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CONFORMAL PROJECTION OF THE SPHERE WITHIN A SQUARE

BY

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CONFORMAL PROJECTION OF THE SPHERE WITHIN A SQUARE

By Oscar S. Adams, *Senior Mathematician, United States Coast and Geodetic Survey*

The problem of projecting the entire sphere conformally within a square, with the poles on one of the diagonals and symmetrically placed with respect to the center, has been in consideration from time to time for several years. Of course, it is an easy matter to get an expression that will accomplish the result, but in most cases the amount of the computation required is excessive. It was desired to develop a method of attack that would be both interesting analytically and easy to compute. Almost any solution would fulfill the first requirement, but it took some thought to develop a method that fulfilled the second.

The earliest attempt at the inversion of an elliptic integral was made by Gauss. He started with the integral,

$$u=\int_0^z \frac{dz}{(1-z^4)^{1/2}},$$

and inverted it by assuming the functional relation, $z=\sin \operatorname{lem} u$.

This function he called the lemniscate sine of u. This procedure was, of course, based upon the analogy of the inversion of the trigonometric integrals into the trigonometric functions. In Gauss's Nachlass, in volume 3 of his works, there are given without demonstration a great many properties of this function.

Now, with this function, we can at once get an expression for the projection desired. If λ denotes longitude, p the polar distance or colatitude, and i denotes as usual $\sqrt{-1}$, we can assume the expression,

$$\sin \operatorname{lem} u=\left(i\,\frac{e^{-i\lambda}-\tan\frac{p}{2}}{e^{-i\lambda}+\tan\frac{p}{2}}\right)^{1/4}$$

This expression will give us the projection sought, but the amount of computation required would be very great. It would be difficult to separate the real and imaginary parts, and also no table of the lemniscate functions has been computed. It would not be difficult to express the above equation in terms of the Jacobi elliptic functions, but that would not materially shorten the computations. Also, it would not be difficult to compute a table for the lemniscate functions from their expression in terms of the Jacobi functions by the aid of Legendre's table of elliptic integrals, but such procedure is not required, as will appear when the method of computation employed is developed.

If we have the arc distances and the azimuths of the intersections of the network of meridians and parallels computed from a point on the Equator, we can make use of the lemniscate integral developed into a series as the basis of our definition of the projection. Let us call the arc distance d and the azimuth c. The formulas for computing these quantities from a point on the Equator and the resulting values of c and d are given in the table on page 6. Developing the lemniscate integral in series we get

$$\int_0^z \frac{dz}{(1-z^4)^{1/2}} = z + \frac{1}{2} \cdot \frac{1}{5} z^5 + \frac{1}{2} \cdot \frac{3}{4} \cdot \frac{1}{9} z^9 + \frac{1}{2} \cdot \frac{3}{4} \cdot \frac{5}{6} \cdot \frac{1}{13} z^{13} + \cdots$$

Now, in this series if we set

$$z = e^{\frac{ic}{4}} \tan^{1/4} \frac{d}{2},$$

we get

$$x + i\,y = e^{\frac{ic}{4}} \tan^{1/4} \frac{d}{2} + \frac{1}{2} \cdot \frac{1}{5}\, e^{\frac{5ic}{4}} \tan^{5/4} \frac{d}{2} + \cdots$$

From this expression, by separating the real and imaginary parts, we get

$$x = \tan^{1/4} \frac{d}{2} \cos \frac{c}{4} + \frac{1}{2} \cdot \frac{1}{5} \tan^{5/4} \frac{d}{2} \cos \frac{5c}{4} + \cdots$$

and

$$y = \tan^{1/4} \frac{d}{2} \sin \frac{c}{4} + \frac{1}{2} \cdot \frac{1}{5} \tan^{5/4} \frac{d}{2} \sin \frac{5c}{4} + \cdots$$

By use of the table of c and d values we could compute the coordinates of the projection from these formulas. However, these two series converge very slowly, and the number of terms required for the computation of some of the coordinates would be very great. Computation by means of such series is a laborious process, and great care has to be used in employing the functions of the multiple arcs to see that the correct sign is used in each case. We prefer to avoid such computations whenever possible.

It should be noted that the relation of the projection to the axes of coordinates is not necessarily the same in the two expressions just given, and they both differ from the relation that was finally used for the computation. The fact that the functional relation is expressed in terms of the complex variable renders the projection conformal, and the relation of the projection to the axes of coordinates is a matter of no importance. This can be assumed in whatever way is most convenient for the particular case in consideration.

We shall now give an account of the method that was actually employed in the computation of the projection. If we have a meridian stereographic projection of the sphere extending over the whole complex plane, we can imagine the plane to be cut along the positive

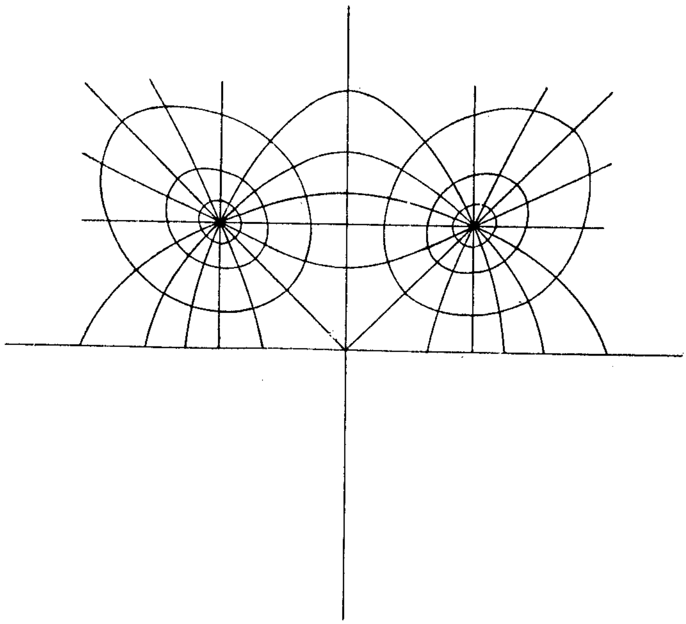

FIG. 1.—Diagram showing stereographic projection on the whole plane contracted to a conformal projection on the half plane

real axis, and then this cut-off part turned clockwise about the origin until it coincides with the negative real axis. While this is being done the whole of the projection must change so as to keep it still conformal. We shall then have a conformal map such as is indicated in Figure 1. The analytical expression of this map is given by the equation

$$x+i\,y=e^{\frac{ic}{2}}\tan^{1/2}\frac{d}{2}.$$

Instead of following this procedure we decided to map the whole sphere conformally on the half sphere in a similar way. We can think of this as a cutting along the Equator halfway around and then a shrinking of the surface until it actually covers just one-half of the sphere. (See fig. 2.) This contraction must, of course, take place in such a way as to preserve the conformality. This is accomplished analytically by the functional relation

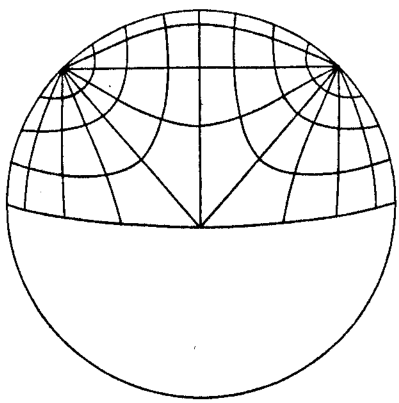

FIG. 2.—Diagram showing surface of sphere contracted onto a hemisphere

$$e^{ir}\tan\frac{q}{2}=e^{\frac{ic}{2}}\tan^{1/2}\frac{d}{2}.$$

In this expression the absolute value of the two sides must be equal as well as the argument of the two, neglecting, of course, any number of times 2π. Hence we have

$$\tan\frac{q}{2}=\tan^{1/2}\frac{d}{2}$$

and

$$r=\frac{c}{2}.$$

A table of the values of q and r was computed (see p. 8). This gives us the spherical coordinates for the conformal map of the whole sphere on the half sphere.

Now, on this sphere, if we make use of Lieutenant Guyot's elliptic isometric coordinates, we can compute the required projection without excessive work. For an explanation of the elliptic isometric coordinates see United States Coast and Geodetic Survey Special Publication No. 112, Elliptic Functions Applied to Conformal World Maps, page 90 et seq. With the focii placed on the Equator of this sphere as indicated in Figure 3, we have for the determination of a and b,

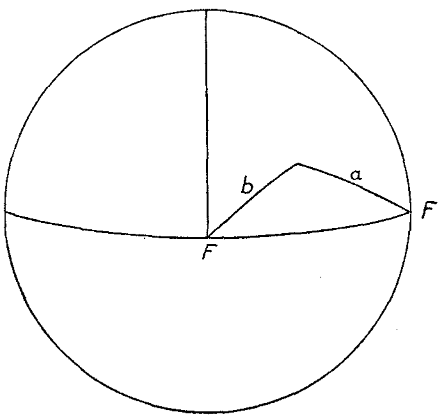

FIG. 3.—Diagram showing positions of arcs *a* and *b*

$$\cos a=\sin q \cos r$$

$$b=q.$$

Then, from the values of a and b, we compute a table of values of m and n (see p. 10) by means of the functional relations

$$\sin m=\sqrt{2}\cos\frac{a+b}{2}$$

$$\sin n=\sqrt{2}\sin\frac{a-b}{2}.$$

After these values are computed the coordinates of the projection are given directly by the relation,

$$x=F\left(m,\frac{1}{\sqrt{2}}\right)$$

$$y=F\left(n,\frac{1}{\sqrt{2}}\right).$$

By means of Legendre's table of the first elliptic integral for $k=\frac{1}{\sqrt{2}}$ we can very readily compute the coordinates for the map. In the interpolations it is better to have the values of m and n in degrees and

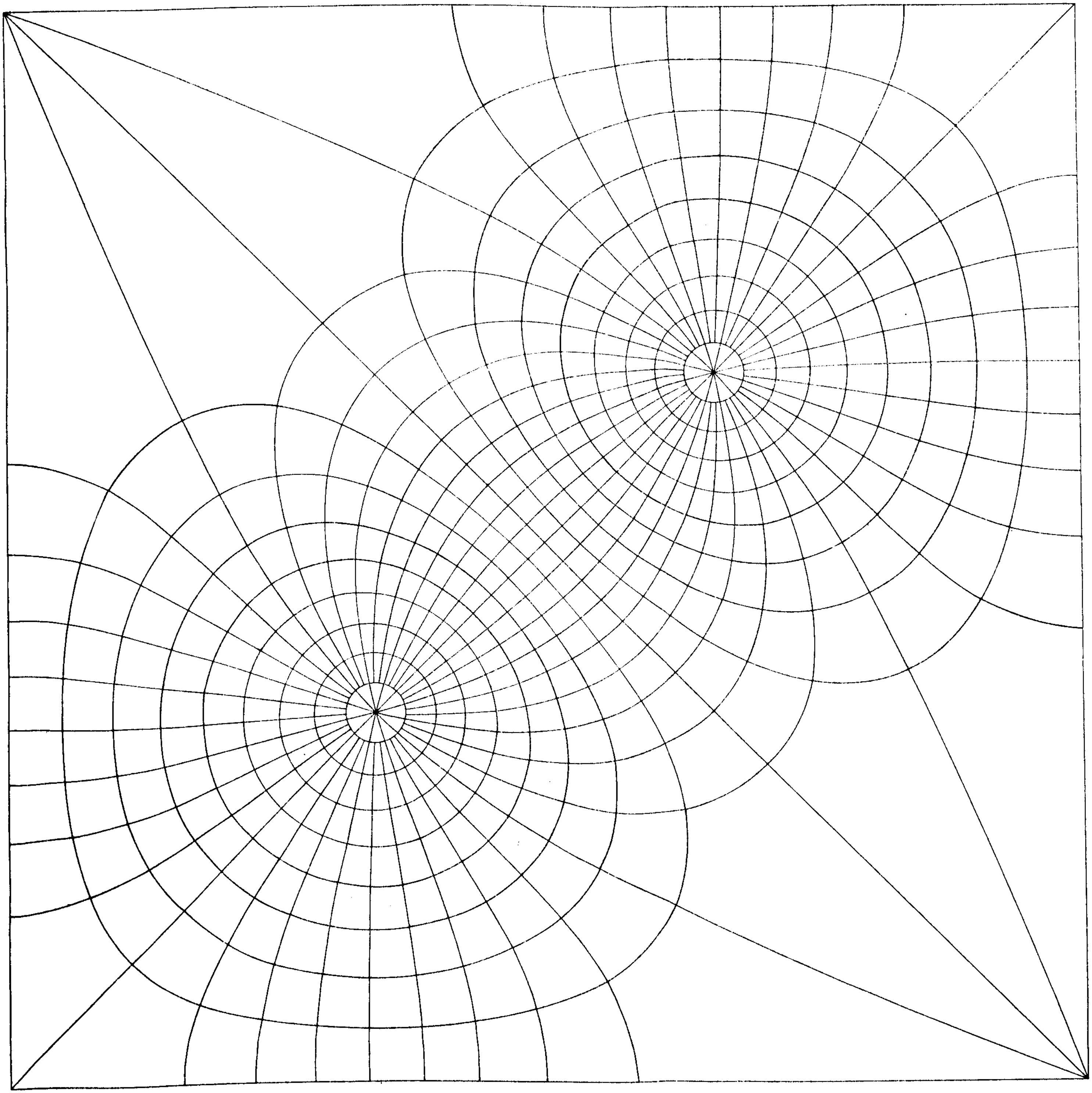

FIG. 4.—Conformal projection of the sphere within a square

(24693—29. Face p. 4.)

fractions of a degree. To facilitate the reduction of minutes and seconds to fractions of a degree, a table of such reductions is given on page 12. The table of the coordinates is given on page 13.

The coordinates for the projection were computed by straight line interpolation in a five-place table; that is, without considering second differences. The values are given to five decimal places but, of course, there is some uncertainty in the fifth place due to the method of interpolation. The coordinates are in any case accurate enough for the construction of the projection. Greater accuracy could be obtained by using a table given to more decimal places and by the use of the higher differences.

Even with this method of attack the amount of computation required was considerable. However, the calculations were all simple trigonometrical computations except, of course, the interpolations in the table of elliptic integrals. The final computations had no further check than that furnished by the symmetry of the projection when it was finally plotted. It is quite astonishing that errors of a few tenths of a millimeter could be detected in this way. For the coordinates as given the axes are drawn through the center of the square and each one parallel to its respective side. The coordinates are given for one quarter of the projection; that is, for that part bounded by the right-hand side and by the two diagonals. The rest of the projection is constructed by considerations of symmetry.

This same method of conformally mapping the whole sphere on the hemisphere could be used as a basis in the construction of maps with the poles placed in any desired position within the square. The position of the poles on the hemisphere would have to be arranged accordingly, and a new table of q and r values would have to be computed to bring about the desired result. The table of these values as computed could be used as the basis for the computation of a conformal projection within the rhombus employed in the rhombic projections in Special Publication No. 112, previously referred to in this discussion. The poles could be placed either on the long diagonal or on the short one but symmetrically in either case with respect to the center of the rhombus.

This method of mapping the whole sphere on the hemisphere was used by Lagrange to compute the map of the whole sphere within the unit circle, an illustration of which is given in Special Publication No. 57, General Theory of Polyconic Projections. It is thus seen that the process has possibilities of usefulness in many directions, and the example given in this publication is of interest as illustrating the general procedure in such cases.

Table of c and d values

[$\cot c = \cos \lambda \cot \phi$; $\cos d = \sin \lambda \cos \phi$]

Longitude in degrees	Latitude 0°		Latitude 10°		Latitude 20°	
	c	*d*	*c*	*d*	*c*	*d*
	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″
0	0 00 00.0	90 00 00.0	10 00 00.0	90 00 00.0	20 00 00.0	90 00 00.0
10	0 00 00.0	80 00 00.0	10 09 03.9	80 09 12.4	20 17 00.8	80 36 31.4
20	0 00 00.0	70 00 00.0	10 37 39.3	70 18 59.4	21 10 22.2	71 15 10.0
30	0 00 00.0	60 00 00.0	11 30 30.2	60 30 04.6	22 47 45.2	61 58 32.4
40	0 00 00.0	50 00 00.0	12 57 44.9	50 43 35.6	25 24 49.6	52 50 29.2
50	0 00 00.0	40 00 00.0	15 20 23.3	41 01 35.2	29 31 12.6	43 57 29.6
60	0 00 00.0	30 00 00.0	19 25 31.4	31 28 29.8	36 03 08.6	35 31 52.9
70	0 00 00.0	20 00 00.0	27 16 23.4	22 16 07.4	46 46 51.0	27 59 27.3
80	0 00 00.0	10 00 00.0	45 26 18.8	14 06 21.6	64 29 40.0	22 16 07.4
90	------------	0 00 00.0	90 00 00.0	10 00 00.0	90 00 00.0	20 00 00.0
100	180 00 00.0	10 00 00.0	134 33 41.2	14 06 21.6	115 30 20.0	22 16 07.4
110	180 00 00.0	20 00 00.0	152 43 36.6	22 16 07.4	133 13 09.0	27 59 27.3
120	180 00 00.0	30 00 00.0	160 34 28.6	31 28 29.8	143 56 51.4	35 31 52.9
130	180 00 00.0	40 00 00.0	164 39 36.7	41 01 35.2	150 28 47.4	43 57 29.6
140	180 00 00.0	50 00 00.0	167 02 15.1	50 43 35.6	154 35 10.4	52 50 29.2
150	180 00 00.0	60 00 00.0	168 29 29.8	60 30 04.6	157 12 14.8	61 58 32.4
160	180 00 00.0	70 00 00.0	169 22 20.7	70 18 59.4	158 49 37.8	71 15 10.0
170	180 00 00.0	80 00 00.0	169 50 56.1	80 09 12.4	159 42 59.2	80 36 31.4
180	180 00 00.0	90 00 00.0	170 00 00.0	90 00 00.0	160 00 00.0	90 00 00.0

Longitude in degrees	Latitude 30°		Latitude 40°		Latitude 50°	
	c	*d*	*c*	*d*	*c*	*d*
	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″
0	30 00 00.0	90 00 00.0	40 00 00.0	90 00 00.0	50 00 00.0	90 00 00.0
10	30 22 52.5	81 21 03.0	40 25 56.8	82 21 20.6	50 25 52.7	83 35 28.9
20	31 34 00.2	72 46 14.2	41 45 47.9	74 48 39.9	51 44 39.7	77 18 00.0
30	33 41 24.2	64 20 28.0	44 05 43.1	67 28 44.4	53 59 41.2	71 15 10.0
40	37 00 16.2	56 10 27.0	47 36 21.3	60 30 04.6	57 16 03.3	65 35 43.8
50	41 55 48.4	48 26 21.2	52 32 46.6	54 04 04.9	61 39 33.2	60 30 04.6
60	49 06 23.8	41 24 34.7	59 12 37.0	48 26 21.2	67 14 22.3	56 10 27.0
70	59 21 27.6	35 31 52.9	67 49 26.4	43 57 29.6	73 59 13.6	52 50 29.2
80	73 15 37.5	31 28 29.8	78 18 28.5	41 01 35.2	81 42 35.6	50 43 35.6
90	90 00 00.0	30 00 00.0	90 00 00.0	40 00 00.0	90 00 00.0	50 00 00.0
100	106 44 22.5	31 28 29.8	101 41 31.5	41 01 35.2	98 17 24.4	50 43 35.6
110	120 38 32.4	35 31 52.9	112 10 33.6	43 57 29.6	106 00 46.4	52 50 29.2
120	130 53 36.2	41 24 34.7	120 47 23.0	48 26 21.2	112 45 37.7	56 10 27.0
130	138 04 11.6	48 26 21.2	127 27 13.4	54 04 04.9	118 20 26.8	60 30 04.6
140	142 59 43.8	56 10 27.0	132 23 38.7	60 30 04.6	122 43 56.7	65 35 43.8
150	146 18 35.8	64 20 28.0	135 54 16.9	67 28 44.4	126 00 18.8	71 15 10.0
160	148 25 59.8	72 46 14.2	138 14 12.1	74 48 39.9	128 15 20.3	77 18 00.0
170	149 37 07.5	81 21 03.0	139 34 03.2	82 21 20.6	129 34 07.3	83 35 28.9
180	150 00 00.0	90 00 00.0	140 00 00.0	90 00 00.0	130 00 00.0	90 00 00.0

Table of c and d values—Continued

Longitude in degrees	Latitude 60°						Latitude 70°					
	c			*d*			*c*			*d*		
	°	′	″	°	′	″	°	′	″	°	′	″
0	60	00	00.0	90	00	00.0	70	00	00.0	90	00	00.0
10	60	22	42.0	85	01	08.7	70	16	48.9	86	35	42.5
20	61	32	07.5	80	09	12.4	71	07	05.8	83	16	56.2
30	63	26	05.8	75	31	21.0	72	30	17.1	80	09	12.4
40	66	08	28.8	71	15	10.0	74	25	14.2	77	18	00.0
50	69	38	21.9	67	28	44.4	76	49	55.8	74	48	39.9
60	73	53	52.4	64	20	28.0	79	41	09.2	72	46	14.2
70	78	49	47.2	61	58	32.4	82	54	14.5	71	15	10.0
80	84	16	29.6	60	30	04.6	86	23	00.8	70	18	59.4
90	90	00	00.0	60	00	00.0	90	00	00.0	70	00	00.0
100	95	43	30.4	60	30	04.6	93	36	59.2	70	18	59.4
110	101	10	12.8	61	58	32.4	97	05	45.5	71	15	10.0
120	106	06	07.6	64	20	28.0	100	18	50.8	72	46	14.2
130	110	21	38.1	67	28	44.4	103	10	04.2	74	48	39.9
140	113	51	31.2	71	15	10.0	105	34	45.8	77	18	00.0
150	116	33	54.2	75	31	21.0	107	29	42.9	80	09	12.4
160	118	28	52.5	80	09	12.4	108	52	54.2	83	16	56.2
170	119	37	18.0	85	01	08.7	109	43	11.1	86	35	42.5
180	120	00	00.0	90	00	00.0	110	00	00.0	90	00	00.0

Longitude in degrees	Latitude 80°						Latitude 90°					
	c			*d*			*c*			*d*		
	°	′	″	°	′	″	°	′	″	°	′	″
0	80	00	00.0	90	00	00.0	90	00	00.0	90	00	00.0
10	80	08	56.1	88	16	19.4	90	00	00.0	90	00	00.0
20	80	35	31.0	86	35	42.5	90	00	00.0	90	00	00.0
30	81	19	04.1	85	01	08.7	90	00	00.0	90	00	00.0
40	82	18	26.5	83	35	28.9	90	00	00.0	90	00	00.0
50	83	32	01.1	82	21	20.6	90	00	00.0	90	00	00.0
60	84	57	41.9	81	21	03.0	90	00	00.0	90	00	00.0
70	86	32	55.8	80	36	31.4	90	00	00.0	90	00	00.0
80	88	14	46.4	80	09	12.4	90	00	00.0	90	00	00.0
90	90	00	00.0	80	00	00.0	90	00	00.0	90	00	00.0
100	91	45	13.6	80	09	12.4	90	00	00.0	90	00	00.0
110	93	27	04.2	80	36	31.4	90	00	00.0	90	00	00.0
120	95	02	18.1	81	21	03.0	90	00	00.0	90	00	00.0
130	96	27	58.9	82	21	20.6	90	00	00.0	90	00	00.0
140	97	41	33.5	83	35	28.9	90	00	00.0	90	00	00.0
150	98	40	55.9	85	01	08.7	90	00	00.0	90	00	00.0
160	99	24	29.0	86	35	42.5	90	00	00.0	90	00	00.0
170	99	51	03.9	88	16	19.4	90	00	00.0	90	00	00.0
180	100	00	00.0	90	00	00.0	90	00	00.0	90	00	00.0

Table of r and q values

$$\left[r=\frac{c}{2};\ \tan\frac{q}{2}=\tan^{\frac{1}{2}}\frac{d}{2}\right]$$

Longitude in degrees	Latitude 0°						Latitude 10°						Latitude 20°					
	r			q			r			q			r			q		
	°	′	″	°	′	″	°	′	″	°	′	″	°	′	″	°	′	″
0	0	00	00.0	90	00	00.0	5	00	00.0	90	00	00.0	10	00	00.0	90	00	00.0
10	0	00	00.0	84	58	51.1	5	04	32.0	85	03	30.4	10	08	30.4	85	17	18.6
20	0	00	00.0	79	50	38.8	5	18	49.6	80	00	35.1	10	35	11.1	80	29	54.1
30	0	00	00.0	74	27	27.9	5	45	15.1	74	44	09.8	11	23	52.6	75	32	56.0
40	0	00	00.0	68	39	20.8	6	28	52.4	69	05	45.0	12	42	24.8	70	21	30.5
50	0	00	00.0	62	12	18.4	7	40	11.6	62	54	22.4	14	45	36.3	64	51	19.2
60	0	00	00.0	54	44	08.2	9	42	45.7	55	55	20.9	18	01	34.3	59	01	24.2
70	0	00	00.0	45	33	22.8	13	38	11.7	47	50	52.2	23	23	25.5	53	03	38.5
80	0	00	00.0	32	57	17.4	22	43	09.4	38	45	28.8	32	14	50.0	47	50	52.2
90	-	-	-	0	00	00.0	45	00	00.0	32	57	17.4	45	00	00.0	45	33	22.8
100	90	00	00.0	32	57	17.4	67	16	50.6	38	45	28.8	57	45	10.0	47	50	52.2
110	90	00	00.0	45	33	22.8	76	21	48.3	47	50	52.2	66	36	34.5	53	03	38.5
120	90	00	00.0	54	44	08.2	80	17	14.3	55	55	20.9	71	58	25.7	59	01	24.2
130	90	00	00.0	62	12	18.4	82	19	48.4	62	54	22.4	75	14	23.7	64	51	19.2
140	90	00	00.0	68	39	20.8	83	31	07.6	69	05	45.0	77	17	35.2	70	21	30.5
150	90	00	00.0	74	27	27.9	84	14	44.9	74	44	09.8	78	36	07.4	75	32	56.0
160	90	00	00.0	79	50	38.8	84	41	10.4	80	00	35.1	79	24	48.9	80	29	54.1
170	90	00	00.0	84	58	51.1	84	55	28.0	85	03	30.4	79	51	29.6	85	17	18.6
180	90	00	00.0	90	00	00.0	85	00	00.0	90	00	00.0	80	00	00.0	90	00	00.0

Longitude in degrees	Latitude 30°						Latitude 40°						Latitude 50°					
	r			q			r			q			r			q		
	°	′	″	°	′	″	°	′	″	°	′	″	°	′	″	°	′	″
0	15	00	00.0	90	00	00.0	20	00	00.0	90	00	00.0	25	00	00.0	90	00	00.0
10	15	11	26.2	85	39	47.0	20	12	58.4	86	10	09.6	25	12	56.4	86	47	26.3
20	15	47	00.1	81	17	10.4	20	52	54.0	82	20	16.6	25	52	19.8	83	36	38.3
30	16	50	42.1	76	50	10.5	22	02	51.6	78	30	55.8	26	59	50.6	80	29	54.1
40	18	30	08.1	72	17	52.2	23	48	10.6	74	44	09.8	28	38	01.6	77	30	40.4
50	20	57	54.2	67	41	56.3	26	16	23.3	71	04	44.4	30	49	46.6	74	44	09.8
60	24	33	11.9	63	09	55.6	29	36	18.5	67	41	56.3	33	37	11.2	72	17	52.1
70	29	40	43.8	59	01	24.2	33	54	43.2	64	51	19.2	36	59	36.8	70	21	30.4
80	36	37	48.8	55	55	20.9	39	09	14.2	62	54	22.4	40	51	17.8	69	05	45.0
90	45	00	00.0	54	44	08.2	45	00	00.0	62	12	18.4	45	00	00.0	68	39	20.8
100	53	22	11.2	55	55	20.9	50	50	45.8	62	54	22.4	49	08	42.2	69	05	45.0
110	60	19	16.2	59	01	24.2	56	05	16.8	64	51	19.2	53	00	23.2	70	21	30.4
120	65	26	48.1	63	09	55.6	60	23	41.5	67	41	56.3	56	22	48.8	72	17	52.1
130	69	02	05.8	67	41	56.3	63	43	36.7	71	04	44.4	59	10	13.4	74	44	09.8
140	71	29	51.9	72	17	52.2	66	11	49.4	74	44	09.8	61	21	58.4	77	30	40.4
150	73	09	17.9	76	50	10.5	67	57	08.4	78	30	55.8	63	00	09.4	80	29	54.1
160	74	12	59.9	81	17	10.4	69	07	06.0	82	20	16.6	64	07	40.2	83	36	38.3
170	74	48	33.8	85	39	47.0	69	47	01.6	86	10	09.6	64	47	03.6	86	47	26.3
180	75	00	00.0	90	00	00.0	70	00	00.0	90	00	00.0	65	00	00.0	90	00	00.0

Table of r and q values—Continued

Longitude in degrees	Latitude 60°						Latitude 70°					
	r			*q*			*r*			*q*		
	°	′	″	°	′	″	°	′	″	°	′	″
0	30	00	00.0	90	00	00.0	35	00	00.0	90	00	00.0
10	30	11	21.0	87	30	25.9	35	08	24.4	88	17	48.5
20	30	45	33.8	85	02	30.3	35	33	32.9	86	38	07.3
30	31	43	02.9	82	38	14.4	36	15	08.6	85	02	30.3
40	33	04	14.4	80	29	54.1	37	12	37.1	83	36	38.3
50	34	49	11.0	78	30	55.8	38	24	57.9	82	20	16.6
60	36	56	56.2	76	50	10.5	39	50	34.6	81	17	10.4
70	39	24	53.6	75	32	56.0	41	27	07.2	80	29	54.1
80	42	08	14.8	74	44	09.8	43	11	30.4	80	00	35.1
90	45	00	00.0	74	27	27.9	45	00	00.0	79	50	38.8
100	47	51	45.2	74	44	09.8	46	48	29.6	80	00	35.1
110	50	35	06.4	75	32	56.0	48	32	52.8	80	29	54.1
120	53	03	03.8	76	50	10.5	50	09	25.4	81	17	10.4
130	55	10	49.0	78	30	55.8	51	35	02.1	82	20	16.6
140	56	55	45.6	80	29	54.1	52	47	22.9	83	36	38.3
150	58	16	57.1	82	38	14.4	53	44	51.4	85	02	30.3
160	59	14	26.2	85	02	30.3	54	26	27.1	86	38	07.3
170	59	48	39.0	87	30	25.9	54	51	35.6	88	17	48.5
180	60	00	00.0	90	00	00.0	55	00	00.0	90	00	00.0

Longitude in degrees	Latitude 80°						Latitude 90°					
	r			*q*			*r*			*q*		
	°	′	″	°	′	″	°	′	″	°	′	″
0	40	00	00.0	90	00	00.0	45	00	00.0	90	00	00.0
10	40	04	28.0	89	08	09.4	45	00	00.0	90	00	00.0
20	40	17	45.5	88	17	48.5	45	00	00.0	90	00	00.0
30	40	39	32.0	87	30	25.9	45	00	00.0	90	00	00.0
40	41	09	13.2	86	47	26.3	45	00	00.0	90	00	00.0
50	41	46	00.6	86	10	09.6	45	00	00.0	90	00	00.0
60	42	28	51.0	85	39	47.0	45	00	00.0	90	00	00.0
70	43	16	27.9	85	17	18.6	45	00	00.0	90	00	00.0
80	44	07	23.2	85	03	30.4	45	00	00.0	90	00	00.0
90	45	00	00.0	84	58	51.1	45	00	00.0	90	00	00.0
100	45	52	36.8	85	03	30.4	45	00	00.0	90	00	00.0
110	46	43	32.1	85	17	18.6	45	00	00.0	90	00	00.0
120	47	31	09.0	85	39	47.0	45	00	00.0	90	00	00.0
130	48	13	59.4	86	10	09.6	45	00	00.0	90	00	00.0
140	48	50	46.8	86	47	26.3	45	00	00.0	90	00	00.0
150	49	20	28.0	87	30	25.9	45	00	00.0	90	00	00.0
160	49	42	14.5	88	17	48.5	45	00	00.0	90	00	00.0
170	49	55	32.0	89	08	09.4	45	00	00.0	90	00	00.0
180	50	00	00.0	90	00	00.0	45	00	00.0	90	00	00.0

Table of m and n values

[For method of computing *m* and *n* see p. 4]

Longitude in degrees	Latitude 0°						Latitude 10°						Latitude 20°					
	m			*n*			*m*			*n*			*m*			*n*		
	°	′	″	°	′	″	°	′	″	°	′	″	°	′	″	°	′	″
0	90	00	00.0	−90	00	00.0	72	49	45.0	−72	49	45.0	65	22	22.4	−65	22	22.4
10	90	00	00.0	−65	19	22.1	78	51	52.5	−62	50	50.8	70	25	26.8	−58	28	20.5
20	90	00	00.0	−53	54	05.2	81	17	40.3	−52	56	34.3	73	23	23.3	−50	39	48.8
30	90	00	00.0	−44	03	57.0	82	18	57.7	−43	39	57.1	74	55	31.3	−42	37	25.8
40	90	00	00.0	−34	34	18.9	82	38	47.7	−34	31	33.3	75	27	27.6	−34	24	26.2
50	90	00	00.0	−24	43	41.7	82	27	51.7	−25	02	18.2	75	08	53.8	−25	52	17.3
60	90	00	00.0	−13	50	10.2	81	41	39.7	−14	40	57.1	73	54	24.3	−16	50	20.0
70	90	00	00.0	−0	47	12.4	79	55	18.2	−2	47	13.7	71	20	42.0	−7	15	24.3
80	90	00	00.0	17	09	53.5	75	43	27.0	11	19	47.1	66	40	39.2	2	20	57.9
90	90	00	00.0	90	00	00.0	64	56	35.8	24	44	16.2	59	09	36.8	10	03	42.3
100	41	38	17.7	41	38	17.7	49	39	58.6	26	51	01.9	49	52	02.3	13	23	27.8
110	32	20	00.6	32	20	00.6	38	29	22.3	23	00	22.1	41	08	00.9	13	05	45.7
120	25	21	51.7	25	21	51.7	30	32	09.1	18	34	58.2	33	49	38.3	11	03	32.2
130	19	51	24.9	19	51	24.9	24	21	44.4	14	24	34.9	27	46	31.2	8	21	58.5
140	15	10	57.3	15	10	57.3	19	14	46.1	10	31	52.4	22	38	45.5	5	26	18.2
150	11	01	27.3	11	01	27.3	14	48	04.8	6	52	36.0	18	10	29.7	2	25	03.1
160	7	11	26.7	7	11	26.7	10	46	17.3	3	21	53.5	14	10	26.0	−0	39	47.5
170	3	33	01.5	3	33	01.5	7	04	44.2	−0	04	53.3	10	30	28.9	−3	49	00.2
180	0	00	00.0	0	00	00.0	3	32	12.0	−3	32	12.0	7	04	48.4	−7	04	48.4

Longitude in degrees	Latitude 30°						Latitude 40°						Latitude 50°					
	m			*n*			*m*			*n*			*m*			*n*		
	°	′	″	°	′	″	°	′	″	°	′	″	°	′	″	°	′	″
0	59	25	11.6	−59	25	11.6	54	12	33.8	−54	12	33.8	49	27	05.6	−49	27	05.6
10	63	32	44.9	−54	05	24.6	57	33	08.8	−50	00	43.9	52	04	27.0	−46	12	28.3
20	66	20	54.0	−47	55	06.8	59	57	14.6	−45	08	10.7	54	00	16.3	−42	27	02.9
30	67	57	28.2	−41	14	49.5	61	24	20.5	−39	46	14.5	55	12	32.1	−38	18	35.3
40	68	32	44.2	−34	15	16.2	61	57	19.0	−34	05	38.0	55	40	40.0	−33	55	24.0
50	68	11	16.6	−27	00	35.5	61	57	09.5	−28	14	40.8	55	24	36.3	−29	26	12.7
60	66	49	26.5	−19	35	02.2	60	22	24.8	−22	23	31.2	54	24	18.2	−25	00	46.5
70	64	15	30.3	−12	11	24.9	58	09	47.3	−16	46	37.6	52	40	21.5	−20	53	32.0
80	60	14	24.5	−5	21	53.6	54	57	33.7	−11	45	15.8	50	14	47.9	−17	08	17.2
90	54	44	08.2	0	00	00.0	50	51	22 1	−7	44	14 7	47	13	38.3	−14	06	27.8
100	48	15	51.5	3	09	18.9	46	07	23.3	−5	01	47.1	43	46	23.6	−11	52	12.3
110	41	40	54.9	4	07	59.9	41	09	59.3	−3	40	15.1	40	04	40.8	−10	33	05.7
120	35	37	48.6	3	32	56.3	36	18	13.1	−3	27	49.8	36	20	23.5	−10	01	27.4
130	30	10	13.6	2	05	59.5	31	44	22.6	−4	07	36.4	32	40	59.8	−10	11	08.0
140	25	23	24.3	0	04	28.4	27	33	55.5	−5	24	49.4	29	13	08.8	−10	55	40.8
150	21	08	45.6	−2	17	03.0	23	46	16.2	−7	11	15.7	25	59	16.4	−12	08	03.9
160	17	19	26.7	−4	52	11.1	20	15	40.9	−9	13	47.3	23	00	30.3	−13	43	14.5
170	13	50	40.6	−7	39	16.6	17	05	57.5	−11	35	34.3	20	17	10.5	−15	37	44.7
180	10	38	14.4	−10	38	14.4	14	12	57.1	−14	12	57.1	17	49	25.9	−17	49	25.9

Table of m and n values—Continued

Longitude in degrees	Latitude 60°		Latitude 70°	
	m	*n*	*m*	*n*
	° ′ ″	° ′ ″	° ′ ″	° ′ ″
0	45 00 00.0	−45 00 00.0	40 46 08.5	−40 46 08.5
10	46 56 12.2	−42 37 43.0	42 02 29.7	−39 13 26.1
20	48 23 44.8	−39 52 53.4	43 00 16.6	−37 27 20.5
30	49 20 58.7	−36 50 02.9	43 38 15.1	−35 30 30.0
40	49 40 28.2	−33 43 51.9	43 53 32.4	−33 29 33.2
50	49 29 44.1	−30 30 43.2	43 48 02.8	−31 26 08.9
60	48 46 24.3	−27 21 41.3	43 21 24.2	−29 25 34.5
70	47 31 55.2	−24 24 23.0	42 34 47.7	−27 32 16.6
80	45 48 59.2	−21 46 19.3	41 30 03.9	−25 50 20.9
90	43 41 47.6	−19 34 05.6	40 09 43.3	−24 23 19.3
100	41 15 55.7	−17 52 29.1	38 36 50.6	−23 13 54.0
110	38 37 48.6	−16 43 48.6	36 54 49.6	−22 23 48.9
120	35 53 50.7	−16 07 54.5	35 07 12.3	−21 53 46.9
130	33 09 43.1	−16 02 43.2	33 17 21.4	−21 43 38.3
140	30 29 56.2	−16 25 08.1	31 28 22.8	−21 52 32.1
150	28 00 33.5	−17 06 16.8	29 42 15.0	−22 17 14.5
160	25 36 26.6	−18 18 45.7	28 03 29.2	−23 02 03.2
170	23 25 23.7	−19 45 43.1	26 31 54.3	−23 59 32.5
180	21 28 14.5	−21 28 14.5	25 10 02.1	−25 10 02.1

Longitude in degrees	Latitude 80°		Latitude 90°	
	m	*n*	*m*	*n*
	° ′ ″	° ′ ″	° ′ ″	° ′ ″
0	36 42 12.2	−36 42 12.2	32 45 54.4	−32 45 54.4
10	37 19 47.8	−35 45 30.9	32 45 54.4	−32 45 54.4
20	37 48 33.4	−34 59 56.7	32 45 54.4	−32 45 54.4
30	38 07 37.3	−34 09 30.7	32 45 54.4	−32 45 54.4
40	38 16 28.0	−33 10 50.7	32 45 54.4	−32 45 54.4
50	38 14 54.0	−32 11 26.9	32 45 54.4	−32 45 54.4
60	38 03 53.0	−31 14 03.6	32 45 54.4	−32 45 54.4
70	37 41 31.0	−30 18 18.6	32 45 54.4	−32 45 54.4
80	37 11 00.2	−29 28 16.3	32 45 54.4	−32 45 54.4
90	36 32 36.7	−28 44 44.6	32 45 54.4	−32 45 54.4
100	35 47 39.1	−28 09 01.5	32 45 54.4	−32 45 54.4
110	34 51 45.6	−27 36 31.1	32 45 54.4	−32 45 54.4
120	34 03 59.0	−27 24 35.1	32 45 54.4	−32 45 54.4
130	33 08 27.8	−27 16 49.3	32 45 54.4	−32 45 54.4
140	32 12 37.0	−27 18 48.0	32 45 54.4	−32 45 54.4
150	31 17 59.7	−27 30 16.1	32 45 54.4	−32 45 54.4
160	30 26 04.0	−27 50 42.7	32 45 54.4	−32 45 54.4
170	29 38 10.9	−28 19 26.4	32 45 54.4	−32 45 54.4
180	28 55 35.9	−28 55 35.9	32 45 54.4	−32 45 54.4

Minutes and seconds of arc as decimal parts of a degree

Minutes	Decimal part of a degree	Minutes	Decimal part of a degree	Seconds	Decimal part of a degree	Seconds	Decimal part of a degree
1	0. 016667	31	0. 516667	1	0. 000278	31	0. 008611
2	0. 033333	32	0. 533333	2	0. 000556	32	0. 008889
3	0. 050000	33	0. 550000	3	0. 000833	33	0. 009167
4	0. 066667	34	0. 566667	4	0. 001111	34	0. 009444
5	0. 083333	35	0. 583333	5	0. 001389	35	0. 009722
6	0. 100000	36	0. 600000	6	0. 001667	36	0. 010000
7	0. 116667	37	0. 616667	7	0. 001944	37	0. 010278
8	0. 133333	38	0. 633333	8	0. 002222	38	0. 010556
9	0. 150000	39	0. 650000	9	0. 002500	39	0. 010833
10	0. 166667	40	0. 666667	10	0. 002778	40	0. 011111
11	0. 183333	41	0. 683333	11	0. 003056	41	0. 011389
12	0. 200000	42	0. 700000	12	0. 003333	42	0. 011667
13	0. 216667	43	0. 716667	13	0. 003611	43	0. 011944
14	0. 233333	44	0. 733333	14	0. 003889	44	0. 012222
15	0. 250000	45	0. 750000	15	0. 004167	45	0. 012500
16	0. 266667	46	0. 766667	16	0. 004444	46	0. 012778
17	0. 283333	47	0. 783333	17	0. 004722	47	0. 013056
18	0. 300000	48	0. 800000	18	0. 005000	48	0. 013333
19	0. 316667	49	0. 816667	19	0. 005278	49	0. 013611
20	0. 333333	50	0. 833333	20	0. 005556	50	0. 013889
21	0. 350000	51	0. 850000	21	0. 005833	51	0. 014167
22	0. 366667	52	0. 866667	22	0. 006111	52	0. 014444
23	0. 383333	53	0. 883333	23	0. 006389	53	0. 014722
24	0. 400000	54	0. 900000	24	0. 006667	54	0. 015000
25	0. 416667	55	0. 916667	25	0. 006944	55	0. 015278
26	0. 433333	56	0. 933333	26	0. 007222	56	0. 015556
27	0. 450000	57	0. 950000	27	0. 007500	57	0. 015833
28	0. 466667	58	0. 966667	28	0. 007778	58	0. 016111
29	0. 483333	59	0. 983333	29	0. 008056	59	0. 016389
30	0. 500000			30	0. 008333		

Projection coordinates

Longitude in degrees	Latitude 0°		Latitude 10°		Latitude 20°		Latitude 30°		Latitude 40°	
	x	y	x	y	x	y	x	y	x	y
0	1.85407	−1.85407	1.43625	−1.43625	1.26297	−1.26297	1.12966	−1.12966	1.01684	−1.01684
10	1.85407	−1.26183	1.58091	−1.20584	1.37966	−1.10878	1.22156	−1.01429	1.08881	−0.92854
20	1.85407	−1.01028	1.63880	−0.98995	1.44955	−0.94211	1.28526	−0.88534	1.14142	−0.82877
30	1.85407	−0.80724	1.66498	−0.79924	1.48610	−0.77847	1.32228	−0.75120	1.17361	−0.72219
40	1.85407	−0.62182	1.67307	−0.62095	1.49884	−0.61869	1.33589	−0.61578	1.18585	−0.61272
50	1.85407	−0.43832	1.66861	−0.44398	1.49143	−0.45925	1.32760	−0.48018	1.18579	−0.50300
60	1.85407	−0.24266	1.64979	−0.25767	1.46183	−0.29602	1.29617	−0.34515	1.15070	−0.39581
70	1.85407	−0.01373	1.60658	−0.04865	1.40125	−0.12683	1.23766	−0.21357	1.10211	−0.29492
80	1.85407	0.30183	1.50522	0.19839	1.29281	0.04101	1.14775	−0.09371	1.03284	−0.20588
90	1.85407	1.85407	1.25318	0.43849	1.12395	0.17604	1.02807	0.00000	0.94612	−0.13525
100	0.75893	0.75893	0.92138	0.47725	0.92544	0.23479	0.89245	0.05508	0.84873	−0.08783
110	0.57939	0.57939	0.69721	0.40695	0.74896	0.22957	0.75979	0.07217	0.74961	−0.06409
120	0.44996	0.44996	0.54564	0.32715	0.60766	0.19362	0.64204	0.06196	0.65496	−0.06047
130	0.35005	0.35005	0.43164	0.25283	0.49431	0.14628	0.53881	0.03665	0.56821	−0.07206
140	0.26654	0.26654	0.33908	0.18433	0.40042	0.09499	0.45043	0.00113	0.49044	−0.09456
150	0.19301	0.19301	0.25978	0.12017	0.31988	0.04220	0.37327	−0.03987	0.42087	−0.12562
160	0.12567	0.12567	0.18856	0.05867	0.24864	−0.01157	0.30468	−0.08504	0.35733	−0.16144
170	0.06199	0.06199	0.12371	−0.00142	0.18392	−0.06664	0.24281	−0.13380	0.30066	−0.20303
180	0.00000	0.00000	0.06175	−0.06175	0.12373	−0.12373	0.18620	−0.18620	0.24939	−0.24939

Longitude in degrees	Latitude 50°		Latitude 60°		Latitude 70°		Latitude 80°		Latitude 90°	
	x	y	x	y	x	y	x	y	x	y
0	0.91694	−0.91694	0.82602	−0.82602	0.74178	−0.74178	0.66265	−0.66265	0.58744	−0.58744
10	0.97163	−0.85046	0.86527	−0.77856	0.76691	−0.71151	0.67474	−0.64449	0.58744	−0.58744
20	1.01247	−0.77502	0.89516	−0.72435	0.78603	−0.67717	0.68400	−0.62996	0.58744	−0.58744
30	1.03820	−0.69371	0.91483	−0.66517	0.79868	−0.63971	0.69016	−0.61395	0.58744	−0.58744
40	1.04827	−0.60929	0.92155	−0.60583	0.80377	−0.60131	0.69303	−0.59540	0.58744	−0.58744
50	1.04252	−0.52513	0.91785	−0.54519	0.80194	−0.56250	0.69252	−0.57670	0.58744	−0.58744
60	1.02101	−0.44352	0.90293	−0.48667	0.79307	−0.52492	0.68895	−0.55872	0.58744	−0.58744
70	0.98425	−0.36869	0.87744	−0.43245	0.77760	−0.48993	0.68174	−0.54133	0.58744	−0.58744
80	0.93343	−0.30136	0.84252	−0.38459	0.75624	−0.45866	0.67391	−0.52577	0.58744	−0.58744
90	0.87121	−0.24747	0.79986	−0.34487	0.72985	−0.43212	0.65958	−0.51229	0.58744	−0.58744
100	0.80139	−0.20792	0.75157	−0.31459	0.69963	−0.41105	0.64518	−0.50125	0.58744	−0.58744
110	0.72820	−0.18469	0.69994	−0.29408	0.66670	−0.39590	0.62736	−0.49124	0.58744	−0.58744
120	0.65566	−0.17530	0.64716	−0.28342	0.63228	−0.38684	0.61219	−0.48756	0.58744	−0.58744
130	0.58599	−0.17825	0.59504	−0.28188	0.59745	−0.38378	0.59464	−0.48518	0.58744	−0.58744
140	0.52109	−0.19131	0.54495	−0.28854	0.56320	−0.38646	0.57707	−0.48578	0.58744	−0.58744
150	0.46138	−0.21258	0.49863	−0.30076	0.53011	−0.39392	0.55995	−0.48931	0.58744	−0.58744
160	0.40698	−0.24062	0.45441	−0.32235	0.49954	−0.40745	0.54374	−0.49562	0.58744	−0.58744
170	0.35778	−0.27448	0.41453	−0.34835	0.47139	−0.42489	0.52885	−0.50447	0.58744	−0.58744
180	0.31360	−0.31360	0.37914	−0.37914	0.44635	−0.44635	0.51564	−0.51564	0.58744	−0.58744