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• **RESEARCH PAPER** •

Quantum cryptanalysis on some Generalized Feistel Schemes

Xiaoyang DONG[1], Zheng LI[2] & Xiaoyun WANG[1,2*]

[1]*Institute for Advanced Study, Tsinghua University, P. R. China*
{xiaoyangdong,xiaoyunwang}@tsinghua.edu.cn;
[2]*Key Laboratory of Cryptologic Technology and Information Security, Ministry of Education, Shandong University, P. R. China*
lizhengcn@mail.sdu.edu.cn

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Abstract Post-quantum cryptography has attracted much attention from worldwide cryptologists. In ISIT 2010, Kuwakado and Morii gave a quantum distinguisher with polynomial time against 3-round Feistel networks. However, generalized Feistel schemes (GFS) have not been systematically investigated against quantum attacks. In this paper, we study the quantum distinguishers about some generalized Feistel schemes. For d-branch Type-1 GFS (CAST256-like Feistel structure), we introduce $(2d-1)$-round quantum distinguishers with polynomial time. For $2d$-branch Type-2 GFS (RC6/CLEFIA-like Feistel structure), we give $(2d+1)$-round quantum distinguishers with polynomial time. Classically, Moriai and Vaudenay proved that a 7-round 4-branch Type-1 GFS and 5-round 4-branch Type-2 GFS are secure pseudo-random permutations. Obviously, they are no longer secure in quantum setting.
Using the above quantum distinguishers, we introduce generic quantum key-recovery attacks by applying the combination of Simon's and Grover's algorithms recently proposed by Leander and May. We denote n as the bit length of a branch. For (d^2-d+2)-round Type-1 GFS with d branches, the time complexity is $2^{(\frac{1}{2}d^2-\frac{3}{2}d+2)\cdot\frac{n}{2}}$, which is better than the quantum brute force search (Grover search) by a factor $2^{(\frac{1}{4}d^2+\frac{1}{4}d)n}$. For $4d$-round Type-2 GFS with $2d$ branches, the time complexity is $2^{\frac{d^2n}{2}}$, which is better than the quantum brute force search by a factor $2^{\frac{3d^2n}{2}}$.

Keywords Generalized Feistel Schemes, Simon, Grover, Quantum Key-recovery, Quantum Cryptanalysis

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1 Introduction

It is well known that several public key cryptosystem standards, such as RSA and ECC, have been broken by Shor's algorithm [1] with a quantum computer. Recently, researchers find that quantum computing not only impacts the public key cryptography, but also could break many secret key schemes, which includes the key-recovery attacks against Even-Mansour ciphers [2], distinguishers against 3-round Feistel networks [3], key-recovery and forgery attacks on some MACs and authenticated encryption ciphers [4], key-recovery

* Corresponding author (email: xiaoyunwang@tsinghua.edu.cn)

Table 1 Results on Type-1 (CAST256-like) GFS in quantum settings

Branches	Distinguisher	Key-recovery Rounds	Complexity (log)	Trivial Bound (log)
$d \geqslant 3$	Round $2d-1$	$r_0 = d^2 - d + 2$	$(\frac{1}{2}d^2 - \frac{3}{2}d + 2) \cdot \frac{n}{2}$	$\frac{(d^2-d+2)n}{2}$
		$r > r_0$	$(\frac{1}{2}d^2 - \frac{3}{2}d + 2) \cdot \frac{n}{2} + \frac{(r-r_0)n}{2}$	$\frac{rn}{2}$

attacks against FX constructions [5], and others. So to study the security of more classical and important cryptographic schemes against quantum attacks is urgently needed. At Asiacrypt 2017, NIST [6] reports the ongoing competition for post-quantum cryptographic algorithms, including signatures, encryptions and key-establishment. The ship for post-quantum crypto has sailed, cryptographic communities must get ready to welcome the post-quantum age.

In a quantum computer, the adversaries could make quantum queries on some superposition quantum states of the relevant cryptosystem, which is the so-called quantum-CPA setting [7]. It is known that Grover's algorithm [8] could speed up brute force search. Given an m-bit key, Grover's algorithm allows to recover the key using $\mathcal{O}(2^{m/2})$ quantum steps. It seems that doubling the key-length of one block cipher could achieve the same security against quantum attackers. However, Kuwakado and Morii [2] identified a new family of quantum attacks on certain generic constructions of secret key schemes. They showed that the Even-Mansour ciphers could be broken in polynomial time by Simon algorithm [9], which could find the period of a periodic function in polynomial time in a quantum computer. The following works by Kaplan *et al.* [4] revealed that many other secret key schemes could also be broken by Simon algorithm, such as CBC-MAC, PMAC, GMAC and some CAESAR candidates.

Feistel block ciphers [10] are extremely important and extensively researched cryptographic schemes. It adopts an efficient Feistel network design. Historically, many block cipher standards such as DES, Triple-DES, MISTY1, Camellia and CAST-128 [11] are based on Feistel design. At CRYPTO 1989, Zheng *et al.* [12] summarised some generalized Feistel schemes (GFS) as Type-1/2/3 GFS. Many block ciphers are based on GFS designs. CAST-256 is based on Type-1 GFS, CLEFIA and RC6 are based on Type-2 GFS, MARS is based on Type-3 GFS, so Type-1/2/3 GFS are also denoted as CAST256-like Feistel scheme, RC6/CLEFIA-like Feistel scheme, and MARS-like Feistel scheme [13]. Chinese standard block cipher SMS4 is based on a different contracting Feistel scheme, we denote it as SMS4-like GFS.

In a seminal work, Luby and Rackoff [14] proved that a three-round Feistel scheme is a secure pseudo-random permutation. However, Kuwakado and Morii [3] introduced a quantum distinguisher attack on 3-round Feistel ciphers, that could distinguish the cipher and a random permutation in polynomial time. At Asiacrypt 2000, Moriai and Vaudenay [13] studied some generalized Feistel schemes (GFS) and proved a 7-round 4-branch CAST256-like GFS and 5-round 4-branch RC6/CLEFIA-like GFS are secure pseudo-random permutations. Quantum distinguishers against those generalized Feistel schemes are missing.

In this paper, we study the quantum distinguisher attacks on Type-1 GFS (CAST256-like), Type-2 GFS (RC6/CLEFIA-like) and others. For d-branch Type-1 GFS, we introduce $(2d-1)$-round quantum distinguishers with polynomial time. For $2d$-branch Type-2 GFS (RC6/CLEFIA-like Feistel structure), we give $(2d+1)$-round quantum distinguishers with polynomial time. Classically, Moriai and Vaudenay [13] proved that a 7-round 4-branch Type-1 GFS and 5-round 4-branch Type-2 GFS are secure pseudo-random permutations. Obviously, they are no longer secure in quantum setting. Denote the branch size as n. We introduce generic quantum key-recovery attacks on Type-1 and Type-2 GFS by applying the combination of Simon's and Grover's algorithms recently proposed by Leander and May. As shown in Table 1, for (d^2-d+2)-round Type-1 GFS with d branches, the time complexity is $2^{(\frac{1}{2}d^2-\frac{3}{2}d+2)\cdot\frac{n}{2}}$, which is better than the quantum brute force search (Grover search) by a factor $2^{(\frac{1}{4}d^2+\frac{1}{4}d)n}$. As shown in Table 2, for $4d$-round Type-2 GFS with $2d$ branches, the time complexity is $2^{\frac{d^2n}{2}}$, which is better than the quantum brute force search by a factor $2^{\frac{3d^2n}{2}}$.

Table 2 Results on Type-2 (RC6/CLEFIA-like) GFS in quantum settings

Branches	Distinguisher	Key-recovery Rounds	Complexity (log)	Trivial Bound (log)
$2d \geqslant 4$	Round $2d+1$	$r_0 = 4d$	$\frac{d^2}{2}n$	$2d^2n$
		$r > r_0$	$\frac{d^2+(r-r_0)d}{2}n$	$\frac{rdn}{2}$

2 Notations

- x_j^0 the jth branch in the input;
- x_j^i the jth branch in the output of ith round, $i \geqslant 1$, $j \geqslant 1$;
- d the branch number of CAST256-like GFS;
- $2d$ the branch number of RC6/CLEFIA-like GFS;
- n the bit length of a branch;
- R^i the ith ($i \geqslant 1$) round function of Type-1 (CAST256-like) GFS, the input and output are n-bit string, n-bit key is absorbed by R^i;
- R_j^i the jth ($1 \leqslant j \leqslant d$) round function in the ith ($i \geqslant 1$) round function of Type-2 (RC6/CLEFIA -like) GFS, the input and output are n-bit string, n-bit key is absorbed by R_j^i.

3 Related works

Our quantum attacks are based the two popular quantum algorithms, i.e. Simon algorithm [9] and Grover algorithm [8].

3.1 Simon's problem

Given a boolen function f $\{0,1\}^n \rightarrow \{0,1\}^n$, that is known to be invariant under some n-bit XOR period a, find a. In other words, find a by given: $f(x) = f(y) \leftrightarrow x \oplus y \in \{0^n, a\}$.

Classically, the optimal time to solve the problem is $\mathcal{O}(2^{n/2})$. However, Simon [9] gives a quantum algorithm that provides exponential speedup and only requires $\mathcal{O}(n)$ quantum queries to find a. The algorithm includes five quantum steps:

I. Initializing two n-bit quantum registers to state $|0\rangle^{\otimes n}|0\rangle^{\otimes n}$, one applies Hadamard transform to the first register to attain an equal superposition:

$$H^{\otimes n}|0\rangle|0\rangle = \frac{1}{\sqrt{2^n}} \sum_{x \in \{0,1\}^n} |x\rangle|0\rangle. \quad (1)$$

II. A quantum query to the function f maps this to the state

$$\frac{1}{\sqrt{2^n}} \sum_{x \in \{0,1\}^n} |x\rangle|f(x)\rangle.$$

III. Measuring the second register, the first register collapses to the state:

$$\frac{1}{\sqrt{2}}(|z\rangle + |z \oplus a\rangle).$$

IV. Applying Hadamard transform to the first register, we get:

$$\frac{1}{\sqrt{2}} \frac{1}{\sqrt{2^n}} \sum_{y \in \{0,1\}^n} (-1)^{y \cdot z}(1 + (-1)^{y \cdot a})|y\rangle.$$

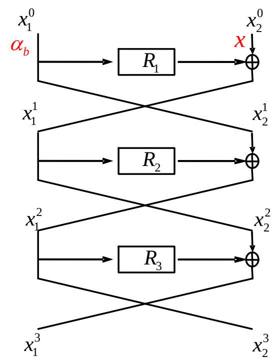

Figure 1 3-round quantum distinguisher

V. The vectors y such that $y \cdot a = 1$ have amplitude 0. Hence, measuring the state yields a value y that $y \cdot a = 0$.

Repeat $\mathcal{O}(n)$ times, one obtains a by solving a system of linear equations.

Kuwakado and Morii [3] introduced a quantum distinguish attack on 3-round Feistel scheme by using Simon algorithm. As shown in Figure 1, α_0 and α_1 are arbitrary constants:

$$\begin{aligned} f : \{0,1\} \times \{0,1\}^n &\rightarrow \{0,1\}^n \\ b, x &\mapsto \alpha_b \oplus x_2^3, \text{ where } (x_1^3, x_2^3) = E(\alpha_b, x), \\ f(b,x) &= R_2(R_1(\alpha_b) \oplus x)). \end{aligned}$$

f is periodic function that $f(b,x) = f(b \oplus 1, x \oplus R_1(\alpha_0) \oplus R_1(\alpha_1))$. Then using Simon's algorithm, one can get the period $s = 1||R_1(\alpha_0) \oplus R_1(\alpha_1)$ in polynomial time.

3.2 Grover's algorithm

The task is to find a marked element from a set X. We denote by $M \subseteq X$ the subset of marked elements. Classically, one solve the problem with time $|X|/|M|$. However, in a quantum computer, the problem is solve with high probability in time $\sqrt{|X|/|M|}$ using Grover's algorithm. The steps of the algorithm is as follows:

I. Initializing a n-bit register $|0\rangle^{\otimes n}$. One applies Hadamard transform to the first register to attain an equal superposition:

$$H^{\otimes n}|0\rangle = \frac{1}{\sqrt{2^n}} \sum_{x \in \{0,1\}^n} |x\rangle = |\varphi\rangle. \quad (2)$$

II. Construct an oracle $\mathcal{O}$: $|x\rangle \xrightarrow{\mathcal{O}} (-1)^{f(x)}|x\rangle$, where $f(x) = 1$ if x is the correct state, and $f(x) = 0$ otherwise.

III. Apply Grover iteration for $R \approx \frac{\pi}{4}\sqrt{2^n}$ times:

$$[(2|\varphi\rangle\langle\varphi| - I)\mathcal{O}]^R|\varphi\rangle \approx |x_0\rangle.$$

IV. return x_0.

Later, Brassard *et al.* [15] generalized the Grover search as amplitude amplification.

Theorem 1. (Brassard, Hoyer, Mosca and Tapp [15]). Let $\mathcal{A}$ be any quantum algorithm on q qubits that uses no measurement. Let $\mathcal{B} : \mathbb{F}_2^q \rightarrow \{0,1\}$ be a function that classifies outcomes of $\mathcal{A}$ as good or bad. Let $p > 0$ be the initial success probability that a measurement of $\mathcal{A}|0\rangle$ is good. Set $k = \lceil \frac{\pi}{4\theta} \rceil$,

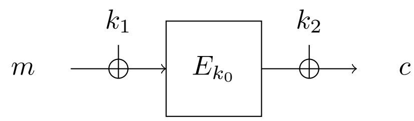

Figure 2 FX constructions

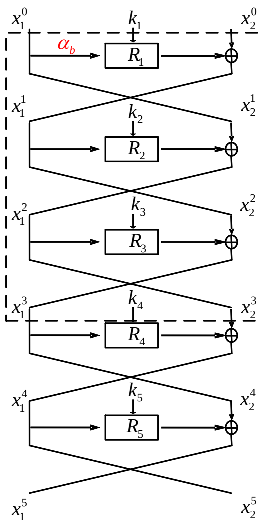

Figure 3 Quantum key-recovery attacks on 5-Round Feistel structures

where θ is defined via $sin^2(\theta) = p$. Moreover, define the unitary operator $Q = -\mathcal{A}S_0\mathcal{A}^{-1}S_{\mathcal{B}}$, where the operator $S_{\mathcal{B}}$ changes the sign of the good state

$$|x\rangle \mapsto \begin{cases} -|x\rangle \text{ if } \mathcal{B}(x) = 1, \\ \ \ |x\rangle \text{ if } \mathcal{B}(x) = 0, \end{cases}$$

while S_0 changes the sign of the amplitude only for the zero state $|0\rangle$. Then after the computation of $Q^k\mathcal{A}|0\rangle$, a measurement yields good with probability a least max{1-p, p}.

Assuming $|\varphi\rangle = \mathcal{A}|0\rangle$ is the initial vector, whose projections on the good and the bad subspace are denoted $|\varphi_1\rangle$ and $|\varphi_0\rangle$. The state $|\varphi\rangle = \mathcal{A}|0\rangle$ has angle θ with the bad subspace, where $sin^2(\theta) = p$. Each Q iteration increase the angle to 2θ. Hence, after $k \approx \frac{\pi}{4\theta}$, the angle roughly equals to $\pi/2$. Thus, the state after k iterations is almost orthogonal to the bad subspace. After measurement, it produces the good vector with high probability.

3.3 Combining Simon and Grover algorithms

At Asiacrypt 2017, Leander and May [5] gave a quantum key-recovery attack on FX-construction shown in Figure 2: $Enc(x) = E_{k_0}(x + k_1) + k_2$. They introduce the function $f(k, x) = Enc(x) + E_k(x) = E_{k_0}(x + k_1) + k_2 + E_k(x)$. For the correct key guess $k = k_0$, we have $f(k, x) = f(k, x + k_1)$ for all x. However, for $k \neq k_0$, $f(k, \cdot)$ is not periodic. They *combine Simon and Grover algorithm* to attack FX ciphers (such as PRINCE [16], PRIDE [17], DESX [18]) in the quantum-CPA model with complexity roughly 2^{32}.

Then Dong *et al.* [19] and Hosoyamada et al. [20] independently applied Leander *et al.*'s [5] attack to generic feistel constructions. As shown in Figure 3, they append 2-round feistel networks under the 3-round quantum distinguisher in Figure 1 to give a quantum key-recovery attack on 5-round feistel construction.

Suppose the state size is n, then the length of k_i is $n/2$. The following functions is defined:

$$f(b, x_{R_0}) = R_2(k_2, x_2^0 \oplus R_1(k_1, \alpha_b)) = \alpha_b \oplus x_2^3 = \alpha_b \oplus R_4(k_4, R_5(k_5, x_2^5) \oplus x_1^5) \oplus x_2^5, \tag{3}$$

where $b \in \mathbb{F}_2$, $\alpha_b \in \mathbb{F}_2^{n/2}$ is arbitrary constant and $\alpha_0 \neq \alpha_1$, $(x_1^5||x_2^5) = Enc(\alpha_b||x_2^0)$. It is easy to verify that $f(b, x_2^0) = f(b \oplus 1, x_2^0 \oplus R_1(k_1, \alpha_0) \oplus R_1(k_1, \alpha_1))$. Therefore, with the right key guess (k_4, k_5), $f(b, x_2^0) = \alpha_b \oplus R_4(k_4, R_5(k_5, x_2^5) \oplus x_1^5)$ has a nontrivial period $s = 1||R_1(k_1, \alpha_0) \oplus R_1(k_1, \alpha_1)$. However, if the guessed (k_4, k_5) is wrong, $f(b, x_2^0)$ is a random function and not periodic with high probability.

Theorem 2. [19] Let g: $\mathbb{F}_2^n \times \mathbb{F}_2^{n/2+1} \mapsto \mathbb{F}_2^{n/2}$ with

$$(k_4, k_5, y) \mapsto f(y) = f(b, x) = \alpha_b \oplus R_4(k_4, R_5(k_5, x_2^5) \oplus x_1^5) \oplus x_2^5,$$

where α_0, α_1 are two arbitrary constants, $(x_1^5||x_2^5) = Enc(\alpha_b||x)$. Given quantum oracle to g and Enc, (k_4, k_5) and $R_1(k_1, \alpha_0) \oplus R_1(k_1, \alpha_1)$ could be computed with $n + (n+1)(n+2+2\sqrt{n/2+1})$ qubits and about $2^{n/2}$ quantum queries.

Under the right key guess k_4, k_5, $g(k_4, k_5, y) = g(k_4, k_5, y \oplus s)$. Let, h: $\mathbb{F}_2^n \times \mathbb{F}_2^{(n/2+1)^l} \mapsto \mathbb{F}_2^{(n/2)^l}$ with

$$(k_4, k_5, y_1, ..., y_l) \mapsto g(k_4, k_5, y_1)||...||g(k_4, k_5, y_l). \tag{4}$$

Let U_h be a quantum oracle that maps

$$|k_4, k_5, y_1, ..., y_l, \mathbf{0}, ..., \mathbf{0}\rangle \mapsto |k_4, k_5, y_1, ..., y_l, h(k_4, k_5, y_1, ..., y_l)\rangle. \tag{5}$$

Similar to the work [5], Dong and Wang [19] constructed the following quantum algorithm $\mathcal{A}$.

1. Preparing the initial $(n + (n/2+1)l + nl/2)$-qubit state $|\mathbf{0}\rangle$.
2. Apply Hadamard $H^{\otimes n+(n/2+1)l}$ on the first $n + (n/2+1)l$ qubits resulting in

$$\sum_{k_4, k_5 \in \mathbb{F}_2^{n/2}, y_1, ..., y_l \in \mathbb{F}_2^{n/2+1}} |k_4, k_5\rangle|y_1\rangle...|y_l\rangle|\mathbf{0}\rangle, \tag{6}$$

   where we omit the amplitudes $2^{-(n+(n/2+1)l)/2}$.

3. Applying U_h to the above state, we get:

$$\sum_{k_4, k_5 \in \mathbb{F}_2^{n/2}, y_1, ..., y_l \in \mathbb{F}_2^{n/2+1}} |k_4, k_5\rangle|y_1\rangle...|y_l\rangle|h(k_4, k_5, y_1, ..., y_l)\rangle. \tag{7}$$

4. Apply Hadamard to the qubits $|y_1\rangle...|y_l\rangle$ of the above state, we get:

$$|\varphi\rangle = \sum_{k_4, k_5 \in \mathbb{F}_2^{n/2}, u_1, ..., u_l, y_1, ..., y_l \in \mathbb{F}_2^{n/2+1}} |k_4, k_5\rangle(-1)^{\langle u_1, y_1\rangle}|u_1\rangle...(-1)^{\langle u_l, y_l\rangle}|u_l\rangle|h(k_4, k_5, y_1, ..., y_l)\rangle. \tag{8}$$

If the guessed k_4, k_5 is right, after measurement of $|\varphi\rangle$, the period s is orthogonal to all the $u_1, ..., u_l$. According to Lemma 4 of [5], choosing $l = 2(n/2 + 1 + \sqrt{n/2+1})$ is enough to compute a unique s .

Without measurement and considering the superposition $|\varphi\rangle$, Dong and Wang [19] introduced a classifier $\mathcal{B}$:

Classifier $\mathcal{B}$. Define $\mathcal{B} : \mathbb{F}_2^{n+(n/2+1)l} \mapsto \{0, 1\}$ that maps $(k_4, k_5, u_1, ..., u_l) \mapsto \{0, 1\}$.

1. Let $\overline{U} = \langle u_1, ..., u_l\rangle$ be the linear span of all u_i. If $dim(\overline{U}) \neq n/2$, output 0. Else, use Lemma 4 of [5] to compute the unique period s.
2. Check $g(k_4, k_5, y) = g(k_4, k_5, y \oplus s)$ for a random given y. If the identity holds, output 1. Else output 0.

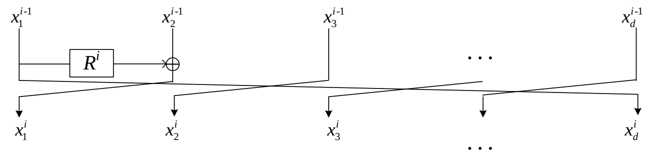

Figure 4 Round i of CAST256-like GFS with d branches

Classifier $\mathcal{B}$ partitions $|\varphi\rangle$ into a good subspace and a bad subspace: $|\varphi\rangle = |\varphi_1\rangle + |\varphi_0\rangle$, where $|\varphi_1\rangle$ and $|\varphi_0\rangle$ denotes the projection onto the good subspace and bad subspace, respectively. For the good one $|x\rangle$, $\mathcal{B}(x) = 1$.

Classifier $\mathcal{B}$ defines a unitary operator $S_{\mathcal{B}}$ that conditionally change the sign of the quantum states:

$$|k_4, k_5\rangle|u_1\rangle...|u_l\rangle \mapsto \begin{cases} -|k_4, k_5\rangle|u_1\rangle...|u_l\rangle \text{ if } \mathcal{B}(k_4, k_5, u_1, ..., u_l) = 1, \\ \quad |k_4, k_5\rangle|u_1\rangle...|u_l\rangle \text{ if } \mathcal{B}(k_4, k_5, u_1, ..., u_l) = 0. \end{cases} \tag{9}$$

The complete amplification process is realized by repeatedly for t times applying the unitary operator $Q = -\mathcal{A}S_0\mathcal{A}^{-1}S_{\mathcal{B}}$ to the state $|\varphi\rangle = \mathcal{A}|0\rangle$, i.e. $Q^t\mathcal{A}|0\rangle$.

Initially, the angle between $|\varphi\rangle = \mathcal{A}|0\rangle$ and the bad subspace $|\varphi_0\rangle$ is θ, where $sin^2(\theta) = p = \langle\varphi_1|\varphi_1\rangle$. When p is smaller enough, $\theta \approx arcsin(\sqrt{p}) \approx 2^{-\frac{n}{2}}$. According to Theorem 1, after $k = \lceil\frac{\pi}{4\theta}\rceil = \lceil\frac{\pi}{4\times 2^{-\frac{n}{2}}}\rceil$ Grover iterations Q, the angle between resulting state and the bad subspace is roughly $\pi/2$. The probability P_{good} that the measurement yields a good state is about $sin^2(\pi/2) = 1$.

The whole attack needs $(n + (n/2+1)l + nl/2) = n + (n+1)(n+2+2\sqrt{n/2+1})$ qubits. About $k = \lceil\frac{\pi}{4\times 2^{-\frac{n}{2}}}\rceil = 2^{n/2}$ quantum queries are required to recover k_4, k_5. Thus, in our quantum cryptanalysis on GFS, the first step is to find new quantum distinguishers, and then give a similar quantum key-recovery attacks by appending several rounds to the distinguishers.

4 Quantum cryptanalysis on Type-1 (CAST256-like) GFS

4.1 Quantum distinguishers on Type-1 (CAST256-like) GFS

As shown in Figure 4, the input of the cipher is divided into d branches, i.e. x_j^0 for $1 \leqslant j \leqslant d$, each of which has n-bit, so the blocksize is $d \times n$. R^i is the round function that absorbs n-bit secret key and n-bit input. We construct the corresponding quantum distinguisher on the $(2d-1)$-round cipher.

The intermediate state after the ith round is x_j^i for $1 \leqslant j \leqslant d$, especially the output of the $(2d-1)$th round is denoted as $x_1^{2d-1}||x_2^{2d-1}||...||x_d^{2d-1}$. For the input of round function R^d, we compute its symbolic expression with x_j^0 for $1 \leqslant j \leqslant d$:

$$R^{d-1}(R^{d-2}(...R^3(R^2(R^1(x_1^0) \oplus x_2^0) \oplus x_3^0)... \oplus x_{d-2}^0) \oplus x_{d-1}^0) \oplus x_d^0. \tag{10}$$

Similarly, the output of round function R^d is $x_1^0 \oplus x_2^{2d-1}$. Thus, we get the following equation:

$$R^d(R^{d-1}(R^{d-2}(...R^3(R^2(R^1(x_1^0) \oplus x_2^0) \oplus x_3^0)... \oplus x_{d-2}^0) \oplus x_{d-1}^0) \oplus x_d^0) = x_1^0 \oplus x_2^{2d-1}. \tag{11}$$

In Equation (11), let $x_1^0 = \alpha_b(b = 0, 1,\ \alpha_0, \alpha_1$ are arbitrary constants, $\alpha_0 \neq \alpha_1)$, $x_d^0 = x$, and all of $x_1^0, x_2^0, ..., x_d^0$ be constants, we get

$$R^d(R^{d-1}(R^{d-2}(...R^3(R^2(R^1(\alpha_b) \oplus x_2^0) \oplus x_3^0)... \oplus x_{d-2}^0) \oplus x_{d-1}^0) \oplus x) = \alpha_b \oplus x_2^{2d-1}. \tag{12}$$

Denote $h(\alpha_b) = R^{d-1}(R^{d-2}(...R^3(R^2(R^1(\alpha_b) \oplus x_2^0) \oplus x_3^0)... \oplus x_{d-2}^0) \oplus x_{d-1}^0)$, then Equation (12) becomes $R^d(h(\alpha_b) \oplus x) = \alpha_b \oplus x_2^{2d-1}$. We construct function f as following:

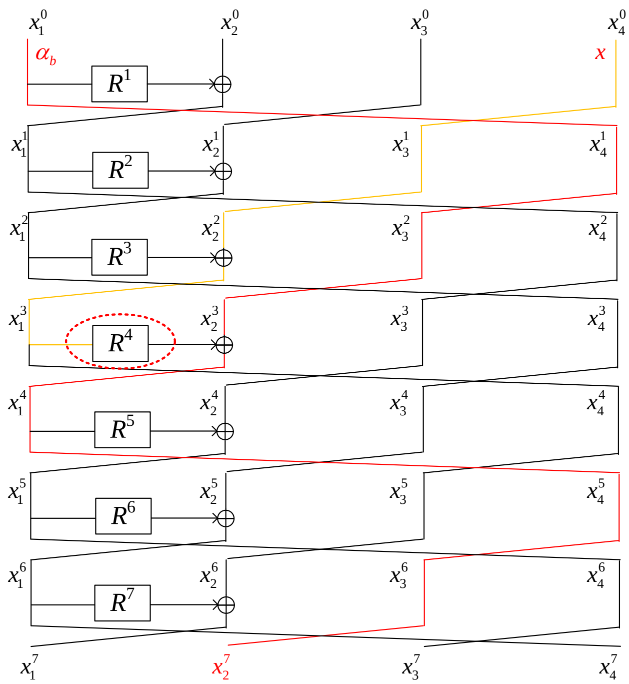

Figure 5 7-round distinguisher on CAST256-like GFS with $d = 4$

$$\begin{aligned} f : \{0,1\} \times \{0,1\}^n &\rightarrow \{0,1\}^n \\ b, x &\mapsto \alpha_b \oplus x_2^{2d-1}, \text{ where } x_1^{2d-1}||x_2^{2d-1}||...||x_d^{2d-1} = E(\alpha_b, x), \\ f(b,x) &= R^d(h(\alpha_b) \oplus x). \end{aligned}$$

So $f(0,x) = f(1, x \oplus h(\alpha_0) \oplus h(\alpha_1)) = R_d(h(\alpha_0) \oplus x)$, $f(1,x) = f(0, x \oplus h(\alpha_0) \oplus h(\alpha_1)) = R_d(h(\alpha_1) \oplus x)$. Thus $f(b,x) = f(b \oplus 1, x \oplus h(\alpha_0) \oplus h(\alpha_1))$. Therefore, function f satisfies Simon's promise with $s = 1||h(\alpha_0) \oplus h(\alpha_1)$.

Example case of Type-1 (CAST256-like) with $d = 4$:

When $d = 4$, we get 7-round quantum distinguisher as shown in Figure 5. Thus, $h(\alpha_b) = R^3(R^2(R^1(\alpha_b) \oplus x_2^0) \oplus x_3^0)$, where x_2^0 and x_3^0 are constants.

4.2 Quantum key-recovery attacks on Type-1 (CAST256-like) GFS

We first study the quantum key-recovery attack on CAST256-like GFS with $d = 4$ branches. Following the similar idea that combines Simon's and Grover's algorithms to attack Feistel structure [19] shown in Section 3.3, we append 7 rounds under the 7-round distinguisher to launch the attack. As shown in Figure 6, there are $4n$-bit key needed to be guessed by Grover's algorithm, which are highlighted in the red boxes of round functions. Thus, the 14-round quantum key-recovery attack needs about 2^{2n} time and $\mathcal{O}(n^2)$ qubits. If we attack $r > 14$ rounds, we need guess $4n + (r-14)n$ key bits by Grover's algorithm. Thus, the the time complexity is $2^{2n + \frac{(r-14)n}{2}}$.

Generally, for $d \geqslant 3$, we could get $(2d-1)$-round quantum distinguisher. We append $d^2 - 3d + 3$ rounds under the quantum distinguisher to attack $r_0 = d^2 - d + 2$ rounds CAST256-like GFS. Similarly, we need to guess $(\frac{1}{2}d^2 - \frac{3}{2}d + 2)n$-bit key by Grover's algorithm. Thus, for r_0 rounds, the time complexity is $(\frac{1}{2}d^2 - \frac{3}{2}d + 2) \cdot \frac{n}{2}$ queries, and $\mathcal{O}(n^2)$ qubits are needed. If we attack $r > r_0$ rounds, we need guess $(\frac{1}{2}d^2 - \frac{3}{2}d + 2)n + (r - r_0)n$ key bits by Grover's algorithm. Thus, the time complexity is $2^{(\frac{1}{2}d^2 - \frac{3}{2}d + 2) \cdot \frac{n}{2} + \frac{(r-r_0)n}{2}}$.

If we use the quantum brute force search (Grover search) to recover the key, for r-round d-branch cipher, totally, rn-bit key need to be found, the complexity is $2^{rn/2}$. Thus, our attack is better than the quantum brute force search (Grover search) by a factor $2^{rn/2 - ((\frac{1}{2}d^2 - \frac{3}{2}d + 2) \cdot \frac{n}{2} + \frac{(r-r_0)n}{2})} = 2^{(\frac{1}{4}d^2 + \frac{1}{4}d)n}$.

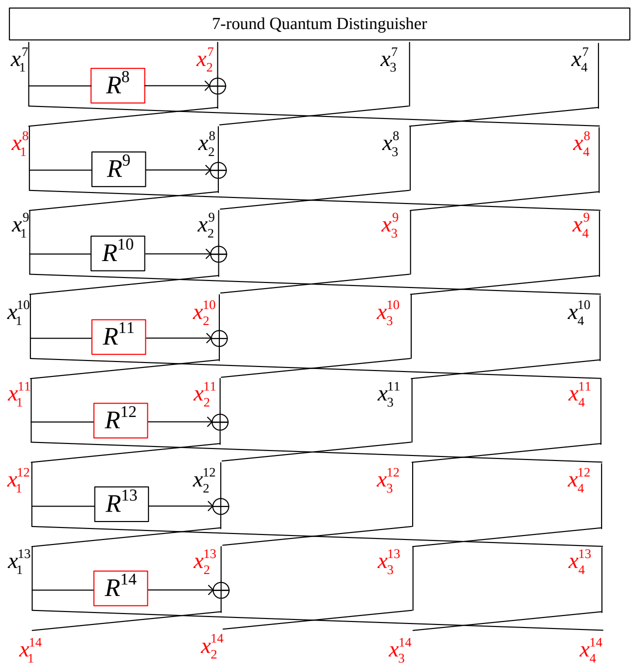

Figure 6 14-round quantum key-recovery attack on CAST256-like GFS with $d = 4$

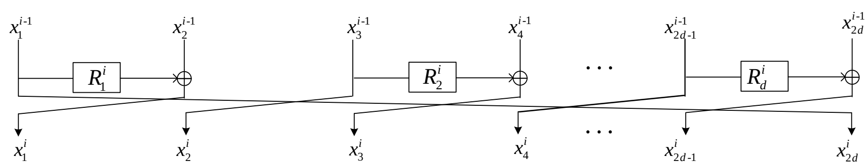

Figure 7 Round i of RC6/CLEFIA-like GFS with $2d$ branches

5 Quantum cryptanalysis on Type-2 (RC6/CLEFIA-like) GFS

5.1 Quantum distinguishers on Type-2 (RC6/CLEFIA-like) GFS

As shown in Figure 7, the input of the cipher is divided into $2d$ branches, i.e. x_j^0 for $1 \leqslant j \leqslant 2d$, each of which has n-bit, so the blocksize is $2d \times n$. R_l^i $(1 \leqslant l \leqslant d)$ is the jth round function in ith round that absorbs n-bit secret key and n-bit input. We construct the corresponding quantum distinguisher on the $(2d+1)$-round cipher.

The intermediate state after the ith round is x_j^i for $1 \leqslant j \leqslant 2d$, especially the output of the $(2d+1)$th round is denoted as $x_1^{2d+1}||x_2^{2d+1}||...||x_{2d}^{2d+1}$.

Case study, $2d = 4$**:**

As shown in Figure 8 with $2d = 4$, for the input of round function R_1^4 about x_j^0 for $1 \leqslant j \leqslant 4$, we compute its symbolic expression:$R_1^3(R_1^2(R_1^1(x_1^0) \oplus x_2^0) \oplus x_3^0) \oplus R_2^1(x_3^0) \oplus x_4^0$. The output of R_1^4 can be expressed as $x_1^0 \oplus x_4^5 \oplus R_2^2(R_2^1(x_3^0) \oplus x_4^0)$. Through R_1^4, we obtain the following equation

$$R_1^4(R_1^3(R_1^2(R_1^1(x_1^0) \oplus x_2^0) \oplus x_3^0) \oplus R_2^1(x_3^0) \oplus x_4^0) = x_1^0 \oplus x_4^5 \oplus R_2^2(R_2^1(x_3^0) \oplus x_4^0). \tag{13}$$

Let $x_1^0 = \alpha_b$, $x_2^0 = x$, x_3^0, x_4^0 be constants, it becomes

$$R_1^4(R_1^3(R_1^2(R_1^1(\alpha_b) \oplus x) \oplus x_3^0) \oplus R_2^1(x_3^0) \oplus x_4^0) = \alpha_b \oplus x_4^5 \oplus R_2^2(R_2^1(x_3^0) \oplus x_4^0). \tag{14}$$

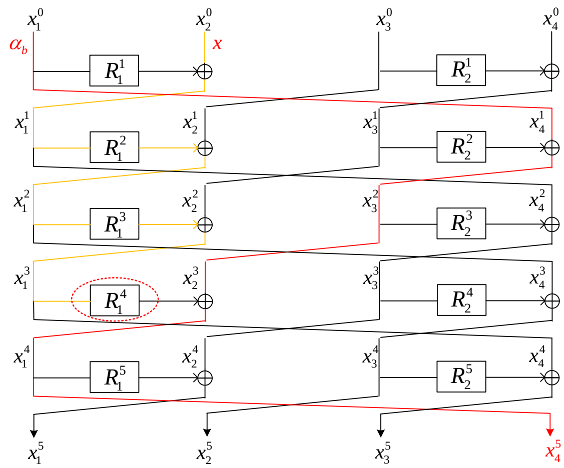

Figure 8 5-round distinguisher on RC6/CLEFIA-like GFS with $2d = 4$

$$\begin{aligned} f_4 : \{0,1\} \times \{0,1\}^n &\rightarrow \{0,1\}^n \\ b, x &\mapsto \alpha_b \oplus x_4^5 \oplus R_2^2(R_2^1(x_3^0) \oplus x_4^0), \text{ where } x_1^5||x_2^5||x_3^5||x_4^5 = E(\alpha_b, x), \\ f_4(b,x) &= R_1^4(R_1^3(R_1^2(R_1^1(\alpha_b) \oplus x) \oplus x_3^0) \oplus R_2^1(x_3^0) \oplus x_4^0). \end{aligned}$$

Thus $f_4(b,x) = f_4(b \oplus 1, x \oplus R_1^1(\alpha_0) \oplus R_1^1(\alpha_1))$. Therefore, function f_4 satisfies Simon's promise with $s = 1||R_1^1(\alpha_0) \oplus R_1^1(\alpha_1)$.

Case study, $2d = 6$**:**

As shown in Figure 9 with $2d = 6$, for the input of round function R_1^6 about x_j^0 for $1 \leqslant j \leqslant 6$, we compute its symbolic expression:$R_1^5(R_1^4(R_1^3(R_1^2(R_1^1(x_1^0) \oplus x_2^0) \oplus x_3^0) \oplus R_2^1(x_3^0) \oplus x_4^0) \oplus R_2^2(R_2^1(x_3^0) \oplus x_4^0) \oplus x_5^0) \oplus R_2^3(R_2^2(R_2^1(x_3^0) \oplus x_4^0) \oplus x_5^0) \oplus R_3^1(x_5^0) \oplus x_6^0$.

The output of R_1^6 can be expressed as $x_1^0 \oplus x_6^7 \oplus R_3^2(R_3^1(x_5^0) \oplus x_6^0) \oplus R_2^4(R_2^3(R_2^2(R_2^1(x_3^0) \oplus x_4^0) \oplus x_5^0) \oplus R_3^1(x_5^0) \oplus x_6^0)$. Through R_1^4, we obtain the following

$$\begin{aligned} &R_1^6[R_1^5(R_1^4(R_1^3(R_1^2(R_1^1(x_1^0) \oplus x_2^0) \oplus x_3^0) \oplus R_2^1(x_3^0) \oplus x_4^0) \oplus R_2^2(R_2^1(x_3^0) \oplus x_4^0) \oplus x_5^0) \oplus R_2^3(R_2^2(R_2^1(x_3^0) \oplus x_4^0) \oplus x_5^0) \\ &\quad \oplus R_3^1(x_5^0) \oplus x_6^0] = x_1^0 \oplus x_6^7 \oplus R_3^2(R_3^1(x_5^0) \oplus x_6^0) \oplus R_2^4(R_2^3(R_2^2(R_2^1(x_3^0) \oplus x_4^0) \oplus x_5^0) \oplus R_3^1(x_5^0) \oplus x_6^0). \end{aligned} \tag{15}$$

Let $x_1^0 = \alpha_b$, $x_2^0 = x$, x_3^0, x_4^0, x_5^0, x_6^0 be constants, it becomes

$$\begin{aligned} &R_1^6[R_1^5(R_1^4(R_1^3(R_1^2(R_1^1(\alpha_b) \oplus x) \oplus x_3^0) \oplus R_2^1(x_3^0) \oplus x_4^0) \oplus R_2^2(R_2^1(x_3^0) \oplus x_4^0) \oplus x_5^0) \oplus R_2^3(R_2^2(R_2^1(x_3^0) \oplus x_4^0) \oplus x_5^0) \\ &\quad \oplus R_3^1(x_5^0) \oplus x_6^0] = \alpha_b \oplus x_6^7 \oplus R_3^2(R_3^1(x_5^0) \oplus x_6^0) \oplus R_2^4(R_2^3(R_2^2(R_2^1(x_3^0) \oplus x_4^0) \oplus x_5^0) \oplus R_3^1(x_5^0) \oplus x_6^0). \end{aligned} \tag{16}$$

$$\begin{aligned} f_6 : \{0,1\} \times \{0,1\}^n &\rightarrow \{0,1\}^n \\ b, x &\mapsto \alpha_b \oplus x_6^7 \oplus R_3^2(R_3^1(x_5^0) \oplus x_6^0) \oplus R_2^4(R_2^3(R_2^2(R_2^1(x_3^0) \oplus x_4^0) \\ &\qquad \oplus x_5^0) \oplus R_3^1(x_5^0) \oplus x_6^0), \text{where } x_1^5||x_2^5||x_3^5||x_4^5||x_5^5||x_6^5 = E(\alpha_b, x), \\ f_6(b,x) &= R_1^6[R_1^5(R_1^4(R_1^3(R_1^2(R_1^1(\alpha_b) \oplus x) \oplus x_3^0) \oplus R_2^1(x_3^0) \oplus x_4^0) \\ &\qquad \oplus R_2^2(R_2^1(x_3^0) \oplus x_4^0) \oplus x_5^0) \oplus R_2^3(R_2^2(R_2^1(x_3^0) \oplus x_4^0) \oplus x_5^0) \oplus R_3^1(x_5^0) \oplus x_6^0]. \end{aligned}$$

Thus $f_6(b,x) = f_6(b \oplus 1, x \oplus R_1^1(\alpha_0) \oplus R_1^1(\alpha_1))$. Therefore, function f_6 satisfies Simon's promise with $s = 1||R_1^1(\alpha_0) \oplus R_1^1(\alpha_1)$.

Similarly, for the $2d$-branch version, we can get corresponding function f_{2d} satisfies Simon's promise with $s = 1||R_1^1(\alpha_0) \oplus R_1^1(\alpha_1)$ at $2d$th round.

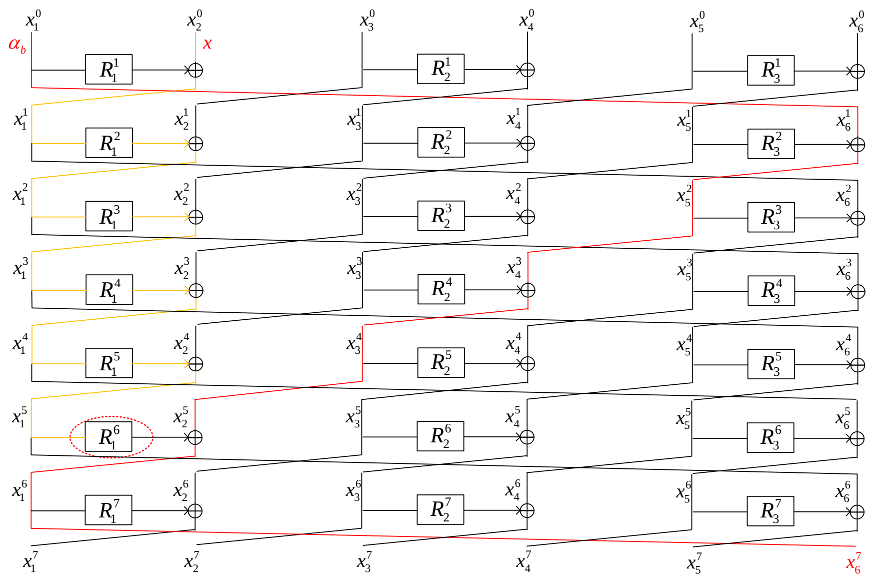

Figure 9 7-round distinguisher on RC6/CLEFIA-like GFS with $2d = 6$

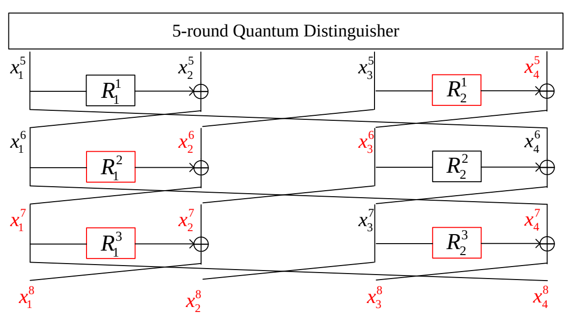

Figure 10 8-round quantum key-recovery attack on RC/CLEFIA-like GFS with $2d = 4$

5.2 Quantum key-recovery attacks on Type-2 (RC6/CLEFIA-like) GFS

Firstly, we study the quantum key-recovery attack on RC6/CLEFIA-like GFS with $2d = 4$ branches. Similarly, combining Simon's and Grover's algorithms shown in Section 3.3, three rounds are appended under the 5-round distinguisher to launch the attack. As shown in Figure 10, there are $4n$-bit key needed to be guessed by Grover's algorithm, which are highlighted in the red boxes of round functions. Thus, the 8-round quantum key-recovery attack needs about 2^{2n} queries and $\mathcal{O}(n^2)$ qubits. If we attack $r > 8$ rounds, we need guess $4n + (r-8) \times 2n$ key bits by Grover's algorithm. Thus, the time complexity is $2^{2n+\frac{(r-8)\times 2n}{2}} = 2^{(r-6)n}$.

Then, for the case of $2d = 6$, we append 5 rounds after the 7-round distinguisher to launch the 12-round quantum key-recovery attack as shown in Figure 11. $9n$ key bits highlighted in red need to be guessed by Grover's algorithm. Thus, the time complexity is $2^{\frac{9n}{2}}$ and $\mathcal{O}(n^2)$ qubits are needed. When we attack $r > 12$ rounds, $9n + (r-12) \times 3n$ key bits need to be guessed by Grover's algorithm. So the time complexity is $2^{\frac{9n}{2}+\frac{(r-12)\times 3n}{2}} = 2^{\frac{(r-9)3n}{2}}$.

Generally, for $2d \geqslant 4$, we could get $(2d+1)$-round quantum distinguisher. We append $2d-1$ rounds under the quantum distinguisher to attack $r_0 = 4d$ round RC/CLEFIA-like GFS. Similarly, we need to guess d^2n-bit key by Grover's algorithm. Thus, for r_0 rounds, the time complexity is $\frac{d^2n}{2}$ queries, and $\mathcal{O}(n^2)$ qubits are needed. If we attack $r > r_0$ rounds, we need guess $d^2n + (r - r_0)dn$ key bits by Grover's

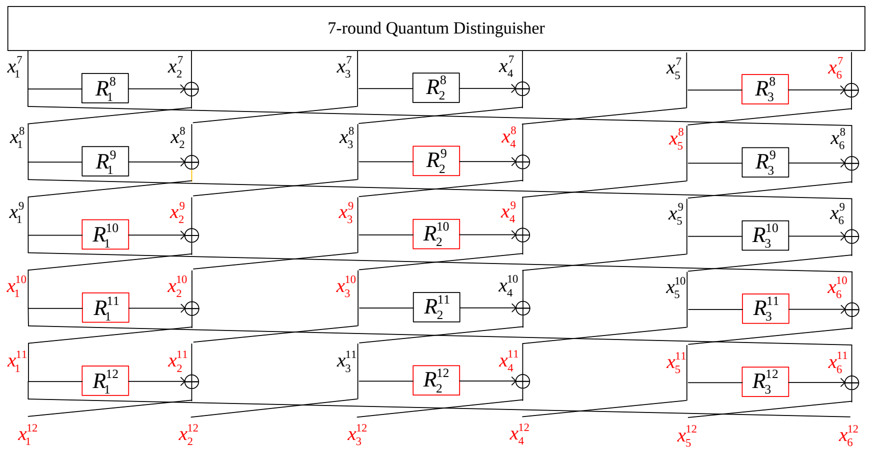

Figure 11 12-round quantum key-recovery attack on RC/CLEFIA-like GFS with $2d = 6$

algorithm. Thus, the time complexity is $2^{\frac{d^2+(r-r_0)d}{2}n}$.

If we use the quantum brute force search (Grover search) to recover the key, for r-round $2d$-branch cipher, totally, rdn-bit key need to be found, the complexity is $2^{rdn/2}$. Thus, our attack is better than the quantum brute force search (Grover search) by a factor $2^{rdn/2-\frac{d^2+(r-r_0)d}{2}n} = 2^{\frac{3d^2n}{2}}$.

6 Conclusion

This paper studies quantum distinguishers and quantum key-recovery attacks on two generalized Feistel schemes (GFS): Type-1 (CAST256-like) and Type-2 (RC6/CLEFIA-like) GFS. For d-branch Type-1 GFS, we introduce $(2d-1)$-round quantum distinguishers with polynomial time. For $2d$-branch Type-2 GFS, we give $(2d+1)$-round quantum distinguishers with polynomial time. Classically, Moriai and Vaudenay [13] proved that a 7-round 4-branch Type-1 GFS and 5-round 4-branch Type-2 GFS are secure pseudo-random permutations. Obviously, they are no longer secure in quantum setting.

Using the above quantum distinguishers, we introduce generic quantum key-recovery attacks by applying the combination of Simon's and Grover's algorithms recently proposed by Leander and May. We denote n as the bit length of a branch. For (d^2-d+2)-round Type-1 GFS with d branches, the time complexity is $2^{(\frac{1}{2}d^2-\frac{3}{2}d+2)\cdot\frac{n}{2}}$, which is better than the quantum brute force search (Grover search) by a factor $2^{(\frac{1}{4}d^2+\frac{1}{4}d)n}$. For $4d$-round Type-2 GFS with $2d$ branches, the time complexity is $2^{\frac{d^2n}{2}}$, which is better than the quantum brute force search by a factor $2^{\frac{3d^2n}{2}}$.

Open discussion: The Chinese standard block cipher SMS4 is based on a different contracting Feistel scheme, we denote it as SMS4-like GFS. For the 4-branch case, we could find a 5-round quantum distinguisher that works with $\mathcal{O}(n)$. However, Zhang and Wu [21] proved that 7-round 4-branch SMS4-like GFS is a pseudo-random permutation. So our quantum distinguisher does not violate Zhang and Wu's claim. It will be interesting to find quantum distinguisher with more rounds.

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Conflict of interest The authors declare that they have no conflict of interest.

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