complexity results were known about these puzzles. This result can be seen as a strengthening of the existing result that the restricted Rush Hour puzzles are PSPACE-complete, which we also give a simpler proof. Finally, we strengthen the existing result that the pushing-blocks puzzle Sokoban is PSPACE-complete, by showing that it is PSPACE-complete even if no barriers are allowed.

Block Pushing Puzzles

Michael Hoffmann

(This is joint work with Erik Demaine, Martin Demaine, and Joseph O'Rourke)

We consider a wide class of pushing-block puzzles similar to the classic Sokoban. The puzzles consist of unit square blocks on an integer lattice; **all** blocks are movable. The robot may move horizontally and vertically in order to reach a specified goal position. The puzzle variants differ in the number of blocks that the robot can push at once, ranging from at most one (PUSH-1) up to arbitrarily many (PUSH-*). Other variations were introduced to make puzzles more tractable, in which blocks must slide their maximal extent when pushed (PUSH-PUSH), and in which the robot's path must not cross itself (PUSH-X). We prove that all of these puzzles are NP-hard. While the PUSH-X puzzles are clearly in NP, this question is still open for most of the other variants. Note that if the condition that all blocks are movable is dropped (as, e.g., in Sokoban), most of the puzzles are known to be PSPACE-complete.

Traveling Salesmen in the Age of Competition

Sándor Fekete

(This is joint work with Aviezri Fraenkel and Matthias Schmitt)

We propose the "Competing Salesmen Problem" (CSP), a 2-player competitive version of the classical Traveling Salesman Problem. This problem arises when we are considering two competing salesmen instead of just one. The concern for a shortest tour is replaced by the necessity to reach any of the customers before the opponent does. In particular, we consider the situation where players are taking turns, moving one edge at a time within a graph $G = (V, E)$. The set of customers is given by a subset $V_C \subseteq V$ of the vertices. At any given time, both players know of their opponent's position. A player wins if he is able to reach a majority of the vertices in V_C before the opponent does.

We give a number of positive results for special cases of the problem, in particular, for the case where G is a tree. We also point out some of the difficulties: Even if both players start at the same vertex, the starting player may lose, and there may be draws. For directed graphs, we show that the problem is PSPACE-complete.

Assembling Molecules in Atomix is Hard

Markus Holzer

(The results presented are from a joint work with Stefan Schwoon from the Technische Universität München)

Atomix is a solitaire game invented by Günter Krämer in 1990 and first published by Thalion Software. The game takes place on a rectangular finite two-dimensional grid, the board. Every cell of the board is either a wall, contains an atom, or is free. Walls cannot be changed, and atoms can be of different types. A move consists of pushing an atom along the x-axis or the y-axis. When an atom is pushed, it continues moving until it reaches a wall or another atom. The game is won when all atoms are arranged in a given "molecule" goal pattern. Formally the Atomix problem is defined as follows: Given an Atomix board and a molecule, is there a sequence of moves to assemble the atoms on the board to form the given molecule? Recently, Hüffner, Edelkamp, Fernau, and Niedermeier (2001) have shown that Atomix is NP-hard and contained in PSPACE, while the exact complexity was stated as an open problem. In this paper we solve this open problem and improve their result showing the following theorem: Atomix on an $n \times n$ board is PSPACE-complete.

To this end we show that Atomix game puzzles can simulate deterministic or nondeterministic finite automata. In particular, we construct an Atomix instance that is solvable if and only if the non-emptiness intersection problem for finite automata, a problem known to be PSPACE-complete by Galil (1976) and Kozen (1977), has a solution. Observe that most importantly the above given theorem shows that there are Atomix instances which have superpolynomially long optimal solutions. Although most constructions used in the PSPACE-completeness proof are much like those in the popular game, it is a tedious exercise to construct a particular Atomix instance having an exponentially long optimal solution. We give an easy construction of Atomix game levels whose optimal solutions meet the worst case. To this end we realize a pseudo n-bit counter in

ATOMIX. The main part of the construction consists of the simulation of a single bit, i.e., a device that stores a bit, changes it accordingly, and produces a carry bit if triggered by an increment event. In order to obtain an easy ATOMIX game level, we implement a pseudo-counting process. This means that the stored 0 bit can, but need not, be changed to 1 and producing no-carry by an increment event, while a 1 bit must be changed to 0 and produces a carry bit. This slight difference to ordinary counting will allow us to construct simple ATOMIX game levels.

On the Lengths of Symmetry Breaking-Preserving Games on Graphs

Wolfgang Slany

Given a graph G, we consider a game where two players, A and B, alternatingly color edges of G in red and in blue respectively. Let $L_{sym}(G)$ be the maximum number of moves in which B is able to keep the red and the blue subgraphs isomorphic, if A plays optimally to destroy the isomorphism. This value is a lower bound for the duration of any *avoidance* game on G under the assumption that B plays optimally. We prove that if G is a path or a cycle of odd length n, then $\Omega(\log n) \leq L_{sym}(G) \leq O(\log^2 n)$. The lower bound is based on relations with Ehrenfeucht-Fraïssé games from model theory. We also consider complete graphs and prove that $L_{sym}(K_n) = O(1)$.

Jenga

Uri Zwick

Jenga is a popular block game played by two players. Each player in her turn has to remove a block from a stack, without toppling the stack, and then add it the top of the stack. We analyze the game mathematically and describe the optimal strategies of both players. We show that physics', that seems to play a dominant role in this game, does not really add much to the complexity of the (idealized) game, and that Jenga is, in fact, a Nim-like game. In particular, we show that a game that starts with n full layers of blocks is a win for the first player if and only if $n = 2$, or $n \equiv 1, 2 \pmod 3$ and $n \geq 4$. We also suggest some several natural extensions of the game. In these extensions, physics does play an interesting role.

Average Time for Trivial-Pursuit-Like Games

Cyril Banderier

Given a random walk on a graph, the cover time is the first time (number of steps) that every vertex has been hit (covered) by the walk. Define the *marking time* for the walk as follows. When the walk reaches vertex v_i, a coin is flipped and with probability p_i the vertex is *marked* (or colored). We study the time that every vertex is marked. (When all the p_i's are equal to 1, this gives the usual cover time problem.) General formulas are given for the marking time of a graph. Connections are made with the generalized coupon collector's problem. Asymptotics for small p_i's are given (this corresponds to the average time for a Trivial-Pursuit-like game with relatively ignorant players!). Techniques used include combinatorics of random walks, theory of determinants, analysis and probabilistic considerations.

Solution of David Gale's Lion and Man Problem

Jiří Sgall

We analyze a pursue-and-evasion game, including almost optimal bounds on the number of moves needed to win. This solves a problem from Richard Guy's list of open problems in game theory.

Living Up to Your Potential (Function)

Joel Spencer

In analyzing two person perfect information (so-called Paul-Carole) games random play by Carole leads to a potential function. This potential function can also be used by Paul. Applications are to the Liar Game, the Tenure Game and various generalizations.

Optimization under Linear Side Conditions Using Inverse Monotone Matrices

Stefan Pickl

This talk is concerned with the optimization of a nonlinear time-discrete model exploiting the special structure of the underlying cost game and the property of inverse matrices. The costs are interlinked by a system of linear inequalities. It is shown that, if the players cooperate, i.e., minimize the sum of all the costs, they achieve a Nash Equilibrium. In order to determine Nash Equilibria, the simplex method can be applied with respect to the dual problem. A general extension of such problems is presented.

Error Analysis in Game Trees

Ulf Lorenz

Game tree search deals with the problems that arise, when computers play two-person-zero-sum-games such as chess, checkers, othello etc. The greatest success of game tree search so far, was the victory of the chess machine 'Deep Blue' vs. G. Kasparov, the best human chess player in the world at that time. In spite of the enormous popularity of computer chess and in spite of the successes of game tree search in game playing programs, we do not know much about a useful theoretical background that could explain the usefulness of (selective) search in adversary games.

We introduce a combinatorial model, which allows us to model errors of a heuristic evaluation function, with the help of coin tosses. As a result, we can show that searching in a game tree will be 'useful' if, and only if, there are at least two leaf-disjoint strategies which prove the root value. In addition, we show that the number of leaf-disjoint strategies, contained in a game tree, determines the order of the quality of a heuristic minimax value. The model is integrated into the context of average-case analyses.

Processor Scheduling for BFS/DFS Searches in Game Spaces

Alejandro López-Ortiz

(This is joint work with Sven Schuierer)

In 1994 Kao et al. showed that heuristic searches are sometimes equivalent to well understood on-line searching strategies in robotics. In particular they showed that given a problem with two heuristic approaches, the optimal scheduling on a single processor corresponds to the "cow path problem" of exploring two rays in search for a target at an unknown distance. In this case the search succeeds when an agent reaches the point target. This corresponds to one of the heuristic approaches succeeding in finding a solution to the problem.

In general, given a processor scheduling strategy S the competitive ratio is the ratio of the time needed by the processors to find the target using S and the time needed if the successful heuristic had never been pre-empted.

In this talk we study a distributed computing setting with m heuristics and p processors. We provide a processor scheduling strategy with competitive ratio of $(m/p-1)(m/(m-p))^{m/p}$. A matching lower bound is shown, hence the strategy proposed is optimal. This corresponds to the on-line search of m rays with p robots. The same general strategy can be applied for general trees, with an iterative deepening scheme at depths $1, 2, 4, \ldots, 2^k, \ldots$. This is optimal to within a constant factor in the competitive ratio. This shows that neither BFS nor DFS are optimal search strategies (absent any other information). Interestingly iterative deepening combines characteristics from both DFS and BFS.

Man vs. Machine in Chess and Go

Jürg Nievergelt

Man and machines, information processors of vastly differing characteristics and abilities, have long been matched as competitors in various games of strategy. We survey the development and state of the art in computer chess and computer Go, and analyze the reasons for the success of computer chess and the comparatively slow progress in computer Go.

Xiangqi and Combinatorial Game Theory

Rudolf Fleischer

(This is joint work with Samee Ullah Khan, HKUST)

We explored whether combinatorial game theory (CGT) is suitable for analyzing endgame positions in Xiangqi (Chinese Chess). We discovered some of the game values that can also be found in the analysis of International Chess (like integers, $*$, and some infinitesimal values), but we also experienced the limitations of CGT when applied to a loopy and non-separable game like Xiangqi.

European Tenure Game

Benjamin Doerr

We study a variant of Joel Spencer's Tenure Game. These are the rules: The game starts with all d chips on level one. Each round Paul splits the chips into two groups. Carole now promotes one group (all chips move up one level), and downgrades the other (all chips move down to the first level). Paul's aim is to get a chip to a possibly high level.

Thus the only difference in rules to the original tenure game is that here chips are not fired (removed from the game), but downgraded instead. Nevertheless, the resulting game is very different: Pusher can get a chip to level $\log_2 d + \log_2 \log_2 d$ at least (instead of $\log_2 d$ only in the original game). A second noteworthy difference concerns the proof. For the original game Spencer invented a generic method coined "Randomization, Derandomization and Antirandomization" to prove matching upper and lower bounds by giving (therefore optimal) strategies for the players. For our problem, this approach seems to fail. It can even be shown that a good potential function assigning weights (potential) to the individual chip can not exist. Nevertheless, we succeed in giving a lower bound of $\lfloor \log_2 d + \log_2 \log_2 d + 1 - o(1) \rfloor$ and a nearly matching upper on of $\lfloor \log_2 d + \log_2 \log_2 d + 1.73 + o(1) \rfloor$. A key idea in the proof is to (mis)use Spencer's original potential function.

A Halflier's Game

Ioana Dumitriu

(This is joint work with Joel Spencer)

In Ulam's game Paul tries to find one of n possibilities with q Yes-No questions, while responder Carole is allowed to lie a fixed number k of times. We consider a variant in which Carole must say yes when that is the correct answer (whence the "halflie"). We show that this variation allows Paul to distinguish between roughly 2^k as many possibilities as in Ulam's game.

Exhaustive Search for Combinatorial Games in Amazons

Raymond Georg Snatzke

Amazons is a young, abstract, strategic, two player game, where the first player unable to move loses. We present a database for small Amazons positions. The database holds the canonical combinatorial game theory values for every position, its thermograph and the corresponding move for every canonical option. Currently the database contains results for positions with a maximum of 22 live squares and for up to 2 black and 2 white amazons.

Such a database is useful to find values and structures in games that were unknown before and hard or impossible to find or verify by hand. In Amazons we were able to prove the existence of nimbers, of fractions down to 1/64 and of various infinitesimals, but these results also suggest that there is no easy construction for most of these values. The database also demonstrates how complex canonical forms in Amazons can be and that many Amazons positions have properties and values that are totally counterintuitive.

Computer Amazons: Shooting The Last Arrow

Theodore Tegos

Games provide a fruitful research area for Artificial Intelligence. For years, chess was considered the most significant field of computer game playing. However, in recent years the interest has shifted towards games that pose greater challenges than chess, such as Go and Amazons. The characteristic of such games is the large number of available moves in each position. Amazons offers an ideal test bed for existing and new search ideas.

This talk presents the work behind Antiope, a strong Amazons program. The strength of Antiope lies in the use of endgame databases. One type of databases store values related to Combinatorial Game Theory. This theory has been almost exclusively the domain of the mathematics community. This talk presents some CGT related results for Amazons and the first investigation of adding Combinatorial Game Theory to a high-performance game-playing program.

Combinatorial Game Values of Capturing Races in Go

Martin Müller

Basic capturing races or semeai in the game of Go have very simple values when considered as combinatorial games. These values are integers, integers plus a dame, or switches constructed from these values.

On the other hand, class 2 semeai can contain unsettled eye shapes, which lead to many more interesting types of positions. In this paper we develop a complete analysis of class 2 semeai and their values as combinatorial games.

Application of Proof-Number Search Variants to Lines of Action

Jos Uiterwijk

(This is joint work with Mark Winands and Jaap van den Herik)

We have tested ab and various proof-number (PN) search variants, including PN, PN2 and PDS, in Lines of Action (LOA) endgame positions. It turns out that PN2 and PDS are the best to solve hard problems. Each of them has a practical disadvantage: PN2 is restricted by the working memory, whereas PDS is slow in searching. Therefore, a new algorithm, called PDS-PN, is developed. It is a two-level search, performing PDS at the first level and a PN search at the second. Experiments reveal that PDS-PN is competitive with PN2 in terms of speed but is not restricted in working memory.

PushPush-1 is PSPACE-complete

Erik D. Demaine

(This is joint work with Robert Hearn, Michael Hoffmann, and Markus Holzer)

PushPush-1 is a pushing-blocks puzzle specified by a rectangular box containing movable blocks and a single robot, each occupying a unit square in the grid. The robot may walk around in empty space, or push one block; in the latter case, the block slides maximally until it hits another block. It was known that deciding whether the robot can reach a specified position is NP-hard, as are several related puzzles. We prove that PushPush-1 is PSPACE-complete by two proofs: the first proof is based on Markus Holzer and Stefan Schwoon's proof that Atomix is PSPACE-complete, and the second proof is based on a nondeterministic constraint logic developed with Robert Hearn to prove that sliding-block puzzles are PSPACE-complete. Both proofs use key PushPush gadgets developed with Martin Demaine and Joseph O'Rourke.

The Complexity of Clickomania

J. Ian Munro

We study a popular puzzle game known variously as Clickomania and Same Game. Basically, a rectangular grid of blocks is initially colored with some number of colors, and the player repeatedly removes a chosen connected monochromatic group of at least two square blocks, and any blocks above it fall down. We show that one-column puzzles can be solved, i.e., the maximum possible number of blocks can be removed, in linear time for two colors, and in polynomial time for an arbitrary number of colors. On the other hand, deciding whether a puzzle is solvable (all blocks can be removed) is NP-complete for two columns and five colors, or five columns and three colors.

The Edge Coloring Game on Trees

Ulrich Faigle

(This is joint work with P. L. Erdös, W. Hochstättler, and W. Kern)

The edge coloring game on a graph involves two players, A and B, that take turns coloring the edges of a graph feasibly. The problem is to decide how many colors suffice for A to have a strategy that always guarantees A a legal move when it is A's turn. The talk reviews some results on the analogous game with respect to vertex coloring and concentrates on edge-coloring trees of maximal vertex degree 3.

Battleship and other Games on Interval Graphs

Alejandro López-Ortiz

(This is joint work with Therese Biedl, Broňa Brejová, Erik D. Demaine, Angèle M. Hamel, and Tomáš Vinař)

We design efficient competitive algorithms for discovering information hidden by an adversary. Specifically, consider a game in a given interval graph G in which the adversary chooses an independent set X in G. Our goal is to discover this hidden independent set X by making the fewest queries of the form "is point p covered by an interval in X?" Our interest in this problem stems from two applications: experimental gene discovery with PCR technology and the game of Battleship (in a 1-dimensional setting). We provide adaptive algorithms for both the verification scenario (given an independent set, is it X?) and the discovery scenario (find X without any information). Under some assumptions on the interval graph, these algorithms use an asymptotically optimal number of queries on every instance.

One Dimensional Phutball

J. P. Grossmann

We consider Philosopher's Football (Phutball) played in one dimension. The game is surprisingly difficult to analyze, and contains many counter-intuitive positions including one in which the only winning move is to jump backwards. We give a complete solution to a restricted version called "Oddish Phutball" in which stones may only be placed at odd distances from the ball. The solution makes use of a potential function which gives the number of stones that a player must place before they can jump to a winning position. We show how to compute the potential function and we show that the player with the lower potential must win.

4 Open Problems Session

Collected by E. D. Demaine, R. Fleischer, A. Fraenkel, and R. J. Nowakowski

For more unsolved problems in combinatorial games see the list of unsolved problems given on pp. 183–189 of AMS *Proc. Sympos. Appl. Math.* **43**(1991), called PSAM **43** below, and its successors on pp. 475–491 of *Games of No Chance,* Cambridge University Press, 1996, hereafter referred to as GONC and in *More Games of No Chance,* Cambridge University Press,2002. An on-line version can be found on David wolfe's site http://www.gac.edu/ wolfe/papers-games/ . Fraenkel's Bibliography of papers about combinatorial games can be found in MGONC or on-line as a dynamic survey in the Electronic Journal of Combinatorics. http://www.combinatorics.org/

1. A **Subtraction game** is played with a heap of counters by two players moving alternately. There is a subtraction set $S = \{s_1, s_2, \ldots, s_k\}$ and on each turn a player may take away s_i counters, for some $i = 1, 2, \ldots, k$, from the heap. I. Althöfer and A. Flammenkamp propose a slight variant: the winner is the first player to make the heap size non-positive rather than the loser is the first player who cannot move where the heap size must be non-negative. A $\mathcal{P}-$position is one in where the **P**revious player has no good move and an $\mathcal{N}-$position one where the **N**ext player has a winning move. The end positions are $\mathcal{P}-$positions and recursively, a position is an $\mathcal{N}-$position if the player to move can move to a $\mathcal{P}-$position, otherwise it is an $\mathcal{N}$-position.

In either variant, the sequence of P and N positions must eventually be periodic since the determination of status of a heap of size n depends on only the k positions $n - s_i$, $i = 1, 2, \ldots, k$ and there are only 2^s possible sequences of length $s = \max\{s \in S\}$. Althöfer and A. Flammenkamp ask:

For a fixed $c > 0$ is it possible to find a subtraction game with maximum $s = \max\{s \in S\}$ and period length 2^{cs}?

The same question can be asked about *partizan* subtraction games, in which each player is assigned an individual subtraction set. See Fraenkel & Kotzig [1987]. For subtraction sets S and T, $S > T$ if for all large enough n, S can always beat T. It is known that this order is non-transitive (i.e., there exists S, T and U with $S > T > U > S$). David Wolfe asks:

Is there an efficient way of comparing subtraction sets?

[For more see Subtraction Games in Winning Ways Berlekamp, Conway, Guy, AKPeters, 2001, pp 83–86, and in the Impartial Games article in GONC.]

2: **Crash** is played on a strip of squares with heaps of red chips and heaps of blue counters. The heaps are called towers. There are two players, Left and Right. Left, on her turn, chooses a heap of blue counters, picks up any number, say k (possibly all), of the counters and distributes then one per square moving to the right. If any of these squares contain red counters then these are removed from the board. Right on his turn, chooses a subset of some red heap and distributes them to the left and removes any blue counters from these squares. The loser is the first player to lose all their counters.

Juerg Nievergelt asks *for a solution* to this game. He remarks that the game with two towers of 3 each has been solved.

J. Nievergelt *et al*, Crash! Mathematik und Kombinatorisches Chaos prallen aufeinander, Informatik Spektram **22** #1, pp. 45–48, Feb. 1999.

3: A dyadic box in d dimensions is of the form $\prod_{i=1}^{d} I_i$ where I_i is an interval $[a/2^s, (a+1)/2^s]$, $0 \leq a < 2^s$. J. Spencer asks

For $d-$dimensions, in how many ways can 2^n dyadic boxes of size $1/2^n$ be chosen so that the $d-$dimensional unit cube be packed?

In 2-dimensions, there are 12 dyadic boxes of size $1/2^2$, 4 $1 \times 1/2^2$ rectangles, 4 $1/2^2 \times 1$ rectangles and the 4 $1/2 \times 1/2$ squares and there are 7 ways of covering the square. Let the number of ways be $f_d(n)$. It is known that $f_2(n) \sim c\alpha^{2n}$. Is it true that $\lim_{n\to\infty} f_d(n)^{1/2^n} = \infty$?

4: Subset sums. Bob Hearn asks:

Given a set S of n positive real numbers is there a set T of m positive real numbers satisfying the following conditions?

(a) any subset sum of S can be represented as a subset sum of T; and

(b) the numbers in T grow exponentially, i.e., each member of T is strictly larger than the sum of all the smaller numbers in T.

Note that if S consists of integers then T can be the powers of 2, from 1 to $2^{\lfloor \log_2(\max s \in S) \rfloor}$

[Note: Part (a) was solved by Stefan Schwarz during the workshop.]

5: Given a directed graph with weighted edges and a distinguished vertex v. The first player chooses an edge directed out from v and thereafter the players, in turn, choose an edge directed out from the terminal vertex of the path. The game ends when a directed cycle is formed. The first player gets a payoff equal to the sum of the weights on the cycle. Her goal is to maximize this payoff and the second players wants to minimize it. Uri Zwick asks:

Can the optimal strategies can be found in polynomial time?

[Polynomial in terms of the binary representation of the weights.]

In a related question, M. Müller notes that if the graph were the positions of a game given in a database then repeating a position would be a draw.

How quickly can one player show that the game is a draw when the second player thinks they have a win?

In the single player case, this is equivalent to finding a Hamiltonian cycle.

6: The **Dreidel** game is played by children at Hannuka. Each child starts with $\$n$. There is a four-sided dreidl (die) marked with N for NISHT (nothing); G for GANZ (everything); H for HALB (half); and S for SHTEL (put in).

VERSION 1: The rules of play are: (1) at the beginning everybody puts \$1 into the pot; (2) While there is money in the pot, each player takes a turn spinning the dreidl. If it comes up N pass; G, take the whole pot; H, take half the pot (round up if odd number is encountered); S, put in \$2. When there is nothing in the pot go back to (1). Play until one player has everyone's money.

VERSION 2: The game finishes when one child has run out of money, i.e., puts their last \$1 into the pot.

C. Banderier asks:

What is the expected length of the game?

He remarks that:

Conjecture 1: D. Zeilberger has a \$25 conjecture that if there are 2 children each with $\$n$ then the expected length is $O(n^2)$;

Conjecture 2: the time to the first ruin, with k players, is $O(n^2)$;

Conjecture 3: Let T be the time to the first ruin, with k players, then

$$\frac{E^2[T]}{V[T]} = \frac{2}{k+1}$$

7: Domineering is played on an $m \times n$ checkerboard. A domino covers exactly 2 squares of the board. Left places her dominoes vertically and Right horizontally. No dominoes may overlap. E. Berlekamp asks:

Is the hottest position in Domineering the 3×3 *square minus a corner?*

This has a value of $\pm 3/2$.

Domineering on $2 \times n$ and $3 \times n$ boards is relatively well understood but not for any wider boards. Berlekamp asks:

What is the maximum value of $(4 \times n) - 2(2 \times n)$*? Is it bounded above by 1?*

The difference is positive since a $4 \times n$ board reduces to 2 $2 \times n$ boards if Left refuses to place a domino across the center line.

8: H. Landman asks:

What is the right search technique for combinatorial games in practice?

We would want that at least each of the Left Stop, Right Stop, Mean Value, and Temperature of the games be computed by the search and accurately bounded during the search. We would also want that the search be maximally efficient (in the sense that alpha-beta is, of visiting as few nodes as possible). Note that we can get the stops by running two alpha-beta searches, one for Left starting and one for Right starting, but that these don't help too much with the temperature. Recent efforts in this direction have been Kao 1999 and Mueller 2000 (Decomposition Search), but they are partial steps and haven't reached the full goal.

9: J. Trump asks about the *complexity of* **Go.** Robson has proved that Go is EXPTIME complete but this was under the simple ko rules — a ko cannot be taken back immediately. What is the complexity under the Super-Ko rule which bans any repetition of a board position? Note that kos can make a game tree have exponential depth so the Super-Ko rule may have an effect.

10: Cash Nim is the game of Nim played with \$1 coins. At the begining of the game, there are n coins distributed in k piles. The winner gets $\$10^n$, the loser gets to keep all the coins that he has removed from the heaps. The player who can win at normal nim [strategy: the winner plays so that she leaves the nim-sum or XOR of the heap sizes to be 0] will still want to win since this gives the largest payoff. M. Albert and R. J. Nowakowski ask

What is the maximum payoff the loser can guarantee himself?

Note that the game with heaps of 15, 12, 3 is a second player win but the first player can take 7 from the 15 heap and the only winning reply is to take 1 from the 12 heap.

[The references can be found in Fraenkel's Bibliography]

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