Improved Circuit-based PSI via Equality Preserving Compression

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Abstract. Circuit-based private set intersection (circuit-PSI) enables two parties with input set X and Y to compute a function f over the intersection set $X \cap Y$, without revealing any other information. State-of-the-art protocols for circuit-PSI commonly involves a procedure that securely checks whether two input strings are equal and outputs an additive share of the equality result. This procedure is typically performed by generic two party computation protocols, and its cost occupies quite large portion of the total cost of circuit-PSI. In this work, we propose *equality preserving compression* (EPC) protocol that compresses the length of equality check targets while preserving equality using homomorphic encryption (HE) scheme, which is secure against the semi-honest adversary. This can be seamlessly applied to state-of-the-art circuit-PSI protocol frameworks. We demonstrate by implementation that our EPC provides $10-40\%$ speed-up for circuit-PSI with set size from 2^{16} to 2^{20}, on LAN network. We believe that EPC protocol itself can be independent interest, which can be applied to other application than PSI.

Keywords: Private Set Intersection, Circuit-based Private Set Intersection, Homomorphic Encryption

1 Introduction

A two-party functionality of private set intersection (PSI) enables two parties P_0 and P_1 having respective input set X and Y to compute the intersection $X \cap Y$, without revealing any other information beyond the original set cardinality $|X|$ and $|Y|$ to each other.

There are many real-world applications related to PSI, and some of them only requiring the intersection set may find an efficient solution from PSI alone. However, there is another variant of PSI that outputs only $f(X \cap Y)$ for some target function f rather than the intersection set $X \cap Y$, and this would be more desirable for other applications. One typical but a popular example is PSI-Cardinality that computes cardinality of the intersection, where $f(X \cap Y) = |X \cap Y|$. Indeed these kinds of PSI are receiving growing attention from industry, for example, Google [21, 26] and Facebook [6] explored some variants PSI including PSI-Cardinality-with-Sum that computes the cardinality and the sum of associated values over the intersection set.

This PSI-with-computation notion is generalized to the *circuit-PSI* functionality, which outputs the intersection information in secret-shared form, instead of the intersection set itself. More precisely, for each element $x \in X$, circuit-PSI outputs each party random bits s_0 and s_1 respectively, such that $s_0 \oplus s_1 = 1$ if and only if $x \in X \cap Y$ (of course 0 otherwise). This is used as a general-purpose preprocessing, in the sense that two parties use the shares to perform target computation on the intersection. Notable example would be PSI-Threshold that only reveals whether the cardinality of $X \cap Y$ is larger than some threshold.

The work of Pinkas *et al.* [31] proposed a novel construction of circuit-PSI protocol which has linear communication complexity in the input set size. After that, several following works [7,34] have proposed improved instantiation of the framework and those works indeed shows the state-of-the-art performance for circuit-PSI.

To generate final bits s_0 and s_1 in circuit-PSI, the framework involves $O(N)$ times of private *equality share generation* (ESG) that takes an input string from each party and outputs Boolean shares of the equality result between two strings for $N = |X| = |Y|$. This is one of the main differences of circuit-PSI from plain PSI, where the latter one typically uses private *equality test* that simply outputs the equality result itself. For private equality test, there are many efficient methods such as *oblivious pseudo-random functions* (OPRFs) [16,23,34]. However it is not directly applicable for ESG, and the most of circuit-PSI protocols perform ESG by other costly methods such as generic two party computations (2PC).

Not too long ago, ESG occupied the largest part of circuit-PSI cost; about 96% and 91% of the total communication in circuit-PSI protocols of [31] and [7] respectively. Although such burden of ESG procedure is greatly reduced thanks to remarkable speed-up of OT extensions [12,37], it still takes quite large portion of circuit-PSI protocol, which is the main reason of performance gap of plain PSI and circuit-PSI.

1.1 Our Contribution

Our work starts with an observation that all known methods for *equality share generation* (ESG) have complexity linear in the input bit-length. Some works [31, 34] simply exploited two party GMW protocol [19] by evaluating equality check circuit composed of $\ell - 1$ AND gates, and it naturally results in complexity linear in ℓ. After then [7, 15] proposed more efficient protocols that have improved communication burden, but it still suffered from linear complexity in ℓ.

- With a purpose of reducing workload of ESG, we propose a functionality what we call *equality preserving compression* (EPC) that converts two large integers into smaller integers, while preserving the equality condition. Then we construct a homomorphic encryption (HE) based efficient protocol realizing the EPC functionality with semi-honest security. Asymptotically it compresses ℓ-bit input integers into $O(\log \ell)$-bits, with $\tilde{O}(\ell)$ computational and communication complexity.

- We then combine our EPC into the circuit-PSI framework of [31]. Our EPC protocol *perfectly* preserve equality, in other words with zero failure probability, and hence the correctness analysis for previous circuit-PSI protocols remains exactly same. Moreover, it provides concrete improvement since it decreases the heavy ESG part input in the logarithmic scale. We check the concrete effect of EPC by implementation, and observe $10 - 40\%$ speed-up over LAN network environment. See Table 2 for details.

1.2 Related Works

Plain PSI. The early proposal of PSI is based on Diffie-Hellman (DH) [25], and this still serve as a basis of modern PSIs with considerably low communication cost but high computational cost. Recently many OPRF-based (plain) PSI protocols [8, 23, 28, 29, 34] have been reported with rather low computational cost, at the cost of communication burden.

PSI-with-functionality. Toward PSI with additional functionality, Google [21, 26] provides PSI-with-computation protocol stem from DH-based PSI, which is tailored for specific target functionality that reveals computing cardinality of the intersection and summing all associated values of the intersection sets. After then Facebook [6] further developed this to a protocol that letting two parties have additive shares of intersected elements, with the purpose of supporting general computation over the intersection set.

Circuit-PSI. As a more generalized concept, circuit-PSI is firstly proposed by [20] and then continuous improvements have been reported [11, 30, 32]. In particular [30] has a similarity with our paper, as their main idea called permutation-based hashing is to cut-off the length of item while preserving equality, with a purpose of reducing the cost for equality check. However, the technique is only applicable to the initial hashing routine (will be explained by cuckoo/simple hashing later), and not compatible with the currently best framework of circuit-PSI due to Pinkas *et al.* [31] based on *oblivious programmable PRF* (OPPRF). As OPPRF-based circuit-PSI framework shows the best performance, whose details are presented later in Section 3. We note that, despite the similarity of their names, construction of OPPRF is quite different from OPRF, and hence OPRF-based PSI protocol does not implies OPPRF-based circuit-PSI protocol. Indeed, we are aware of only one work [34] that constructs plain PSI and circuit-PSI from the same underlying idea. There is another concept of PSI-with-computation [15] different to circuit-PSI, which improves the efficiency of PSI-with-computation while additionally reveals the cardinality of intersection set as well as the desired function evaluation $f(X \cap Y)$.

HE in PSI field. There are also HE-based PSI approaches [9, 10], which mainly focused on extremely unbalance-sized set cases. The first work [10] considered plain PSI, and the main usage of HE is to solve the private set membership (PSM) problem by evaluating inclusion polynomial; $x \in Y$ is equivalent to

$F(x) = \prod_{y \in Y}(x-y) = 0$, which is quite different to our use of HE. The following work [9] extended this protocol to PSI having associated value and strengthened the security to malicious setting, but HE is applied in the similar sense to the previous work. The authors of [9] leaved a short mention on circuit-PSI as a combination of their HE-based PSM protocol with the final equality share generation. As the circuit-PSI protocol was not the main interest of the paper, the authors merely mentioned that the final task can be done by a 2PC without detailed analysis.

1.3 Roadmap

In Section 2, we recall the preliminaries including oblivious transfer and homomorphic encryption, and in Section 3, we present the state-of-the-art circuit-PSI framework due to [31]. In Section 4, we propose an equality preserving compression functionality concept and efficient protocol for that. Then in Section 5, we combine our proposed EPC protocol with the OPPRF-based circuit-PSI protocol to improve efficiency, and provide experimental results in Section 6.

2 Preliminary

2.1 Notations

We write vectors as bold lowercase letters, and matrices as bold uppercase letters. For any real number x, we denote $\lfloor x \rceil$ by the round-off to integer. The i-th component of a vector $\mathbf{v}$ is denoted by v_i, and i,j-th entry of a matrix M is denoted by $m_{i,j}$. For an integer k, a set $\{1, \cdots, k\}$ is denoted by $[k]$. The logarithm function log is assumed to have base 2 unless specially denoted by $\log_w$ with base w. For any statement T that can be determined by true or false (Boolean), we denote $\mathbf{1}(T)$ be the truth value for the equality, i.e., it is 1 if T is true and 0 else.

2.2 Oblivious Transfers

A 1-out-of-n oblivious transfer (OT) of ℓ-bit input messages $(n,1)$-OT_ℓ takes as input n messages $m_1, \cdots, m_n \in \{0,1\}^\ell$ from the sender and a choice index $c \in [n]$ from the receiver, and outputs m_c to the receiver and nothing to the sender. We also use a notion of 1-out-of-2 *correlated-OT* (COT) of ℓ-bit input messages $(2,1)$-COT_ℓ, where the sender inputs a correlation $d \in \{0,1\}^\ell$ and the receiver inputs a choice bit $b \in \{0,1\}$. Then the functionality outputs to the sender r and $d+r$ for a randomly chosen $r \in \{0,1\}^\ell$, and to the receiver $b \cdot d + r$. We write m times of $(n,1)$-$(\mathsf{C})\mathsf{OT}_\ell$ calls by $(n,1)$-$(\mathsf{C})\mathsf{OT}_\ell^m$.

There are protocols called OT-extension (OTe) that efficiently extend small numbers of *base* OTs to large numbers of OTs. Assuming that such small numbers of base OTs are done, the most typical IKNP OTe protocols execute $(2,1)$-OT_ℓ and $(2,1)$-COT_ℓ with communication $\lambda + 2\ell$ [22] and $\lambda + \ell$ [3] bits per

one call. Recently another breakthrough line of OT extensions [5, 12, 37] are proposed, which greatly reduces communication overhead of IKNP-style OT-extension, while preserving similar computational cost to IKNP. For sufficiently many OT and COT calls, for example more than 2^{20} calls, Silent OTe allows one to execute $(2,1)$-OT_ℓ and $(2,1)$-COT_ℓ with nearly $2\ell+1$ and $\ell+1$ bit communication per one call, respectively.

GMW protocol or Gate evaluation. For a bit $x \in \{0,1\}$, we say $x_0 \in \{0,1\}$ and $x_1 \in \{0,1\}$ satisfying $x = x_0 \oplus x_1$ be 2-party additive Boolean shares, or simply Boolean shares of x. Consider two bits x and y are shared as x_i and y_i by two party P_0 and P_1. Then two parties can privately compute Boolean shares of gate evaluations on input x and y using OT. Note that Boolean shares for XOR $x \oplus y$ can be easily computed by $x_i \oplus y_i$ by each party's own. Boolean shares for AND gate can be evaluated by $(2,1)$-COT_1^2 [13,19]. For the underlying idea, observe that $(2,1)$-COT_1 with the sender's input correlation bit d and the receiver's input choice bit b essentially computes Boolean shares of $b \wedge d$. To evaluate an AND gate, two parties execute a correlated-OT with input x_i and y_{1-i} to have Boolean shares of $a = x_i \wedge y_{1-i}$, and then with input y_i and x_{1-i} to have Boolean shares of $b = x_{1-i} \wedge y_{1-i}$. Then the party P_i outputs $x_i \wedge y_i \oplus a_i \oplus b_i$ and the other party P_{1-i} outputs $x_{1-i} \wedge y_{1-i} \oplus a_{1-i} \oplus b_{1-i}$, which are Boolean shares of $x \wedge y = (x_0 \oplus x_1) \wedge (y_0 \oplus y_1)$.

2.3 RLWE-based Homomorphic Encryption

A homomorphic encryption (HE) scheme is an encryption scheme that supports a ring-structured plaintext $\mathcal{M}$, and homomorphic arithmetic operations between ciphertexts that acts on inner plaintext. We especially exploit a ring learning with errors (RLWE) based HE scheme, BFV scheme [14].

For simplicity, we restrict our description for RLWE-based HE using power-of-2 cyclotomic rings of integers, which is widely used in several HE libraries. Let $\mathcal{R} := \mathbb{Z}[X]/(X^n+1)$ be a polynomial quotient ring where n is a power-of-2 integer. This scheme supports a plaintext space $\mathcal{R}_p := \mathcal{R}/p\mathcal{R} = \mathbb{Z}_p[X]/(X^n+1)$ for some plaintext modulus prime integer p, and the corresponding ciphertext space is $\mathcal{R}_q^2$ for some $q \gg p$.

BFV Scheme. We will briefly review the BFV homomorphic encryption scheme. The IND-CPA security of BFV is based on the hardness assumption of the RLWE problem. For more details, we refer to [4, 14].

Key Generation. Given a security parameter $\lambda > 0$, fix integers n, P (P be a positive integer that will be used in the evaluation key generation), and distributions $\mathcal{D}_{key}$, $\mathcal{D}_{err}$ and $\mathcal{D}_{enc}$ over $\mathcal{R}$ in a way that the resulting scheme is secure against any adversary with computational resource of $O(2^\lambda)$. Typically $\mathcal{D}_{key}$ is chosen by ternary coefficient polynomials in $\mathcal{R}$, and $\mathcal{D}_{err}$ and $\mathcal{D}_{enc}$ are chosen by a discrete Gaussian distribution of appropriate standard deviation σ.

1. Sample $a \leftarrow \mathcal{R}_q$, $s \leftarrow \mathcal{D}_{key}$, and $e \leftarrow \mathcal{D}_{err}$. Then the secret key is defined as $\mathsf{sk} = (1, s) \in \mathcal{R}^2$, and the corresponding public key is defined as $\mathsf{pk} = (b, a) \in \mathcal{R}_q^2$, where $b = [-a \cdot s + e]_q$.
2. Sample $a' \leftarrow \mathcal{R}_q$ and $e' \leftarrow \mathcal{D}_{err}$. Then the evaluation key is defined as $\mathsf{evk} = (b', a') \in \mathcal{R}_q^2$, where $b' = [-a' \cdot s + e' + Ps']_q$ for $s' = [s^2]_q$.

Encryption. Given a public key pk and a plaintext $m \in \mathcal{R}$, Sample $r \leftarrow \mathcal{D}_{\mathsf{Enc}}$ and $e_0, e_1 \leftarrow \mathcal{D}_{err}$. Then compute $\mathsf{Enc}(\mathsf{pk}, 0) = [r \cdot \mathsf{pk} + (e_0, e_1)]_q$ and $\mathsf{Enc}^{\mathsf{BFV}}(\mathsf{pk}, m) = [\mathsf{Enc}(\mathsf{pk}, 0) + (\Delta_{\mathsf{BFV}} \cdot [m]_p, 0)]_q$, where $\Delta_{\mathsf{BFV}} = \lfloor q/p \rfloor$.

Decryption. Given a secret key $\mathsf{sk} \in \mathcal{R}^2$ and a ciphertext $\mathsf{ct} \in \mathcal{R}_q^2$, $\mathsf{Dec}^{\mathsf{BFV}}(\mathsf{sk}, \mathsf{ct}) = \left\lfloor \frac{p}{q} [\langle \mathsf{sk}, \mathsf{ct} \rangle]_q \right\rceil$.

The ciphertext of BFV scheme is $(b(x), a(x))$ satisfying $b(x) = -a(x) \cdot s(x) + e(x)$. The $e(x)$ part is called as *noise term* of ciphertext. We note that infinite norm of noise term of ct in decryption function should be bounded by $\frac{q}{2p}$ for correctness of decryption.

Addition. Given ciphertexts ct_1 and ct_2 in $\mathcal{R}_q^2$, their sum is defined as $\mathsf{ct}_{\mathsf{Add}} = [\mathsf{ct}_1 + \mathsf{ct}_2]_q$.

Multiplication. Given ciphertexts $\mathsf{ct}_1 = (b_1, a_1)$ and $\mathsf{ct}_2 = (b_2, a_2)$ in $\mathcal{R}_q^2$ and an evaluation key evk, their product is defined as $\mathsf{ct}_{\mathsf{Mult}} = \left[(d_0, d_1) + \left\lfloor P^{-1} \cdot d_2 \cdot \mathsf{evk} \right\rceil\right]_q$, where (d_0, d_1, d_2) is defined by $\left[\left\lfloor \frac{p}{q}(b_1 b_2, a_1 b_2 + a_2 b_1, a_1 a_2) \right\rceil\right]_q$.

Batching. BFV scheme basically supports encryptions of plaintext ring $\mathcal{R}_p$ element, and homomorphic addition and multiplication over $\mathcal{R}_p$. As a useful notion for batching multiple data in one ciphertext, one can use a ring isomorphism $\mathcal{R}_p \cong \mathbb{F}_{p^d}^{n/d}$ where d is the smallest integer such that $p^d = 1 \mod 2n$ and $\mathbb{F}_{p^d}$ is a finite field of order p^d. Using this isomorphism, one can perform slot-wise encryption and operation of n/d elements in $\mathbb{F}_{p^d}$ by single instruction on the ciphertext. It is worth to note when the plaintext modulus p and the polynomial quotient n satisfies

$$p = 1 \mod 2n, \tag{1}$$

which provides n slots of $\mathbb{Z}_p$ element. This can be achieved only with somewhat restrictive parameters, but the underlying plaintext slot $\mathbb{Z}_p$ is much simpler than extension fields $\mathbb{F}_{p^d}$ so that one can fully enjoy the power of batching. In this regard, we refer this case by *full batch* and indeed our paper mainly focus on full batch HE parameters.

Security Notions. For security, we consider the standard IND-CPA security that requires two ciphertexts of different messages are (computationally) indistinguishable given an encryption oracle. The IND-CPA security of RLWE-based

HE literally comes from the hardness of ring learning with errors (RLWE) problem. For concrete parameter setting of IND-CPA security, the bit-size of ciphertext modulus $\log q$ and polynomial ring dimension n, and error distribution $\mathcal{D}_{err}$ should be selected to secure against various lattice reduction attacks.

3 Circuit-based PSI

The definition circuit-based PSI (circuit-PSI) functionality to generate Boolean additive shares is given as Figure 1. After circuit-PSI, the results can be used for one's desired function evaluation. In the rest of this section, we describe the abstract framework of [31] which continues to the following improvements [7,34]. Then we especially review the equality share generation method of each work which occupies the largest part of the total cost, from which we can observe the input bit-length ℓ equality share generation plays the most crucial role for complexity.

Parameters: A receiver with an input set X of size N and a sender with an input set Y of size N.

Functionality: The functionality sends to the receiver an injective indexing function $\iota : X \to [M]$ for some $M \geq N$ and a vector $\mathbf{s}_0 \in \{0,1\}^M$, and to the sender a vector $\mathbf{s}_1 \in \{0,1\}^M$ such that $s_{0,i} \oplus s_{1,i} = \mathbf{1}(\iota^{-1}(i) \in X \cap Y)$ for $i \in \iota(X)$, and $s_{0,i} \oplus s_{1,i} = 0$ for $i \notin \iota(X)$.

Fig. 1: $\mathcal{F}_{\mathsf{CPSI}}$. (Ideal) Functionality of circuit-PSI

3.1 OPPRF-based Circuit-PSI Framework

Let the receiver $\mathcal{R}$ holds a set X and the sender $\mathcal{S}$ holds a set Y of the same size N. The framework consists of the following three main stages.

Step 1. Hashing. For $\varepsilon > 0$, each party creates a hash table with $M = (1+\varepsilon)\cdot N$ bins, but with different hashing method. The receiver applies cuckoo hashing with d hash functions $h_1, \cdots, h_d : \{0,1\}^* \to [M]$ on input X. More precisely, for a suitable choice of ε, there is a cuckoo hashing algorithm that stores every element $x \in X$ in $h_j(x)$-th bin for some $j \in [d]$ with overwhelming probability, while ensuring that at most one element is stored in each bin. This yields a simple representation of the cuckoo hash table: $T_X[h_j(x)] = x$. Note that the mapping from $x \in X$ to $h_j(x)$ determines the indexing function ι in the circuit-PSI definition of Figure 1.

On the other hand, the sender creates a simple hash table with the same hash functions on input Y, which stores each $y \in Y$ in every bin $h_j(x)$ for every

$j \in [d]$. Naturally each bin can hold more than one element, and hence the i-th bin of the simple hash table $T_Y[i]$ is indeed a set. It is known that that for $M = O(N)$ hash table size, the number of elements in each bin is $O(\log(N))$.

Since $h_j(x) \neq h_j(y)$ for some j implies $x \neq y$, two parties only need to compare each elements of the same bin of each hash tables. Since the cuckoo hash table T_X ensures at most one element of $x \in X$ per each bin, circuit-PSI reduces to the problem that securely outputs an additive share of $\mathbf{1}(T_X[i] \in T_Y[i])$ for each bin i, which is essentially a private set membership (PSM) problem. Here the receiver has to fill the empty bin in T_X with dummy value to prevent additional information leakage.

Step 2. Bin Tagging. This step further reduces the aforementioned PSM problem into an equality share generation (ESG) problem between two parties, where each party inputs a vector $\mathbf{v}$ and $\mathbf{v}^*$ of length M respectively, and is given as output a Boolean vector of additive share of $\mathbf{1}(v_i = v_i^*)$.

This is realized by a functionality called *oblivious programmable pseudo-random function* (OPPRF) [24] where the sender obliviously computes a PRF F on receiver's input while the sender can *program* F with values (y_i, z_i) so that $F(y_i) = z_i$. The formal definition of OPPRF is given as Figure 2. [31] is the first work that applies OPPRF functionality for this purpose, and then [7] and [34] developed more efficient OPPRF protocols to improve the performance of circuit-PSI.

Parameters: A sender with input $L = \{(y_i, z_i)\}$ where $y_i \in \{0,1\}^*$ and $z_i \in \{0,1\}^\ell$, and a receiver with input $X = \{x_i\}$ with $x_i \in \{0,1\}^*$.

Functionality: The functionality samples a random function $F : \{0,1\}^* \to \{0,1\}^\ell$ such that $F(y) = z$ for each $(y, z) \in L$, and sends $F(X) := \{F(x) : x \in X\}$ to the receiver.
After then, upon an input y of the sender, the functionality outputs $F(y)$ to the sender.

Fig. 2: $\mathcal{F}_{\mathsf{OPPRF}}$. (Ideal) Functionality of oblivious programmable PRF

To convert PSM problem to ESG problem, two parties execute a protocol for OPPRF functionality with the following input. The sender who has a simple table samples a random tag value $v_i \in \{0,1\}^\ell$ for each i-th bin, and generate the input set L obtained by concatenating each $y \in Y$ with the tag of the bins where y is stored, namely

$$L = \left\{ \left(y || h_j(y), v_{h_j(y)} \right) \right\}_{y \in Y, j \in [d]} = \left\{ (y' || i, v_i) \right\}_{i \in [M], y' \in T_Y[i]}.$$

The receiver feeds its input set by $\tilde{T}_X = \{T_X[i]||i\}_{i \in [M]}$. After the execution of OPPRF protocol, the receiver assigns

$$v_i^* = F\left(T_X[i]||i\right) \in \{0,1\}^\ell$$

in each hash address i to construct a vector $\mathbf{v}^*$ of length M. From the definition of OPPRF functionality, it holds that $v_i = v_i^*$ if the element $T_X[i]$ is in the set $T_Y[i]$, otherwise v_i^* is a random element. Therefore the original PSM-related problem is translated into equality share generation problem between $\mathbf{v}$ from the sender and $\mathbf{v}^*$ from the receiver.

Remark 1 (Failure Probability). Note that there is a failure probability of $2^{-\ell}$ where the random element v_i^* is same to v_i despite $T_X[i]$ is not in $T_Y[i]$. The length of tag ℓ should be chosen so that the overall failure probability is smaller than $2^{-\sigma}$ where σ is statistical security parameter. Since there are M bins, it should hold that $2^{-\sigma} > 1 - (1 - 2^{-\ell})^M$, which is sufficient with

$$\ell > \sigma + \lceil \log M \rceil. \tag{2}$$

One exception is OPPRF of [7] that requires $\ell > \sigma + \lceil \log 4M \rceil$, and this comes from different structure of their OPPRF. For the detailed explanation, see Appendix A.

Step 3. Equality Share Generation. In this step two parties finally generate Boolean shares of $\mathbf{1}(v_i = v_i^*)$, whose definition is formally given as Figure 3.

Parameters: A sender with an input string a and a receiver with an input string b.
Functionality: The functionality outputs bits s_0 and s_1 such that $s_0 \oplus s_1 = \mathbf{1}(a = b)$ to each party respectively.

Fig. 3: $\mathcal{F}_{\mathsf{ESG}}$. (Ideal) Functionality of equality share generation

There are several known methods [7,15] to perform this step in semi-honest model. Most of methods take an approach that evaluates the equality check circuit on ℓ-bit string, composed of $\ell - 1$ AND gate evaluations. One may exploit Yao's garbled circuit protocol [38] for evaluation, and it requires $2\lambda = 256$ bits communication per one AND gate evaluation, with only a single round of interactions. Meanwhile, GMW protocol requires only 2 bits communication per one AND gate[1], at the cost of $\log(\ell)$ rounds of interactions. We consider the low communication benefit of GMW protocol is larger than the smaller round complexity of Yao's protocol. In the remaining of the paper, we implicitly assume that ESG is executed by GMW protocol.

3.2 Applications of Circuit-PSI

Below we present some typical but popular applications of circuit-PSI. We would like to remark that the overheads for these applications are significantly small compared to circuit-PSI cost, as also remarked in [31].

[1] Thanks to recent improvements on OT extension [12,37].

Private Intersection Cardinality and Threshold. These applications would be most direct consequences of circuit-PSI. The cardinality of intersection set (*PSI-Ca*) can be obtained by evaluating a Hamming distance circuit that requires less than M AND gates on circuit-PSI outputs. Moreover, by augmenting one comparison circuit to the Hamming distance circuit (less than $M + \log M$ AND gates), we can let the parties know whether the cardinality is larger than some threshold t (*PSI-Th*).

Private Sum over Intersection. Assume the sender having set X additionally holds an associated values $\{v_x \in \mathbb{G} : x \in X\}$ for some additive group $\mathbb{G}$, and we want to let the receiver having set Y knows the sum of associated values over the intersection set, namely $V = \sum_{x \in X \cap Y} v_x$. This is sometimes called *PSI-Sum*[2]. For that we adapt a method of [15]: The sender samples $\mathbf{r} \in \mathbb{G}^M$ that sums to $\sum r_i = 0$. Then two parties execute OT upon the choice bit $s_{1,i}$ from the receiver, and two messages r_i for $s_{0,i}$ choice and $r_i + v_{\iota^{-1}(i)}$ for $1 - s_{0,i}$ choice from the sender, where $v_{\iota^{-1}(i)} = 0$ for $i \notin \iota(X)$. The receiver adds all received value to have $\sum r_i + V = V$, without knowing any other information since each summand is masked by random value r_i. This can be easily tweaked to let the sender know $V = \sum_{x \in X \cap Y} v_x$, by letting the sender samples $\mathbf{r}$ such that $\sum r_i = R$ for a sender-side chosen $R \in \mathbb{G}$. Then from the same protocol the receiver ends with $R + V$, and finally sends back the value to the sender so that the sender recover $V = (R + V) - R$.

Remark 2. Circuit-PSI can handle the case where both parties hold associated value sets so that parties perform further computations over those sets. However it is somewhat complicated as it requires some modification of OPPRF application (of Step 2. Bin Tagging). Thus we simply refer Section 6 of [31] for details.

4 Equality Preserving Compression

The final equality share generation procedure occupies the largest part of the total cost in circuit-PSI protocol, and the input bit-length ℓ of equality share generation plays an important role. In this section, we present a procedure that converts the equality share generation target inputs into another values whose size is asymptotically logarithm to the original input bit-length, while the equality results remain unchanged. More formally, we define the 2-party functionality *equality preserving compression* (EPC) $\mathcal{F}_{\mathsf{EPC}}$ that takes an integer $v \in \mathbb{Z}_t$ from the sender and another integer $v^* \in \mathbb{Z}_t$ from the receiver. The functionality outputs each party a random integer r and r^* in another modulus ring $\mathbb{Z}_p$, where it holds that $v = v^*$ in $\mathbb{Z}_t$ *if and only if* $r = r^*$ in $\mathbb{Z}_p$ for $p < t$.

[2] Some protocols [15, 21] outputs both cardinality and summation. It should be remarked that circuit-PSI based protocol can selectively exposes cardinality or summation, or even both.

Parameters: A sender with an input $v \in \mathbb{Z}_t$ and a receiver with an input $v^* \in \mathbb{Z}_t$, and the target size p.

Functionality: The functionality sends a random $r \in \mathbb{Z}_p$ and $r^* \in \mathbb{Z}_p$ to the sender and receiver respectively, such that $v = v^*$ in $\mathbb{Z}_t$ if and only if $r = r^*$ in $\mathbb{Z}_p$.

Fig. 4: $\mathcal{F}_{\mathsf{EPC}}$. (Ideal) Functionality of equality preserving compression

4.1 A Basic Protocol

Our protocol starts from the following simple observation on word decomposition. For any base w, the w-base decomposition of v and v^* by $v = \sum_{i=0}^{u-1} v_i \cdot w^i$ and $v^* = \sum_{i=0}^{u-1} v_i^* \cdot w^i$ where $u := \lceil \log_w t \rceil$ and $v_i, v_i^* \in [0, w)$ satisfies

$$v = v^* \iff D := \sum_{i=0}^{u-1} (v_i - v_i^*)^2 = 0 \text{ in } \mathbb{Z}. \tag{3}$$

Note that $D \leq u \cdot (w-1)^2 \approx \log_w t \cdot (w-1)^2$, which has much smaller size than the original size t.

Based on this idea, we consider a simple protocol that privately computes D and output a random element $r \in \mathbb{Z}_p$ and $r^* := r + D \in \mathbb{Z}_p$ by Figure 5. However, the correctness may fail without any condition on the word base w and p, since it may happen that $r = r^* \in \mathbb{Z}_p$ despite of $v \neq v^*$ if D is divisible by p. To avoid this, the word base w has to be chosen so that D is always less than p, namely

$$p > u \cdot (w-1)^2. \tag{4}$$

We note that $u \cdot (w-1)^2 = O(\log t)$, this protocol asymptotically realizes $\mathcal{F}_{\mathsf{EPC}}$ for $p = O(\log t)$.

4.2 Optimizations and Full Protocol

Upon the basic protocol above, we specially focus on BFV scheme to utilize batching property. Furthermore, we achieve huge speed-up from a simple decomposition of $D = \sum (v_i - v_i^*)^2$ by totally removing homomorphic ciphertext multiplication. On security aspect, we use noise flooding to ensure function privacy of homomorphic encryption. A full protocol description that puts everything together is presented by Figure 6, and below we provide some details for each technique.

Batching with RLWE-based HE. As reivewed in Section 3, two parties have to perform $O(N)$-many times of equality checks in circuit-PSI. In this regard, we can exploit batch property of BFV scheme to perform multiple calls of $\mathcal{F}_{\mathsf{EPC}}$, on some conditions on target size p and HE parameters. To recall, for the given RLWE dimension n, we can encrypt n/d number of $\mathbb{F}_{p^d}$ elements in

Parameters: A sender with input $v \in \mathbb{Z}_t$ and a receiver with input $v^* \in \mathbb{Z}_t$ and the target size p.

Protocol:

1. Sender generates a homomorphic encryption secret key sk, and decomposes in w-base $v \in \mathbb{Z}_t$ to $\{v_i\}_{0 \le i < u}$ for $u = \lceil \log_w t \rceil$. After that sender encrypts each v_i using sk, and sends them to receiver.
2. Receiver picks a random integer $r \in \mathbb{Z}_p$, and decomposes $v^* \in \mathbb{Z}_t$ to $\{v_i^*\}_{0 \le i < u}$. Then receiver homomorphically compute $r + \sum_{i=0}^{u-1}(v_i - v_i^*)^2$, and sends the resulting ciphertext back to sender.
3. Sender decrypts the received ciphertext using sk, to obtain $r^* = r + \sum_{i=0}^{u-1}(v_i - v_i^*)^2 \in \mathbb{Z}_p$.

Fig. 5: A basic protocol for $\mathcal{F}_{\mathsf{EPC}}$ functionalities

one ciphertext for the smallest integer d such that $p^d = 1 \bmod 2n$. This means that using smaller p gives better compression ratio, but makes the number of slots in a single ciphertext smaller. For example, $p > 2n$ is necessary to use full batch (i.e n slots).

Removing Ciphertext Multiplications. In most of HE schemes, homomorphic multiplication takes much larger time than scalar multiplication. To remove homomorphic multiplications, we let the sender additionally sends one more ciphertext which is an encryption of $\sum_{i=0}^{u-1} v_i^2$. In this case, the receiver can compute D by

$$D = \sum_{i=0}^{u-1} v_i^2 - 2 \cdot \sum_{i=0}^{u-1} v_i \cdot v_i^* + \sum_{i=0}^{u-1} v_i^{*2}.$$

As the receiver knows v_i^* values, it can compute $\sum_{i=0}^{u-1} v_i^{*2}$ part and then the receiver only needs to perform scalar multiplications and additions to obtain an encryption of D.

Remark 3 (Additive HE). This optimization opens possibility to apply *additive homomorphic encryption* (AHE) schemes such as Paillier scheme [27], but the performance of RLWE-based AHE is still better when we use small plaintext space and batching technique. See Appendix B for more detailed argument.

Realizing Function Privacy. For the security proof, we need to ensure the function privacy from the return ciphertext from receiver to sender. For that we apply randomization and noise flooding method, whose detail will be presented in the next subsection. Concretely this can be realized by letting receiver randomize the resulting ciphertext by homomorphically adding a fresh encryption of zero, and add large enough error to apply noise flooding method before send the computation result back to sender.

(In-)efficiency of Binary Case: $w = 2$. The extreme case $w = 2$ deserves to be considered independently, as it obviously results in the smallest output (exactly $\log t$), although requires the largest number of ciphertexts communication. In fact, we found in literature [18] a similar idea using (3) especially for bit decomposition, which further exploits a computational convenience of bit decomposition: Note that $(x-y)^2 = x \oplus y$ for binary x and y, and $x \oplus y$ can be computed by outputting x if $y = 0$ and $1 - x$ otherwise, which does not require even scalar multiplications.

Therefore, one may think $w = 2$ as an appealing choice due to these advantages, while sacrificing some communications. However, we would like to remark that the batching efficiency has to be importantly considered also, and this extreme case indeed has quite poor batching efficiency. It is because the desired plaintext modulus $p \approx \log t$ becomes smaller than a typical choice of the ring dimension $n \geq 4096$ of RLWE-based HE. For example, our interest t is less than 64 bits, and it can be easily checked that small primes of size ≈ 64 have order at least 32 in $\mathbb{Z}_{2n}$ for $n = 4096$. This means that we are only able to batch $128 = 4096/32$ elements in one ciphertexts. On the contrary, we can take full 4096 slots by taking larger word size w that provides $p > 2n$, which are indeed used for our experiments in Section 6.

4.3 Security and Cost Analysis

In this section, we will discuss about security of our protocol with correctness proof. We also analyze the computational and communication costs. Before that, we need to recall some details of RLWE-based HE scheme. We will focus on BFV scheme [14], but it does not mean that our method is restricted to this scheme.

Randomizing BFV Ciphertexts. Recall that a BFV encryption of a message $m(x)$ is of the form

$$\left(-a(x) \cdot s(x) + \frac{q}{p} \cdot m(x) + e(x), a(x)\right) \in \mathcal{R}_q^2.$$

As secret key owner can recover not only $m(x)$ but also $e(x)$. For this reason, we need to add additional noise $e^*(x)$ such that $|e_i^*| > 2^\sigma \cdot B$ for the function privacy of homomorphic encryption scheme. Here B is upper bound of $e(x)$'s coefficients and σ is the statistical security parameter. This method is called *noise flooding* and this idea is firstly proposed by [17].

Noise Analysis. For the concrete choice of homomorphic encryption parameter, we need to analyze the noise term in our HE-based EPC protocol. Here we will consider the infinity norm $||f(x)||$ which is defined as $\max_i |f_i|$ and the expansion factor of ring $\mathcal{R}$ is defined as $\delta_R = \max\{||f(x) \cdot g(x)||/(||f(x)|| \cdot ||g(x)||) : f(x), g(x) \in \mathcal{R}\}$. In addition, we assume that the noise term of $\mathsf{ctxt}_{k,i}$ in Figure 6 is bounded by B_{fresh}.

Parameters: A sender with input $\mathbf{v} \in \mathbb{Z}_t^M$ and a receiver with input $\mathbf{v}^* \in \mathbb{Z}_t^M$ and the target size p.

Protocol:

1. [**Setup**] Two parties agree on a proper HE parameter (n, q) that supports plaintext space $\mathbb{Z}_p^n$, and satisfies IND-CPA security. Then the sender samples a key pair $(\mathsf{sk}, \mathsf{pk})$, and sends the public key pk to the receiver. The sender pads $\mathbf{v}$ by 0 and the receiver pads $\mathbf{v}^*$ by 1, until they have length divisible by n, say $\gamma \cdot n$. Two parties also agree on word base w satisfying $p > \lceil \log_w t \rceil \cdot (w-1)^2$, and define $u = \lceil \log_w t \rceil$.
2. [**Encryption**] Sender performs the following for $0 \leq k < \gamma$:
   (a) Decompose each v_{nk+j} into $\sum_{i=0}^{u-1} v_{j,i} \cdot w^i$ for $1 \leq j \leq n$.
   (b) Batch them into $\mathbf{m}_{k,i} = (v_{j,i})_{1 \leq j \leq n} \in \mathbb{Z}_p^n$ for $0 \leq i < u$.
   (c) Define $\mathbf{m}_{k,u} = (\sum_{i=0}^{u-1} v_{j,i}^2)_{1 \leq j \leq n} \in \mathbb{Z}_p^n$
   (d) Encrypt $\{\mathbf{m}_{k,i}\}$ into $\{\mathsf{ctxt}_{k,i}\}$ using sk and send those ciphertexts to the receiver.
3. [**Compute D and Masking**] Receiver performs the following for $0 \leq k < \gamma$:
   (a) Decompose each v^*_{nk+j} into $\sum_{i=0}^{u-1} v^*_{j,i} \cdot w^i$ for $1 \leq j \leq n$.
   (b) Batch them into $\mathbf{m}^*_{k,i} = (v^*_{j,i})_{1 \leq j \leq n} \in \mathbb{Z}_p^n$ for $0 \leq i < u$.
   (c) Define $\mathbf{m}^*_{k,u} = (\sum_{i=0}^{u-1} v^{*2}_{j,i})_{1 \leq j \leq n} \in \mathbb{Z}_p^n$
   (d) Compute a ciphertext $\mathsf{ctxt}_{k,d} = \mathsf{ctxt}_{k,u} \oplus \sum_{i=0}^{u-1} \left(\mathsf{ctxt}_{k,i} \odot 2\mathbf{m}^*_{k,i}\right) \oplus \mathbf{m}^*_{k,u}$
   (e) Sample a random vector $\mathbf{r}_k^* \in \mathbb{Z}_p^n$.
   (f) Generate an encryption $\mathsf{ctxt}_{fp,k}$ (using pk) of zero of error size B_{fp} which is large enough for function privacy.
   (g) Send back $\mathsf{ctxt}_k := \mathsf{ctxt}_{k,d} \oplus \mathsf{ctxt}_{fp,k} \oplus \mathbf{r}_k^*$ to the sender.
4. [**Decryption**] Sender decrypts ctxt_k to have $\mathbf{r}_k \in \mathbb{Z}_p^n$ for $0 \leq k < \gamma$.
5. [**Finalize**] Sender outputs $\mathbf{r} \in \mathbb{Z}_p^M$ by concatenating every $\mathbf{r}_k$ and cutting the last $\gamma \cdot n - M$ dummy elements. Receiver outputs $\mathbf{r}^* \in \mathbb{Z}_p^M$ by performing the same with $\mathbf{r}_k^*$.

Fig. 6: A full protocol Π_{BEPC} for M batch calls of $\mathcal{F}_{\mathsf{EPC}}$ functionalities

Lemma 1 (Noise growth during homomorphic scalar multiplication). *For the given BFV ciphertext $(b(x), a(x))$ with noise term $e(x)$ such that $||e(x)|| < B$, the result ciphertext of homomorphic scalar multiplication has noise term $e^*(x)$ such that $||e^*(x)|| < \delta_R \cdot p \cdot B + \delta_R \cdot p^2$.*

Proof. In case of homomorphic scalar multiplication, it can be done by multiplying a polynomial $c(x)$ to each $a(x)$ and $b(x)$. Each coefficient of $c(x)$ is bounded by the plaintext modulus p. For the $a^*(x) = c(x) \cdot a(x)$ and $b^*(x) = c(x) \cdot b(x)$,

$$
\begin{aligned}
b^*(x) + a^*(x) \cdot s(x) &= \left\lfloor \frac{q}{t} \right\rceil \cdot (m(x) \cdot c(x)) + e(x) \cdot c(x) \\
&= \left\lfloor \frac{q}{p} \right\rceil \cdot ([m(x) \cdot c(x)]_p + p \cdot I(x)) + e(x) \cdot c(x) \\
&= \left\lfloor \frac{q}{p} \right\rceil \cdot [m(x) \cdot c(x)]_p + \left(\frac{q}{p} + \epsilon \right) \cdot p \cdot I(x) + e(x) \cdot c(x) \\
&= \left\lfloor \frac{q}{p} \right\rceil \cdot [m(x) \cdot c(x)]_p + \epsilon \cdot p \cdot I(x) + e(x) \cdot c(x) \mod q
\end{aligned}
$$

Therefore, $||e^*(x)|| = ||\epsilon \cdot p \cdot I(x) + e(x) \cdot c(x)|| \leq \delta_R \cdot p^2 + \delta_R \cdot p \cdot B$. □

Furthermore, homomorphic addition between two ciphertext with noise bound B_1 and B_2 returns ciphertext with noise bound $B_1 + B_2 + 2p$. Finally, homomorphic addition between ciphertext with noise bound B and plaintext returns ciphertext with noise bound $B + 2p$.

From now on, we can analyze the noise term in our HE-based EPC protocol. This analysis gives us concrete HE parameter choices. If we see Figure 6, the receiver have to compute following (at 3-(d)):

$$
\mathsf{ctxt}_{k,d} = \mathsf{ctxt}_{k,u} \oplus \sum_{i=0}^{u-1} \left(\mathsf{ctxt}_{k,i} \odot 2\mathbf{m}^*_{k,i} \right) \oplus \mathbf{m}^*_{k,u}.
$$

By Lemma 1, the noise term of output ciphertext $\mathsf{ctxt}_{k,d}$ will be bounded by $B^* = 2u \cdot (\delta_R \cdot p \cdot B_{\mathsf{fresh}} + \delta_R \cdot p^2) + B_{\mathsf{fresh}} + 4p$. After that we need to add encryption of zero of error size $B_{fp} = 2^\sigma B^*$ for statistic security parameter σ for the function privacy. At last, receiver needs to add random vector $\mathbf{r}^*_k$ to the ciphertext. So, for the correct BFV decryption at the decryption phase, the ciphertext modulus q should satisfies the following inequality:

$$
\frac{q}{p} > (2^\sigma + 1) \cdot \left(2u \cdot (\delta_R \cdot p \cdot B_{\mathsf{fresh}} + \delta_R \cdot p^2) + B_{\mathsf{fresh}} + 4p \right) + 2p.
$$

Recall that we have $u = O(\log t)$ for the target size $p = O(\log t)$, and therefore we asymptotically have $q = O(\log^4 t)$ where t is input size.

Theorem 1. *The protocol Π_{BEPC} of Figure 6 realizes M times of $\mathcal{F}_{\mathsf{EPC}}$ functionality calls in a semi-honest model if*

$$
q > p \cdot (B_{fp} + B^* + 2p)
$$

where $B^* = 2u \cdot (\delta_R \cdot p \cdot B_{\mathsf{fresh}} + \delta_R \cdot p^2) + B_{\mathsf{fresh}} + 4p$ and $B_{fp} = 2^\sigma B^*$ for a statistical security parameter σ.

Proof. It is already explained that the condition for q provides the correctness and the function privacy required for our protocol.

For the sake of simplicity, we forget batching for a while and assume each parties has integer v and v^* in $\mathbb{Z}_t$. During the protocol execution, the receiver has input v^* and a random output r^*, and its view consists of the public key pk and the ciphertexts of v_i (decomposed value) and $\sum v_i^2$. This can be simulated by replacing all ciphertexts to encryptions of zero, which is indistinguishable from the real execution thanks to the IND-CPA security of HE.

The sender has input v and its view is a ciphertext of $D + r$ and it outputs the plaintext $D + r \in \mathbb{Z}_p$ by decrypting the ciphertext. This can be simulated by encrypting the output $r' \in \mathbb{Z}_p$ of ideal functionality, since from the function privacy the sender cannot know any other information than the decryption result, and the distribution of $D + r$ is identical to the distribution of r' (uniform over $\mathbb{Z}_p$). □

Asymptotic Cost Analysis. As the ciphertext modulus q is determined as above, we can estimate the total costs. Let $\gamma = \lceil M/n \rceil$ and $u = \log_w t$ by following notations of Π_{BEPC}. For computational cost, our protocol requires $\gamma(u+2)$ encryptions, γu homomorphic scalar multiplications, $2\gamma(u+3)$ homomorphic additions, and γ decryptions for M numbers of EPC calls. Such HE operations including homomorphic scalar multiplication can be done by $O(1)$ numbers of $\mathcal{R}_q$ operations that is roughly translates into $O(n \log n \log q)$ bit operations [4]. By approximating $\gamma n \approx M$, we conclude that amortized computational cost per EPC call is $O(\log t \cdot \log n \cdot \log q)$ bit operations as $u = O(\log t)$. In case of secure RLWE parameters, $n \propto \log q$ roughly holds for the fixed computational security parameter λ. Since $q = O(\log^4 t)$, we conclude that the computational cost per one EPC is $\tilde{O}(\log t)$. Toward communication cost, the sender sends $\gamma(u+1)$ fresh ciphertexts to the receiver and the receiver returns γ ciphertexts after HE oepration to the sender. The size of fresh ciphertexts is $\gamma(u+1)(n \log q + \lambda)$ and the size of returned ciphertexts is $2\gamma n \log q$. Then the total communication cost is $\gamma n(u+3) \log q + \gamma(u+1)\lambda$ bits. We again approximate $\gamma n \approx M$ and divide the total cost by M to see amortized cost for one EPC call. Then it results in approximately $(u+3) \log q \approx (\log_w t + 3) \log q$ bits communication and asymptotically $\tilde{O}(\log t)$ for one EPC call.

5 Application to Circuit-PSI Framework

Our equality preserving compression (EPC) of the previous section can be seamlessly augmented to the OPPRF-based circuit-PSI framework described in Section 3 as Figure 7.

Since EPC perfectly preserve equality (without failure probability), all previous works' analysis for correctness (or failure probability) are still valid. Moreover, as Theorem 1 shows that EPC is secure against semi-honest adversary, the semi-honest security Π_{CPSI} is also guaranteed.

Theorem 2. *The protocol Π_{CPSI} of Figure 7 realizes the $\mathcal{F}_{\mathsf{CPSI}}$ functionality in a semi-honest model in the hybrid model of $\mathcal{F}_{\mathsf{OPPRF}}, \mathcal{F}_{\mathsf{EPC}}$ and $\mathcal{F}_{\mathsf{ESG}}$.*

Parameters: A receiver with an input set X of size N and a sender with an input set Y of size N, and compression target bit-length ℓ_{c}.

Protocol:

1. [**Hashing**] Both parties agree on hash functions $h_1, \cdots, h_d$, and table size parameter ε. The receiver construct a cuckoo hash table T_X from X, and the sender constructs a simple hash table T_Y from Y using hash functions $h_1, \cdots, h_d$ into $M = (1+\varepsilon) \cdot N$ bins. The receiver define the address mapping X to T_X by ι.
2. [**Bin Tagging**] The sender samples uniformly random tags $\mathbf{v} \in \mathbb{Z}_{2^\ell}^M$ and sends $L = \{(y'||i, v_i)\}_{i\in[M], y'\in T_Y[i]}$ to $\mathcal{F}_{\mathsf{OPPRF}}$. The receiver sends $\tilde{T}_X = \{T_X[i]||i\}_{i\in[M]}$ to $\mathcal{F}_{\mathsf{OPPRF}}$, and receives $\mathbf{v}^* \in \mathbb{Z}_{2^\ell}^M$ from $\mathcal{F}_{\mathsf{OPPRF}}$.
3. [**Equality Preserving Compression**] The sender sends $\mathbf{v}$ and the receiver sends $\mathbf{v}^*$ to $\mathcal{F}_{\mathsf{EPC}}$, and receives $\mathbf{r} \in \mathbb{Z}_{2^{\ell_{\mathsf{c}}}}^M$ and $\mathbf{r}^* \in \mathbb{Z}_{2^{\ell_{\mathsf{c}}}}^M$ from $\mathcal{F}_{\mathsf{EPC}}$ respectively.
4. [**Equality Share Generation**] For $1 \le i \le M$, the sender sends r_i and the receiver sends r_i^* to $\mathcal{F}_{\mathsf{ESG}}$, and receives $\mathbf{s}_{0,i} \in \{0,1\}$ and $\mathbf{s}_{1,i} \in \{0,1\}$ from $\mathcal{F}_{\mathsf{ESG}}$ respectively.

Fig. 7: Π_{CPSI}. Protocol of our circuit-PSI: OPPRF-based framework + EPC

Effect of EPC. In asymptotic complexity view, the overall cost remains same since EPC itself takes $\tilde{O}(\ell)$ complexities. Thus, we have to figure out concrete costs to see the effect of EPC. We already observe that known methods for ESG has linear cost in ℓ_{c}, and in particular GMW protocol requires about $2M\ell_{\mathsf{c}}$ (correlated) OTs. For EPC from ℓ-bit to ℓ_{c}-bit, the word-size w should be maximally taken so that $2^{\ell_{\mathsf{c}}} \approx \frac{(w-1)^2}{\log w} \cdot \ell$, and it determines the corresponding chunk size $u = \ell / \log w$. Then EPC takes u times homomorphic operations including encryption, scalar multiplication, decryption, and addition with communication of u number of ciphertext. We point that n times of EPC calls can be done at once, thanks to batching property. Thus, as ESG input bit-length reduces from ℓ to ℓ_{c} thanks to EPC, we can save $2n(\ell - \ell_{\mathsf{c}})$ the number of OTs from $\approx u$ times of HE operations. More precisely, we have the trade-off below:

$$2n(\ell - \ell_{\mathsf{c}}) \times \text{ correlated-OTs}$$
$$\updownarrow$$
$$u \times \text{ HE encryptions, scalar-mults, additions, and Ciphertext Comm.}$$

One may think that HE operations are incomparably slow than OT, and hence this trade-off provides no benefit. However, we would like emphasize that our HE operations only consist of scalar multiplication and additions: HE scalar multiplication is just two polynomial multiplication with degree n which is quite fast compare to multiplication between encrypted data. For a concrete example, we may take $n = 4096$, $\ell = 61$, $\ell_{\mathsf{c}} = 19$, and $u = 8$, which is one of exploited parameters in later experiment section. This reduces $344,064$ number

of chosen-message OTs at the cost of 8 homomorphic scalar multiplication, 10 homomorphic additions, and 8 HE ciphertext communications.

Round Complexity. On round complexity view, one may think EPC requires additional one communication round than vanilla OPPRF-based framework. However, we remark that ESG stage takes $O(\log \ell)$ rounds for input length ℓ when performed by GMW protocol. Since EPC reduces ESG input length into $\ell_{\mathsf{c}} = O(\log \ell)$, EPC indeed brings asymptotic improvement on the round complexity when GMW protocol is used.

Offline Tag Encryption. The tag vector $\mathbf{v}$ sampled by the sender in the bin tagging step is independent to the input set of the protocol, so it can be sampled before the input set is known, in other words in offline phase. This observation brings negligible improvement in the original framework without equality preserving compression, as it only shifts the random $\mathbf{v}$ sampling time to offline. Meanwhile, it has a notable effect when combined with our equality preserving compression, as the server can perform the encryption phase of Π_{BEPC} in offline phase. Then the online phase of the protocol performs only HE operations, which leads to faster online execution. However, separating online/offline phase is not our interest, and then this is not applied for our experiments in Section 6.

6 Performance Evaluation

In this section, we evaluate the performance of several instantiation of our circuit-PSI protocol of Section 5. More precisely, we first discuss concrete parameter selections of sub-protocols, especially with respect to the compression target ℓ_{c}. Then we evaluate the performances of several combinations of our EPC protocol and previous ESG protocols. Finally, we provide full circuit-PSI protocol costs evaluation by attaching previous hashing and OPPRF steps, and some consequences of our protocols.

Throughout this section, we assume computational security parameter $\lambda = 128$ and statistical security parameter $\sigma = 40$. For experiments, we use a single machine equipped with 3.50GHz Intel Xeon processors with 128GBs of RAM. The network environments are simulated by `linux tc` command. LAN represents 5Gbps bandwidth with 0.6ms RTT, and WAN denotes 100Mbps bandwidth with 80ms RTT. All experiments are executed with a single thread on each party in order to be consistent with previous works. For implementation, we use SEAL [35] library for homomorphic encryption, libOTe library [33] for IKNP and Silver OTe [12], and emp-ot library [36] for Ferret OTe [37].

6.1 Parameter Selections

OPPRF output length ℓ. In our circuit-PSI framework, the output of OPPRF is directly fed into EPC or ESG. The OPPRF output length is taken by $\ell =$

$\sigma + \lceil \log M \rceil$ to ensure failure probability less than $2^{-\sigma}$ (See Equation 2) where $M = (1+\varepsilon) \cdot N$ is cuckoo/simple hash table size with d hash functions. We use $\varepsilon = 0.27$ and $d = 3$ by following previous works [31, 34], and then OPPRF output length is given by $\ell = \sigma + 1 + \lceil \log N \rceil$.

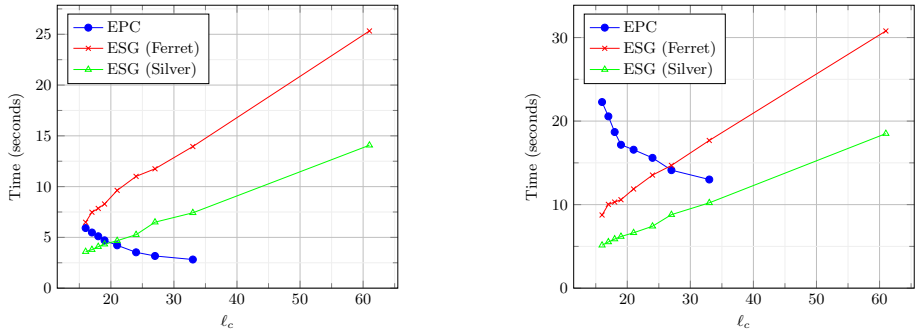
HE parameters. First of all, we fix HE ring dimension $n = 2^{12}$ which is the minimal one supporting depth-1 scalar multiplication. For the choice of HE plaintext modulus p, it is quite obvious that the full batch case would be the most efficient. Thus we take p to support full batch, whose concrete choice is a prime integer satisfying $p = 1 \mod 2n$. The minimal prime satisfying $p = 1 \mod 2n$ is $p = 40961$, and hence the minimal possible ℓ_{c} is $\lceil \log(40961) \rceil = 16$. For $\ell_{\mathsf{c}} > 16$, there are several primes p such that $p = 1 \mod 2n$, and we choose maximal p among them for each ℓ_{c}. We then choose the word-base w by the maximal one satisfying the correctness condition $p > u \cdot (w-1)^2$, where $u = \lceil \ell / \log w \rceil$ is the number of chunks. Then we have several ℓ_c for the same chunk number u. Since EPC costs are mainly determined by u rather than ℓ_c, it is convenient to arrange parameters with respect to the chunk size u. To minimize the total cost, we choose the minimal ℓ_c for each u.

It remains to determine HE ciphertext modulus q. We first take an initial modulus q' by the minimal one where our protocol is correct, and then the final modulus q is augmented by σ-bit margin on q' for function privacy. It empirically holds that $\log q \approx \sigma + 2 \log p + \log n$. To finalize parameter selection, we have to consider concrete attack cost of resulting parameters. We found that small chunk number $u \leq 3$ leads too large ciphertext modulus q that makes the parameters has far less than $\lambda = 128$-level of security. Therefore, we only conduct experiments with chunk number $u \geq 4$. More detailed HE and EPC parameters are presented in Appendix C.

6.2 Choice of ℓ_{c} with ESG

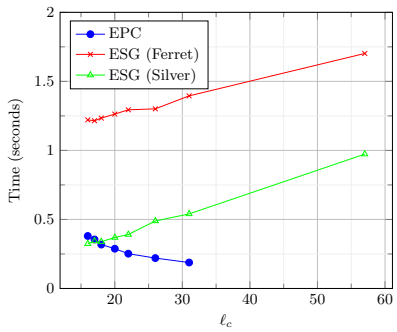
First recall that state-of-the-art OTe like Silver [12] and Ferret [37] already have extremely low communication cost. As a consequence, combining EPC with ESG rather leads to larger communication cost than sole ESG. Thus, we have to weigh the gain from EPC on computational cost and the loss from EPC on communication cost. It clearly depends on the network environment, and hence we conduct several experiments for several ℓ_{c} over different network environments. Some results are visualized in Figure 8. Generally, smaller ℓ_{c} leads to heavier EPC and lighter ESG, but the trade-off rate and the optimal point differ by network environment. We select the optimal ℓ_{c} giving minimal total running time, and the results are summarized by Table 1.

As expected, EPC effect is positive on LAN network as it reduces the computation burden, but negative in WAN network due to the communication cost growth. One can see that Silver [12] is always better than Ferret [37], but the both cases deserve to consider since the performance gain of Silver comes from so far non-standard assumption.

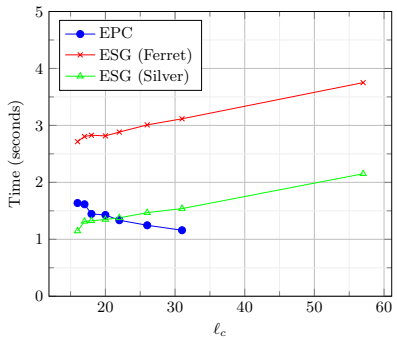


(a) $N = 2^{20}$ over LAN

(b) $N = 2^{20}$ over WAN



(c) $N = 2^{16}$ over LAN



(d) $N = 2^{16}$ over WAN

Fig. 8: Timing result of EPC and ESG on several network bandwidths and OTe protocols.

Another ESG Considerations. We also consider IKNP-OTe [22] for GMW protocol. It is obvious that IKNP is less competitive than Silver/Ferret for WAN, since it requires much larger communication; it requires more than 2500MB communication for $N = 2^{20}$ [31], while Silver/Ferret requires only 38MB. One may at least expect that it could be the best for LAN setting, but our internal experiments show that IKNP and Silver have similar performance even on LAN environment. Thus we decide to omit IKNP results in tables.

We also found another ESG method from [7], which uses another novel idea not based on GMW protocol. Our EPC can also be applied to this ESG method to reduce communication, but we also have to consider timing costs for a rigorous comparison. According to the original paper [7], the method takes (less than[3]) 9.27 seconds on 3Gbps network, 1107MB communication for $N = 2^{20}$. For a fair comparison, we need to run the proposed method in our machines, but we found no publicized implementation. Furthermore, our own implementation of

ESG $N = 2^{16}$	ℓ_c	OTe	Ours (w/ EPC)		Prev. (No EPC)	
			Time	Comm.	Time	Comm.
LAN	20	Silver	**0.657**	12.09	0.973	2.35
	26	Ferret	**1.515**	10.82	1.702	2.22
WAN	22	Silver	2.712	11.15	**2.151**	2.35
	22	Ferret	4.218	11.03	**3.751**	2.22

ESG $N = 2^{18}$	ℓ_c	OTe	Ours (w/ EPC)		Prev. (No EPC)	
			Time	Comm.	Time	Comm.
LAN	23	Silver	**2.288**	44.38	3.673	9.34
	23	Ferret	**3.887**	44.25	6.5	9.21
WAN	23	Silver	6.843	44.38	**5.52**	9.34
	21	Ferret	**8.83**	55.48	9.35	9.21

ESG $N = 2^{20}$	ℓ_c	OTe	Ours (w/ EPC)		Prev. (No EPC)	
			Time	Comm.	Time	Comm.
LAN	21	Silver	**8.896**	186.5	14.07	38.24
	16	Ferret	**12.42**	236.8	25.33	38.10
WAN	21	Silver	22.56	186.5	**18.50**	38.24
	21	Ferret	**28.02**	186.4	30.79	38.10

Table 1: Performances of ESG with/without EPC, provided with the best choice of ℓ_c. Communications in MB, and timings in seconds.

their method results in quite larger numbers than originally reported number in [7].

For these reasons, we can only leave some comments. Most importantly, since Silver/Ferret-based GMW has significantly lower communication, we presume that their ESG method is not competitive for WAN. However, there is still a possibility that this method is the most competitive one for LAN. Hence, we leave as a future work to further examine the effect of EPC on this ESG method and compare with Silver/Ferret-based GMW.

6.3 Impact on Circuit-PSI

Toward a complete circuit-PSI protocol, we only have to attach hash step and OPPRF step before ESG. For that, we note that the choice of OPPRF has no relation with the post ESG and EPC phase. It implies that, regardless of the choice of OPPRF, the absolute amount of effect of EPC remains same. However,

[3] We only find a timing report for a procedure that contains the ESG as a subroutine, denoted by PSM_2 in the original paper [7]. The exact timing for ESG would be smaller, but we have no further clue to guess it.

Circuit-PSI $N = 2^{16}$	OTe	Ours (w/ EPC)		Prev. (No EPC)		OPPRF [7]	
		Time	Comm.	Time	Comm.	Time	Comm.
LAN	Silver	**1.233**	21.96	1.549	12.21	0.576	9.87
	Ferret	**2.091**	20.69	2.278	12.09		
WAN	Silver	5.264	21.01	**4.703**	12.21	2.552	
	Ferret	6.770	20.89	**6.303**	12.09		

Circuit-PSI $N = 2^{18}$	OTe	Ours (w/ EPC)		Prev. (No EPC)		OPPRF [7]	
		Time	Comm.	Time	Comm.	Time	Comm.
LAN	Silver	**4.118**	83.77	5.503	48.73	1.83	39.39
	Ferret	**5.717**	83.64	8.33	48.6		
WAN	Silver	13.66	83.77	**12.34**	48.73	6.82	
	Ferret	**15.65**	94.87	16.17	48.6		

Circuit-PSI $N = 2^{20}$	OTe	Ours (w/ EPC)		Prev. (No EPC)		OPPRF [7]	
		Time	Comm.	Time	Comm.	Time	Comm.
LAN	Silver	**16.65**	344.0	21.82	195.7	7.75	157.5
	Ferret	**20.17**	394.3	33.08	195.6		
WAN	Silver	45.39	327.0	**40.96**	195.7	22.46	
	Ferret	**50.48**	343.9	53.25	195.6		

Table 2: Resulting circuit-PSI performances obtained by attaching OPPRF protocol of [7] before ESG. Communications in MB, and timings in seconds.

it is still important to consider full circuit-PSI cost; if ESG part occupies only a little portion of full circuit-PSI, our EPC leads to tiny improvement on circuit-PSI.

To argue that our EPC has a meaningful effect on circuit-PSI protocols, we implement one of state-of-the-art OPPRF protocols due to [7] as an example. The details are presented in Table 2. Our experiments indicates that EPC brings $10-40\%$ speed-up over LAN environment, and small speed-up on WAN environment when ESG is done with Ferret OTe.

We end with a final remark. According to our implementation, OPPRF step occupies $20-60\%$ of full circuit-PSI running time, as the rightmost column of Table 2 shows. Regarding this, we found further advances on OPPRF has been reported, and hence OPPRF has smaller portion in total circuit-PSI [16, 34]. Thus the relative benefit of EPC becomes larger, which makes our EPC technique more valuable.

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A Relaxed OPPRF [7]

The OPPRF (Oblivious Programmable PRF) functionality picks a random function F where the sender can program the value $F(x)$ by desired z. In [7], the authors proposed an extended notion called *relaxed OPPRF* functionality, which considers several random functions $F_1, \cdots, F_d$ and the sender programs $x \in X$ so that $F_i(x) = z$ with at least one $i \in [d]$. This converts a bin-wise PSM (Private Set Membership) problem $T_X[i] \in T_Y[i]$ to another PSM problem that checking $z \in \{F_1(x), \cdots, F_d(x)\}$. After then, they apply the standard table OPRF [24] to further converts this into ESG problem. Rigorously speaking, this consecutive execution of relaxed OPPRF and table OPPRF does not exactly fit to OPPRF functionality definition, since the sender cannot program it's desired values. However, in this work, it only matters that two parties can attach some tags for each bin to convert PSM problem into ESG problem, and we simply say the consecutive execution by OPPRF.

For the failure probability, we note that the authors uses $d = 3$ random functions for relaxed OPPRF that succeeds with probability $(1 - 2^{-\ell})^{3M}$, and then post-OPPRF succeeds with probability $(1 - 2^{-\ell})^{M}$. This results in the condition $\ell > \sigma + \lceil \log 4M \rceil$.

B Comparison with Paillier Additive HE

As our protocol only perform scalar multiplications, one may consider to use another *additive HE* (AHE), for example Paillier [27] scheme. Paillier scheme supports plaintext space $\mathbb{Z}_P$ for some integer P, and the corresponding ciphertext space is $\mathbb{Z}_{P^2}$. Here P is typically taken quite large ($\geq 2^{1024}$) to ensure certain security level, and a naive application of Protocol Π_{EPC} outputs huge random numbers in $\mathbb{Z}_P$. This can be circumvented by applying well-known *smudging* technique [2] where we take a sufficiently large random masking r so that r statistically hides the information of d, and each party take the final modulus reduction by p on each output $d + r$ and r.

However, we argue that RLWE-based AHE is still better for circuit-PSI purpose, where the encryption target message size is much less than 32-bit. RLWE-based AHE can supports plaintext space $\mathbb{Z}_p^N$ for rather small p, and the corresponding ciphertext space is taken $\mathcal{R}_q^2$ where $\log q = O(\log p)$. Then the amortized encryption cost per one message is $2 \log q$. For our interest message size, RLWE ciphertext modulus $q \approx 2^{100}$ suffices so that one message is encrypted into less than 200 bits,. However, Paillier AHE encrypts a message into a quite large ciphertext of $2 \log P \geq 2048$ bits, and the amortized cost is less inefficient than RLWE-base AHE.

C HE and EPC Parameters

Below shows detailed parameter information that is used in our experiment. For all cases, ring dimension in HE scheme is fixed with 4096. And, this parameter

satisfied 128 security based on homomorphic encryption standard document [1] (except the last row, as maximal possible $\log q$ for 4096 dimension is 109).

N	u	p	$\log q$	w	ℓ_c
2^{16}	10	40961	84	65	16
	9	114689	86	113	17
	8	188417	88	154	18
	7	1032193	92	385	20
	6	4169729	96	834	22
	5	67094289	104	3663	26
	4	2147377153	114	23170	31
2^{18}	10	40961	84	65	16
	9	114689	86	113	17
	8	417793	90	229	19
	7	1032193	92	385	20
	6	8380417	98	1182	23
	5	134176769	106	5181	27
	4	4294959105	116	32768	32
2^{20}	11	40961	84	62	16
	10	114689	86	108	17
	9	188417	88	145	18
	8	417793	90	229	19
	7	2056193	94	542	21
	6	16760833	100	1672	24
	5	134176769	106	5181	27
	4	8589852673	118	46341	33

Table 3: HE and EPC parameters in our evaluations.