04351 Abstracts Collection

Spatial Representation: Discrete vs. Continuous Computational Models

— Dagstuhl Seminar —

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Abstract. From 22.08.04 to 27.08.04, the Dagstuhl Seminar 04351 "Spatial Representation: Discrete vs. Continuous Computational Models" was held in the International Conference and Research Center (IBFI), Schloss Dagstuhl. During the seminar, several participants presented their current research, and ongoing work and open problems were discussed. Abstracts of the presentations given during the seminar as well as abstracts of seminar results and ideas are put together in this paper. The first section describes the seminar topics and goals in general. Links to extended abstracts or full papers are provided, if available.

Keywords. Domain theory, formal topology, constructive topology, domain representation, space-time, quantum gravity, inverse limit construction, matroid, geometry, descriptive set theory, Borel hierarchy, Hausdorff diffrence hierarchy,Wadge degree, partial metric, fractafold, region geometry, oriented projective geometry

04351 – Summary: Spatial Representation: Discrete vs. Continuous Computational Models

Topological notions and methods are used in various areas of the physical sciences and engineering, and therefore computer processing of topological data is important. Separate from this, but closely related, are computer science uses of topology: applications to programming language semantics and computing with exact real numbers are important examples. The seminar concentrated on an important approach, which is basic to all these applications, i.e. spatial representation.

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Spatial Representation: Discrete vs. Continuous Computational Models
http://drops.dagstuhl.de/opus/volltexte/2005/174

Keywords: Domain theory, formal topology, constructive topology, domain representation, space-time, quantum gravity, inverse limit construction, matroid, geometry, descriptive set theory, Borel hierarchy, Hausdorff difference hierarchy,Wadge degree, partial metric, fractafold, regiongeometry, oriented projective geometry

Joint work of: Kopperman, Ralph; Panangaden, Prakash; Smyth, Mike; Spreen, Dieter; Webster, Julian

Full Paper: http://drops.dagstuhl.de/opus/volltexte/2005/171

Synthetic Computability Theory

Andrej Bauer (University of Ljubljana, SLO)

I will discuss an axiomatic approach to computability theory in which a computational model (Turing machines) is never mentioned.

Instead, the classical concepts of computability theory are replaced by set-theoretic and topological notions. For example, the computably enumerable sets are just the open subsets of the (discrete!) natural numbers. It turns out that the basic results of computability theory can be derived from just a few simple axioms.

Keywords: Axiomatic computability theory

Continuous Semantics for Termination Proofs

Ulrich Berger (University of Wales - Swansea, GB)

We prove a general strong normalization theorem for higher type rewrite systems based on Tait's strong computability predicates and a strictly continuous domain-theoretic semantics. The theorem applies to extensions of Goedel's system T, but also to various forms of bar recursion for which termination was hitherto unknown.

Keywords: Higher-order term rewriting, termination, domain theory

Full Paper: http://drops.dagstuhl.de/opus/volltexte/2005/130

Computability on Non-Separable Banach Spaces

Vasco Brattka (University of Cape Town, ZA)

While there is a well-established concept of a computable normed space in the separable case, one can prove that there is no way to represent non-separable normed spaces on Turing machines such that both operations, vector space subtraction and the norm, become computable.

However, in certain cases one can at least keep one of these computability properties. We discuss two generalized concepts of computability for normed spaces which allow either the norm or the subtraction to become computable. Using the first concept we can prove a computable version of Landau's Theorem for sequence spaces which suggests that priority should be given to the computability of the norm, i.e. to the first concept.

Keywords: Computable analysis

Compactness in apartness spaces?

Douglas Bridges (University of Canterbury - Christchurch, NZ)

A major problem in the constructive theory of apartness spaces is that of finding a good notion of compactness. Such a notion should (i) reduce to "complete plus totally bounded" for uniform spaces and (ii) classically be equivalent to the usual Heine-Borel-Lebesgue property for the apartness topology. The constructive counterpart of the smallest uniform structure compatible with a given apartness, while not constructively a uniform structure, offers a possible solution to the compactness-definition problem. That counterpart turns out to be interesting in its own right, and reveals some additional properties of an apartness that may have uses elsewhere in the theory.

Keywords: Apartness, constructive, compact uniform space

Joint work of: Bridges, Douglas; Ishihara, Hajime; Schuster, Peter; Vîţă, Luminiţa

Full Paper: http://drops.dagstuhl.de/opus/volltexte/2005/117

Random Causal Sets

Graham R. Brightwell (London School of Economics, GB)

The aim of this talk is to describe various models of random partially ordered sets, and to discuss whether they may be appropriate in a theory of quantum gravity.

The aim is to treat the (discrete) random partial order as the fundamental object, instead of the (continuous) space-time manifold that seems to be the appropriate model for large-scale behaviour.

The first model is obtained by taking 4-dimensional Minkowski space, which comes equipped with a measure and a partial order, and taking n random points in the space or, almost equivalently, taking points according to a Poisson process with the appropriate density. Not surprisingly, the resulting finite partial order retains some of the structure of the underlying space-time. However, the drawback of the model is that it can hardly be thought that the random partial order is a more fundamental object than the manifold itself.

Another family of models has recently been studied by Rafael Sorkin and collaborators in the context of quantum gravity. These are "sequential growth" models, where the $(n+1)$-element partial order is obtained from the n-element partial order by adding a maximal element according to some stochastic rule. Examples of such a rule include: "put the new element above each existing element with fixed probability p, then take the transitive closure", or "put the new element above exactly two randomly chosen existing element, then take the transitive closure". The first of these examples corresponds to a model of random orders that has been studied in the combinatorial community; unfortunately neither it nor any variant exhibits manifold-like behaviour, although some of the properties are tantalising analogues to those of the "real" universe.

In the talk, details are given of some of the global properties of the various random models, and an explanation is given of a result of Rideout and Sorkin showing that the sequential growth models arise naturally from some physically reasonable axioms.

Keywords: Quantum gravity, causal sets, Minkowski space, random poset

Discrete classical vs. continuous quantum data in abstract quantum mechanics

Bob Coecke (Oxford University, GB)

"Quantum" stands for for the concepts (both operational and formal) which had to be added to classical physics in order to understand otherwise unexplainable observed phenomena such as the structure of the spectral lines in atomic spectra. While the basic part of classical mechanics deals with the (essentially) reversible dynamics, quantum required adding the notions of "measurement" and (possibly non-local) “correlations” to the discussion. Crucially, all this comes with a “probabilistic calculus”. The corresponding mathematical formalism was considered to have reached maturity in [von Neumann 1932], but there are some manifest problems with that formalism:

(i) While measurements are applied to physical systems, application of their formal counterpart (i.e. a self-adjoint linear operator) to the vector representing that state of the system in no way reflects how the state changes during the act of measurement. Analogously, the composite of two self-adjoint operators has no physical significance while in practice measurements can be effectuated sequentially. More generally, the formal types in von Neumann’s formalism do not reflect the nature of the corresponding underlying concept at all!

(ii) Part of the problem regarding the measurements discussed above is that in the von Neumann formalism there is no place for storage, manipulation and exchange of the classical data obtained from measurements. Protocols such as quantum teleportation involving these cannot be given a full formal description.

(iii) The behavioral properties of quantum entanglement which for example enable continuous data exchange using only finitary communication are hidden in the formalism.

In [Abramsky and Coecke 2004] we addressed all these problems, and in addition provided a purely categorical axiomatization of quantum mechanics. The concepts of the abstract quantum mechanics are formulated relative to a strongly compact closed category with biproducts (of which the category FdHilb of finite dimensional Hilbert spaces and linear maps is an example). Preparations, measurements, either destructive or not, classical data exchange are all morphisms in that category, and their types fully reflect their kinds. Correctness properties of standard quantum protocols can be abstractly proven.

Surprisingly, in this seemingly purely qualitative setting even the quantitative Born rule arises, that is the rule which tells you how to calculate the probabilities. Indeed, each such category has as endomorphism Hom of the tensor unit an abelian semiring of ‘scalars’, and a special subset of these scalars will play the role of weights: each scalar induces a natural transformation which propagates through physical processes, and when a ‘state’ undergoes a ‘measurement’, the composition of the corresponding morphisms gives rise to the weight. Hence the probabilistic weights live within the category of processes.

See also: J. von Neumann. Mathematische Grundlagen der Quantenmechanik. Springer-Verlag (1932). English translation in Mathematical Foundations of Quantum Mechanics. Princeton University Press (1955).

See also: S. Abramsky and B. Coecke. A categorical semantics of quantum protocols. In the proceedings of LiCS’04 (2004).

See also: An extended version is available at arXiv:quant-ph/0402130 A more reader friendly version entitled ‘Quantum information flow, concretely, abstractly’ is at http://www.vub.ac.be/CLEA/Bob/Papers/QPL.pdf

Keywords: Category theory, strong compact closure, quantum information-flow

Joint work of: Abramsky, Samson; Coecke, Bob

Extended Abstract: http://drops.dagstuhl.de/opus/volltexte/2005/131

Full Paper: http://drops.dagstuhl.de/opus/volltexte/2005/131

A Hausdorff compactification of the Samborski function space

Martin H. Escardo (University of Birmingham, GB)

If X is a compact Hausdorff space and $\mathbb{R}$ is the set of compact non-empty intervals of the extended real line, then the Scott continuous functions from X to $\mathbb{R}$ under the Lawson topology have the Samborski function space, described in Jimmie Lawson's talk, as a densely embedded subspace.

Keywords: Interval domain, Scott continuous function, Lawson topology, Samborski function space

Dihomotopy Classes of Dipaths in the Geometric Realization of a Cubical Set: from Discrete to Continuous and back again

Lisbeth Fajstrup (Aalborg University, DK)

The geometric models of concurrency - Dijkstra's PV-models and V. Pratt's Higher Dimensional Automata - rely on a translation of discrete or algebraic information to geometry. In both these cases, the translation is the geometric realisation of a semi cubical complex, which is then a locally partially ordered space, an lpo space. The aim is to use the algebraic topology machinery, suitably adapted to the fact that there is a preferred time direction. Then the results - for instance dihomotopy classes of dipaths, which model the number of inequivalent computations should be used on the discrete model and give the corresponding discrete objects. We prove that this is in fact the case for the models considered: Each dipath is dihomottopic to a combinatorial dipath and if two combinatorial dipaths are dihomotopic, then they are combinatorially equivalent. Moreover, the notions of dihomotopy (LF., E. Goubault, M. Raussen) and d-homotopy (M. Grandis) are proven to be equivalent for these models - hence the Van Kampen theorem is available for dihomotopy. Finally we give an idea of how many spaces have a local po-structure given by cubes. The answer is, that any cubicalized space has such a structure after at most one subdivision. In particular, all triangulable spaces have a cubical local po-structure.

Keywords: Cubical Complex, Higher Dimensional Automaton, Ditopology

Extended Abstract: http://drops.dagstuhl.de/opus/volltexte/2005/132

Fractafolds, their geometry and topology: a test bed for spatial reprsentation. PartII: The Sorkin Model, The Mallios-Raptis-Zapatrin Differential Calculus, Differential Graded Bi-Algebras and Approximations to the Exterior Calculus

Jonathan Gratus (University of Wales - Bangor, GB)

The long term goal of this project is to construct a geometry of fractals and "fractafolds". Since the standard differential geometry applies to smooth manifolds only, it is necessary to construct a new "geometry" which can be applied to fractals and fractafolds but in which the standard geometry can be recovered in the special case of a smooth manifolds. Many fractals are obtained by the limit of simple spaces, using an iterative function sequence. We hope that we can construct a differential geometry for each of the simple spaces. And then by taking the limit we would obtain the required geometry. We use a combination of the Sorkin model and the Mallios Raptis and Zapatrin MRZ calculus to construct a finite differential geometry for each of the sequence of simple spaces.
In my talk,

I shall review the Sorkin model and look at its limit.

I also shall review the MRZ calculus an look at its limit.

It turns out that the limit of the Sorkin model is too big and the limit of the MRZ calculus too small.

I give two suggestions for the alternative limits which may have the correct properties.

There is also a problem in applying the MRZ calculus directly to the Sorkin model, especially when taking refinements in 2 or more dimension. I propose a extension to the MRZ calculus called a differential graded bi-algebras. These can be used to avoid this problem.

Keywords: Fractals, iterative function sequence, differential geometry, Sorkin model

A Cartesian Closed Extension of the Category of Locales

Reinhold Heckmann (AbsInt - Saarbrücken, D)

We present a Cartesian closed category ELOC of equilocales, which contains the category LOC of locales as a reflective full subcategory.

The embedding of LOC into ELOC preserves products and all exponentials of exponentiable locales.

Keywords: Locale, Cartesian closed category

Extended Abstract: http://drops.dagstuhl.de/opus/volltexte/2005/133

Full Paper: http://drops.dagstuhl.de/opus/volltexte/2005/133

Refinement is complete for implementations

Michael Huth (Imperial College London, GB)

A modal transition system specifies a set of implementations, those labelled transition systems that co-inductively refine it. As refinement is transitive, refinement implies reverse containment of implementations. We demonstrate that refinement indeed equals reverse containment of implementations. This result is shown through a game semantics for refinement on finite trees and an approximation argument based on an omega-algebraic bifinite domain, developped with Jagadeesan & Schmidt, whose elements "are" all equivalence classes of modal transition systems under refinement. We give a characterization of this result in terms of semantic self-minimization of certain Hennessy-Milner formulas and state the existence of semantic minimization for all of Hennessy-Milner logic as an open problem.

Keywords: Refinement, game semantics, domain theory

See also: A full paper has been submitted to a journal and is currently under review.

Auxiliary relations and sandwich theorems

Ralph Kopperman (City University of New York, USA)

A well-known topological theorem due to Katětov states:

Suppose (X, τ) is a normal topological space, and let $f : X \to [0, 1]$ be upper semicontinuous, $g : X \to [0, 1]$ be lower semicontinuous, and $f \leq g$. Then there is a continuous $h : X \to [0, 1]$ such that $f \leq h \leq g$.

We show a version of this theorem for many posets with auxiliary relations. In particular, if P is a Scott domain and $f, g : P \to [0, 1]$ are such that $f \leq g$, and f is lower continuous and g Scott continuous, then for some h, $f \leq h \leq g$ and h is both Scott and lower continuous.

As a result, each Scott continuous function from P to $[0, 1]$, is the sup of the functions below it which are both Scott and lower continuous.

Keywords: Adjoint, auxiliary relation, continuous poset, pairwise completely regular (and pairwise normal) bitopological space, upper (lower) semicontinuous, Urysohn relation

Joint work of: God, Chris; Jung, Achim; Knight, Robin; Kopperman, Ralph

Full Paper: http://drops.dagstuhl.de/opus/volltexte/2005/134

Hofmann-Mislove theorem for general order and topological structures

Martin Kovar (Techn. University - Brno, CZ)

We study and characterize the posets which satisfy a properly generalized version of the Hofmann-Mislove theorem. For that purpose, we need to generalize some notions (like compactness, prime elements, the spectrum, Scott- open filter, etc.) and adjust them for more general posets than frames and the corresponding localic structures. We also study the topology-like structures induced by the order and prove a slightly relaxed topological-like version of the Hofmann-Mislove theorem which holds for more general spaces, in particular also for those which are not sober. The utility of this modification of the Hofmann-Mislove theorem may be demonstrated on the study of the maximality of compact topologies.

Keywords: Posets, upper interval topology, Scott topology, wide filters, compactness, prime elements, sobriety, de Groot dual, maximal compact topology

On Maximality of Compact Topologies

Martin Kovar (Techn. University - Brno, CZ)

Using some advanced properties of the de Groot dual and some generalization of the Hofmann-Mislove theorem, we solve in the positive the question of D. E. Cameron: Is every compact topology contained in some maximal compact topology?

Keywords: De Groot dual, compact saturated set, wide Scott open filter, maximal compact topology

Full Paper: http://drops.dagstuhl.de/opus/volltexte/2005/118

The Hofmann-Mislove Theorem for general posets

Martin Kovar (Techn. University - Brno, CZ

In this paper we attempt to find and investigate the most general class of posets which satisfy a properly generalized version of the Hofmann-Mislove theorem. For that purpose, we generalize and study some notions (like compactness, the Scott topology, Scott open filters, prime elements, the spectrum etc.), and adjust them for use in general posets. Then we characterize the posets satisfying the Hofmann-Mislove theorem by the relationship between the generalized Scott closed prime subsets and the generalized prime elements of the poset. The theory become classic for distributive lattices. Remark that the topologies induced on the generalized spectra in general need not be sober.

Keywords: Posets, generalized Scott topology, Scott open filters, (filtered) compactness, saturated sets, prime elements, prime subsets

Full Paper: http://drops.dagstuhl.de/opus/volltexte/2005/119

The Hofmann-Mislove Theorem for general topological structures

Martin Kovar (Techn. University - Brno, CZ)

In this paper we prove a modification of Hofmann-Mislove theorem for a topological structure similar to the minusspaces of de Groot, in which the empty set "need not be open". This will extend, in a slightly relaxed form, the validity of the classical Hofmann-Mislove theorem also to some of those spaces, whose underlying topology need not be (quasi-) sober.

Keywords: Compact saturated set, Scott open filter, (quasi-) sober space

Full Paper: http://drops.dagstuhl.de/opus/volltexte/2005/120

The de Groot dual for general collections of sets

Martin Kovar (Techn. University - Brno, CZ)

A topology is de Groot dual of another topology, if it has a closed base consisting of all its compact saturated sets. Until 2001 it was an unsolved problem of J. Lawson and M. Mislove whether the sequence of iterated dualizations of a topological space is finite. In this paper we generalize the author's original construction to an arbitrary family instead of a topology. Among other results we prove that for any family $\mathfrak{C} \subseteq 2^X$ it holds $\mathfrak{C}^{dd} = \mathfrak{C}^{dddd}$. We also show similar identities for some other similar and topology-related structures.

Keywords: Saturated set, dual topology, compactness operator

Full Paper: http://drops.dagstuhl.de/opus/volltexte/2005/121

The Construction of Finer Compact Topologies

Hans-Peter Albert Künzi (University of Cape Town, ZA)

It is well known that each locally compact strongly sober topology is contained in a compact Hausdorff topology; just take the supremum of its topology with its dual topology. On the other hand, examples of compact topologies are known that do not have a finer compact Hausdorff topology.

This led to the question (first explicitly formulated by D.E. Cameron) whether each compact topology is contained in a compact topology with respect to which

all compact sets are closed. (For the obvious reason these spaces are called maximal compact in the literature.)

While this major problem remains open, we present several partial solutions to the question in our talk.

For instance we show that each compact topology is contained in a compact topology with respect to which convergent sequences have unique limits. In fact each compact topology is contained in a compact topology with respect to which countable compact sets are closed. Furthermore we note that each compact sober T_1-topology is contained in a maximal compact topology and that each sober compact T_1-topology which is locally compact or sequential is the infimum of a family of maximal compact topologies.

Keywords: Maximal compact, KC-space, sober, US-space, locally compact, sequential, sequentially compact

Joint work of: Künzi, Hans-Peter A.; van der Zypen, Dominic

Extended Abstract: http://drops.dagstuhl.de/opus/volltexte/2005/122

Approximate and Quasicontinuous Maps and Generalized Derivatives

Jimmie D. Lawson (Louisiana State University, USA)

A continuous domain is a set equipped with a special kind of partial order.

One typically thinks of the members of the set as representing approximations, perhaps computable approximations, to some idealized mathematical construct or precise quantity. The system of approximations is ordered by the order of information content, better approximations being higher in the order, with the maximal or top elements representing the quantities being approximated. If one applies these ideas to the domain of continuous functions of a real variable and their approximations, one obtains naturally an enlarged family of maximal elements which admits a differential calculus extending the familiar one. We call this the generalized derivative. This type of extension is motivated by the so-called viscosity solutions of partial differential equations.

Keywords: Domains, domain environments, lsc functions, quasicontinuous functions, maximal points, generalized function, viscosity solution

What do partial metrics represent?

Steve G. Matthews (University of Warwick, GB)

Partial metrics were introduced in 1992 as a metric to allow the distance of a point from itself to be non zero. This notion of self distance, designed to extend metrical concepts to Scott topologies as used in computing, has little intuition for the mainstream Hausdorff topologist.

The talk will show that a partial metric over a set can be represented by a metric over that set with a so-called 'base point'.

Thus we establish that a partial metric is essentially a structure combining both a metric space and a skewed view of that space from the base point. From this we can deduce what it is that partial metrics are really all about.

Keywords: Metric, partial metric, base point

Joint work of: Kopperman, Ralph; Matthews, Steve; Pajoohesh, Homeira

Full Paper: http://drops.dagstuhl.de/opus/volltexte/2005/123

Algebras over Stably Compact Spaces

M. Andrew Moshier (Chapman University - Orange, USA)

Free algebras in a topological category are useful for modelling various computationally motivated data types. Notably semi-lattices model non-determinism. In this talk, we establish two related results. [1] The embedding of stably compact spaces and perfect maps into stably compact spaces and continuous maps has a left adjoint. [2] In the category of stably compact spaces and perfect maps, free algebras for finitely axiomatizable theories exist. Combining the two results, we have a construction of free "perfect" algebras over stably compact spaces, i.e., algebras in which the operations are perfect, but the homomorphisms are permitted to be continuous.

As an application, we also discuss the construction of free inflationary and deflationary semi-lattices (aka power domains) over stably compact spaces, showing that the Hoare powerdomain is indeed free over the entire category of stably compact spaces and continuous maps, whereas the Smyth powerdomain is only free in the more limited sense.

Keywords: Stably compact spaces, perfect maps, free algebras, power domains

Generalized partial metrics

Homeira Pajoohesh (University College Cork, IRL)

Partial metrics are metrics except that the distance from a point to itself need not be 0. These are useful in modelling partially defined information, which often appears in computer science.

We generalize this notion to study "partical metrics" whose values lie in a value lattice which may be other that the reals. Then each topology arises from such a generalized metric, and for each continuous poset, there is such a generalized metric whose topology is the Scott topology, and whose dual is the lower topology. These are both corollaries to our result that a bitopological space is pairwise completely regular if and only if there is such a generalized metric whose topology is the first topology, and whose dual topology is the second.

Then we study their completion and state the relation between spherical completion and round ideal completion.

Keywords: Partial metrics, generalized partial metrics, value lattice, continuous posets, Scott topology, lower topology, bitopological space, completions

A domain of spacetime intervals in general relativity

Prakash Panangaden (McGill University - Montreal, CDN)

We prove that a globally hyperbolic spacetime with its causality relation is a bicontinuous poset whose interval topology is the manifold topology. This implies that from only a countable dense set of events and the causality relation, it is possible to reconstruct a globally hyperbolic spacetime in a purely order theoretic manner. The ultimate reason for this is that globally hyperbolic spacetimes belong to a category that is equivalent to a special category of domains called interval domains.

We obtain a mathematical setting in which one can study causality independently of geometry and differentiable structure, and which also suggests that spacetime emanates from something discrete.

Keywords: Causality, spacetime, global hyperbolicity, interval domains, bicontinuous posets, spacetime topology

Joint work of: Martin, Keye; Panangaden, Prakash

Full Paper: http://drops.dagstuhl.de/opus/volltexte/2005/135

Computational Topology and Spline Surfaces

Thomas J. Peters (Univ. of Connecticut, USA)

The main objective of this talk is to show how spline surfaces are used for approximate representations of manifolds, with applications in engineering and molecular modeling.

Intersection algorithms are fundamental to forming complex geometric models from splines and work on the I-TANGO project (Intersections – Topology, Accuracy and Numerics for Geometric Objects) will be reported, both completed results and work in progress. Finally, the use of knot theory in both engineering and biological applications will be discussed. Supporting animations will be presented to aid intuition.

Keywords: Computational topology, spline, approximation

Extended Abstract: http://drops.dagstuhl.de/opus/volltexte/2005/124

A Category of Discrete Closure Spaces

John L. Pfaltz (University of Virginia, USA)

Discrete systems such as sets, monoids, groups are familiar categories.

The internal strucutre of the latter two is defined by an algebraic operator.

In this paper we describe the internal structure of the base set by a closure operator. We illustrate the role of such closure in convex geometries and partially ordered sets and thus suggestthe wide applicability of closure systems.

Next we develop the ideas of closed and complete functions over closure spaces. These can be used to establish criteria for asserting that "the closure of a functional image under f is equal to the functional image of the closure". Functions with these properties can be treated as categorical morphisms. Finally, the category "CSystem" of closure systems is shown to be cartesian closed.

Keywords: Category, closure, antimatroid, function

Full Paper: http://drops.dagstuhl.de/opus/volltexte/2005/125

A geometry of information, I: Nerves, posets and differential forms

Timothy Porter (University of Wales - Bangor, GB)

The main theme of this workshop is ‘Spatial Representation: Continuous vs. Discrete’. Spatial representation has two contrasting but interacting aspects (i) representation *of* spaces’ and (ii) representation *by* spaces. In this paper we will examine two aspects that are common to both interpretations of the theme, namely nerve constructions and refinement. Representations change, data changes, spaces change. We will examine the possibility of a ‘differential geometry’ of spatial representations of both types, and in the sequel give an algebra of differential forms that has the potential to handle the dynamical aspect of such a geometry. We will discuss briefly a conjectured class of spaces, generalising the Cantor set which would seem ideal as a test-bed for the set of tools we are developing.

Keywords: Chu spaces, nerves, differential forms

Joint work of: Gratus, Jonathan; Porter, Timothy

Full Paper: http://drops.dagstuhl.de/opus/volltexte/2005/126

A geometry of information, II: Sorkin models, and biextensional collapses

Timothy Porter (University of Wales - Bangor, GB)

In this second part of our contribution to the workshop, we look in more detail at the Sorkin model, its relationship to constructions in Chu space theory, and then compare it with the Nerve constructions given in the first part.

Keywords: Chu space, Sorkin model, Nerve

Joint work of: Jonathan Gratus; Porter, Timothy

Full Paper: http://drops.dagstuhl.de/opus/volltexte/2005/127

Deadlocks and Dihomotopy in Mutual Exclusion Models

Martin Raussen (Aalborg University, DK)

Parallel processes in concurrency theory can be modelled in a geometric framework. A convenient model are the Higher Dimensional Automata of V. Pratt and E. Goubault with cubical complexes as their mathematical description. More abstract models are given by (locally) partially ordered topological spaces, the directed (d-spaces) of M.Grandis and the flows of P. Gaucher. All models invite to use or modify ideas from algebraic topology, notably homotopy.

In specific semaphore models for mutual exclusion, we have developed methods and algorithms that can detect deadlocks and unsafe regions and give information about essentially different schedules using higher dimensional "geometric" representations of the state space and executions (directed paths) along it.

Keywords: Mutual exclusion, deadlock detection, dihomotopy

Extended Abstract: http://drops.dagstuhl.de/opus/volltexte/2005/136

Representation of Real Numbers as Continued Radicals

Tom Richmond (Western Kentucky Univ. - Bowling Green, USA)

A nested radical with terms $a_1, a_2, \ldots, a_N$ is an expression of form $\sqrt{a_N + \cdots + \sqrt{a_2 + \sqrt{a_1}}}$. The limit as N approaches infinity of such an expression, if it exists, is called a continued radical. We consider the set of real numbers $S(M)$ representable as a continued radical whose terms $a_1, a_2, \ldots$ are all from a finite set M of nonnegative real numbers. We give conditions on the set M for $S(M)$ to be (a) an interval, and (b) homeomorphic to the Cantor set.

Keywords: Continued radical

Joint work of: Johnson, Jamie; Richmond, Tom

Extended Abstract: http://drops.dagstuhl.de/opus/volltexte/2005/128

Overlap Algebras, That Is, Towards a Justification of Definitions in Formal Topology and the Basic Picture

Giovanni Sambin (Universitá di Padova, I)

A careful analysis from the predicative perspective of the notion of topological space $(X, \mathcal{OX})$, where X is the set of points and $\mathcal{OX}$ the family of open subsets, leads to the introduction of a second set S and a function $\mathsf{ext} : \mathcal{PX} \leftarrow \mathcal{S}$ such that $\mathsf{ext}\,(a) \subseteq X (a \in S)$ is a base for open subsets. By looking at ext as a relation $x \Vdash a \equiv x \in \mathsf{ext}\,(a)$, one is lead to consider basic pairs, that is structures $(X, \Vdash, S)$ where X, S are two sets and $\Vdash$ any relation between them. In this setting, one can define basic notions of topology, like open, closed, cover, positivity, continuity, ..., and see that they have a deep structural justification, based on symmetry and logical duality. In particular, the notion of overlap $\between$ is dual to that of inclusion $\subseteq$, and that of positivity $\ltimes$ is dual to that of cover $\lhd$. I have given the name "the basic picture" to the mathematical theory resulting from this.

Standard topology, either pointwise or pointfree, is obtained by adding consideration of convergence.

However, I claim that the raison d'etre of the basic picture is not just ancillary to that of topology, but that it has an intrinsic mathematical motivation. With this aim in mind, I follow the algebraic approach and introduce the new notion of overlap algebra. This is an algebraic description of the power of a set in which overlap $\between$ is also considered. The presence of overlap increases the expressive power considerably, to the point that most of the basic picture, and hence of topology itself, can be formulated in algebraic terms.

This seems to confirm that the basic picture has a deep motivation, which has an algebraic nature and hence is independent of the predicative foundation adopted.

Keywords: Formal topology, basic picture, constructive and predicative foundation

Co-countably based Spaces and Their Relationship to Complexity Theory

Matthias Schröder (University of Edinburgh, GB)

We call a topological space co-countably-based iff it is sequential and the Scott topology on the lattice of its opens has a countable base.

Representations of spaces are the key device in Type-2 Theory of Effectivity (TTE) for defining computability on non-countable spaces. 'Almost-compact' representations are defined in such a way that they allow the measurement of time complexity in terms of a *discrete* property of the input (and in terms of the output precision).

We show that a sequential Hausdorff space has an almost-compact admissible representation if and only if it is co-countably based.

Moreover, we state some closure properties of the category of co-countably based spaces.

Keywords: Topological spaces, type-2 theory of effectivity

The projective spectrum as a distributive lattice

Peter Schuster (Universität München, D)

The spectrum of a commutative ring is the collection of prime ideals endowed with the Zariski topology. To arrive at a point-free description of this topological space, one can define the corresponding distributive lattice directly from the ring, without any talk of prime ideals. This idea, which presumably is due to Joyal, has recently been reactivated to start a first-order treatment of commutative algebra.

By continuing this road, we bring the projective spectrum of a graded ring into the point-free realm. More specifically, we present a distributive lattice whose prime filters correspond to the homogeneous prime ideals of the graded ring. This also is the prime example of a non-affine scheme in the context of formal topology, and turns out to be a particular case of a fairly general glueing method for finitely many distributive lattices. We furthermore prove the projective form of the formal Hilbert Nullstellensatz, from which we achieve a concrete description of the projective lattice as a by-product.

Keywords: Projective spectrum, graded ring, distributive lattice, formal topology, first-order concepts

Joint work of: Coquand, Thierry; Lombardi, Henri; Schuster, Peter

Towards a descriptive set theory in domain-like structures

Victor Selivanov (Pedagogical University - Novosibirsk, RUS)

To my knowledge, so far there was no systematic attempt to develop a descriptive set theory (DST) in domain-like structures, in parallel to the well-known DST in Polish spaces. Some early papers by Adrian Tang and by the author considered only a couple of particular questions related to this problem.

This talk is devoted to the development of the Borel hierarchy, Hausdorff difference hierarchy and Wadge degrees in Ershov's φ-spaces, which are topological counterparts of algebraic directed-complete partial orderings. The difference hierarchy is understood rather comprehensively. We give an application of the developed theory to a natural classification problem about infinitary Boolean operations on Cantor space which are well-known in classical DST. We also show that there exist natural classes of φ-spaces where the Wadge reducibility behaves much better than in some classical spaces like the space of reals.

Keywords: Descriptive set theory, domain theory, Wadge reducibility

Topological Domains and the Probabilistic Powerdomain

Alex Simpson (University of Edinburgh, GB)

In previous talks I have introduced the notion of *topological predomain*. The category of topological predomains and continuous functions enjoys all the usual closure conditions of categories of dcpos in domain theory, and more.

For quite abstract reasons, topological predomains are closed under the construction of a space of continuous probabilistic valuations, which thus acts as a candidate probabilistic powerdomain.

There is also a second notion of probabilistic powerdomain based on a computationally natural definition of probabilistic process over a topological predomain. In the talk I discuss the (open) question of relating the two notions, and relate it to the (open) problem of whether the sobrification of a qcb (quotient of countably-based) space is again qcb.

Keywords: Topological predomain, closure properties, valuations, probabilistic process

Distance in Graphs and Hyperconvexity

Michael B. Smyth (Imperial College London, GB)

We consider distance in digital spaces, in its topological and categorical aspects. Concretely, the spaces are (not necessarily reflexive) graphs.

Grahs are viewed as closure spaces, and in order to induce the right closure structure, we require a distance function different from the usual graph metric.

In the second part we look at the category-theoretic aspect, specifically injectivity. The injective objects can be characterized as the hyperconvex spaces, suitably generalized.

Keywords: Closure space; semi-metric; hyperconvex

Is spacetime a past-finite poset?

Rafael D. Sorkin (Syracuse University, USA)

The causal set – mathematically a locally finite ordered set or "poset" – is a candidate discrete substratum for spacetime. I will introduce this idea and describe some aspects of causal set kinematics, dynamics, and phenomenology, including, as time permits, a notion of fractal dimension, a stochastic growth dynamics, and an idea for explaining some of the puzzling large numbers of cosmology. I will also mention some questions of mathematical interest that have arisen in this connection.

Keywords: Causal set, spacetime, kinematics, dynamics, fractal dimension, stochastic growth dynamics, cosmology

Uniform approximation in domains

Dieter Spreen (Universität Siegen, D)

In his search for maximal cartesian closed full subcategories of continuous domains A. Jung discovered the class of FS-domains. These are pointed directed-complete partial orders coming with a directed family of continuous self-maps which are below the identity and have the identity as their least upper bound. In addition, for each of these maps there is a finite set of domain elements separating domain elements from their images.

In order to characterize FS-domains Kummetz as well as Jung and Sünderhauf considered domains coming with such a directed family of self-maps which need not have finite separating sets.

Also in this more general case the family of self-maps furnishes the domain with a uniform way of approximating the domain elements. It also gives rise to the introduction of natural quasi-uniformities. Here, two such quasi-uniformities are studied.

First necessary and sufficient conditions are derived for when the topologies induced by the quasi-uniformities and their associated uniformities, respectively, are the Scott and the Lawson topologies. It turns out that the conditions thus obtained force the given self-maps to be what Jung and Sünderhauf called approximating. This leads to a further characterization of the FS-domains.

For any topological space there is always a quasi-uniformity generating the given topology, namely the Pervin quasi-uniformity. It turns out that the quasi-uniformities studied here are coarser than the Pervin quasi-uniformity with respect to the Scott topology exactly if the domain is FS. In one case it even has to be an algebraic FS-domain, i.e., an SFP-domain.

Keywords: FS-domains, SFP- domains, quasi-uniformity, Pervin quasi-uniformity, Scott topology, Lawson topology

Spatio-Temporal Mereotopology

John Stell (University of Leeds, GB)

Theories of space based on a Boolean algebra of regions equipped with a relation of connection have been been axiomatized in various ways and have been applied to areas such as qualitative reasoning in artificial intelligence. Most work on these mereotopological theories has taken regions to be purely spatial, but we can consider how the theory can be developed so as to provide an account of spatio-temporal regions. One approach is that of Muller which introduces additional primitive relations of temporal connection and temporal precedence. In this talk I will present an alternative account based on an historical closure operator, and a pair of further operators, called pre-history and post-history, which form a Galois connection. This significantly simplifies several aspects of Muller's axiomatization.

Keywords: Region, Boolean algebra

Joint work of: Stell, John; West, Matthew

Contractible Digraphs, Fixed Points and the Cop-robber Games

Rueiher Tsaur (Imperial College London, GB)

In this paper, a non-recursive definition of "dismantlability" is defined for discrete structures, thus unifies various definitions of dismantlable structures.

Keywords: Dismantlability

Dyadic subbases : representing spaces as sequences with bottoms

Hideki Tsuiki (Kyoto University, J)

We explain topological properties of the embedding-based approach to computability on topological spaces. With this approach, he considered a special kind of embedding of a topological space into Plotkin's T^ω, which is the set of infinite sequences of $T = \{0, 1, \perp\}$.

We show that such an embedding can also be characterized by a dyadic subbase, which is a countable subbase $S = (S_0^0, S_0^1, S_1^0, S_1^1, \ldots)$ such that S_n^j ($n = 0, 1, 2, \ldots; j = 0, 1$ are regular open and S_n^0 and S_n^1 are exteriors of each other. We survey properties of dyadic subbases which are related to efficiency properties of the representation corresponding to the embedding.

Keywords: Dyadic subbase, embedding, computation over topological spaces, Plotkin's T^ω

Full Paper: http://drops.dagstuhl.de/opus/volltexte/2005/137

Stable Compactification and the Causal Boundary of Minkowski Space Time (Work in progress)

Steven J. Vickers (University of Birmingham, GB)

In a sober topological space X, the points can be recovered from the lattice $\mathcal{O}(X)$ of opens as the completely prime filters - the subsets F of $\mathcal{O}(X)$ that are filters (upper closed, non-empty and closed under binary intersections), and completely prime (if a union of a family is in F, then the family meets F). In fact this is the definition of sober. It covers a wide range of useful topological spaces, including all Hausdorff ones.

Smyth has modified this idea in his "stable compactification". He takes a space X whose topology $\mathcal{O}(X)$ comes equipped with an additional "strong" order $<$ with certain properties. The usual intuition is that $U < V$ if U is included in V "with a bit to spare" - U can be expanded slightly in a uniform way and still be in V. He then defines a "proximal filter" of $\mathcal{O}(X)$ to be a certain kind of filter F in $\mathcal{O}(X)$, and generates a topology from subbasic opens

$$\{ F \mid U \in F \}$$

for $U \in \mathcal{O}(X)$.

One aspect of the interest of stably compact spaces is that they are equivalent to compact Hausdorff spaces equipped with a certain kind of partial order (on the points, this time, not the opens). The points are (classically) the same, but the opens are different. The opens in the stably compact space are the opens in the compact Hausdorff spaces that are also upper closed in the order.

An ordered space of central importance in physics is Minkowski space-time M, equipped with its causal order (space-time point x is *causally before* y if y is within or on the future light cone starting from x).

M is Hausdorff but not compact, so the usual equivalence breaks down. However, physicists have found ways in which they wish to compactify M by adjoining points at infinity - one point each at future time-like infinity, space-like infinity, and past time-like infinity, and families of points at future and past null infinities. [Geroch, Kronheimer and Penrose] The new points are the "causal boundary" of M.

Since this now is an ordered compact Hausdorff space, it corresponds to a stably compact space and I set out to investigate whether this could be obtained by a Smyth stable compactification of the original space M equipped with its *causal* topology T_c, in which the opens are the sets that are open in the Euclidean topology T_e but also future closed.

The GKP construction is done in two kinds of steps.

The first lies in taking the "past indecomposable sets" of M. These correspond to the original points of M, together with future time-like and null infinities and a space-like infinity.

A dual step takes the future indecomposable sets, and the second kind of step is to glue these together, identifying the copies of M and identifying the space-like infinities.

I have shown that the past indecomposable sets are in bijection with the irreducible closed sets for (M, T_c), as used in soberifying (M, T_c), except for the one corresponding to space-like infinity. (This would correspond to the empty closed set, which is not irreducible.)

In effect, the past part of the GKP construction soberifies. The future infinities that it adjoins are directed joins of points of M. The dual, future part, while obviously soberifying dually, also makes the space compact by adjoining a bottom point in the past time-like infinity, and contributes towards making it stably compact by adjoining the past null infinities.

However, neither soberification adjoins any space-like infinities.

A Smyth stable completion, as a one-step construction, is in that respect simpler than the GKP construction with its glueing of two separate parts. It will inevitably soferify, but a question of interest is whether it can adjoin space-like infinities.

The strong order under investigation is that for which $U < V$ if for some $\epsilon > 0$, the union of the epsilon-balls round points of U is included in V.

Other than the space-like infinity, I have identified the points of the GKP compactification as proximal filters of T_c with respect to $<$. Further analysis needs to be done. However, it already seems clear that there are more than one point at space-like infinity.

References:

Smyth: "Stable compactification I", J. London Math. Soc. (2), vol 45 (1992), pp. 321-340.

GKP: See Hawking and Ellis "The large scale structure of space-time", CUP 1973.

Keywords: Minkowski space-time, sober, “stably compact”

Aspects of apartness

Luminita Simona Vita (University of Canterbury - Christchurch, NZ)

Starting with a primitive notion of pre–apartness on Boolean algebras, we develop a constructive approach (in Bishop’s style) to the classical structures of proximity spaces. We also show how our model may be linked to similar structures like the Boolean Connection Algebras used in the theory of Region Connection Calculus.

Keywords: Apartness, proximity spaces

Partial metrization of continuous domains

Pawel Waszkiewicz (University of Krakow, PL)

We present three conditions that are necessary and sufficient for partial metrization of continuous domains.

Keywords: Partial metric, measurement, radially convex metric, abstract basis, continuous poset

Finite spherical space

Julian Webster (Imperial College London, GB)

Oriented matroids are, roughly, combinatorial Euclidean geometries, and can serve as a basis for spatial representation in several areas of computer science: e.g. digital geometry/topology, GIS, computational geometry.

Although finite, oriented matroids are in practice far too large to feasibly store, and background Euclidean space is needed to compute their structure. I am interested in trying to push the combinatorial method further by developing finite algebraic models that have a Euclidean geometry - an oriented matroid structure.

In this regard, symmetry of the n-sphere seems to have an important role. One of the most basic aspects of oriented matroid theory is their close relationship with the geometry/topology of the n-sphere. Moreover, the recent work of Borovik, Gelfand and White on Coxeter Matroids (a generalization of matroids based on Coxeter groups) seems highly relevant to the investigation.

Keywords: Oriented matroids, matroids, Coxeter groups, the Symmetric group, spherical geometry

From Algebraic Lattices to Approximable Concepts - connecting the "dots"

Guo-Qiang Zhang (Case Western Reserve University, USA)

This talk provides an overview of several widely used mathematical structures, both old and recent, such as algebraic lattices, join-semilattices with bottom, Scott continuous closure operators, algebraic closure systems, information systems, formal contexts and approximable concepts, and so on (more can be added to this list). The first kind of "dots" are these structures and the connections are appropriately formulated morphisms corresponding to Scott continuous functions; but they may look different in each individual case. We introduce, for example, a notion of join-semilattice morphisms which by ideal completion correspond to Scott continuous functions on the corresponding algebraic lattices. The second kind of "dots" are the categories thus formed and the connections are functors between these categories. All the aforementioned structures give rise to equivalent categories through these functors and hence all are cartesian closed. Two "dots" of the third kind are discussed here: domain theory and formal concept analysis, so the dots represent different subject areas. The connection between the last two "dots" is in fact the key motivation for this work, along a line of recent work by the speaker and others to explore the interplay between the two areas, a particular case of which is based on the new notion of approximable concepts.

Keywords: Domain theory, topology, algebraic lattices, formal concept analysis, cartesian closed categories

Joint work of: Hitzler, Pascal; Krötzsch, Markus; Zhang, GQ (Guo-Qiang)