Information-based nonlinear approximation: an average case setting

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Nonlinear approximation has usually been studied under deterministic assumptions and complete information about the underlying functions.

We assume only *partial information*, e.g., function values at some points, and we are interested in the *average case* error and complexity of approximation. We show that the problem can be essentially split into two independent problems related to average case nonlinear (restricted) approximation from complete information, and average case unrestricted approximation from partial information.

The results are then applied to two special problems. The first problem is the average case piecewise polynomial approximation in $C([0,1])$ based on function values with respect to r-fold Wiener measure. In this case, to approximate with average error ε it is necessary and sufficient to know the function values at $\Theta\left(\left(\varepsilon^{-1}\ln^{1/2}(1/\varepsilon)\right)^{1/(r+1/2)}\right)$ equidistant points and use $\Theta\left(\varepsilon^{-1/(r+1/2)}\right)$ adaptively chosen break points in piecewise polynomial approximation.

The second problem is the average case approximation of infinite sequences $f = (f_1, f_2, \ldots) \in l_p$ by sequences with finitely many nonzero coefficients, with respect to l_q-norm, $1 \le p \le q < \infty$. Information consists of some coordinates f_i of f, and the successive coordinates are independent, $f_i \sim a_i Z$ with $|a_i| = \Theta(i^{-r})$, $r > 1/q$. In this case, we need to know $n = \Theta\left(\varepsilon^{-1/(r-1/q)}\right)$ coordinates and use as many nonzero coefficients in approximation. The natural (nonadaptive) approximation $(f_1, \ldots, f_n, 0, 0, \ldots)$ is (almost) optimal.

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