DeciPipes: Helping Students to "Get the Point"

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describes the use of "decipipes" as a manipulative to help overcome decimal misconceptions. As you read this article note how students explain their thinking about decimals.

Background

As highlighted in a previous issue of APMC, difficulties with decimal magnitude leave students unable to apply number sense to solve problems (Roche, 2010). Many students fail to correctly interpret the use of the decimal point and over-generalise a prior concept to create a procedure to deal with decimal numbers. Work in the 1980s showed that students were remarkably consistent in the use of their preferred strategy (Nesher & Peled, 1986). One of these strategies is termed "longer is larger", whereby the number is seen to "start again" after the decimal point. With this, 0.37 is regarded as larger than 0.6 because 37 > 6. Another is termed "shorter is larger". Here, 0.4 is regarded as larger than 0.85. Some students give mathematically-based explanations for this strategy such a confusion with denominators. Others do the opposite of how they would order whole numbers, but do not really know why. The role of zero as a placeholder is especially problematic. Students often ignore zeroes to the left of other digits so 0.7 and 0.07 are seen as being identical amounts. Fuller explanations of student difficulties can be found in the work from Melbourne University (e.g., Steinle & Stacey, 1998).

The main problem is not the existence of these student-invented strategies, but that students continue to operate with them long after they have received new teaching. This article will review theory around why student intuitions concerning decimals are resistant to change and present the use of the Decipipes equipment as one way of helping students "get the point".

Durability and change

Students do not simply replace old systems of thinking when new ones are presented, but allow them to dwell side-by-side for considerable lengths of time. Enduring change only occurs when the student is convinced that their thinking needs changing (Siegler, 2000). New knowledge presented in class can be compartmentalised as being true in some situations but prior knowledge being true in others, even when this approach is contradictory from an adult perspective.

Students do not readily abandon well-established patterns of behaviour. This is especially true where surface features of new situations cue them into applying a strategy that was previously successful (McNeil & Alibali, 2005). One such cue with decimal numbers is the familiarity of the digits used. When a student sees a number such as 0.42, it looks like 42 and may be treated as such. Another is the language cue associated with money: shown $1.75, we often say, "one dollar seventy-five", shown 1.75, students may be primed to process this as "one" and "seventy-five" as well.

Presenting counter-examples to students does not guarantee their acceptance of the reasons for the need for conceptual change. Many of our students are not actively listening to our explanations, partly because they are not convinced that they need to re-think their understanding of decimal notation.

If mathematical issues arise in "student-time" rather than in "teacher-time" then they can be addressed as students notice them. Providing students with an engaging task can promote profitable discussion between peers. As teachers, we often want to prepare students for what they will encounter but sometimes "just in time" can be more powerful than "just in case". If the student agenda of completing a task overlaps with the teacher agenda of introducing new mathematics, then both parties invest in the communication process.

Students need to make strong mathematical links between the mathematics they already know and any new symbolic form (Goldin & Shteingold, 2001). Helping students to make such links allows them to see that they are still building upon existing knowledge even though this construction is different to that which they had previously thought correct. Interaction with equipment can create opportunities for mathematical discussion to shape this process. It is crucial that students see through the equipment to the concept it is modelling (Stacey, Helme, Archer & Condon, 2001). We should ask ourselves whether equipment is simply an aid to solving a localised task or if it enabling a deeper engagement with mathematical issues. The answer is usually found by considering how it is being used rather than being an inherent quality of the materials.

Measurement is a great context to address decimal magnitude as relative differences are transparent. Students can easily verify which object is longer and this in turn forces engagement with the symbols used to record those lengths (Lamon, 2001).

Decipipes

Decipipes are a representational model that can be used to help students develop conceptual understanding of decimal place

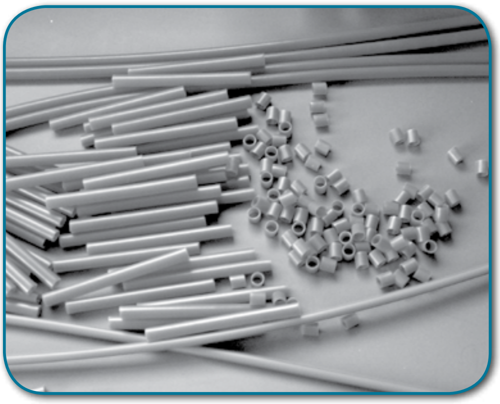

Figure 1. Decipipes.

value. They provide a non-standard tool for representing length, which in turn can be represented using conventional decimal notation. They are conceptually identical to Linear Arithmetic Blocks (Helme & Stacey, 2000). A long piece of hollow blue tubing represents a length of 1 unit, with the smaller pieces being tenths or hundredths. The representational pieces are deliberately the same colour so that any measurement is seen as one length made up of sub-units. The solid red rods are non-representational so an empty red rod is not a "zero". The hollow blue pipes can be fitted onto the red pipes and do not easily fall off again. This means that lengths represented by a combination of blue pieces can be readily moved from the initial site of measurement to show others. Physically moving a representation of the measurement to another place is an important part of the abstraction process. The decipipes representation starts being treated as an object in its own right. The advantage of this is that the representation both suggests how decimal symbols are used and allows for symbols to be modelled in a physical form.

The unit of "1" is deliberately not 1 m, as it was found that many students read 1.42 m as "one metre and 42 centimetres". While correct, this language reinforces the view that numbers "start again" after the decimal point. Many of these students will also interpret 1.4 m as 1 m and 4 cm.

Decipipes can also be used to model addition problems. Students can discover why 0.4 + 0.12 is not 0.16, for example, and work with situations where "exchanges" occur, such as 0.87 + 0.45. They are not suitable for multiplicative work as the commutative law cannot easily be modelled by using them. An area-based model is preferable.

It is not intended that an entire class would operate with decipipes at one time. Instead, a group of students who are being introduced to decimal notation or a sub-group of the class with difficulties with decimals would work with them. This allows for monitoring of the mathematical discourse around their use and keeps the classroom safe. Teachers should incorporate one or two lessons with decipipes into a unit on decimal numbers. Multiple representations and varying situations will help ensure that the students learn about decimals, and not just decipipes.

Working with decipipes

This section will describe how the decipipes have been used in classrooms. I often start by writing an addition problem such as "1.1 + 1.12 = ☐" on the board and asking students to talk about the answer. I have found that most students with either the "longer is larger" or "shorter is larger" strategy give the answer as 2.13. I "park" this result without comment and introduce the decipipes. Students will come back to their answers in their own time and ask for it to be changed. Student self-identification of a previous error is a good indication of learning.

I think that the best way to introduce the decipipes is to tell the students that they are going to be measuring things using the new equipment. The unit they will be using is the length of the longest blue pipe which will now be called "1". Students start to measure an object and quickly realise that they will have to decode what the smaller pieces represent—an example of "just in time" learning. I have found that students typically ignore the one-hundredths pieces until they feel confident in their ability to measure and represent answers using tenths. This only takes two or three examples. Decoding that these smaller pieces are in fact one-hundredths is part of their learning. They then start using all of the pieces in order to get better accuracy in their measurements. Students who measure and record in pairs have tactile, visual and verbal verification of their results.

Find an object for students to measure that that is clearly longer than one-tenth but shorter than two-tenths (e.g., Figure 2).

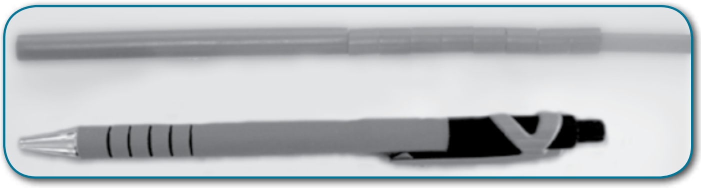

Figure 2. The pen is longer than one tenth but shorter than two tenths.

This creates student engagement with the hundredths pieces. When students place one tenth-piece and eight hundredth-pieces, they see the length is represented by nine pieces, of two sizes. They tell their partner that they used one tenth and eight hundredths to get the length. They are now self-primed to record this measurement as 0.18. This may be compared with another item of say, 0.3 units. These comparisons of both objects and their decimal lengths promote student engagement with the issue of how a shorter number can represent the larger magnitude in a more powerful manner than a whiteboard explanation.

Examples of work

Figure 3 shows some of the work of two girls named Grace and Wini. When given written tasks, Grace had been operating with the "longer is larger" procedure with 100% consistency, while Wini used the "shorter is larger" method for 90% of the examples given to her (Moody, 2008).

They had measured objects of their choice from around a room and recorded these onto A3 paper as shown in Figure 3.

At Site 1, the girls had measured the height of a chair. Their initial recording shows how they wrote $\frac{4}{10}$ and $\frac{2}{100}$ and then crossed these fractions out to write 0.42. They had connected the equipment to the fractions and then the fractions to the

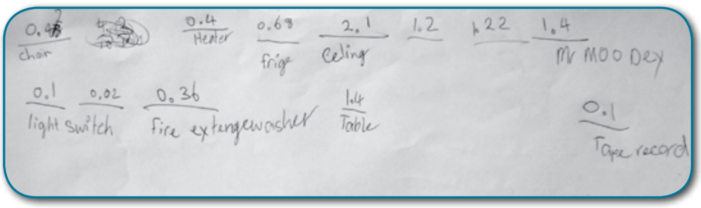

Figure 3. Work sample by students aged 9–10.

decimal symbols. Encouraging students to verbalise the place value of each digit as they record these measurements may be helpful. We often write the place value columns above decimals but students saying the values while seeing the relative contrast in proportions may be more effective.

At Site 2, the girls measured each other's heights. They did this because of an argument as to who was taller since they knew that they were very similar in height. For Grace, 1.22 should have been a lot taller than 1.2 as 22 > 2. The opposite was true for Wini, 1.2 is shorter than 1.22 and so in her system should be larger. Their eyes told them a different story and their actions proved that two small pieces were all that separated their heights. Neither would have thought that rounding 1.22 to 1.2 made any sense until they had seen how close the representations of these numbers really were.

At Site 3, they measured both the light fitting and the moveable switch. Previously, both girls had said that "the zero doesn't mean anything" when comparing decimals and so thought that 0.3 was smaller than 0.09 for example. They now understood that 0.1 was larger than 0.02. That Grace and Wini had to find a way to communicate to others in the group that they had used two hundredths and not two tenths made them engage with the use of zero as a placeholder.

Having students collect data in pairs allows for discussion around the decisions made to record these measurements and also provides material for the teacher to use with other students. Students are asked to consider the symbolic forms presented by others and so have to start thinking about what these represent.

> Teacher: So look at what these guys did, they did the actual switch part of a light switch and they got zero point zero two. So what does that zero mean Tame?
>
> Tame: That there was no wholes (sic).
>
> Teacher: And what does that zero mean Ripeka?
>
> Ripeka: No tenths.

Tame had measured a sink bench using a one and a hundredth piece.

Teacher: Tame, can you tell me what the sink bench was?

Tame: 1.01.

Teacher: Can someone tell me what Tame did to measure the bench?

Aroha: One whole and one hundredth.

Teacher: OK, so what does that zero tell us?

Aroha: That there's no tenths.

From discussing examples of measurements such as these, this group of students was able to deal with decimals that had zero as a placeholder despite no formal teaching.

At some point students need to generalise the process and imagine what would be rather than examining what is. The desire to produce even more accuracy in measurements initiated this discussion in the next lesson.

Aroha: So we use ones, tenths and hundredths…

Tame: What about thousandths?

Wini: Yeah we could have thousandths, what would they be like?

Tame: Real big… No, real small, they would be like [shows with thumb and forefinger].

Teacher: Would thousandths be big or little? [They had no physical model of one-thousandth; they could only make sense of it by extending their mental schema.]

Aroha: They'd only be tiny little, they'd be really little.

Teacher: How could you make them?

Wini You'd have to cut this down to about there [pointing on hundredths pieces].

Tame: You'd have to have heaps and heaps, like a thousand of them.

Learning

I have found that much of the learning takes place as students engage with an immediate aspect of a task (such as how to show $\frac{2}{100}$ in decimal form) rather than in more general discussions. These are often more effective after students have had their own small "wrestles" with such issues and now can see that their experiences form part of a larger picture.

You have some confidence that students have re-organised their thinking when they choose to use the correct place-value language to explain their decisions. Grace, who had previously always operated using a "longer is larger" system, had this to say when asked to explain why she now thought that 0.8 was larger than 0.75: "It's larger, it has an extra tenth; when I first started I thought that [pointing to 0.75] was the highest because of seventy-five." Wini, who had previously operated with a "shorter is larger" system, explained that 0.555 was larger than 0.55 because it had "five thousandths more".

Students have often come back to the "parked" addition problem of $1.1 + 1.12 = \square$ after 20–30 minutes of work with the decipipes and described why they now believe that the correct answer to be 2.22, and just as importantly, why it is not 2.13. Their justifications almost always explicitly refer to tenths and hundredths, something absent from previous explanations.

Conclusion

A first response to what we know about the persistence of students' intuitive strategies for working with decimal numbers is to recognise that even the clearest of teacher explanations may be insufficient. A clear plan of action that helps students tie together what they already know with information from a new set of experiences can effect enduring change in thinking however. The use of decipipes can form part of a bridging process from prior concepts of measurement and fractions to understanding decimal notation. Students have the support of a representational model to help them make decisions about the appropriate recording of measurements. Once students have started to make sense of how we use decimal notation they should be provided with other situations to work with so that this initial understanding does not remain dependent upon one tool.

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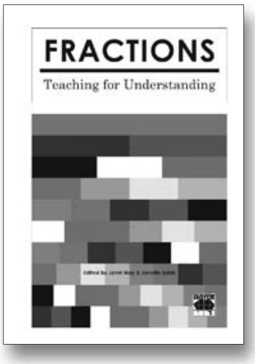

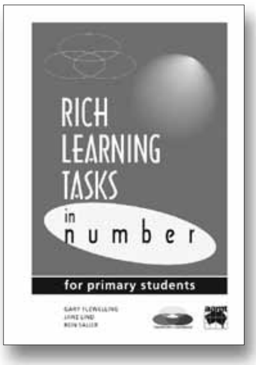

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