

10481 Abstracts Collection

Computational Counting

— Dagstuhl Seminar —

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Abstract. From November 28 to December 3 2010, the Dagstuhl Seminar 10481 “Computational Counting” was held in Schloss Dagstuhl – Leibniz Center for Informatics. During the seminar, several participants presented their current research, and ongoing work and open problems were discussed. Abstracts of the presentations given during the seminar as well as abstracts of seminar results and ideas are put together in this paper. The first section describes the seminar topics and goals in general. Links to extended abstracts or full papers are provided, if available.

Keywords. Computational complexity, counting problems, holographic algorithms, statistical physics, constraint satisfaction

10481 Executive Summary – Computational Counting

From November 28 to December 3 2010, the Dagstuhl seminar 10481 “Computational Counting” was held in Schloss Dagstuhl – Leibniz Center for Informatics. 36 researchers from all over the world, with interests and expertise in different aspects of computational counting, actively participated in the meeting.

Keywords: Computational complexity, counting problems, holographic algorithms, statistical physics, constraint satisfaction

Joint work of: Bürgisser, Peter; Goldber, Leslie Ann; Jerrum, Mark

Extended Abstract: <http://drops.dagstuhl.de/opus/volltexte/2011/2944>

Uniform Derandomization from Pathetic Lower Bounds

Eric Allender (Rutgers Univ. - Piscataway, US)

It’s long been known that derandomization follows from strong lower bounds; the contribution of this paper is to show that derandomization also sometimes follows from very weak lower bounds.

The problem of multiplying a sequence of 3-by-3 matrices is complete for Arithmetic NC¹. If this problem can be shown to require constant-depth arithmetic circuits of size at least $n^{1+\epsilon}$ for some $\epsilon > 0$, then for every d , black-box identity testing for depth d arithmetic circuits can be performed in subexponential time. (Related results also hold in the setting of Boolean circuit complexity.)

Keywords: Arithmetic Circuits, Derandomization

Joint work of: Allender, Eric; Arvind, V; Wang, Fengming

Full Paper:

<http://www.eccc.uni-trier.de/report/2010/069/>

See also: Proc. 14th International Workshop on Randomization and Computation (RANDOM/APPROX 2010), Lecture Notes in Computer Science 6302, pp. 380-393, 2010.

Induced subgraph polynomials

Markus Blaeser (Universität des Saarlandes, DE)

Many graph polynomials can be written as a weighted sum over certain substructures of a given graph.

Furthermore, most of these polynomials have the property that if they are #P-hard to evaluate in a single point, then they are #P-hard to evaluate at almost all points.

The proofs of all of these results are rather similar:

First one looks for certain nice graphs transformations.

Then one uses interpolation to reduce the evaluation at the known hard point to other points.

We introduce the notion of vertex or edge induced subgraph polynomials. We show that every vertex or edge induced subgraph polynomial with one additional property ("supports cloning") is #P-hard to evaluate almost everywhere provided it is #P-hard to evaluate at one point.

This provides a unified framework for proving #P-hardness which can be applied to the Tutte polynomial, the independent set polynomial, the interlace polynomial, and the extended bivariate chromatic polynomial.

Joint work of: Blaeser, Markus; Hoffmann, Christian

On the approximation complexity hierarchy

Magnus Bordewich (University of Durham, GB)

We present an extension of Ladner's Theorem to the approximation complexity hierarchy. In 1975 Ladner proved that if $P \neq NP$, then there exists an incomplete problem A which is neither in P nor NP -complete.

Here we show that if $\text{RP} \neq \text{NP}$, then there is a counting problem π_A which neither has a *fully polynomial randomised approximation scheme* (FPRAS), nor is as hard to approximate as $\#\text{SAT}$.

This work is motivated by recent results showing that approximately counting H -colouring problems appears to fall into three complexity groups. Those problems which admit an FPRAS, those which are ‘AP-interreducible’ with $\#\text{SAT}$ and an intermediate class of problems all AP-interreducible with $\#\text{BIS}$ (counting independent sets in bipartite graphs). It has been asked whether this intermediate class in fact collapses into one of the former two classes, or whether it truly occupies some middle ground. Moreover, supposing it does occupy some middle ground, does it capture all the ground between?

Our results reveal that there are counting problems whose approximation complexity lies between FPRASable and $\#\text{SAT}$, under the assumption that $\text{NP} \neq \text{RP}$. Indeed, there are infinitely many complexity levels between. Moreover we show that if $\#\text{BIS}$ is genuinely in the middle ground (neither having an FPRAS, nor as hard to approximate as $\#\text{SAT}$), then there are problems that do not admit an FPRAS, are not equivalent in approximation complexity to $\#\text{BIS}$ and are not ‘AP-interreducible’ with $\#\text{SAT}$, thus also occupy the middle ground.

The proof is based upon Ladner’s original proof that there are classes between P and NP, and suffers the same drawback that the problems constructed are not natural. In particular our constructed problems are not H -colourings. The question of the approximation complexity of $\#\text{BIS}$ remains open.

Keywords: Ladner’s Theorem, FPRAS, approximation complexity, $\#\text{BIS}$

The state of the art in geometric complexity theory

Peter Buegisser (Universität Paderborn, DE)

Geometric complexity theory is an approach towards the fundamental lower bound problems in complexity theory based on algebraic geometry and representation theory. Originally, it has been proposed for the permanent versus determinant problem. Recently, the overall framework has been applied and further developed for the tensor rank problem (complexity of matrix multiplication). While no good lower bounds have been obtained so far with these methods, we have now a much clearer understanding of the mathematical difficulties to be overcome for advancing.

Keywords: Geometric complexity theory, permanent versus determinant, tensor rank, matrix multiplication, orbit closures, group representations, Kronecker coefficients

Approximation of #CSP and universal algebra

Andrei A. Bulatov (Simon Fraser University - Burnaby, CA)

The universal algebra approach has been quite useful for studying the complexity of #CSP. In this talk we discuss its applicability to the approximation complexity of this problem. We present several results and observations in favor of the algebraic approach.

Holographic Algorithms and Complexity of Counting Problems

Jin-yi Cai (University of Wisconsin - Madison, US)

An introduction to Holographic Algorithms and Holographic Reductions applied to the Complexity of Counting Problems.

Keywords: Holographic Algorithms, Holographic Reductions, Holant Problems, Complexity of Counting Problems, Dichotomy Theorems

Full Paper:

<http://pages.cs.wisc.edu/~jyc/>

The Complexity of Approximately Counting Stable Roommate Assignments

Prasad Chebolu (University of Liverpool, GB)

We investigate the complexity of *approximately counting* stable roommate assignments in two models: (i) the k -attribute model, in which the preference lists are determined by dot products of “preference vectors” with “attribute vectors” and (ii) the k -Euclidean model, in which the preference lists are determined by the closeness of the “positions” of the people to their “preferred positions”. Exactly counting the number of assignments is #P-complete, since Irving and Leather demonstrated #P-completeness for the special case of the stable marriage problem. We show that counting the number of stable roommate assignments in the k -attribute model ($\#k$ - *ATTRIBUTE SR*, $k \geq 4$) and the 3-Euclidean model (k - *Euclidean SR*, $k \geq 3$) is interreducible, in an approximation-preserving sense, with counting independent sets (of all sizes) ($\#IS$) in a graph, or counting the number of satisfying assignments of a Boolean formula ($\#SAT$).

This means that there can be no FPRAS for any of these problems unless NP=RP. As a consequence, we infer that there is no FPRAS for counting stable roommate assignments ($\#SR$) unless NP=RP.

Utilizing previous results by the authors, we give an approximation-preserving reduction from counting the number of independent sets in a bipartite graph ($\#BIS$) to counting the number of stable roommate assignments both in the 3-attribute model and in the 2-Euclidean model. $\#BIS$ is complete with respect to approximation-preserving reductions in the logically-defined complexity class $\#RHH_1$. Hence, our result shows that an FPRAS for counting stable roommate assignments in the 3-attribute model would give an FPRAS for all of $\#RHH_1$. We also show that the 1-attribute stable roommate problem always has either one or two stable roommate assignments, so the number of assignments can be determined exactly in polynomial time.

Keywords: Computational Complexity, $\#P$ -Completeness, Approximation Preserving Reductions, Stable Roommate Problem

Joint work of: Chebolu, Prasad; Goldberg, Leslie Ann; Martin, Russell

Holographic Algorithms and Graph Polynomials

Radu Curticapean (Universität des Saarlandes, DE)

The scope of our talk is twofold:

1. We observe the existence of holographic algorithms with runtime $(2^{2^g})poly(n)$ for graphs of genus g on n vertices, relaxing the restriction to planar input graphs. As a continuation of this, we obtain algorithms for problems on graphs that can be embedded on surfaces of fixed genus such that certain vertices lie on the outer face.
2. We apply results from the field of holographic algorithms to the study of graph polynomials: In particular, we show that almost all evaluations of both the factorial and the geometric cover polynomial are $\#P$ -hard on planar graphs, tightening a previous result from (Bläser, Dell). We also show hardness of the cycle-chain sum on planar graphs of maximum degree 4 at almost all points. The hardness results are complemented by holographic algorithms that solve restricted evaluation problems in polynomial time.

Keywords: Holographic Algorithms, Graph Polynomials

The Exponential Time Hypothesis for Counting Problems

Holger Dell (HU Berlin, DE)

In this talk, we refine the usual $P \neq \#P$ assumption for proving hardness of counting to the exponential time setting. The counting exponential time hypothesis ($\#ETH$) is that the number of satisfying assignments of n -variable 3CNF formulas cannot be computed in time $\exp(o(n))$. We discuss this hypothesis, the sparsification lemma, and the isolation lemma. Then we use ETH and $\#ETH$ as hardness assumptions to get lower bounds for the permanent.

Keywords: Exponential time complexity, sparsification lemma, satisfiability, permanent

The complexity of $\#CSP$

Martin Dyer (University of Leeds, GB)

In the counting constraint satisfaction problem ($\#CSP$), we wish to know how many ways there are to satisfy a given system of constraints, where the constraints are defined by a fixed set of relations. Andrei Bulatov has shown that this class of problems has a dichotomy, depending on the form of the relations. Each problem is either solvable in polynomial time or is $\#P$ -complete, with no intermediate cases. However, his proof is long, and requires a good understanding of universal algebra. We will describe a simpler proof of the result, based on succinct representations for relations in a class which has a polynomial-time decision algorithm. This leads to a new criterion for the dichotomy, which is different from Bulatov's, but can be shown to be equivalent to it.

The talk will explain the significance of the result and outline the proof. It will use no universal algebra, and will include a brief introduction to the counting constraint satisfaction problem.

We will conclude the talk by sketching a proof that the dichotomy is decidable, answering the major question left open by Bulatov.

Joint work of: Dyer, Martin; Richerby, David

The Complexity of Modular Counting in CSPs

John Faben (University of London, GB)

We look at the complexity of determining the number of solutions to CSP counting problems in modular counting classes. These classes were first introduced by Valiant in 1979, where he showed that counting matchings is polynomial time modulo 2, but hard in general. We give a brief discussion of the interaction between the various $\#_kP$ counting classes. Eg, if we know a problem is complete in the class $\#_mP$ and $\#_nP$, what does this tell us about $\#_{mn}P$.

We give dichotomy theorems for all k in the case where the relations in the CSP are constrained to be Boolean (Generalised Satisfiability) and some progress towards a potential dichotomy for the case of CSP restricted to a single binary relation (Graph Homomorphism).

Keywords: CSP, modular counting, graph homomorphism, complexity

A graph polynomial for independent sets of bipartite graphs

Qi Ge (University of Rochester, US)

We introduce a new graph polynomial that encodes interesting properties of graphs, for example, the number of matchings, the number of perfect matchings, and, for bipartite graphs, the number of independent sets ($\#BIS$).

We analyze the complexity of exact evaluation of the polynomial at rational points and show a dichotomy result—for most points exact evaluation is $\#P$ -hard (assuming the generalized Riemann hypothesis) and for the rest of the points exact evaluation is trivial.

We propose a natural Markov chain to approximately evaluate the polynomial for a range of parameters.

We prove an upper bound on the mixing time of the Markov chain on trees.

As a by-product we show that the “single bond flip” Markov chain for the random cluster model is rapidly mixing on constant tree-width graphs.

Joint work of: Ge, Qi; Stefankovic, Daniel

Approximating the Partition Function of the Ferromagnetic Potts Model

Leslie Ann Goldberg (University of Liverpool, GB)

We provide evidence that it is computationally difficult to approximate the partition function of the ferromagnetic q -state Potts model when $q > 2$. Specifically we show that the partition function is hard for the complexity class $\#RHI_1$ under approximation-preserving reducibility. Thus, it is as hard to approximate the partition function as it is to find approximate solutions to a wide range of counting problems, including that of determining the number of independent sets in a bipartite graph. Our proof exploits the first order phase transition of the “random cluster” model, which is a probability distribution on graphs that is closely related to the q -state Potts model. (Joint work with Mark Jerrum)

Keywords: Approximate counting, Potts model, Tutte polynomial

Joint work of: Goldberg, Leslie Ann; Jerrum, Mark

Full Paper:

<http://arxiv.org/abs/1002.0986>

Symmetric Determinantal Representation of Polynomials

Bruno Grenet (ENS - Lyon, FR)

In his seminal paper (Completeness classes in algebra, STOC 1979) Valiant expressed the polynomial computed by an arithmetic formula as the determinant of a matrix whose entries are constants or variables.

Malod and Toda showed how to express a more general class of circuits, named weakly-skew circuits by Toda, as a determinant. We are interested here in expressing formulas and skew circuits as determinant of symmetric matrices. This question is related to the Lax conjecture.

In my talk I will explain some of these constructions, which are valid in any field of characteristic different from 2. An ongoing work of the speaker with Thomassé and Monteil shows that this universality fails in characteristic 2. Furthermore, in characteristic 2 we are led to an almost complete solution to a question of Bürgisser on the VNP-completeness of the partial permanent. In particular, we show that the partial permanent cannot be VNP-complete in a field of characteristic 2 unless the polynomial hierarchy collapses.

Joint work of: Grenet, Bruno; Kaltofen, Erich; Koiran, Pascal; Portier, Natacha

Full Paper:

<http://arxiv.org/abs/1007.3804>

See also: B. Grenet, E.L. Kaltofen, P. Koiran, N. Portier. Symmetric Determinantal Representation of Formulas and Weakly-Skew Circuits. To appear in Proc. STACS 2011.

The exponential time complexity of computing the Tutte polynomial

Thore Husfeldt (Lund University, SE)

I will give a brief overview of results concerning computation of the Tutte polynomial in time exponential in the number of vertices. This includes an algorithm published announced at FOCS 2008 and lower bounds under #ETH announced in ICALP 2010 and IPEC 2010.

See also:

1. Nina Taslaman, Thore Husfeldt: The exponential time complexity of computing the probability that a graph is connected, IPEC 2010.
2. Holger Dell, Thore Husfeldt, Martin Wahlen: Exponential Time Complexity of the Permanent and the Tutte Polynomial. ICALP 2010: 426-437.
3. Andreas Björklund, Thore Husfeldt, Petteri Kaski, Mikko Koivisto: Computing the Tutte Polynomial in Vertex-Exponential Time. FOCS 2008: 677-686.

A max-flow algorithm for positivity of Littlewood-Richardson coefficients

Christian Ikenmeyer (Universität Paderborn, DE)

Littlewood-Richardson coefficients are the multiplicities in the tensor product decomposition of two irreducible representations of the general linear group $GL(n, \mathbb{C})$. They have a wide variety of interpretations in combinatorics, representation theory and geometry. Mulmuley and Sohoni pointed out that it is possible to decide the positivity of Littlewood-Richardson coefficients in polynomial time. This follows by combining the saturation property of Littlewood-Richardson coefficients (shown by Knutson and Tao 1999) with the well-known fact that linear optimization is solvable in polynomial time. We design an explicit combinatorial polynomial time algorithm for deciding the positivity of Littlewood-Richardson coefficients. This algorithm is highly adapted to the problem and it is based on ideas from the theory of optimizing flows in networks.

Keywords: Littlewood-Richardson coefficients, saturation conjecture, flows in network, polynomial time

Joint work of: Ikenmeyer, Christian; Bürgisser, Peter

Full Paper:

<http://math-www.uni-paderborn.de/agpb/work/maxflow.pdf>

See also: FPSAC 2009, Hagenberg, Austria, DMTCS proc. AK, 2009, pp. 267-278

A polynomial-time algorithm for estimating the partition function of the ferromagnetic Ising model on a regular matroid

Mark Jerrum (University of London, GB)

We investigate the computational difficulty of approximating the partition function of the ferromagnetic Ising model on a regular matroid. Jerrum and Sinclair have shown that there is a fully polynomial randomised approximation scheme (FPRAS) for the class of graphic matroids. On the other hand, the authors have previously shown, subject to a complexity-theoretic assumption, that there is no FPRAS for the class of binary matroids, which is a proper superset of the class of graphic matroids. In order to map out the region where approximation is feasible, we focus on the class of regular matroids, an important class of matroids which properly includes the class of graphic matroids, and is properly included in the class of binary matroids. Using Seymour's decomposition theorem, we give an FPRAS for the class of regular matroids.

Keywords: Matroids, Ising model, analysis of algorithms

Joint work of: Goldberg, Leslie Ann; Jerrum, Mark

Full Paper:

<http://arxiv.org/abs/1010.6231>

Complexity of Arithmetic Circuits (a skewed perspective)

Pascal Koiran (ENS - Lyon, FR)

This was an expository talk on arithmetic circuit complexity, with emphasis on the role played by skew and weakly skew circuits.

The talk covered the following 3 topics:

- universality of the determinant.
- depth reduction for arithmetic circuits.
- prospects for lower bounds.

Keywords: Arithmetic circuits, boolean circuits, permanent, determinant, completeness, parallelization, lower bounds

A representation theorem for holonomic sequences

Tomer Kotek (Technion - Haifa, IL)

Holonomic or P-recursive sequences satisfy a linear recurrence relations with polynomial coefficients. Holonomic sequences arise naturally in enumerative combinatorics as the counting functions of various combinatorial objects. In particular, some holonomic sequences are known to have interpretations as counting various lattice paths. In this extended abstract we give a representation theorem for holonomic sequences in terms of counting lattice paths related to words of a regular language.

Joint work of: Kotek, Tomer; Johann A. Makowsky

Complexity Dichotomies of Holant Problems

Pin-Yan Lu (Microsoft Research - Beijing, CN)

We propose and explore a novel alternative framework to study the complexity of counting problems, called Holant Problems. Compared to counting Constraint Satisfaction Problems ($\#CSP$), it is a refinement with a more explicit role for the function constraints. Both graph homomorphism and $\#CSP$ can be viewed as special cases of Holant Problems.

Because the framework is more stringent, previous dichotomy theorems for #CSP problems no longer apply. Indeed, we discover surprising tractable subclasses of counting problems, which could not have been easily specified in the #CSP framework. A number of complexity dichotomy theorems in this framework were proved, and I will introduce some of them in this talk. This notion is motivated by holographic reductions proposed by Valiant. Furthermore, holographic reduction is a main technical tool both for the algorithm design part and hardness proof part.

Some counting problems between NC^1 and LogSpace .

Meena Mahajan (The Institute of Mathematical Sciences - Chennai, IN)

$\#\text{NC}^1$ corresponds to functions computed by polynomial-size logarithmic depth arithmetic circuits. $\#\text{BWBP}$ corresponds to functions that compute the number of paths in bounded width branching programs.

NC^1 equals BWBP in the Boolean world, and this equivalence carries over to the arithmetic world if we allow cancellations (negative constants), giving $\text{GapNC}^1 = \text{GapBWBP}$. However, in the absence of cancellations, we only know that $\#\text{BWBP}$ is contained in $\#\text{NC}^1$.

We describe two results in this landscape:

1. A complete problem for GapNC^1 is computing the product of 3×3 matrices over $0, 1, -1$. Restricting the matrix dimension to 2, and disallowing -1 , and allowing only planar connections, still results in a counting problem that is at least NC^1 hard (under ACC^0 reductions).
2. $\#\text{BWBP}$ is known to be characterised by functions that count accepting paths in nondeterministic finite-state automata NFA. We show that generalising this to visibly pushdown automata characterises $\#\text{NC}^1$. So the difference between $\#\text{BWBP}$ and $\#\text{NC}^1$ (that vanishes when cancellations are allowed) is captured by adding a visible stack to an NFA.

Keywords: Counting accepting paths, bounded width branching programs, visibly pushdown automata

Full Papers:

<http://www.imsc.res.in/~meena/papers/sharpvpl.pdf>

Comment at <http://www.eccc.uni-trier.de/report/1999/012/>

Intriguing graph polynomials

Johann A. Makowsky (Technion - Haifa, IL)

We report about our ongoing study of inter-reducibilities of parameterized numeric graph invariants and graph polynomials.

The purpose of this work is to systematize recent emerging work on graph polynomials and various partition functions with respect to their combinatorial expressiveness and computational complexity.

This will be a revised version of an invited lecture given at CiE 2008.

Joint work of: Makovsky, Johann A.; Averbouch, I., Blaeser, Markus; Dell, V.; Godlin, B.; Kotek, Tomer; Ravve, L.; Tittmann, P.; Zilber, B.

Keywords: Graph polynomials, partition functions, combinatorial counting

Eliminating integer constants from arithmetic circuits

Guillaume Malod (University Paris-Diderot, FR)

Koiran and Perifel mention in [1] the general question of whether a polynomial function which is efficiently computable over the integers in bits is necessarily computable as a polynomial by an arithmetic circuit.

We study a special case of this question, namely we try to determine whether huge integer constants can speed up the computation of a polynomial with small coefficients.

Unfortunately we only get weak or partial results. We show that it is possible to eliminate a fixed number of constants and that proving that classes like VP or VNP cannot eliminate constants would imply the separation of VP and VNP. Only in the case of the big class VPSPACE are we able to show that constants can be eliminated. We will give a sketch of these arguments.

[1] Interpolation in Valiant's theory, to appear in Computational Complexity, <http://perso.ens-lyon.fr/pascal.koiran/publications.html>

Keywords: Circuit complexity, Valiant, polynomials

Joint work of: Malod, Guillaume; Perifel, Sylvain

The Kowalczyk-Cai dichotomy and bounded-degree #CSP

Colin McQuillan (University of Liverpool, GB)

I have worked out, in detail, the complexity of some cases of the bounded-degree constraint satisfaction problem "#CSP₂". This is a nice example (essentially due to Cai, Lu and Xia) of how holographic reductions apply to a natural problem. I will talk about this reduction and some related techniques. I then hope to talk about some extensions of Kowalczyk and Cai's result.

Keywords: Holant, CSP, interpolation

Characterizing VP by constraint satisfaction problems

Stefan Mengel (Universität Paderborn, DE)

We explore the expressivity of constraint satisfaction problems (CSPs) in the arithmetic circuit model and obtain the first natural non-circuit characterization of VP, the class of polynomial families efficiently computable by arithmetic circuits. This class is obtained when bounding the tree-width of the underlying structure and allowing domains of polynomially growing size.

Lower bounds for "explicit" polynomials

Sylvain Perifel (University Paris-Diderot, CNRS, FR)

Raz asks for lower bounds better than $n \log(n)$ for "explicit" polynomials on the size of arithmetic circuits necessary to compute them. "Explicit" here means computed by a uniform family of polynomial size circuits, with small degree and 0-1 coefficients. The difficulty here is to manage the arbitrary constants possibly used by arithmetic circuits over the complex field.

For all k , Lipton, and then Schnorr (1978) construct such polynomials with an n^k lower bound, but their constructions are not uniform. Under GRH, one can build such a polynomial with coefficients in PH. The open question is whether one can do the same with coefficients in #P, which would yield a polynomial in uniform-VNP.

Keywords: Arithmetic circuits, polynomials, lower bounds

Decidability of the #CSP Dichotomy

David Richerby (University of Leeds, GB)

Bulatov (2008) and Dyer and Richerby (2010) have established the following dichotomy for the counting constraint satisfaction (#CSP) problem: for any constraint language Γ , the problem of computing the number of satisfying assignments to constraints drawn from Γ is either in FP or is #P-complete, depending on the structure of Γ . The principal question left open by this research was whether the dichotomy is decidable. We show that it is; in fact, it is in NP.

Keywords: Counting constraint satisfaction problem, complexity dichotomy

Joint work of: Richerby, David; Dyer, Martin

On the Computation of the Betti Numbers of Complex Algebraic Varieties

Peter Scheiblechner (Purdue University, US)

We consider the algorithmic problem of computing the topological Betti numbers of a complex algebraic variety from a set of defining equations. The best known algorithms solving this problem run in double exponential time. We report on partial results towards single exponential time algorithms, in particular concerning the zeroth cohomology and the smooth projective case. We shall give some ideas about these algorithms, describe how good degree bounds yield efficient algorithms, and what is used to prove such bounds.

Keywords: Connected components, Betti numbers, algebraic varieties, cohomology

A combinatorial interpolation technique and scaling limits in sparse random graphs

Prasad Tetali (Georgia Institute of Technology, US)

Inspired by a clever interpolation technique from statistical physics (as introduced and developed by Guerra, Toninelli, Franz and Leone), we introduce a more elementary combinatorial approach to the technique.

We illustrate its power by proving the existence of appropriate scaling limits for the size of a max independent set, the size of a largest properly colorable subgraph, the size of Max-Cut (and other problems) in Erdos-Renyi as well as random regular graph models.

Joint work of: Tetali, Prasad; Bayati, Mohsen; Gamarnik, David

Keywords: K-SAT, independent set, random regular graphs

Phase Transition for the mixing time of Glauber Dynamics on Regular Trees at Reconstruction: Independent Sets and Colorings

Juan Vera (Tilburg University, NL)

We show that the mixing time of the Glauber dynamics for the hard core model with boundary conditions (and for random k -colorings) of the complete tree undergoes a phase transition around the reconstruction threshold. The central element of our result is a general technique that transforms a reconstruction algorithm into a set with poor conductance.

Joint work of: Restrepo, Ricardo; Stefankovic, Daniel; Vera, Juan; Vigoda, Eric; Yang, Linji; Tetali, Prasad

See also: Phase Transition for Glauber Dynamics for Independent Sets on Regular Trees SODA 2011. Phase Transition for the Mixing Time of the Glauber Dynamics for Coloring Regular Trees SODA 2010