Approximation and Visualization of Pareto Frontier in the Framework of Classical Approach to Multi-Objective Optimization

Alexander V. Lotov

Russian Academy of Sciences, Dorodnicyn Computing Centre,
119991, Moscow, 40 Vavilova str., Russia,
lotov05@ccas.ru
and
State University – Higher School of Economics, Moscow, Russia, and
Lomonosov Moscow State University, Russia

Abstract. This paper is devoted to a Pareto frontier generation technique, which is aimed at subsequent visualization of the Pareto frontier in an interaction with the user. This technique known as the Interactive Decision Maps technique was initiated about 30 years ago. Now it is applied for decision support in both convex and non-convex decision problems in various fields, from machinery design to environmental planning. The number of conflicting criteria explored with the help of the Interactive Decision Maps technique is usually between three and seven, but some users manage to apply the technique in the case of a larger number of criteria. Here we outline the main ideas of the technique, concentrating at non-linear problems.

Keywords. Multi-objective optimization, Pareto frontier, visualization

1 Introduction

The classical approach to the multi-objective optimization is usually associated with interactive techniques for multi-criteria decision aid (MCDA), which use generating the Pareto frontier points sequentially in the process of interactive revealing the preferences of the decision maker. However, the classical approach is not restricted to the interactive techniques – it includes techniques of alternative types. Classification of the MCDA techniques in accordance to the role of the decision maker (see Fig. 1) was introduced in [1]. One can find it in the books [2, 3], too.

One class of the alternative techniques is provided by the Pareto frontier generation methods, which are based on approximating the Pareto frontier before the decision maker (or any other user) is involved into the decision process. For this reason they have got the name of *a posteriori* preference techniques. This feature of the generation techniques relates them to the evolutionary multi-objective optimization techniques. The generation techniques were introduced about 50 years ago [4] and stated as a particular field of multi-objective optimization in the end of 1970s [1].
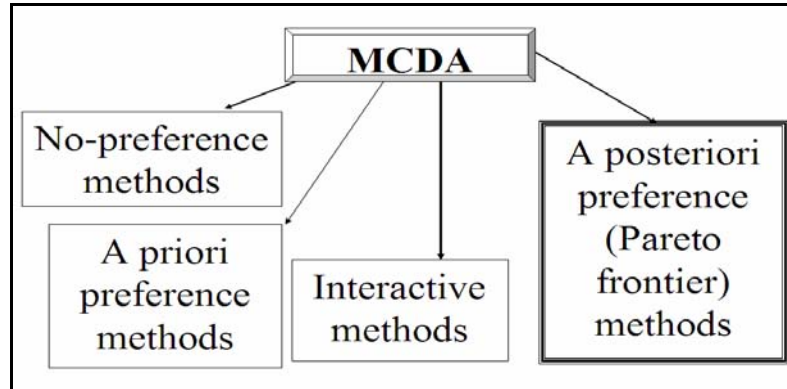
Fig. 1. Classification of the MCDA techniques according to the role of the decision maker

Various techniques for approximating the Pareto frontier were developed since 1955. In this paper we describe the main ideas of a Pareto frontier generation technique, which is aimed at subsequent visualization of the Pareto frontier. The user explores the Pareto frontier in interaction with the computer, but this interaction starts only after the approximation process is over. This technique known now as the Interactive Decision Maps (IDM) technique was introduced about 30 years ago [5, 6]. Its concepts, methods and applications are summarized in the recent book [7]. The main feature of the method consists in approximating the Edgeworth-Pareto Hull of the feasible criterion set instead of the direct approximating the Pareto frontier. The Edgeworth-Pareto Hull (EPH), which plays an important role in the theory of multi-criteria optimization, is the maximal set in criterion space that has the same Pareto frontier as the feasible criterion set. It includes, along with the feasible criterion points, all criterion points dominated by the feasible points. An approximation of the EPH is used for fast interactive visualization of the Pareto frontier as the frontiers of slices of the EPH. To be precise, decision maps (collections of criterion tradeoff curves) are displayed.

Methods for approximating the EPH used in the framework of the IDM technique depend on the model. In the linear case, they are based on the combination of the single-criterion optimization with the Fourier convolution of the linear inequality systems, and the EPH is approximated by a polyhedral set. In the non-linear case, the random search and the statistical evaluation of approximation quality are combined with the local optimization, and the EPH is approximated by a collection of domination cones with vertices located in points, which are close to the Pareto frontier. Recently, concepts of the evolutionary optimization methods were included into the IDM technique, too.

Applications of the IDM technique include national economic planning, water management, national energy planning, machinery design, etc (see [7]). Web applications are developed very intensively now. Experimental application of the IDM technique on Web has started as soon as in 1996 [8]. Its refined version based on Java technology was developed in 2000 in the form of Web application server [9, 10]. The first real-life application of the Web application server is related to supporting of

remote negotiations and decision making in regional water management in the Werra River project, Germany [11]. It is clear, however, that a wide range of applications of such Web tool can be considered.

The concept of the IDM technique is described in the paper. In Section 2 the mathematical description of the technique is provided. Its application in the convex case is outlined. Section 3 describes the main ideas of the approximation methods in the case of the non-linear problems including the methods for statistical evaluation of the quality of an approximation of the EPH. In Section 4 we outline an example of the IDM technique application in the non-linear case.

2   Mathematical Description

We assume that the feasible decision x belongs to a compact set X in the linear metric space R^n and that the vector of criterion values y is a point of the linear metric space R^m. The vector y is given by a vector function f: $R^n \rightarrow R^m$. The feasible set of criterion vectors is $Y=f(X)$, that is,

$$Y = \left\{ y \in R^m : y = f(x),\ x \in X \right\}.$$

Let us assume that the directions of the desired variation in the criteria values are given. To be definite, we assume that this is their decrease. Then, a point $y'' \in R^m$ is preferred to a point $y' \in R^m$ ($y'' \succ y'$, that is, y'' dominates y') if $y'' \leq y'$ и $y'' \neq y'$. In this case, the Pareto (non-dominated) frontier $P(Y)$ of Y is defined as

$$P(Y) = \left\{ y' \in Y : \left\{ y'' \in Y : y'' \leq y', y' \neq y'' \right\} = \varnothing \right\}$$
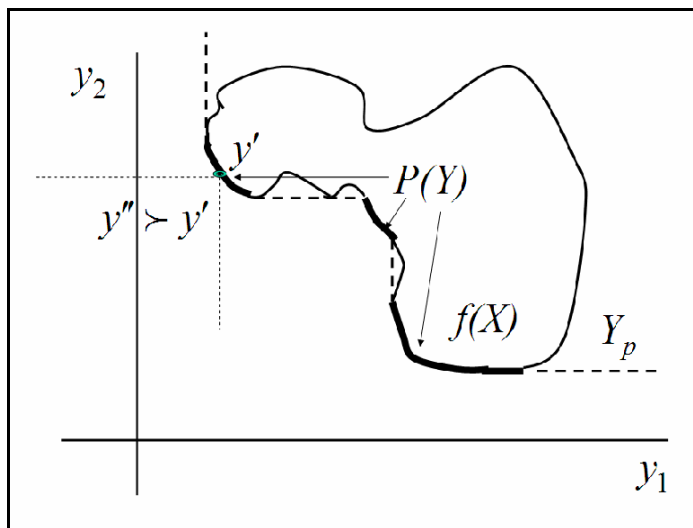

Fig. 2. Illustration of $Y=f(X)$, $P(Y)$ and Y_p. The frontier of the set Y_p is given by the dashed and bold lines. One can easily identify points of the Pareto frontier, which is given by the bold line: the cone of points $y'' : y'' \succ y'$ does not intersect with the set Y_p for a non-dominated criterion point y'.

The Edgeworth-Pareto hull of the set Y is defined in the case under consideration as

$$Y_p = Y + R_+^m = \{y^* \in R^m : y^* = y_1 + y_2,\, y_1 \in Y,\, y_2 \in R_+^m\},$$

where R_+^m is the nonnegative cone of R^m. Importantly, $P(Y_p) = P(Y)$. Moreover, proceeding to Y_p, we get rid of the dominated frontier of Y, which impedes visualization. Visualization of the Pareto frontier is based on the display of the approximation of the EPH using collections of its two-criterion slices (cross-sections). A two-criterion slice of Y_p passing through a point $y^* \in R^m$ is defined as follows. Let us consider a pair of criteria, say u and v. Let z^* be the values of the rest of criteria in the point y^*. Then, a two-criterion slice of the set Y_p, which passes through the point y^* and is related to the pair (u, v), is defined as (we do not care about the order of the criteria)

$$G\big(Y_p, z^*\big) = \big\{(u, v) : (u, v, z^*) \in Y_p\big\}.$$

The slices of the Pareto frontier are displayed as frontiers of the slices of the EPH. A collection of such two-criterion frontiers, for which the value of only one of the rest of criteria can change, constitutes a decision map. Therefore, a decision map can provide information on criterion tradeoffs among two criteria, depending on the values of the third criterion. The influence of other criteria is displayed in the framework of the IDM technique by animation. Since the EPH has already been approximated, modern personal computers spend only a few seconds for computing and displaying hundreds of decision maps. Due to it, animation is possible by successive demonstration of decision maps associated with monotonically changing value of a fourth criterion.

Let us consider an example for a problem characterized by the convex EPH. In Fig. 3 a display of the IDM technique for a large linear model with five linear criteria is provided. The model is used in the DSS for computer aided design of water quality improvement projects in a river basin (see [7], chapter 3; a detailed description of the study is provided in [12]). It is clear that for a linear model the EPH is convex. It is displayed in the following way. Feasible values of two criteria are displayed in the plane, the water pollution in two regions located in the river basin, which remain after the water quality improvement project is completed. The color informs concerning the cost of the project. It means that the lilac color (zero cost) is associated with the point with maximal pollution (upper right corner of the decision map). The cost of US$200 million is associated with the blue color. The feasible values of the pollutions are given by the blue area in the decision map. One can see the Pareto frontier of this area, which is the slice of the Pareto frontier for the whole problem. One can easily understand the tradeoffs among two criteria. The next cost value is US$400 million, and it is associated with the green color. The green area in the decision map displays the pollution values, which are feasible for the cost higher, than US$200 million, but not greater, than US$400 million. The next two areas (yellow and red) inform the user on the effect of larger costs. One can see the Pareto frontiers (tradeoffs) among remaining pollution criteria for all five values of cost under consideration. Jointly, these tradeoff curves inform, however, about the tradeoffs for all three criteria. Indeed, it is fairly clear that first US$200 million are much more efficient than the last US$200 million.
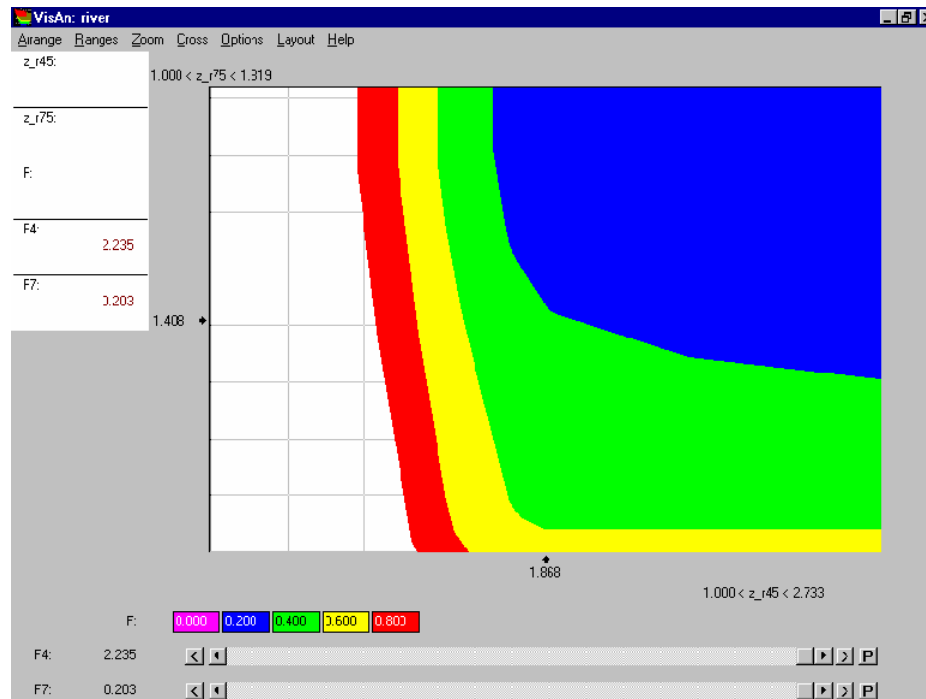
Fig. 3. An example of the IDM technique display

Now let us consider the influence of other two criteria (investments in two regions of the basin, F4 and F7). Influence of these two criteria on decision map is studied with the help of scroll-bars located under the decision map. There are sliders at the scroll-bars located in Fig. 3 at the right ends of the bars (maximal non-dominated values of the criteria). One can move the sliders manually using the computer mouse, studying by this the influence of the decrement of one of the scroll-bar criteria on the decision map. Alternatively, the sliders can be moved automatically, animating by this the decision maps. Since it is impossible to show such a form of the analysis in a paper, we have to advise downloading our software from the Web site

http://www.ccas.ru/mmes/mmeda/soft

and playing with it. Fortunately, a finite number of snapshots of the animation can be given in the paper. The display of the snapshots is provided in the form of a matrix of decision maps illustrated in Fig. 4. Note that in Fig. 4 the decision maps are given using the only color: the values of cost are associated with different shades of green. In addition, only the frontiers of the slices are displayed. Such display helps to estimate the values of particular points of the frontiers more easily and is therefore preferred by many users.
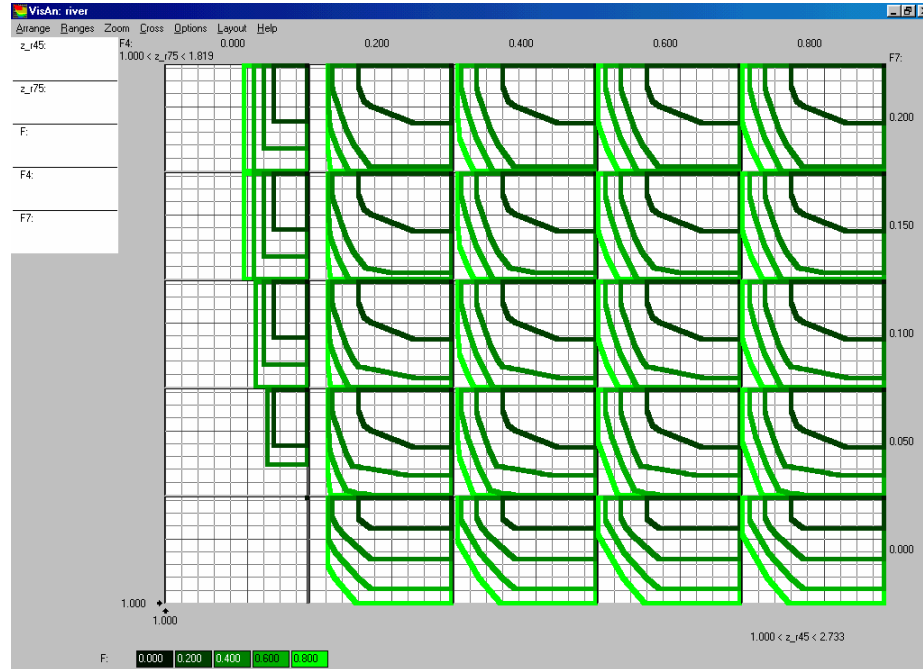
Fig. 4. A matrix of decision maps

Any column of the matrix is associated with a certain value of the fourth criterion F4. Any row is associated with a certain value of the fifth criterion F7. The matrix of decision maps provides an opportunity to study the influence of both F4 and F7 simultaneously.

Usually we advise the user to restrict themselves with five criteria. Indeed, psychological studies prove that seven is the maximal number of objects a normal human being can deal with. Since the criterion tradeoffs are fairly complicated, exploring five criteria simultaneously seems to be natural for human beings. However, since the users apply our technique without our control, they manage to study sometime till nine criteria simultaneously. Technically, the software provides such an opportunity (additional scroll-bars are displayed for any reasonable number of criteria). However, understanding of such complicated relations is not simple, and therefore restricting with five criteria is recommended by us.

By identifying a preferred criterion point (feasible goal) at the Pareto frontier of a suitable decision map, the user can express his/her preferences. Since the identified criterion point is feasible, the user obtains a decision, which results in the identified goal. It is important that the main problem of the goal programming, that is, unfeasibility of the identified goal, is solved here by the display of the Pareto frontier. Therefore, the IDM technique can be considered as the tool for visualization of the goal identification procedure in the goal programming. Such application of the IDM technique has got the name of the Feasible Goals Method (see [7] for details).

Another application of the IDM technique for the convex EPH is related to visualization of tables (relational databases) containing large number (may be,

millions) of rows. Several numerical attributes (columns) are specified to be the selection criteria. Due to it, the rows of the database are associated with the points in the criterion space. Then, the EPH of the convex hull of the criterion points is approximated and the IDM technique is applied for supporting the identification of the goal at the Pareto frontier. However, this time the goal is not feasible, but only reasonable (that is, close to feasible criterion points). Therefore, this method is known as the Reasonable Goals Method (see Chapter 4 of the book [7] as well as the recent paper [10]).

3 Approximation of the EPH in the non-linear case

In the non-linear case the EPH is approximated by the set

$$T_p = \bigcup \ \{ y + R_+^m : y \in T \},$$

where T is a finite set of criterion points. Geometrically speaking, T_p is the union of cones $y + R_+^m$ with the vertices located in points $y \in T$. The set T is named the approximation base. If the points of the set T are close to the Pareto frontier, the set T_p, approximates the EPH (see Fig. 5 where the Pareto frontier $P(Y)$ and the approximation T_p of the EPH are given for the case $m = 2$). The methods for approximating and visualization of the Pareto frontier on the basis of the concepts of the IDM technique were introduced for non-linear models in [13]. The modified versions of the methods are described in Chapter 5 of the book [7].

The methods are based on combination of random multi-objective search with scalar local optimization. One must use an algorithm for generating independent random points distributed uniformly over X and an algorithm for solving local optimization problems for any continuous scalarizing function $\varphi(\ )$ of the vector criterion, i.e., the problems

$$\varphi(f(x)) \to \min \text{ while } x \in X. \tag{1}$$

It is assumed that the algorithm can find a local minimum for any initial point $x^0 \in X$.

Methods for evaluating the quality of an EPH approximation given by the approximation base T play an important role in the methods for approximating the EPH. Therefore, we start with the description of the evaluation methods. They are based on stochastic estimation of the approximation quality and have the following sense. The quality of approximation of the set Y_p for the case $m = 2$ can be measured by the dashed area (Fig. 5) that belongs to Y_p, but does not belong to T_p. However, though we know the approximation, we do not know the set Y_p or its Pareto frontier $P(Y)$. Therefore direct measurement of the dashed area seems to be impossible. However, there exists an indirect way of its measurement. This indirect measure was proposed in [14] for estimating the quality of an approximation of a multi-dimensional non-convex image of a compact set. In [13] it was proposed to apply the same ideas for estimating the quality of an approximation of the EPH.
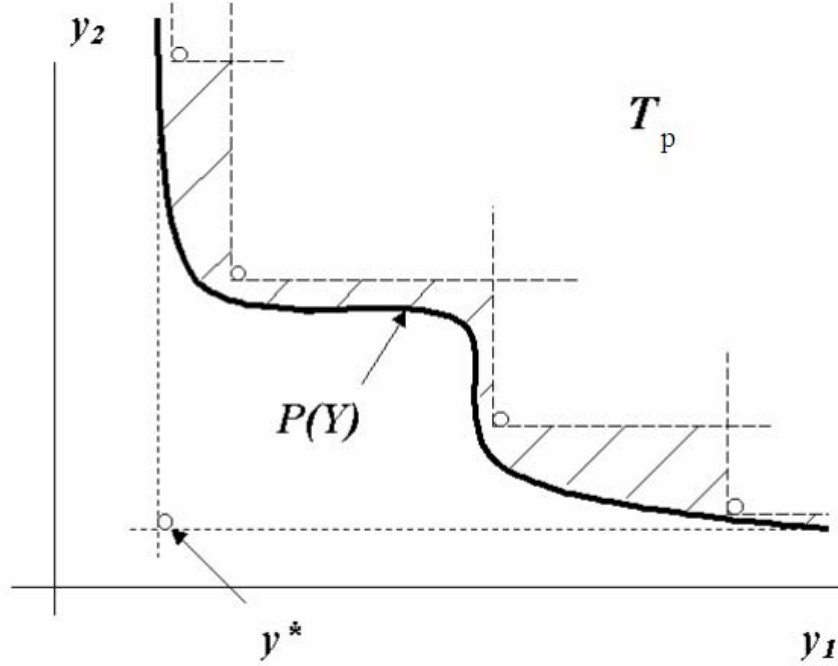
Fig. 5. The Pareto frontier $P(Y)$ and the approximation T_p of the set Y_p

Let us consider the uniform measure μ_X in R^n, that is, $\mu_X(A)$ for any measurable set A from R^n equals to its volume. Let us assume that the set X is measurable and that $\mu_X(X)=1$. Let the function $f\colon R^n \to R^m$ be continuous. Then, for any approximation T_p we can define the value

$$\eta = \mu_X (f^{-1}(T_p \cap Y)), \qquad (2)$$

where $f^{-1}(B)$ is the full pre-image of a subset B of the set Y. The meaning of η is evident: it is the probability that $x \in X$ implies $f(x) \in T_p$, that is,

$$\eta = \Pr \{ \, x{\in}X \implies f(x){\in}T_p \, \}.$$

In turn, the value $1- \eta$ is the probability that, for a point $x \in X$, the point $f(x)$ does not belong to the set T_p, that is, it belongs to the dashed area in Fig. 5. The value of η named *approximation completeness*. It can be used for evaluating the quality of the approximation of Y_p by T_p.

For some $0 < \eta^* < 1$, the condition

$$\eta \geq \eta^* \qquad (3)$$

requires that no less than the η^*-th fraction of the set X is represented by criterion points of T_p. Estimating the quality of the approximation can be formulated as checking the condition (3), which is verified in our method statistically, on the basis of generating random uniformly distributed points in X. Therefore, the condition (3)

can be guaranteed only with certain reliability χ smaller than one. Thus, instead of (3), we verify the statement

$$P\{\eta > \eta^*\} \geq \chi, \qquad (4)$$

where $0 < \chi < 1$. The problem is to pick a sufficient number N of random points $x \in X$ so as to verify estimate (4) for a given value of χ.

Let us consider $H_N = \{x_1, ..., x_N\}$, which is a random sample of uniformly distributed points of X. Let N_+ be the number of elements of the sample, for which $f(x) \in T_p$. Then, the following theorem holds (proof is given in [7]).

Theorem. An estimate of the approximation completeness is given by

$$P\{\eta > \eta^{(N)} - \Delta(\chi, N)\} \geq \chi$$

where $\eta^{(N)} = N_+/N$ and the accuracy $\Delta(\chi,N)$ is defined by the relation

$$\Delta(\chi,N) = (\ln(1-\chi)^{-1} / (2N))^{1/2}. \qquad (5)$$

A comparison of this relation with formula (4) shows that $\eta^{(N)} - \Delta(\chi, N)$ can serve as the value of η^*. Relation (5) can be used to find the required N for given Δ and χ: a sufficient number N of points can be determined as the minimum integer satisfying

$$N(\Delta, \chi) \geq \ln(1-\chi)^{-1} / (2\Delta^2). \qquad (6)$$

For example, for $\Delta = 0.05$ and $\chi = 0.95$, formula (6) yields $N = 600$, and for $\Delta = 0.1$ and $\chi = 0.9$, formula (6) yields $N = 116$.

Note that the value of N does not depend on η. Due to it, in addition to T_p, one can estimate the quality of approximation by a neighborhood of T_p. Let Q_ε be the ε–neighborhood of a set $Q \subset R^m$. We can introduce the estimate of the approximation quality for any $\varepsilon \geq 0$. It is the function $\eta^N(\varepsilon)$ defined for all $\varepsilon \geq 0$ as follows:

$$\eta^N(\varepsilon) = N_+(\varepsilon)/N,$$

where $N_+(\varepsilon)$ is the number of points x of the sample H_N that satisfy $f(x) \in (T_p)_\varepsilon$. The function $\eta^N(\varepsilon)$ is denoted as *sample approximation completeness function*. It is an increasing function of ε, and for some finite value of ε it equals to one. The function $\eta^N(\varepsilon) - \Delta(\chi, N)$ provides an estimate of the confidence interval for the approximation completeness $\eta(\varepsilon)$ for any non-negative value of ε. However, the function $\eta^N(\varepsilon)$ provides information by itself, too, since its mathematical expectation equals to the value of $\eta(\varepsilon)$.

Let us discuss several requirements that must be satisfied by an estimate that can evaluate the quality of approximation of EPH. First, the estimate must be *monotonic*, that is, if an approximation $T_p^{(2)}$ is more precise, than another approximation $T_p^{(1)}$ ($T_p^{(1)} \subset T_p^{(2)}$), then the estimate of $T_p^{(2)}$ must be higher, that of $T_p^{(1)}$. Secondly, the estimate must be *effective*, that is, it must distinguish between good and bad approximations. Finally, it must be *practical*, that is, relatively simple algorithms should exist for its computing in real-life problems.

It is clear that the completeness function is monotonic and practical. However, experimental application of it proved that this estimate is effective only in the case of

relatively small dimensions n. If n is large, the value of $\eta^N(0)$ can be equal to one even for large values of N and rough approximations of the EPH. This effect is related to the fact that the volume of points of the set X, which result in non-dominated criterion points, may be very small. Therefore, it is practically impossible to find points, which images do not belong to the current approximation. For this reason, the approximation completeness function was modified in [7] in the following way: instead of simple testing $f(x) \in T_p$ for the random points from the sample H_N, we first "improve" these points. The "improvement" means that starting with an initial point $x' \in H_N$, we construct such a new point $x'' = \Phi(x') \in X$, which satisfies $f(x'') \leq f(x')$. It means that $f(x'')$ is closer to the Pareto frontier than $f(x')$. The mapping $\Phi: X \rightarrow X$ that describes the "improvement" of the points of the sample can be based, for example, on an algorithm for solving local optimization problem (1) with some scalarizing function.

Several scalarizing functions $\varphi(\ )$ were tested, first of all the weighted Tchebycheff function proposed independently in [2] and [15]

$$\varphi(f(x)) = \max\ \{\lambda_j\ (f_j(x) - y^*_j), j=1,2,..,m\} + \beta \sum_{j=1}^{m} (f_j(x) - y^*_j)\ ,$$

where y^* is the ideal point, that is, the vector of best feasible values for particular criteria, the values of λ_j are positive weights and β is a small positive number. In contrast to the above techniques, in our study the weights were determined by the initial point $x^{(l)} \in H_N$:

$$\lambda_j = 1\ /\ (y_j^{(l)} - y^*_j),\ j=1,2,..,m,$$

where $y^{(l)} = f(x^{(l)})$. Then, we use "improved" points of X instead of the initial points of the sample. It means that the modified approximation completeness is applied. It is the function $\eta_\Phi^N(\varepsilon)$ defined for all $\varepsilon \geq 0$ as follows:

$$\eta_\Phi^N(\varepsilon) = N_+(\varepsilon)/N,$$

where $N_+(\varepsilon)$ is now the number of points x of the sample H_N that satisfy $f(\Phi(x)) \in (T_p)_\varepsilon$. The function $\eta_\Phi^N(\varepsilon)$ is denoted as *generalized sample approximation completeness function*. Since optimization is used to implement the mapping $\Phi: X \rightarrow X$, the function $\eta_\Phi^N(\varepsilon)$ is often denoted as *optimization-based sample approximation completeness,* too.

Note that the function $\eta_\Phi^N(\varepsilon)$ is monotonic in respect to the approximation and is practical since it can be easily calculated in real-life problems. Experiments with non-linear problems with four-five criteria described by hundreds of decision variables proved that this estimate of the approximation quality is effective in this case. If the probability

$$\eta_\Phi(\varepsilon) = \Pr\ \{\ x \in X \implies f(\Phi(x)) \in (T_p)_\varepsilon\ \}$$

exists, it is denoted as the *generalized (optimization-based) approximation completeness*. Then, it is possible to use the values of $\eta_\Phi^N(\varepsilon)$ for constructing the confidence interval for the value $\eta_\Phi(\varepsilon)$ for any $\varepsilon > 0$ as it was done in the case of the ordinary approximation completeness. In addition, $\eta_\Phi^N(\varepsilon)$ provides the unbiased estimate of $\eta_\Phi(\varepsilon)$.

The function $\eta_\Phi^N(\varepsilon)$ can be provided to the user in the graphical form. Note that the value $1 - \eta_\Phi(\varepsilon)$ provides the probability to get a new feasible point outside the set $(T_p)_\varepsilon$. Therefore, it characterizes the efforts needed to improve the current approximation. If the value of $\eta_\Phi^N(\varepsilon)$ is sufficiently close to one for small values of $\varepsilon>0$, then the user may be satisfied with the approximation. However, it is important to remember that the completeness function provides a stochastic approximation of the quality of approximation and does not provide guaranteed estimate of the approximation precision.

Different scalarizing functions were tested as well. They resulted in even more effective estimates. However, they are out of scope of this paper.
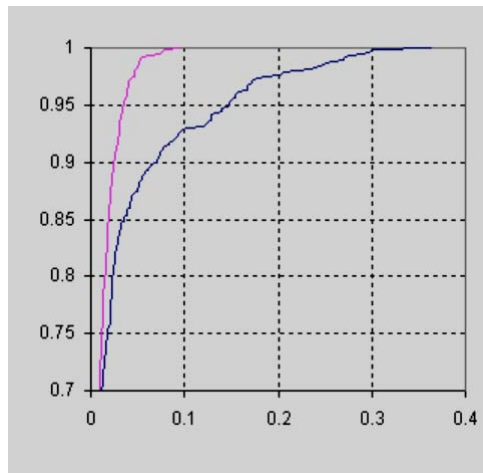

Fig. 6. Graphs of $\eta_\Phi^N(\varepsilon)$ for $\varepsilon >0$

(blue curve for a rough approximation and pink curve for a more precise approximation)

Let us consider an example of generalized approximation completeness function for a problem with five criteria. This problem is described by several extrema characterized by exponential growth. Several approximations of the EPH were constructed. In Fig. 6 two functions $\eta_\Phi^N(\varepsilon)$ for a rough approximation and for a more precise approximation of the EPH are given. The value of $\varepsilon>0$ is measured in relative units based on the ranges of the Pareto frontier. One can see that, in the case of the rough approximation, the acceptable value of the sample completeness (say, 0.95) is achieved for $\varepsilon=0.15$. In the case of the more precise approximation, the acceptable value of the sample completeness (say, 0.95) is already achieved for $\varepsilon= 0.03$. So, it is clear that the estimate makes difference between rough and more precise approximations, i.e., it turned out to be effective in this case.

Since the value $1 - \eta_\Phi(\varepsilon)$ provides the probability to get a new feasible point outside the set $(T_p)_\varepsilon$, it characterizes the efforts needed to improve the current approximation. If the value of $\eta_\Phi^N(\varepsilon)$ is sufficiently close to 1 for relatively small values of $\varepsilon>0$, then the user may be satisfied with the approximation.

After discussing the methods for estimating the quality of the approximation, we can describe the idea of the approximation methods. The methods proposed in [13] are iterative. Estimating of the approximation quality and improvement of the approximation follow each other in the course of an iteration.

At the preliminary iteration, an approximation base T_0 is found. It can be provided by the nondominated points of a collection of responses of relatively few random points of X. The k-th iteration is as follows. It is assumed that the approximation base

T_{k-1} was obtained at the preceding iteration, and so we have got the current approximation of the EPH in the form of the set T_p^{k-1} specified by the base T_{k-1}.

Step 1. The quality of the current approximation T_p^{k-1} is estimated by using one of the several methods for estimating the approximation quality. If the user is satisfied with the result of the estimating, the approximation procedure is terminated (formal stopping rules can be applied, too). Otherwise, the next step is started.

Step 2. Several most distanced (or all) criterion points found at the previous step that do not belong to T_p^{k-1} are included into the list of points of the approximation base. Then, dominated points are excluded from the list. The resulting approximation base is denoted by T_k and used at the $(k+1)$-st iteration. When the number of points in the approximation base is too large or it contains points, which are the result of close decisions, the base is filtered.

Additional steps may be included into the iterations. It turned to be very efficient to squeeze the region of the search after several iterations of the described type as well as to apply genetic algorithms at the very end of the approximation procedure.

After completing the approximation of the EPH, an interactive visualization of the Pareto frontier is started. In the same manner as in the convex case, the decision maker inspects various collections of slices of the EPH and indicates a preferable criterion point (goal) on one of the slices. Next, the computer finds a feasible decision whose response is close to the goal. To this end, the vertex of the goal-containing cone and the point of X generating the vertex are found (for this purpose, the points of the base are usually stored together with the coordinates of the decisions generating them).

Note that, since the responses of random points of X can be calculated in parallel with solving the local optimization problems (1) for different points of the sample, the methods can be implemented on multiprocessor computing systems.

4   An application example

As an example of the IDM technique application in the non-linear case we provide a short description of its use in the course of analysis of cooling equipment for continuous steel casting. The study was carried out as a joint project of Academy of Finland and Russian Academy of Sciences. It is described in details in the report [16] and in the paper [17]. Though continuous steel casting has wide recognition and is used universally, its development continues. In particular, the technology is studied and improved with the help of exploration of mathematical models. In our study we applied the IDM technique in the course of the analysis of the multi-criteria optimization model of steel cooling developed at the University of Jyvaskyla, Finland, [18], which, in turn, is based on the mathematical model of the cooling process [19].

The process of steel casting is illustrated in Fig 7. Molten steel is poured down from the tundish into a water cooled mold, where the strand of steel obtains a solid shell. After the mold exit (point z_1), the strand is supported by rollers and cooled down by water sprays. In this spray cooling region (the so-called secondary cooling region) more heat is extracted. After the water sprays region (point z_2), the strand is cooled down mainly by radiation. At the point z_3 the solidification is completed. Then, the strand is straightened at the unbending point z_4, and then in the cutting point z_5 it is cut up.

The secondary cooling plays a major role in the steel casting process, since the intensity of the water sprays affects highly the solidification rate. First of all, overcooling can lead to the formation of cracks. Moreover, there must be a smooth transition of the surface temperature as the steel passes through in the secondary cooling region. In addition, undercooling of the strand in the process of secondary cooling can result in a liquid pool that is too long. These technological requirements result in constraints that must be imposed on the secondary cooling process. Fortunately, it is quite easy to adjust the intensity of the water sprays.
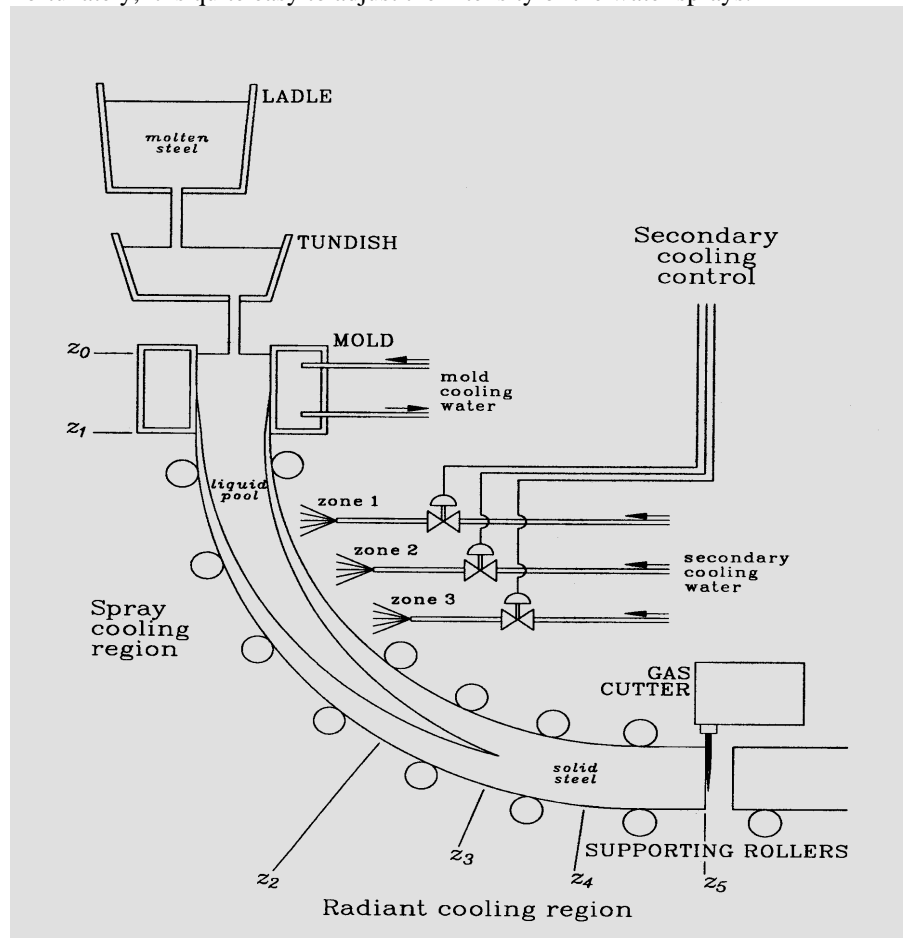

Fig. 7. Steel cooling in the process of continuous steel casting

It is clear that a mathematical model of the cooling process must be non-linear and fairly complicated, especially if one takes into account the fact that the position of the solidification front is not known a priori and depends upon the secondary cooling. Moreover, the solidification front is not sharp, between the solid and the liquid phases there exists a mushy zone, which is considered to be partially solid and partially liquid. The mathematical model [19] that takes these features into account was

implemented in the form of a computational module based on application of the FEM/FDM techniques and used for specification of parameters of the secondary cooling process.

The task of selecting the intensities of the water sprays formulated in the form of a multi-criteria optimization problem in [18] used in total 325 control variables to describe the intensity of water sprays from different sprayers. The technological constraints imposed on the steel cooling process included constraints on the surface temperature of the strand, on the derivative of the surface temperature along the strand, on the temperature after the point z_3 and at the point z_5. Originally, a single optimization criterion J_1 was considered that described deviation from the desired surface temperature of the steel strand. However, it turned out that this criterion is of minor importance since the constraints imposed on the steel cooling process cannot be satisfied simultaneously. For this reason, a multi-criteria problem of minimization of constraints violation was formulated in [18], where four additional criteria J_2 to J_5 were considered that are the penalty criteria introduced to describe the violations of constraints. Namely,

- criterion J_2 is related to the surface temperature;
- criterion J_3 is related to the derivative of the surface temperature along the strand;
- criterion J_4 is related to the temperature after point z_3; and
- criterion J_5 is related to the temperature at point z_5.

Criteria from J_2 to J_5 were considered in the study that applied the IDM technique. So, the EPH was approximated for the non-linear model, which contains 325 decision variables and four criteria. Let us provide one of the decision maps considered in this study (Fig. 8).
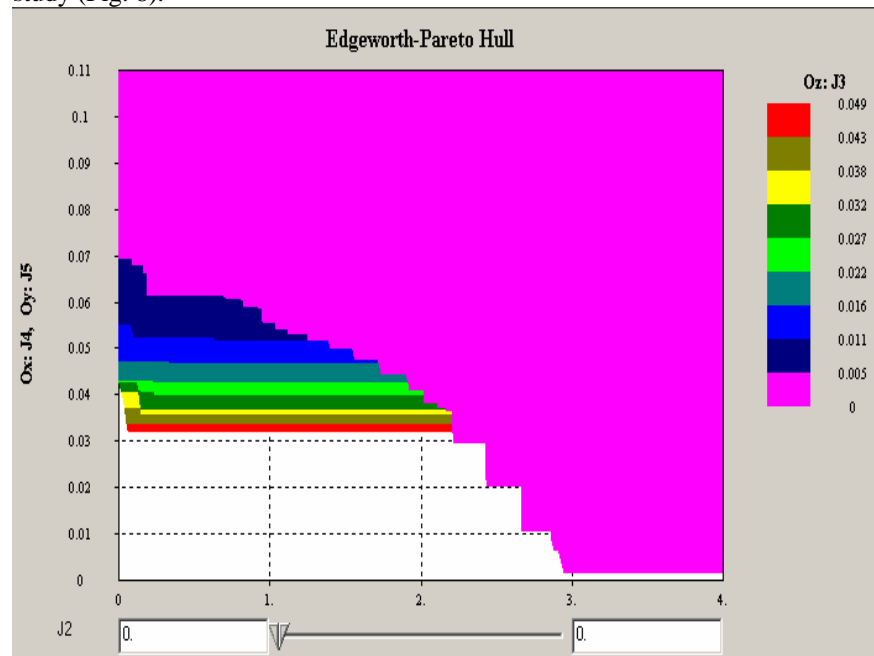

Fig. 8. A decision map for the steel cooling problem

In this figure the values of J_4 are given on the horizontal axis and the values of J_5 are given on the vertical axis. Values of J_3 are given by colors that correspond to value intervals of J_3. The range of the criterion J_3, which values change from 0 to 0.05, is split into nine intervals. The correspondence between the color and the value interval is given in the palette located to the right of the decision map. The slices for smaller values of J_3 are superimposed over the slices for higher values.

Range of J_2 can be specified by two sliders of the scroll-bar located under the decision map. It turned out that positions of the sliders practically did not influence the decision map. Therefore, zero values for both sliders were specified in Fig. 8. It means that the value of J_2 equals zero.

The decision map given in the figure proves that the ideal criterion values cannot be achieved simultaneously, and so there is a conflict among criteria. Look at the large pink area given by the slice that is related to the interval of the criterion J_3 with its minimal values (between zero and 0.005). It is clear that the criteria J_4 and J_5 cannot achieve their minimal (zero) value simultaneously for these values of J_3. Zero value of J_4 can be achieved only while the value of J_5 is about 0.07. At the same time, the smallest value of J_5 (it is close to zero) can be achieved while the value of J_4 is about 3.0. The tradeoff curve between J_4 and J_5 is given in a clear way for the interval of J_3 with minimal values.

The decision map informs on how the increment of the value of J_3 influences the feasible combinations of J_4 and J_5. Fig. 8 proves that, in the model under study, the situation is not getting much better while one turns to larger values of J_3. Though larger values of J_3 provide new feasible criterion points (we can see those points that are not covered by the slices with smaller values of J_3), the increment of slices is fairly small. The minimal value of J_4 for J_5 being about zero is decreased a bit, but not substantially. Say, it is close to 0.032 for the maximal value interval of J_3 in the figure (J_3 is between 0.043 and 0.049, that is, red colored area is discussed). Larger values of J_3 do not help either (additional figures that are not given here prove it).

As the result, the user knows that the control of the intensity of the water sprays cannot solve the conflict between the constraints imposed on the process. Moreover, he/she knows how much are the possible violations of constraints and how the tradeoff curves look. The user (designer in this case) has to find a balance between the constraint violations or propose some changes in the steel casting equipment that will solve the conflict.

4   Summary
The IDM technique helps to explore various multi-criteria optimization problems described by linear or non-linear models. The user has an opportunity to explore the Pareto frontier and select a preferred decision by a simple click at the computer mouse.

5   Acknowledgments

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