On updates with integrity constraints

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Abstract

In his paper "Making Counterfactual Assumptions" Frank Veltman has proposed a new semantics for counterfactual conditionals. It is based on a particular update operation, and we show that it provides a new and interesting way of updating logical databases under integrity constraints which generalizes in particular Winslett's PMA.

1 Introduction

In his paper "Making Counterfactual Assumptions" [Vel05] that is going to appear in the J. of Semantics, Frank Veltman has proposed a new semantics for counterfactual conditionals. The semantics contains a particular update operation that we shall call "V-update" in the sequel.

We shall argue in this paper that beyond counterfactual conditionals, V-update provides a new and interesting way of updating logical databases under integrity constraints. The latter has been much debated in artificial intelligence since the 90ies.

The present paper makes the following contributions.

- We show that V-update is a generalization of Winslett's Possible Models Approach (PMA).
- We show by means of examples that V-update does better than what has been proposed in the field up to now, and discuss its limitations.
- We give a proof system for V-update and study its complexity.
- We study the logical principles that are satisfied by V-update (in particular the Katsuno-Mendelzon postulates for updates).

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2 V-update

We here give a syntactic version of V-update.

First, let $ATM = \{p, q, \ldots\}$ be the (finite) set of atoms, and $LIT = ATM \cup \{\neg p : p \in ATM\}$ the associated set of literals. A *situation* s is a consistent set of literals. SIT is the set of all situations. A *possible world* w is a maximally consistent set of literals. W is the set of all (logically) possible worlds [Vel05, Definition 1]. As ATM is finite we can safely identify sets of literals with conjunctions of literals.

In Veltman's paper, a cognitive state is a couple $\langle Universe, Facts\rangle$ such that $Facts \subseteq Universe$ and $Universe \subseteq W$. $Universe$ is the set of worlds that are compatible with the laws the agent is aware of, and $Facts$ is the set of worlds describing the current beliefs of the agent.

We here suppose that the universe and the facts are described by formulas of propositional logic, noted A, B, C, D, ... The formula C corresponds to the constraints of U, and B corresponds to the beliefs of $Facts$. Let $[\![A]\!]$ associate to every formula A of propositional logic the set of all worlds where A holds. Then $[\![C]\!] = Universe$, $[\![B]\!] = Facts$, and $B \vdash C$ because $Facts \subseteq Universe$.

Veltman defines the notion of a *basis* for the possible world w within a set of constraints C: "A basis for a world $w \in [\![C]\!]$ is a part of w consisting of mutually independent facts which, given the general laws, bring the other facts constituting w in their train" [Vel05, Definition 3]:

- $bases_C(w) = \min_{\subseteq}\{s \in SIT : s \wedge C \not\vdash \bot, s \wedge C \vdash w\}$

Hence a basis for w within C is a minimal situation $s \subseteq w$ such that $s \wedge C$ is consistent and $s \wedge C \vdash w$. (For all other situations s' such that $s' \wedge C \vdash w$ and $s' \subseteq s$ we have $s' = s$.)

Example 1 *$\{p\}$ is a basis of $\{p, q\}$ within $p \to q$.*
$bases_{p\vee q}(\{p, q\}) = \{\{p, q\}\}$
$bases_{p\vee q}(\{\neg p, q\}) = \{\{\neg p\}\}$
$bases_{p\leftrightarrow q}(\{p, q\}) = \{\{p\}, \{q\}\}$
$bases_{(p\wedge q)\to r}(\{p, q, r\}) = \{\{p, q\}\}$
$bases_{(p\vee q)\to r}(\{p, q, r\}) = \{\{p\}, \{q\}\}$

Remark 1 *If there are no constraints (i.e. $C = \top$) then the set of bases of w is just $\{w\}$.*
If $[\![C]\!]$ is a singleton then $bases_C(B) = \{\emptyset\} = \top$.

We can also define the set of bases for the beliefs B:

- $bases_C(B) = \bigcup_{w\in[\![B]\!]} bases_C(w)$

For example, $bases_{p\to q}(p \leftrightarrow q) = \{\{p\}, \{\neg q\}\}$.

Then the *orthogonal* of w under C is:

- $w \downarrow_C A = \max_{\subseteq}\{s' :$ there is $s \in bases_C(w)$ such that $s' \subseteq s$ and $s' \wedge C \wedge A$ is consistent$\}$

Hence $w \downarrow_C A$ is the set of situations s' that are maximal subsets of some basis for w within C such that $s' \wedge C \not\vdash A$.

The *V-contraction* of w by A under constraints C is:

- $w -^{\mathsf{v}}_C A = [\![\bigvee w \downarrow_C A]\!]$
- $w \diamond^{\mathsf{v}}_{IC} A = (w -^{\mathsf{v}}_{IC} A) \cap [\![A]\!]$

Hence $w -^{\mathsf{v}}_C A$ is the set of those worlds $w' \in [\![C]\!]$ such that $s \subseteq w'$ for some $s \in w \downarrow_C A$.

Remark 2 *We have $w -^{v}_C A \subseteq [\![C]\!]$. Hence $C \vdash A$ implies $w \downarrow_C A = \emptyset$.*

Finally, the *V-contraction* and the the *V-update* of beliefs B by A under constraints C are defined as follows.

- $B -^{\mathsf{v}}_C A = \bigcup_{w \in [\![B]\!]} w -^{\mathsf{v}}_C A$
- $B \diamond^{\mathsf{v}}_C A = (B -^{\mathsf{v}}_C \neg A) \cap [\![A]\!]$ [1]

This corresponds to Definition 4 in [Vel05].

The last item is the Levy identity that constructs a revision operation (that is here better called an update operation) from a given contraction operation.

It follows from the above Remark 2 that $B \diamond^{\mathsf{v}}_C A = B \diamond^{\mathsf{v}}_C (A \wedge C)$. This illustrates that if $C \wedge A$ is inconsistent then $B \diamond^{\mathsf{v}}_C A$ is inconsistent, too.

3 No constraints: the PMA

In 1988 Mary-Anne Winslett proposed an operator for the update of logical databases [Win88, Win90], the so-called *Possible Models Approach* (PMA). It is based on minimization of the distance $DIST$ between interpretations, where the distance between two possible worlds w and v is the set of atoms whose truth value differs:

$$DIST(w, v) = \{p : w \in [\![p]\!] \text{ and } v \notin [\![p]\!]\} \cup \{p : w \notin [\![p]\!] \text{ and } v \in [\![p]\!]\}$$

For example suppose $ATM = \{p, q, r\}$, $w = \{p, q, \neg r\}$ and $v = \{p, \neg q, r\}$. Then $DIST(w, v) = \{q, r\}$.

Let A be the formula representing the incoming information (the input). Then the update of a possible world $w \in W$ by $[\![A]\!]$ is defined as:

$$w \diamond^{\mathsf{w}} A = \{u \in [\![A]\!] : \forall u' \in [\![A]\!],\ DIST(w, u') \not\subset DIST(w, u)\}.$$

In other terms, the set $w \diamond^{\mathsf{w}} A$ contains all those elements of $[\![A]\!]$ that are minimal w.r.t. the closeness ordering $\leq_w$, where $\leq_w$ is defined by

$$u \leq_w v \text{ iff } DIST(w, u) \subseteq DIST(w, v).$$

[1] In [Vel05] $B \diamond^{\mathsf{v}}_C A$ is noted in terms of counterfactuals: $\langle [\![C]\!], [\![B]\!] \rangle$[if had been A]. (It is also noted $(\mathbf{1}[\Box C][B])$[if had been A].)

Every $\leq_w$ is a partial pre-order over interpretations. In terms of conditional logics this corresponds to a semantics à la Burgess [Bur81].

Finally, the update of a set of worlds is defined as:

$$B \diamond^{\mathsf{W}} A = \bigcup_{w \in [\![B]\!]} w \diamond^{\mathsf{W}} A$$

Then we have:

Theorem 1 *For all propositional formulas B and A,*

$$B \diamond^{\mathsf{V}}_{\top} A = B \diamond^{\mathsf{W}} A$$

Proof 1 *It suffices to prove that $w \diamond^{\mathsf{W}} A \;=\; (w -^{\mathsf{V}}_{\top} \neg A) \cap [\![A]\!]$.*

Example 2 *For $w = \{\neg p, \neg q\}$ and $A = p \vee q$, we have*

$$[\![A]\!] = \{\{p, q\}, \{\neg p, q\}, \{p, \neg q\}\}$$

Then $DIST(w, \{p, q\}) = \{p, q\}$, $DIST(w, \{\neg p, q\}) = \{q\}$, and $DIST(w, \{p, \neg q\}) = \{p\}$. Thus the models of A which are minimal for distance set inclusion are $\{\{\neg p, q\}, \{p, \neg q\}\}$. Hence we get

$$w \diamond^{\mathsf{W}} (p \vee q) = \{\{\neg p, q\}, \{p, \neg q\}\}$$

If p is read "the butler was the murderer" and q "the gardener was the murderer" then this illustrates that Sherlock Holmes cannot counterfactually suspect the butler, or the gardener, *or both.*

The example illustrates that both for $\diamond^{\mathsf{V}}$ and $\diamond^{\mathsf{W}}$, inclusive disjunctions might be interpreted exclusively. This has been criticized in the literature, and several solutions have been proposed that all try to relax the minimality of change constraint, see [HR99] for an overview.

4 V-update as a new proposal for updating under integrity constraints

V-update gives us a recipe for updating logical databases under integrity constraints. It does much better than the standard proposal [KM91, KM92], which reduces updating under C to updating without integrity constraints by postulating the identity

$$B \diamond_C A = B \diamond_{\top} (A \wedge C)$$

The reason for that is that $\diamond^{\mathsf{V}}$ takes C into account in a more sensible way.

This can be illustrated by Tichy's example, where $C = p \rightarrow q$ and $B = p \wedge q$. We expect that when we update B by $\neg p$, then q does not persist (because it depends on the truth of p through the integrity constraint $p \rightarrow q$). V-update gives us just this: $(p \wedge (p \rightarrow q)) \diamond^{\mathsf{V}}_{p \rightarrow q} \neg p = [\![\neg p]\!]$ (see example 4 in section 6 for the proof). With the standard identity one would get $(p \wedge (p \rightarrow q)) \diamond^{\mathbf{V}}_{p \rightarrow q} \neg p = (p \wedge (p \rightarrow q)) \diamond^{\mathbf{V}}_{\top} (\neg p \wedge p \rightarrow q)$ With insensitivity to syntax the latter becomes $(p \wedge q) \diamond^{\mathbf{V}}_{\top} \neg p$, which means that we have lost the connection between p and q that is embodied in the integrity constraint $p \rightarrow q$: for V-updates (thus for and W-updates) this yields $\neg p \wedge q$.

That V-updates behave better than the standard identity can be illustrated as well by Veltman's version Tichy2.

V-update also behaves better on conditionals with non-false antecedents. For instance,

$$q \diamond^{\mathbf{V}}_{p \rightarrow q} \neg p \ \leftrightarrow \ \neg p$$

while the standard proposal yields

$$(q \wedge (p \rightarrow q)) \diamond^{\mathbf{V}}_{\top} (\neg p \wedge (p \rightarrow q)) \ \leftrightarrow \ \neg p \wedge q.$$

5 A limitation: the lack of causal information

Lifschitz' lamp example [Lif86] (due to M. Ginsberg) illustrates the limitations of V-update. It is similar to Kratzer's King Ludwig example that is discussed in [Vel05].

Example 3 *Let p mean 'switch 1 is up', q 'switch 2 is up', and r 'the light is on'. Suppose there is a circuit such that the light is on exactly when both switches are in the same position. Hence the integrity constraint is $C = (p \leftrightarrow q) \leftrightarrow r$. Let B be $p \wedge q \wedge r$.*

One would expect that $B \diamond^{v}_{C} \neg p \rightarrow \neg r$. As well, we would expect $B \diamond^{v}_{C} \neg p \rightarrow q$, i.e. the second switch does not move. Neither is the case: first,

$$(p \wedge q \wedge r) \diamond^{v}_{((p \leftrightarrow q) \leftrightarrow r)} \neg p \not\models \ \neg r$$

In words, it is not always the case that r gets false. Second,

$$(p \wedge q \wedge r) \diamond^{v}_{((p \leftrightarrow q) \leftrightarrow r)} \neg p \not\models \ q$$

Hence q might change truth value, i.e. the second switch might 'magically' move.

Such examples turn out to be problematic for many approaches to updates under integrity constraints. It has triggered a lot of research in AI on the integration of some notion of causality into logical approaches to updates and reasoning about actions. Basically, some linguistic means is introduced into the language in order to be able to express that in our case p does not causally depend on q, while r does.

It has been argued in [CGH99] that all these approaches are nevertheless unsatisfactory. So it seems that this is still an open problem, as is also pointed out in Veltman's paper.

6 Automated deduction

The first step is to relate bases of worlds and formulas to prime implicants. The latter are defined as follows [Mar00, CZ04]:

- A situation s is an *implicant* of A if $s \vdash A$.
- s is a *prime implicant* of A if
  - s is an implicant of A;
  - for every implicant s' of A, if $s' \subseteq s$ then $s = s'$.

We denote the set of prime implicants of a formula A by $IP(a)$.

Theorem 2 $bases_C(w) = \{s \in IP(C \to w) : s \textit{ is consistent with } C\}$

Theorem 3 $[\![bases_C(B)]\!] = [\![\{s \in IP(C \to B) : s \textit{ is consistent with } C\}]\!]$

Note that $bases_C(B)$ is not always equal to $\{s \in IP(C \to B) : s$ is consistent with $C\}$: if $ATM = \{p, q\}$ then $bases_\top(p) = \{\{p, q\}, \{p, \neg q\}\}$, while $IP(\top \to p) = \{p\}$.

The next theorem is central. It says that V-update of B under constraint C can be reduced to updating $bases_C(B)$ under the empty constraint $\top$.

Theorem 4 $B \diamond^{\mathsf{v}}_C A \ = \ (bases_C(B)) \diamond^{\mathsf{v}}_\top (A \wedge C)$

Proof 2 *As V-updates are done world by world it suffices to prove that* $w \diamond^{\mathsf{v}}_C A \ = \ (bases_C(w)) \diamond^{\mathsf{v}}_\top (A \wedge C)$.

By definition,

$$w \diamond^{\mathsf{v}}_C A \ = \ w -^{\mathsf{v}}_C \neg A \cap [\![A]\!].$$

As noted in Remark 2, we have $w -^{\mathsf{v}}_C \neg A \subseteq [\![C]\!]$. *Therefore*

$$w \diamond^{\mathsf{v}}_C A \ = \ w -^{\mathsf{v}}_C \neg A \cap [\![A \wedge C]\!].$$

By the definition of contraction we then get

$$w \diamond^{\mathsf{v}}_C A \ = \ [\![\bigwedge(w \downarrow_C \neg A)]\!] \cap [\![A \wedge C]\!].$$

By the definition of $\downarrow_C$ *this is*

$$w \diamond^{\mathsf{v}}_C A \ = \ \bigcup_{s \in bases_C(w)} [\![\bigwedge(\max_\subseteq\{s' \subseteq s : s' \wedge C \wedge A \textit{ consistent}\})]\!] \cap [\![A \wedge C]\!].$$

This is nothing but

$$w \diamond^{\mathsf{v}}_C A \ = \ (\bigcup_{s \in bases_C(w)} (s -^{\mathsf{v}}_\top (A \wedge C))) \cap [\![A \wedge C]\!].$$

Hence

$$w \diamond^{\mathsf{v}}_C A \ = \ \bigcup_{s \in bases_C(w)} s \diamond^{\mathsf{v}}_\top (A \wedge C).$$

As we have seen, $\diamond^{\mathsf{v}}_\top$ is just Winslett's PMA update. Therefore, putting everything together we obtain:

Corollary 1

$$\begin{aligned} B \diamond^{\mathsf{v}}_C A \ &= \ (bases_C(B)) \diamond^{\mathsf{w}} (A \wedge C) \\ &= \ \bigcup_{\{s \in IP(C \to B):\ s \wedge C \not\vdash \bot\}} (s \diamond^{\mathsf{w}} (A \wedge C)) \end{aligned}$$

Hence it can be checked whether $B \diamond^{\mathsf{v}}_C A \models D$ by checking whether $s \diamond^{\mathsf{w}} (A \wedge C) \models D$ for every $s \in IP(C \to B)$ such that $s \wedge C$ is consistent.

Remark 3 *Note that* $B \diamond^{\mathsf{v}}_C A \ \neq \ (C \to B) \diamond^{\mathsf{w}} (A \wedge C)$. *To see this it suffices to take* $C = p \vee q \vee r$, $B = p \wedge q \wedge r$, $A = \neg p \vee \neg q \vee \neg r$. *Then* $B \diamond^{\mathsf{v}}_C A = \{\{p \wedge q \wedge \neg r\}, \{p \wedge \neg q \wedge r\}, \{\neg p \wedge q \wedge r\},\}$, *while* $(C \to B) \diamond^{\mathsf{w}} (A \wedge C) = [\![p \vee q \vee r \wedge \neg p \vee \neg q \vee \neg r]\!]$

To sum it up, V-update can be computed by combining computation of prime implicants with deduction in the PMA. Algorithms for the former can be found in [Mar00], and for the latter in [EG92, HR99].

Example 4 *Let* $C = p \rightarrow q$*,* $B = p \wedge q$*, and* $A = \neg p$*. Then* $bases_{p\rightarrow q}(p \wedge q) = \{p\}$*. As* $A \wedge C \leftrightarrow \neg p$*, we obtain that*

$$B \diamond^{v}_{C} A = (bases_{p\rightarrow q}(p \wedge q)) \diamond^{w} (\neg p \wedge (p \rightarrow q)) = p \diamond^{w} \neg p = [\![\neg p]\!].$$

Example 5 *Consider again Lifschitz' lamp, where* $C = (p \leftrightarrow q) \leftrightarrow r$ *and* $B = p \wedge q \wedge r$*.* $IP(((p \leftrightarrow q) \leftrightarrow r) \rightarrow (p \wedge q \wedge r)) = \{p \wedge q, p \wedge r, q \wedge r, \neg p \wedge \neg q \wedge \neg r\}$*. The last element is inconsistent with* C *and is thus eliminated:* $bases_C(B) = \{p \wedge q, p \wedge r, q \wedge r\}$*. Then*

$$(p \wedge q) \diamond^{w} (\neg p \wedge C) = \{\neg p \wedge q \wedge \neg r, \neg p \wedge \neg q \wedge r\}$$

This is also the result of $(p \wedge r) \diamond^{w} (\neg p \wedge C)$ *and of* $(q \wedge r) \diamond^{w} (\neg p \wedge C)$*. Hence* $(p \wedge q \wedge r) \diamond^{v}_{((p\leftrightarrow q)\leftrightarrow r)} \neg p = \{\neg p \wedge q \wedge \neg r, \neg p \wedge \neg q \wedge r\}$*.*

Beyond this it would be interesting to have results on complexity.

7 The status of the KM update postulates

It is worth while investigating which of the standard logical principles for conditionals hold for this operation, as well as which of the Katsuno-Mendelzon postulates for updates are satisfied.

Conjecture 1 *V-update validates the following principles for selection function models à la Stalnaker [Sta68].*

(RU.EA) $\dfrac{A_1 \leftrightarrow A_2}{B \diamond A_1 \leftrightarrow B \diamond A_2}$

(RU.EC) $\dfrac{B_1 \leftrightarrow B_2}{B_1 \diamond A \leftrightarrow B_2 \diamond A}$

(RU.M) $\dfrac{B_1 \rightarrow B_2}{B_1 \diamond A \rightarrow B_2 \diamond A}$

(U.M) $B_1 \diamond A \vee B_2 \diamond A \;. \rightarrow\; (B_1 \vee B_2) \diamond A$

(U.C) $(B_1 \vee B_2) \diamond A \;\rightarrow .\; B_1 \diamond A \vee B_2 \diamond A$

These principles are the update counterparts of standard principles for conditional logics[2]*, cf. [RS97, Her98].*

[2] These are:

(RC.EA) $\dfrac{A_1 \leftrightarrow A_2}{A_1 \Box\!\!\rightarrow C \leftrightarrow A_2 \Box\!\!\rightarrow C}$

(RC.EC) $\dfrac{C_1 \leftrightarrow C_2}{A \Box\!\!\rightarrow C_1 \leftrightarrow A \Box\!\!\rightarrow C_2}$

(RC.M) $\dfrac{C_1 \rightarrow C_2}{A \Box\!\!\rightarrow C_1 \rightarrow A \Box\!\!\rightarrow C_2}$

(C.M) $A \Box\!\!\rightarrow (C_1 \wedge C_2) \;. \rightarrow .\; A \Box\!\!\rightarrow C_1 \wedge A \Box\!\!\rightarrow C_2$

(C.C) $A \Box\!\!\rightarrow C_1 \wedge A \Box\!\!\rightarrow C_2 \;. \rightarrow .\; A \Box\!\!\rightarrow (C_1 \wedge C_2)$

Conjecture 2 *V-update moreover validates the following principles for partial order models à la Burgess [Bur81].*

(U.ID) $B \diamond A \ \rightarrow \ A$

(U.N) $\neg(\bot \diamond A)$

(U.CA) $B \diamond (A_1 \vee A_2) \ \rightarrow . \ B \diamond A_1 \vee B \diamond A_2$

(RU.CSO) $\dfrac{B \diamond A_1 \ \rightarrow \ A_2 \ , \ B \diamond A_2 \ \rightarrow \ A_1}{B \diamond A_1 \ \leftrightarrow \ B \diamond A_2}$

Again, these are the update counterparts of standard principles for conditional logics.[3]

Conjecture 3 *V-update validates*

(U.MP) $(B \wedge A) \ \rightarrow \ B \diamond A$

(U.CS) $(B \wedge A) \diamond A \ \rightarrow \ B$

V-update does not validate CV, which characterizes total order models à la Lewis [Lew73]. In consequence, they do not validate the Katsuno-Mendelzon postulates for updates [KM92].

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[3] These are:

(ID) $A \Box\!\!\rightarrow A$

(C.N) $A \Box\!\!\rightarrow \top$

(CA) $A_1 \Box\!\!\rightarrow C \wedge A_2 \Box\!\!\rightarrow C \ . \rightarrow \ (A_1 \vee A_2) \Box\!\!\rightarrow C$

(CSO) $A_1 \Box\!\!\rightarrow A_2 \wedge A_2 \Box\!\!\rightarrow A_1 \ . \rightarrow . \ A_1 \Box\!\!\rightarrow C \ \leftrightarrow \ A_2 \Box\!\!\rightarrow C$

(MOD_0) $A \Box\!\!\rightarrow \bot \ \rightarrow \ (A \wedge A') \Box\!\!\rightarrow \bot$

(MOD) $\neg A \Box\!\!\rightarrow A \ \rightarrow \ A' \Box\!\!\rightarrow A$

(ASC) $(A \Box\!\!\rightarrow A') \wedge (A \Box\!\!\rightarrow C) \ . \rightarrow \ (A \wedge A') \Box\!\!\rightarrow C$

(RT) $A \Box\!\!\rightarrow A' \wedge (A \wedge A') \Box\!\!\rightarrow C \ . \rightarrow \ A \Box\!\!\rightarrow C$

(CUM) $A \Box\!\!\rightarrow A' \ \rightarrow . \ A \Box\!\!\rightarrow C \leftrightarrow (A \wedge A') \Box\!\!\rightarrow C$

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