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Modeling Reaction Time within a Traffic Simulation Model

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Abstract - **Human reaction time has a substantial effect on modeling of human behavior at a microscopic level. Drivers and pedestrian do not react to an event instantaneously; rather, they take time to perceive the event, process the information, decide on a response and finally enact their decision. All these processes introduce delay. As human movement is simulated at increasingly fine-grained resolutions, it becomes critical to consider the delay due to reaction time if one is to achieve accurate results. Most existing simulators over-simplify the reaction time implementation to reduce computational overhead and memory requirements. In this paper, we detail the framework which we are developing within the SimMobility Short Term Simulator (a microscopic traffic simulator), which is capable of explicitly modeling reaction time for each person in a detailed, flexible manner. This framework will enable modelers to set realistic reaction time values, relying on the simulator to handle implementation and optimization considerations. Following this, we report our findings demonstrating the impact of reaction time on traffic dynamics within several simulation scenarios. The findings indicate that in the incorporation of reaction time within microscopic simulations improves the traffic dynamics that produces more realistic traffic condition.**

I. INTRODUCTION

A large body of literature studies reaction time - the time that elapses from the occurrence of a stimulus to

the action of the response to it. It is generally accepted that mean reaction time lies in the order of 1-2 seconds and varies across the population as a function of several factors, such as the nature of the stimulus and the driver's age and mental state (1), (2). Reaction time affects traffic safety (3) as it limits the time drivers have to respond to various situations they are confronted with. It also has an important effect on traffic flow (4). For example, Krauss *et al*. (5) show that reaction time is instrumental in creating capacity drops. Several authors [e.g. (6, 7)] demonstrate its effect on the stability properties of traffic flow.

Reaction time is a complex phenomenon. Research shows that an individual's reaction time can be decomposed into a sequence of components (8):
- Mental Processing Time;
- Movement Time;
- Device Response Time.

Mental Processing Time (MPT) is the time taken to perceive a stimulus and decide upon a response. For example, it is the time required for drivers to detect that a

pedestrian is directly in front of their car and that they must therefore brake. MPT comprises of sensation, recognition, situational awareness and response selection. The value of each of these components can also depend on external factors. For example, sensing a traffic light can depend on signal intensity, visibility distance, weather, cognitive load, etc.

Once a response has been selected, the responder must undertake the required muscle movement to enact the desired effect. For example, it takes time to lift the foot off the accelerator pedal, move it laterally to the brake and then to depress the pedal. This duration is termed Movement Time. Even after the responder has acted, mechanical devices also take time to engage. The Device Response Time is the time it takes for the vehicle to begin to decelerate and varies based on the vehicle model.

A very close relationship also exists between the reaction time and a given driver's alertness factor. An alert driver will react faster in the same situation than an inattentive driver. The alertness factor also depends on the urgency of the situation and the characteristics of the driver.

Despite its importance, the treatment of reaction time in microscopic traffic simulation models is rather limited. In many of these models, reaction time is an artifact of the simulation step size or the update interval for driving decisions (i.e. acceleration and lane changing). For example, in MITSIMLab (9) driving decisions such as acceleration and lane changing are made at time resolutions that are integer multipliers of the simulation step size; with the specific multiplier set based on the situation (e.g. presence of other vehicles, traffic lights and other objects). The time-based simulation methodology also results in decisions being fed with stale information from the prior time period, and left unchanged till the next decision point. Typically, the simulation step size is 0.1 s, and decision update intervals range from 0.5 s to 1.0 s. Thus, the effective reaction time is artificially tied to the simulation step size and oscillates from 0.1 s to 1.0 s. A similar approach is also implemented in Transmodeler (10).

HUTSIM (11) implements random perception frequency, which dictates how often drivers will update their perception of other vehicles. This again ties reaction times to the most recent perception update, where the frequency of perception updates changes with the situation. HUTSIM also imposes that a driver can perceive only one event at a time, leading to less frequent perception updates (implying longer reaction times) when other activities are occurring (e.g. lane changing).

In Q-PARAMICS (12), driver reaction time is simulated by basing the calculation of the necessary acceleration/deceleration on the speed at which the vehicle in front was travelling at some time in the past. A default mean reaction time of one second is used, and this is modeled by giving each vehicle a memory, so that it carries with it not only its current speed and position, but a record of its speed and position for a specified number of time steps in the past. This is known as the speed memory. It is configurable, and by default set to 3 time steps. However, larger values are recommended for fine-grained time steps, or to achieve greater accuracy. As with many simulation data parameters there is a trade-off between accuracy and memory requirement.

In VISSIM (13), some implicit reaction time is modeled inherently by the action thresholds of the psycho-physical model. But, this has an impact only when the driver changes the regime, e.g. from free to approaching. Within the same regime, e.g. car-following, the reaction time is not modeled and the simulation time step is implicitly used as a constant reaction time. AIMSUN (16) uses a driver reaction time equal to the simulation time step, thus drivers react to leader actions immediately in the next time step. The reaction time is also equal across all drivers.

In summary, the implementation of reaction time in traffic simulation models is limited and in many cases derives its characteristics more from computational convenience and less from behavioral theory. This simplified representation of reaction time and driver behavior resulted in the creation of emergency deceleration regimes or safety headways to avoid crashes. Recent applications of traffic simulation models for safety evaluations (16), mainly using safety surrogate indicators (14) (e.g. frequency of emergency brakes, fraction of vehicles running red lights or time related measures of safety)or probabilistic frameworks (15) rely on these models, is questionable. These assumptions also affect traffic flow characteristics, resulting in increased traffic flow and reduced occurrence of shock waves in the traffic flow which are unrealistically optimistic. Treiber*et al* (18) proposed an improvement of the car-following model to mitigate these limitations by integrating spatial and temporal anticipation in drivers' estimation of variables. These and other improvements regarding drivers' mental behavior (19, 20) allowed for a better representation of different congested traffic states when considering increased reaction times and perception errors. Reaction time, however, has remained invariable and improvements at its variability modeling has been reported as necessary (8, 21), namely for better optimization of traffic and safety management and advanced driver assistance systems.

This paper reports on an explicit implementation of a flexible reaction time model within SimMobility, a new microscopic traffic simulation model currently being developed at SMART (Singapore-MIT Alliance for Research and Technology). We also demonstrate the impact that explicit reaction times have on traffic flow and driver behavior response. The first results of a more variable reaction time model integrated in a complex simulator and its ability to reproduce drivers' behavior are

presented. The rest of this paper is organized as follows: The next section provides a brief overview of the SimMobility Short Term Simulator. Next, the details of the reaction time implementation are described and their properties in terms of memory and computational requirements are discussed. Following, the effects of an explicit reaction time model in various situations are demonstrated with results from simulation experiments. Finally, the conclusion section summarizes the findings of this research and discusses further enhancements.

II. SimMobility short-term simulator

The SimMobility short-term simulator (SimMobilityST) is an agent-based, multimodal microscopic simulator where agents' movements are captured at a very fine resolution. Its massively parallel software architecture makes it possible to simulate a large number of agents at fine time steps (100 ms). These characteristics make SimMobilityST a suitable platform for implementing and evaluating the proposed reaction time model.

SimMobilityST comprises two main components. The Microscopic Movement module is responsible for advancing drivers and pedestrians on the road network according to their respective behavioral models. The Control and Management module simulates the functions of the control center, such as traffic signals, transit control, parking, road pricing etc. The outcomes of these control actions will influence an agent's movement decisions, path choices and other related decisions in the movement simulator. To illustrate their interactions, the

Microscopic Movement module sends infrastructure-based detected data (e.g. loop detector data) to the Control and Management module, which then processes the data to generate the control action plan before returning it to the Microscopic Movement module. Drivers that observed the traffic control state (e.g. traffic lights) take it into account when making their decisions.

The structure of the Microscopic Movement module is detailed in Figure 1. The virtual world is populated during the initialization phase, after which the simulation receives the control information/action plan at every time step. During the initialization phase set of reasonable paths are generated. The route choice model includes two steps: path choice set generation and the choice model. The path choice set generation is conducted with regards to each destination in the pre-defined ODs (i.e., to generate path choice sets from all other nodes to a given destination). The path choice set generation consists of three steps: (1) shortest path calculation; (2) link elimination (link penalty); and (3) random perturbation. Path-size logit model is used as the choice model.

Two kinds of behaviors are simulated: High level decisions, such as route choices, are taken at some decision point (e.g., a bus stop). Lower level movement decisions, such as car following and lane changing, occur while the agent is in movement. While the agent's position is updated at every time step, the movement-related decisions only takes place when specific events occur. Currently, reaction times are implemented within the acceleration model only.
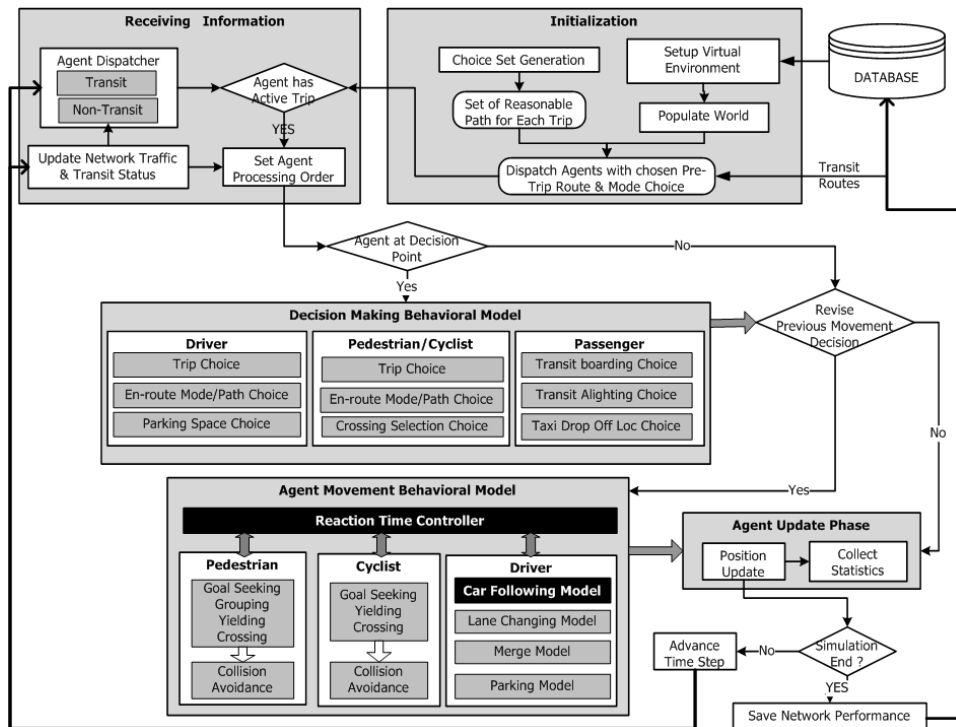

Figure 1 SimMobilityST's Microscopic Movement Module

III. IMPLEMENTATION OF REACTION TIME

An effective implementation of reaction time must achieve a balance between abstraction, encapsulation, flexibility, and performance. First, the implementation must sufficiently abstract the details of reaction time so that modelers can easily apply it to their existing models. Second, this abstraction must also encapsulate common tasks such as responding to new data and modifying perception delay. Third, it must be flexible enough to support dynamic, heterogeneous reaction times among otherwise similar agents. Drivers, for example, should each have their own base reaction time that may change under various conditions. Finally, the system must be algorithmically efficient and impose a minimal memory overhead that is acceptable for practical applications. These requirements are independent of the per-model validity requirements of simulation in general.

As shown in the lower half of Figure 1, a reaction time controller (RTC) exists in the kernel of SimMobilityST. The kernel is responsible for providing an encapsulation of reaction time for the various agent movement and behavior models. The RTC is responsible for creating the various data structures required to maintain past values of model-level parameters. It is exposed to modelers as a template class in C++, which allows them to "wrap" an existing variable with reaction-aware functionality. The RTC exposes an advanced programmers' interface (API) to modelers, allowing them to make fine-tuned adjustments to reaction time without requiring detailed knowledge of the RTC's internal workings. Using this API, reaction times can be configured using three major steps:

1. The variables that should be subject to reaction time are chosen and "marked" with a C++ template.
2. For each marked variable, the distribution of reaction times over the agent population id defined.
3. Variable reaction times for different situations may be defined by modifying the agents' behavioral models.

Variables that are marked for reaction time should not include those that are not directly perceived by the agent or variables for which reaction time will not noticeably affect the results of the simulation. In general, a parsimonious choice of variables that are required to accurately model the effects of reaction time is desired. In the work reported here, the variables that were subjected to reaction time relate both to car following and to the response to traffic lights:

- Speed of the subject vehicle;
- Speed of the leading vehicle;
- Spacing between the subject vehicle and the vehicle in front;
- State(light indication) of the signal;
- Distance of the subject vehicle to the stop line.

The RTC ensures that the correct values of these variables will be available to the relevant behavioral models (i.e. car following), thus, taking drivers' reaction times into account. This is accomplished by keeping track of all marked variables across all agents as the simulation advances.

The second task for enabling reaction time is to specify the distributions for reaction times for each marked variable. Each agent is then assigned a base reaction time value for this distribution. Currently, uniform, normal, and log-normal distributions and constant reaction time may be specified.

Finally, an agent in the simulation may request a change to its own reaction time in response to a particular situation. An example of this is a driver arriving at the scene of a car crash. As the driver's attention shifts to the spectacle, the reaction time may increase. Once the vehicle passed the crash point, the reaction time may gradually returns to normal as the driver's attention is shifted back to the driving task. The current RTC supports such a scenario.

Internally, the RTC functions as a storehouse for the past values of any marked variables. The data structure of the RTC is shown in Figure 2. For each marked variable, a linked list of historical values is maintained and ordered by simulation time. The size of each list is determined by the maximum reaction time, which dictates the highest value the reaction time may be set to for a given variable. A pointer to the currently perceived value based on the valid reaction time is updated as new data arrive from the simulation. This pointer can vary from 0 up to the maximum reaction time, allowing the simulation engine to efficiently modify the reaction time for each agent. Finally, as items in the list pass the maximum reaction time, the RTC removes them from memory.
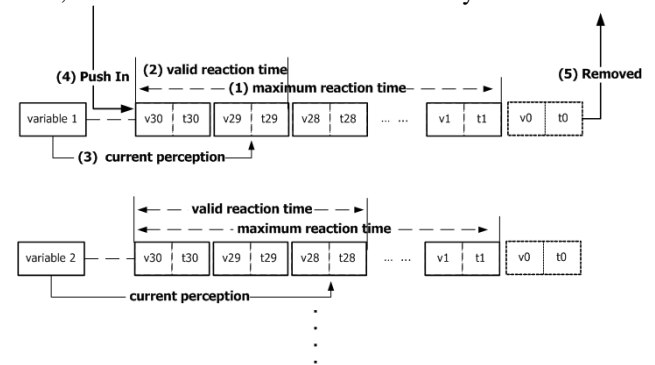

Figure 2 Data Structure of Reaction Time Controller

The major limiting factor of the RTC is the extra memory required to keep all past variable values. This memory overhead per variable can be estimated as:

$$\frac{max\ active\ agent\ number \times max\ RT \times variable\ size}{simulation\ time\ step}$$

For example, in SimMobilityST, the maximum reaction time is set to 3 seconds and the simulation time step is 0.1 second. Thus, each reaction time variable requires that we store the past 30 values. Assuming that the number of agents present in the simulation is 10,000 and the five marked variables listed earlier stored as double precision floating point numbers, the extra memory requirement is about 12 megabytes. Possible optimizations mostly relate to memory usage with the intent to support a larger number of agents and variables and are beyond the scope of this paper.

Figure 2 also provides a visualization of how reaction times may be changed during the simulation run. If reaction time is reduced, the pointer to the currently perceived value moves left. Likewise, if the reaction time increases, the pointer is moved right. Moving a pointer through a linked list is efficient. Elements are only removed if they exceed the maximum reaction time. Thus, modifying the current reaction time imposes no significant overhead.

A. The acceleration model

SimMobilityST implements the acceleration model of Ahmed *(22)*. This model distinguishes between car following and free flow regimes. The driving regime is determined by the time headway between the subject and the vehicle in front. In a car following regime, which is a generalization of the GM model, the driver's acceleration/deceleration is affected by the speeds of the subject and leader vehicle and by the space gap between them:

$$a_n(t) = \alpha \frac{V_n(t-\xi\tau_n)^\beta}{\Delta X_n(t-\xi\tau_n)^\gamma} k_n(t)^\delta \Delta V_n(t-\tau_n)^\rho + \varepsilon_n(t) \qquad (1)$$

Where, $a_n(t)$ is the acceleration/deceleration of the subject vehicle n at time t. $V_n(.)$ is the speed of the subject vehicle. $\Delta X_n(.)$ and $\Delta V_n(.)$ are the clear spacing and speed difference between the subject vehicle and its leader, respectively. $k_n(t)$ is the density of traffic ahead of the subject. α, β, γ, δ and ρ are parameters. The parameter values are different for acceleration and deceleration situations (i.e. when the leader is faster or slower than the subject, respectively). $\xi \in [0,1]$ is a sensitivity lag parameter. $\varepsilon_n(t)$ is a random error term. τ_n is the reaction times.

In a free flow regime, when the lead vehicle does not influence the driver's acceleration behavior, it is assumed that the driver would accelerate/decelerate to attain a desired speed:

$$a_n(t) = \lambda \left[DV_n(t) - V_n(t-\tau_n) \right] + \upsilon_n(t) \qquad (2)$$

Where λ is a constant sensitivity term. $DV_n(t)$ is the desired speed. $\upsilon_n(t)$ is a random error term.

In the simulation, a visibility distance is associated with each traffic signs or signal. Drivers react to signs and signals with a lag defined by the reaction time when they are visible to them. Specifically, drivers that perceive a green traffic light would apply the acceleration that they would otherwise use (i.e. based on the car following or free flow models). Drivers that perceive a red traffic light decelerate to a stop at the stop line, if they determine that the deceleration they need to apply is within an acceptable threshold. If they cannot stop at the stop line, they choose to cross the intersection and apply the acceleration they would otherwise use. The parameter values used in the simulation are those estimated by Ahmed *(22)*.

B. Demonstration

To study the influence of reaction time on traffic dynamics, we simulated several scenarios with our reaction time model. The findings are described below.

C. Scenario 1: a platoon of vehicles

Our first scenario simulated a platoon of 10 vehicles, each with a desired speed of 60km/h. The initial headways between vehicles were 2 s. The first vehicle in the platoon decelerated at 3 m/s^2 to a complete stop, after which it accelerated back to its target speed. Figure 3, Figure 4 and Figure 5 graph the time-space diagrams for the vehicles in the platoon with no reaction time modeled at all and with log-normally distributed reaction times with means of 0.5 seconds and 1 seconds, respectively and standard deviation 0.1 seconds.
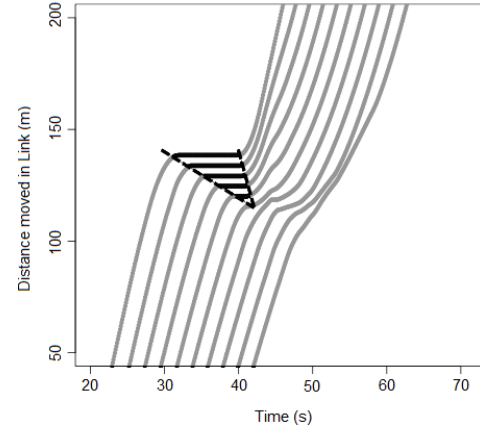

Figure 3 Time-space diagram for simulation with no reaction time

Figure 3, in which reaction times are not implemented, shows that drivers strongly decelerate immediately after the initial deceleration of the platoon leader. This results in unrealistic shock waves with high propagation speeds. The higher reaction times used in Figure 4 and Figure 5, show shock waves that propagate more slowly, especially further upstream in traffic.

5

The figures emphasize this by highlighting stopped vehicles; the overall size of the shockwave increases with greater reaction time. The deceleration values that vehicles apply also differ between the simulations. The average absolute acceleration is doubled (from 1.3 m/s² with no reaction time, to 2.6 m/s² with 1 second reaction time). These results may be significant not only in the prediction of traffic flow, but also if the simulations are intended to evaluate traffic safety.
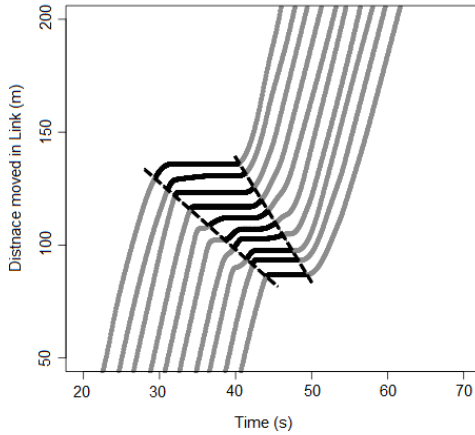

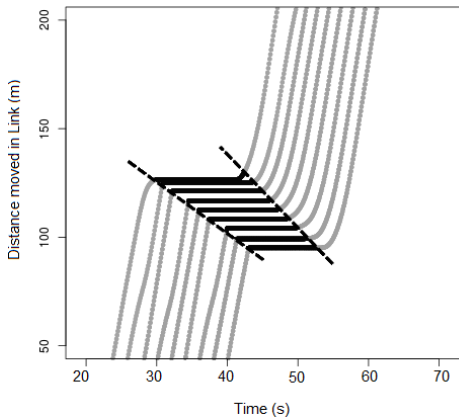Figure 4 Time-space diagram for simulation using reaction times with mean of 0.5 seconds

Figure 5 Time-space diagram for simulation using reaction times with mean of 1.0 seconds

D. Scenario 2: Release of vehicles from the stop line

This scenario considered a long queue of vehicles stopped at a signalized intersection. When the light indication changes from red to green, we observe the crossing times for the various vehicles in the queue. Cars accelerate to a desired speed of 60 km/h. The traffic light operates with fixed cycle length of 200s, and a green phase of 30s for the movement being studied.

Figure 6 depicts the results of simulation runs with different reaction time assumptions. The plots are for the case of no reaction times, mean reaction times of 1 s and

1.5 s, and a case in which the reaction time for the first two vehicles in the queue is 2.5 s, and 1.5 s for the remaining vehicles. Generally, longer reaction times directly reduce the saturation flow of the approach.
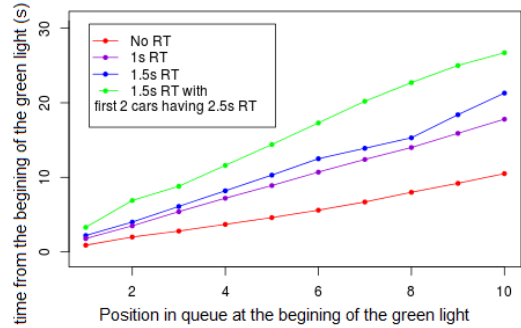

Figure 6 Time to cross the stop line with various reaction time distributions

To further demonstrate the capabilities of the reaction time implementation we consider a setup within this scenario, in which reaction times are dynamic and varies as a function of the simulation state. Specifically, it is assumed that the reaction time is higher initially when the traffic light changes (unexpected stimulus) and decreases beyond some time point (driver are expecting to start moving). Thus, reaction times are given by:

$$\tau_n(t) = \tau_0\left(\alpha + \beta\delta_g(t)\right) \qquad (3)$$

Where τ_0, α and β are parameters. $\delta_g(t)$ is an indicator function that takes the value 1 if the time that elapsed since the beginning of the green light exceeded a certain threshold t_{cr} and 0 otherwise. In the simulations we used the values: $\tau_0 = 1.0$, $\alpha = 0.6$, $\beta = 0.4$ and $t_{cr} = 3.0$. All units are in seconds.

Figure 7 shows the results of this experiment compared to constant reaction times of 0.5 s and 0.75 s. The results show the effect of the dynamic reaction time: the release rate is lower for the first three vehicles, which experience larger reaction times. At this point, the flow rate is lowest with the dynamic reaction time. This rate increases with later vehicles that tend to have shorter dynamic reaction times. Therefore, the entire platoon takes shorter time with the dynamic reaction time compared to the simulation run with 0.75 s reaction time.
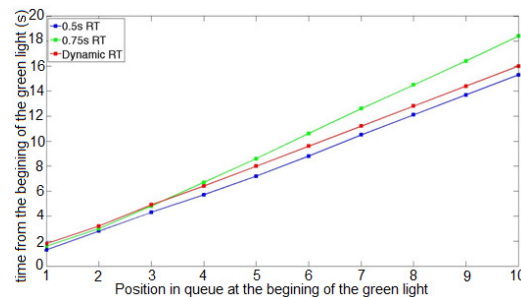

Figure7 Time to cross the stop line with dynamic reaction time

6

E. Scenario 3: Response to a traffic light changing from green to red

Our final scenario considers a vehicle arriving at an intersection as the traffic light changes from green to amber and then to red. Cars were studied in isolation in an attempt to eliminate the effect of car following. Vehicles entered the system 270 meters from the stop line with a target speed of 60 km/h, 16.5 s, 17 s or 17.5 s before the end of the green light. An amber period of 3 s was used.

Table 1: Percentage of drivers crossing the stop line for each signal phase

	No RT	0.5s RT	1s RT
Red	0.0%	1.4%	8.4%
Yellow	34.0%	33.4%	23.4%
Green	66.0%	65.2%	68.2%

We ran this scenario 500 times for each of 0, 0.5 and 1s RT; Table 1 shows the percentage of drivers that crossed the stop line for each corresponding traffic signal color. Note that the "Green" category includes drivers who stopped for the red light and then crossed the stop line when the traffic light subsequently turned green. We observed that 42 out of 500 drivers ran the red light when the reaction time was around 1s.
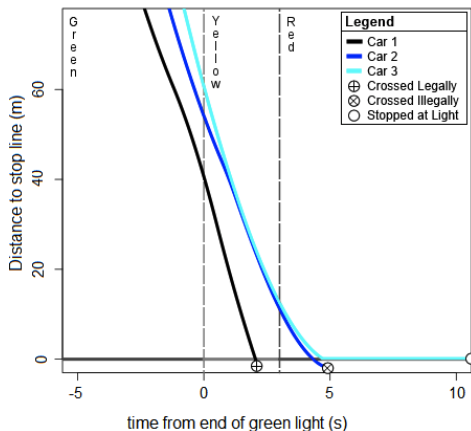

Figure 8 Trajectories of vehicles approaching the intersection at the end of the green light with log normally distributed reaction times with mean 1.0 seconds

Our final scenario considers a vehicle arriving at an intersection as the traffic light changes from green to amber and then to red. Cars were studied in isolation in an attempt to eliminate the effect of car following. Figure 8 is a representative example, depicting the trajectories of three vehicles (Car1, Car2 and Car3), each with a target speed of 60 km/h. Each vehicle entered the system 270 meters from the stop line with start times 17.5 s, 18 s and 18.5 s before the end of the green light. An amber period of 3 s was used. The Y axis indicates the distance of each vehicle from the stop line, with 0 representing the stop line itself. All three drivers have RT=1s and they therefore perceive/react to changes in traffic signal 1s

later. When Car1 perceives the Amber light, he judges based on his current distance to the stop line that he can cross the stop line before the end of Amber. He indeed does this, in spite of the delayed perception. But, when Car2 perceives the amber, his perception delay causes him to misjudge the time it will take him to reach the stop line before the end of amber. Car2, therefore, crosses the stop line when the traffic light is red. He ends up violating the traffic signal. Finally, Car3 is able to react to the traffic signal correctly and come to a stop at the stop line.

IV. Conclusion

The primary advantage of the tunable, per-agent reaction time implemented in SimMobilityST is in the flexibility it provides to capture more realistic behavior in common traffic situations. We have demonstrated this improved realism in regards to shockwaves, deceleration at intersections, and acceleration from rest.

The scenarios' results demonstrate the effect of reaction time implementation on simulation results. The increase in the details of representation will allow for more degrees of freedom in the calibration process, requiring however more detailed data. For example, relationships between visibility factors and reaction time will have to be defined. The integration of a reaction time model in SimMobilityST allows for the analysis of unsafe behaviors, thus extending the scope of applications that microscopic traffic simulations can be reliably used for in this domain.

By encapsulating reaction time functionality into a controller, we allow for the possibility of optimizing the system at a centralized level. In particular, the memory requirements of SimMobilityST, while not excessive, could benefit from some simple optimizations. We are considering three potential improvements. First, variables such as velocity or inter-vehicle distance do not change abruptly. Thus, the reaction time library may compare new values to the previous observation and discard the new value (interpolating between values if necessary) if a certain threshold difference is not reached. Secondly, we may remove the maximum reaction time, instead allowing vehicles to only change their reaction times over the course of several time ticks. Note that this will only affect drivers broadening their reaction times (e.g., as the result of a long journey), so split-second decision-making behavior will be preserved. Finally, we would consider "shared" reaction times for global entities such as traffic lights. In this case, the traffic light will maintain the historical data on behalf of the various driver agents, accounting internally for their individual reaction times.

In the current implementation reaction time is used only by the car following model, but in the future reaction time will be applied to the lane changing and pedestrian movement models as well as various vehicle/pedestrian interaction models. More studies can be performed in

7

future by varying the reaction time in different situations for the same agent.

Finally, the proposed model has to be calibrated with real data along with the full SimMobility framework. Efforts on collecting aggregated (traffic) and disaggregated (trajectories) data in Singapore for different scenarios are in progress. These and existing datasets such as the NGSIM database (*23*), will allow the computation of the true benefits of the proposed framework..Its use for reaction-time calibration will bring new insights on this particular issue.

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