On reducing vibration transmission in a two-dimensional cantilever truss structure using geometric optimization and active vibration control techniques

D. K. Anthony and S. J. Elliott
Institute of Sound and Vibration Research, University of Southampton, Southampton SO17 1BJ, United Kingdom

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Four optimization strategies were used to improve the average vibration isolation between the base and the end of a 10-m long two-dimensional (2D) cantilever truss structure. These were combinations of optimizing the structure geometry and the application of active vibration control (AVC) with optimal actuator positions. A power distribution analysis to investigate the mechanisms by which each strategy achieves reductions in the vibration transmission is reported. The trade-off is also explored between the freedom allowed in the size of the geometric changes and the number of actuators used in an AVC system to achieve a given level of vibration attenuation. © *2001 Acoustical Society of America.* [DOI: 10.1121/1.1381022]

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I. INTRODUCTION

Lightweight truss structures are commonly used in the aerospace industry, and there is considerable interest in the vibration transmission properties of such structures when excited by either acoustic or vibrational sources over frequency bandwidths of a few tens of hertz to tens of kilohertz.[1] The authors recently reported the optimization of a truss structure to minimize the average broadband vibration transmission using (i) the optimization of the structure geometry,[2] (ii) the application of active vibration control (AVC) with optimal actuator placement,[3] and (ii) the combined use of both strategies.[4] The consideration of robust optimization, which was an additional common theme in these references, is not reported here. The results pertinent to this letter are first briefly summarized in Secs. II, III, and IV. Then, a power analysis of the mechanisms by which the optimization has achieved its aim is reported, and last, the trade-off between geometric optimization and the use of AVC is demonstrated.

The specific aim of the optimization study was to minimize the average vibration transmission from the base to the end of a structure, whose unoptimized geometry is shown in Fig. 1(a), over the frequency range 150–250 Hz, realized in 21 frequency steps. The base excitation was modeled as a transverse force applied at midlength to the beam with ends at (0,0) and (0,1), and the reduction in vibration was measured at the beam with ends at (10,0) and (10,1) (subsequently denoted the end beam). The vibration of the end beam was quantified by the average vibrational energy level that arises due to the balance of energy supplied to the beam and the energy dissipated due the beam vibration (by virtue of beam damping). For simplicity, the presence of an AVC actuator on a structure beam was not considered to change its mechanical properties.

II. OPTIMAL ACTIVE VIBRATION CONTROL

One optimization strategy is to apply feedforward active control of vibration (AVC) to the original structure geometry, using double-acting axial actuators. Feedforward AVC can be used for the reduction of vibration in the low- and mid-frequency regions, provided a suitable reference signal exists.[5] The optimization task is to find the actuator positions that achieve the best average vibration reductions over the frequency band considered. Systems using one, two, and three actuators were considered, resulting in 39, 741, and 9139 possible actuator positions combinations, respectively (the end beam cannot accommodate an actuator). It is feasible to computationally evaluate all these actuator combinations, and thus, in this case, an exhaustive search offers the best optimization technique. Figure 1 shows the optimum positions for two actuators on the unoptimized structure geometry, which achieves a frequency-averaged attenuation of 31.1 dB. The attenuation at the individually controlled frequencies is also shown. The average values of attenuation achieved across the ten best-ranking actuator positions using one, two, and three actuators are shown in Table I as AVC(1), AVC(2), and AVC(3). The total control effort required by each control system is the sum of the squared moduli of the actuator forces at each frequency, and provides an indication of the electrical power required to drive an AVC system. It is shown normalized to the total control effort of primary input.[3]

III. GEOMETRIC OPTIMIZATION

An alternative optimization strategy is to redesign the geometry of the structure in order to inherently achieve a better performance. This was considered for the structure by allowing the midspan joints a freedom of ± 0.25 m. This optimization is fully described in Ref. 2, and the results discussed here refer to the broadband results in this reference, where the average energy level was evaluated over a bandwidth from 150 to 250 Hz.

The geometric redesign is a multimodal, highly combinatorial optimization problem that is often accomplished using so-called natural algorithms, such as Genetic algorithms.[6] These were used to solve this highly combinatorial optimization problem. The 36 joint coordinates were coded into binary string chromosomes, and optimized structure designs

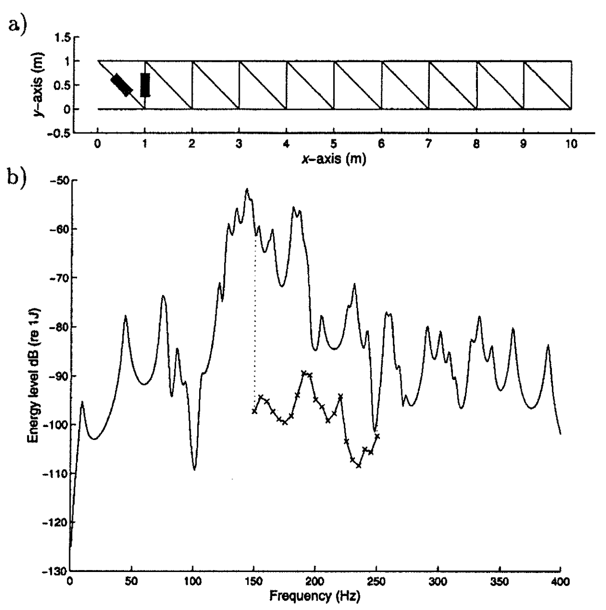

FIG. 1. Truss structure with unoptimized geometry. (a) Optimum actuator positions, (b) energy level of the end beam against frequency without AVC (—), with AVC at frequencies shown −×−×−.

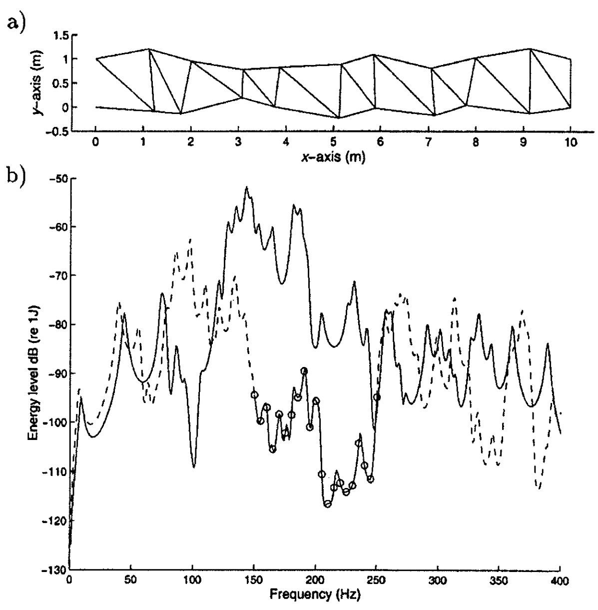

FIG. 2. Geometrically optimized truss structure. (a) Optimized geometry, (b) energy level of the end beam against frequency before optimization (—), after optimization (−−−). The objective function is the average value of the frequency region denoted −O−O−.

were taken to be the best solutions occurring after 15 generations, each of 300 chromosomes. Due to the stochastic nature of the algorithm, different optimized structures were produced using different initial random number seeds for the algorithm. In this study ten such structures were generated. All ten structures had different geometries but showed a similar level of performance. Figure 2 shows the best structure produced by such an optimization, and the changes in energy level in the end beam against frequency before and after optimization. The frequency-averaged reduction in the vibrational energy is 33.3 dB, which is similar to the attenuation achieved for AVC(2), as shown in Table I. Despite its irregular geometry, the structure is relatively easy to construct using modern methods and useful reductions in average vibrational energy have been obtained, by over a factor of 1000, using only geometric redesign. It provides an alternative *passive* solution to AVC, which does not suffer a significant reduction in static strength.[7]

TABLE I. Summary of results for the average performance and control effort for the structures resulting from all the optimization strategies considered. Optimization type key: Geometric, geometric optimization only; AVC, active vibration control only; PTA, passive-then-active optimization strategy; CO, combined optimization strategy.

Optimization type (No. actuators)	Geometric attenuation contribution (dB)	AVC attenuation contribution (dB)	Overall attenuation (dB)	AVC total control effort (normalized)
Geometric	32.6	···	32.6	···
AVC (1)	···	9.2	9.2	350
AVC (2)	···	27.7	27.7	1400
AVC (3)	···	45.9	45.9	2900
PTA (1)	32.6	11.2	43.2[a]	7.4
PTA (2)	32.6	32.2	63.6[a]	9.6
CO (1)	21.0	27.5	48.7[a]	4.3
CO (2)	16.7	60.3	78.4[a]	29

[a]As a consequence of the logarithmic scaling, the addition of the two average components of the overall attenuation does not result in the average overall attenuation.

IV. OPTIMAL COMBINED STRATEGIES

A sensible progression from the two individual optimization strategies reported in the previous sections is to combine them. The optimization of both the geometry and the AVC actuator positions have been previously reported for these frequencies in similar types of structures.[8–10] Reference 10 uses the addition of extra damping to a number of structural beams instead of geometric adjustment. The two combined strategies used were (i) to apply AVC with optimally placed actuator positions to structures whose geometries had previously been optimized (passive-then-active optimization, PTA), and (ii) to optimize the structure geometry and actuator positions simultaneously (combined optimization, CO).

Figure 3 shows the structure that resulted from the PTA optimization strategy; using an exhaustive search to locate the best two-actuator positions on the structure that had previously been geometrically optimized (shown in Fig. 2). (In fact, a slightly better overall performance was achieved using this strategy with another of the ten geometrically optimized structures discussed in Sec. III.) The energy levels in the end beam at the individual frequencies for which the AVC are applied are also shown in Fig. 3. The average overall attenuation for the ten structures, the average attenuation achieved due to the previous geometric optimization and due to the AVC, using one and two actuators, are shown in Table I as PTA(1) and PTA(2). For a two-actuator AVC system it is seen that the average attenuation achieved by geometric optimization and AVC is almost the same. The average attenu-

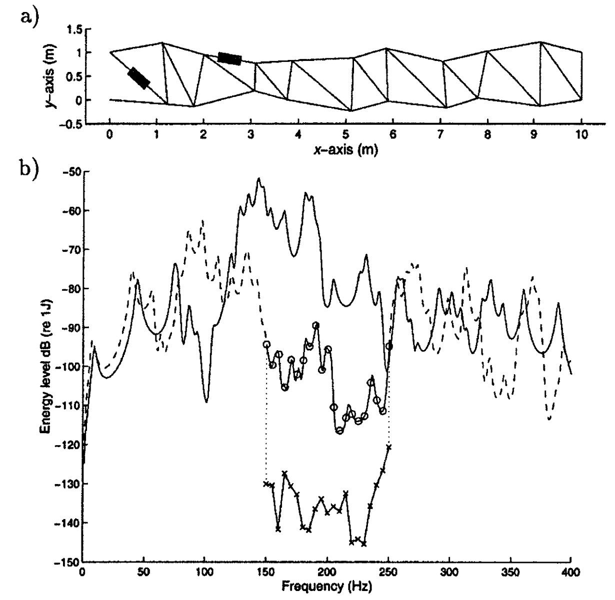

FIG. 3. Geometrically optimized truss structure from Fig. 2. (a) Optimum actuator positions; (b) energy level of the end beam against frequency: before optimization (—); after optimization outside the objective function frequency range (---); value of objective function without AVC, -O-O-; with AVC, -×-×-.

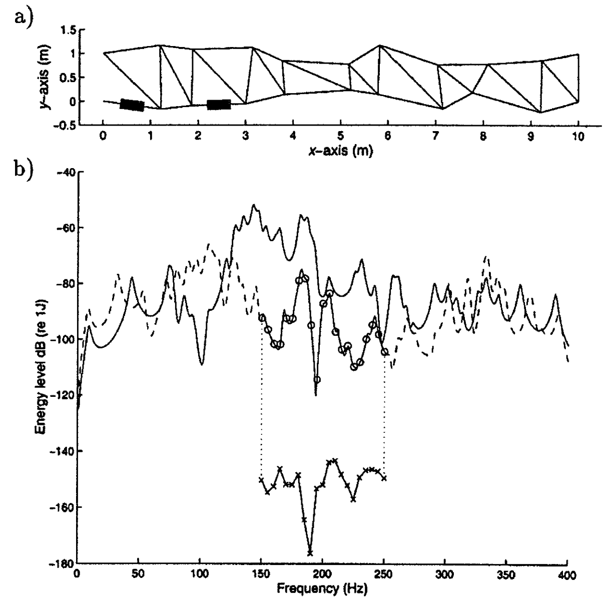

FIG. 4. Truss structure optimized using combined optimization strategy, CO. (a) Optimum geometry and actuator positions; (b) energy level of the end beam against frequency: before optimization (—); after optimization outside the objective function frequency range (---); value of objective function without AVC, -O-O-; with AVC, -×-×-.

ation in the vibration of the end beam using the AVC system on a geometrically optimized structure is of a similar level as that using the unoptimized structure. It is interesting to note that the overall attenuation is approximately equal to the sum of the values of attenuation achieved by optimizing the structure's geometry, and the application of AVC to the unoptimized structure.

The CO (combined optimization) strategy uses the same genetic algorithm optimization employed for the geometric redesign detailed in Sec. III. Again, a chromosome was used to represent the structure geometry, but extended to additionally represent the actuator positions. Ten such structures were designed, with AVC using one and two actuators. The best structure using two actuators is shown in Fig. 4 along with the energy level in the end beam against frequency, with the AVC system operational and nonoperational. The overall AVC attenuation is in excess of 60 dB and, in practice, is likely to be limited due to system noise. The average results across the ten best structures with one and two actuators are shown as CO(1) and CO(2) in Table I. Comparing the amount of reduction achieved by the geometrically optimized structure in Table I, a smaller geometric contribution is seen for the structures using the CO strategy. However, with the CO strategy, far greater reductions are achieved per actuator. It is also apparent from Table I that an AVC system is more efficient when applied to a structure which has also undergone geometric optimization, as the values of total control effort for both the average PTA optimization and the CO strategies are significantly less than for the average performance of the AVC strategy.[11] This is true either on a per-actuator or per-dB attenuation basis.

V. ANALYSIS OF POWER WITHIN OPTIMIZED STRUCTURES

The mechanisms by which the reductions in the energy level in the end beam have been achieved have been investigated by analyzing the energy flow (i.e., power) within the structure. Table II shows the change in input power (due to the change in the mechanical impedance presented to the input force) and the change in the distribution of the power dissipated within the structure between the end beam and the remainder of the structure. The power supplied or dissipated by the actuators, where applicable, were not significant compared with the components given and are ignored here for simplicity. For geometric optimization, the reduction in the energy level of the end beam has been achieved by a 10 dB reduction in the input power, but also by a 22 dB reduction in power redistribution. For the AVC system, the reductions are

TABLE II. Average power components of optimised structures, with and without AVC operational. Passive beams are those not containing an actuator. (Optimization type key the same as for Table I.)

Optimization type (No. actuators)	Primary input power reduction (dB)		Power redistribution between passive beams and the end beam (dB)	
	w/o AVC	with AVC	w/o AVC	with AVC
Geometric	10.7	···	22.3	···
AVC (1)	···	0.6	···	9.6
AVC (2)	···	0.9	···	28.0
AVC (3)	···	1.0	···	46.4
PTA (1)	10.7	10.7	22.3	32.9
PTA (2)	10.7	10.5	22.3	53.9
CO (1)	9.7	9.8	14.4	39.5
CO (2)	10.0	10.4	8.5	69.9

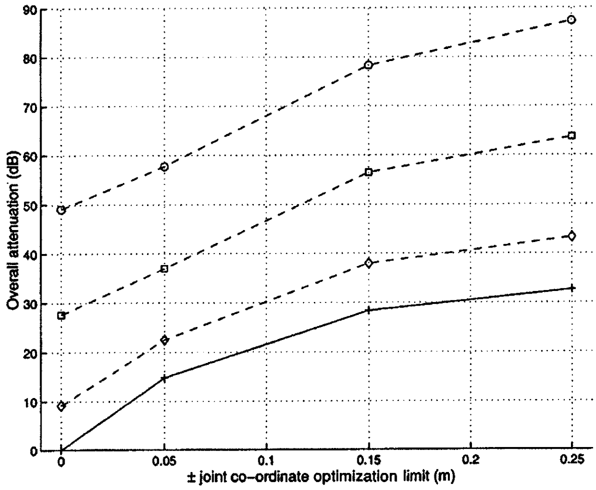

FIG. 5. Average overall attenuation achieved in the end beam against the joint coordinate limit used in the optimization. —, geometric optimization only, PTA (passive-then-active) optimization, with one actuator, ◇; two actuators, □; three actuators, ○.

seen to be chiefly from the power redistribution, with little effect on the input power. For the PTA optimization strategy, the application of AVC augments the gain achieved by redistribution of the preceding geometric optimization, but has no further effect on the reduction in input power. For the CO strategy, similar levels of reduction in input power are achieved as with the geometric optimization, however, a smaller level of power redistribution due to the geometry alone results. The CO strategy produces the most effective use of the AVC, as when operational the largest additional power redistribution results.

Thus, in general, geometric optimization works by reducing the input power to the structure, and also a varying degree of power redistribution. The application of optimal AVC in each case enhances the power redistribution, but is most effective when optimized simultaneously with the structure geometry. Fuller details are given in Ref. 7, where a study of the power in the individual beams is also given. For three optimally placed actuators, a strategy of blocking the flow of energy down the structure is clearly seen, suggesting that these solutions would be more robust to changes in mechanical impedance at the end beam (due to a change in the mass of the load, for example).

VI. TRADE-OFF BETWEEN PASSIVE AND ACTIVE METHODS

It is clear that more than one strategy may be used to provide similar reductions in vibration. When optimizing the geometry, allowing a greater freedom in the possible movement of the joint is expected to affect the attenuation in the vibration level achievable. Likewise, increasing the number of optimally placed actuators in an AVC system is expected to increase the attenuation achievable. This hypothesis was considered for the attenuation achievable using solely geometric redesign, and also the subsequent application of AVC using optimally placed actuator positions (the PTA strategy). The results are presented in Fig. 5, which shows the average overall attenuation achievable over ten structures optimized to reduce vibration in the end beam with different limits on the extent in the variation in the joint coordinates, initially using geometric redesign. The performance is then shown when subsequently applying AVC using one, two, and three optimally placed actuators. The additional attenuation provided by the AVC systems is, again, not significantly affected by the geometric optimization and so, to a first approximation, the two values of attenuation can simply be added. The levelling out of the attenuation as the joint coordinate limits increase indicates that not much more attenuation would result if these limits were increased (although to avoid joint-beam contact, greater freedom is not possible in this case).

Thus, to achieve a value of attenuation in the frequency-averaged vibration reduction of about 30 dB, three options are possible: (i) geometric redesign with a maximum joint coordinate freedom of about ± 0.2 m, (ii) geometric redesign of the geometry with a freedom of ± 0.1 m and the application of AVC with one optimally placed actuator, and (iii) the application of two optimally placed AVC actuators on an unoptimized structure. A trade-off thus exists between passive (geometric redesign) and active (AVC) optimization techniques.

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[11] This comparison of average performance against average total control effort for each optimization strategy was given in a graphical format in Ref. 4.