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THE BOWIE METHOD OF TRIANGULATION ADJUSTMENT

AS APPLIED TO THE FIRST-ORDER NET IN THE WESTERN PART OF THE UNITED STATES

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THE BOWIE METHOD OF TRIANGULATION ADJUSTMENT AS APPLIED TO THE FIRST-ORDER NET IN THE WESTERN PART OF THE UNITED STATES

By OSCAR S. ADAMS, *Senior Mathematician, U. S. Coast and Geodetic Survey*

ABSTRACT

This publication contains a discussion of the new method for the adjustment of a triangulation net as applied in the western part of the United States. The extent of the arcs and their complicated interrelations made it practically impossible to adjust the whole with one set of equations; accordingly a new method of handling such networks was devised employing junction figures and sections of arcs between them. A complete discussion of the procedure in the application of the method in the western half of the United States is given, together with an account of the method of establishing the junction figures. The formation of the equations for the loop closures is given in detail and the method of least squares is employed to determine the best positions to be held in the junction figures.

The tables give the results of the preliminary computations and the equations derived from the loop closures. The formation and solution of the normal equations are also given to serve as the examples for such applications. Lastly the final results derived by the solutions are given also as a table showing the changes in position of previously adjusted junction points.

At the end of the publication there is given a discussion of the facts disclosed by this extensive adjustment of triangulation arcs. This section also includes a discussion regarding the accuracy of the observations as disclosed by the remarkable closures of loops throughout the whole area.

INTRODUCTION

The first-order triangulation of the United States has been extended from year to year with a more or less degree of regularity for the past 80 years or more. The arcs in the western part of the United States have been in process of extension for approximately 50 years. This more or less uniform growth consisted of extending new arcs from those already in existence. As the various arcs were observed they were adjusted by least squares so that the work should be consistent when the results were furnished for engineering applications. When any loop was completed, the total discrepancy in closure was adjusted into the last arc that closed the loop. This quite frequently imposed unfavorable conditions upon the last arc and resulted in distortions of the work that were not justified by the general accord of the observations. For practical reasons this method had to be followed, for otherwise the whole system of triangulation would have to be readjusted every time a new loop was closed. The economic conditions thus necessitated that this plan should be followed until the main chains of triangles should be completed and then a unified adjustment would be undertaken for the whole region.

The final arcs for the framework in the western part of the United States were completed in 1926, and then the general adjustment was carried through. The plan upon which the adjustment should be made was decided upon in 1924 and many of the preliminary computations were made before the final arcs were completed. An undertaking of such proportions could only be carried through by the aid of a large body of men trained in such calculations. Especial mention should be made of a number of members of the division of geodesy; W. F. Reynolds, C. H. Swick, H. C. Mitchell, W. D. Sutcliffe, and Howard S. Rappleye aided in the preparation of the equations for the adjustments, as well as by giving expert advice upon technical points. In the further calculations the main work was done by G. L. Fentress, H. P. Kaufman, F. W. Darling, J. A. Duerksen, W. Shofnos, C. J. Clifford, and W. E. Wood.

GROWTH OF TRIANGULATION NET

In the adjustment of triangulation it is evident that the closure of a loop should be distributed throughout its entire length. This is a very simple matter when the arc consists of a single loop such as that formed by carrying triangulation around an island or around a lake. However, when the system consists of a number of interconnected loops, the solution of the equations of condition for all of the loops in a single set presents serious difficulty on account of the amount of computation that would be required in carrying out the adjustment.

In the United States the geodetic datum was adopted when the triangulation system consisted of a meager skeleton joining the arc near the Atlantic coast to that near the Pacific coast. As this mere outline was supplemented by new work, the discrepancies in the closures of loops were adjusted into the new arcs, the part already adjusted being held fixed. In the earlier work no true geodetic azimuths, derived by applying the Laplace correction to observed astronomic azimuths, were held in the adjustments. Experience had not yet shown that an arc of triangulation tends to swerve in azimuth unless it is held in orientation by equations of condition derived from the observed azimuths corrected for the deflection of the vertical. This departure from true orientation in the previously adjusted arcs at times caused comparatively large discrepancies to appear when new arcs were observed to close loops. As the country became divided up by many closed arcs of triangulation, this method of adjustment became more and more objectionable because often comparatively short arcs were forced to absorb loop closures that were out of all proportion to their lengths, and as a result the corrections that had to be applied to them were unduly large. In all loops the discrepancies disclosed were such as might be expected from the grade of the work when the whole length of the loop was considered, but they were unduly large for adjustment into the new work with old work held fixed. It became evident that at some time a readjustment would have to be made of the whole, or at least part, of the net to relieve this condition of unequal strain. Since the western part of the country was divided up into about the ideal-sized loops, the part from the ninety-eighth meridian arc inclusive was chosen for the first application of the method.

NEED FOR GENERAL ADJUSTMENT

Although this section was sufficiently covered by arcs of triangulation to permit of a general adjustment that would give consistent results which could then be held fixed for all time, yet a serious practical difficulty arose when the adjustment was contemplated. If the adjustment should be made by the method of least squares including the whole region in one adjustment, it would involve the solution of more than 3,000 simultaneous equations. Even with the use of modern calculating machines this would require an enormous amount of work, much more than could be justified economically. There would also be the possibility of some error in the formation or the solution that would fail to be detected until the final results were being computed, and then it would require much extra labor to make the corrections.

The question, however, was an important one at that time, for the arc of first-order triangulation, extending from the northwestern part of the United States along the coast of British Columbia and through southeastern Alaska, was awaiting adjustment, and this could not be undertaken until final values for the geographic positions of the stations from which the new arc would extend could be obtained. This arc was measured and adjusted by the cooperation of this bureau and the Geodetic Survey of Canada. The data carried by this arc into Alaska are the controlling data for the whole of the work in Alaska as well as for the work in western and northwestern Canada. It thus was imperative that the best value possible should be derived for the line in northwest Washington from which the Canadian-Alaskan arc extended.

BOWIE METHOD DEVISED

An economically feasible method for making an adjustment of such an extensive net of triangulation was worked out early in 1924 by Dr. William Bowie, chief of the division of geodesy of the United States Coast and Geodetic Survey. He conceived and formulated a plan by means of which such an extensive system can be adjusted with sufficient rigor at comparatively small cost and in a comparatively short time. By this Bowie method the discrepancies in loop closures of a net formed by arcs of triangulation are distributed in a way somewhat similar to that used in the adjustment of a first-order level net.

The triangulation in the western part of the United States consists of a number of intersecting arcs. In order to make the readjustment of the net by the new method, the unit was taken as the section of an arc between two junction points. Hereafter in this publication this unit will be called simply a "section."

This same method of adjustment can also be applied to an area completely covered by triangulation by selecting loops formed by chains of well-shaped triangles, quadrilaterals, or other figures with the intermediate figures or stations omitted. Such omitted stations can later be adjusted into the general net by supplementary computations.

After the loop closures have been distributed among the various sections of the arcs of triangulation forming the loops, and in this way the most probable geographic positions of the stations used as junction points have been derived, then the sections can be adjusted separately between the junction figures.

At the request of Doctor Bowie, the writer worked out a method for forming the equations and for making the adjustments by the method of least squares which are necessary to render the plan practicable. On first consideration it was contemplated that the adjustment might be made by the use of interrelated equations for the loop closures, but on further investigation it was found that no economical scheme of this kind could be devised. After careful study a plan was perfected for the use of two independent sets of equations, one set for the closures in latitude and the other set for the longitude closures. This plan was found to be both economical and practical and at the same time it gives a very satisfactory distribution of the discrepancies in loop closures.

After working out the mathematical theory of the method, the author was put in entire charge of the execution of the project. From 8 to 12 mathematicians were employed on the adjustments of the sections, the number varying as the availability of the work demanded. As a result, in about 15 months after starting the computations the work had advanced to the point where the adjustment of the loop closures could be made to obtain the most probable positions of the junction points to be held in the junction figures. One of the great advantages of the Bowie method is the fact that a large number of computers can be employed upon the work at the same time. When the plan of the adjustment was initiated not all of the field work of the various arcs to be included was completed. If all of the observational data had been ready, as many computers could have been put upon the computations as there were sections, and the final results could have been obtained in a much shorter time. As the matter stood with this bureau, efforts were made to complete the work as soon as all of the field work was available. The record for accomplishment speaks well both for the Bowie method and for the efficiency of the mathematicians who were employed upon the various computations necessary to carry out the scheme. We believe that this accomplishment deserves the attention of all practical geodesists, showing as it does a new method for bringing about an adjustment of complicated nets of triangulation in a comparatively short time with sufficient rigor to meet all demands, either practical or scientific. The economy both of time and expense is a fact to consider as well as the fact that the method is applicable to adjustments of triangulation that could scarcely be handled in any other way. In actual working days for one man the time employed up to the determination of the junction points was 2,200 days of 7 hours each. For the computations of the sections between junction figures the time consumed was 1,600 days, making a total of 3,800 days of 7 hours each for the whole work of the adjustment of the net.

DESCRIPTION OF THE PLAN OF ADJUSTMENT

In carrying out the scheme of adjustment the first step consisted in laying out junction figures at the various intersections of the arcs. The general aim was to make these figures as simple as possible but to choose them, if possible, so that the various sections radiating from them should be attached to the junction figure by a single line. It was not possible in all cases to attain this ideal, but it was so arranged whenever feasible. The junction figures in some cases consisted of only one or two quadrilaterals, but in other cases they became fairly complicated figures. If the plan had been in mind during the time when the arcs were being observed, no doubt the ideal simple figures could have been observed in almost every case. However, since the work was already done it remained to make the best use of what was at hand.

After the junction figures were laid out, it then became necessary to adopt some length and azimuth to hold in them. The ideal case was a junction figure in which was included a measured base and an observed Laplace azimuth. The figure could then be adjusted by least squares, and the measured base and azimuth could furnish the required control. Each line from which a section started was fixed in length and azimuth by this adjustment and these values were then used when the adjoined section was adjusted.

If either a measured length or an observed Laplace azimuth was wanting in the junction figure, it became necessary to adopt a value for some line of the figure for length or azimuth, or for both in case neither was present in the figure. This was generally done by setting up equations from the nearest control data through each of the radiating sections to some line in the junction figure and then some weighted mean was adopted as the value to be used. The weights were assigned by consideration of the strength of the various connections, a base or azimuth lying near the line having more weight than one farther away. The main object was to adopt control data that would best accord with the nearest bases and Laplace azimuths in the various sections. It was always possible to arrive at some reasonable compromise among the various values derived from the equations in the sections. In some cases where the chosen junction figure had already been adjusted in one of the arcs, the adjusted values of the angles and lengths were held fixed if they agreed reasonably well with the lengths derived from the other sections. Various methods were thus used in fixing these data, with the sole object in mind to adopt values that would best fit in with the bases and Laplace azimuths in the radiating sections.

LAPLACE AZIMUTHS

Since there may be some question as to how the Laplace azimuth values were obtained, it is probably best to give some account of them at this point. A Laplace azimuth is an observed astronomical azimuth that has been corrected for the effect of the deflection of the vertical. This correction can be computed if an astronomical longitude has been observed at the station as well as the astronomical azimuth. If we denote the astronomical value by A and the geodetic value by G, we have the relation,

$$(A-G) \text{ in azimuth} = -[(A-G) \text{ in longitude}] \sin \phi$$

in which ϕ is the latitude of the station. In computing this correction the best available value for the geodetic longitude as derived from the preliminary computations is used. This will not, in most cases, differ from the final value by more than a few tenths of a second of arc, and hence the computed correction will be very nearly correct. After the final value of the geodetic longitude is adopted in the junction figure, a small correction to the azimuth as held can be

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made to take care of the change in the geodetic longitude, and this change in the azimuth can be taken care of in the final adjustment of the sections between the junction figures.

TYPICAL JUNCTION FIGURES

As examples of the various conditions that may arise in the work of establishing the fundamental length and azimuth in junction figures, three diagrams are included showing three typical cases that arose in this adjustment. Figure 1 shows an example of the simplest case in which a measured base (indicated by a heavy line) and a Laplace azimuth (indicated by an arrowhead) are included in a simple figure. In Figure 2 we also have these fundamental data included in the junction figure, but the figure itself is considerably more complicated. Finally, in Figure 3 we have an example of the most complicated case. The length and azimuth had to be carried into the junction figure from four radiating sections. The simple chain of triangles through which the data had to be carried is shown in each case. The junction figure itself was somewhat complicated even after these accessory triangles were omitted. This was the most involved case that arose in the whole adjustment. In any case of the application of this method of adjustment the length and azimuth in the junction figure could be determined in any reasonable way that might be desired; the end in view should be to choose these data so as best to accord with the other fixed data in the adjoining sections. In the present adjustment this aim was kept in view at all times and the actual method of attaining it was varied at times to suit the individual case.

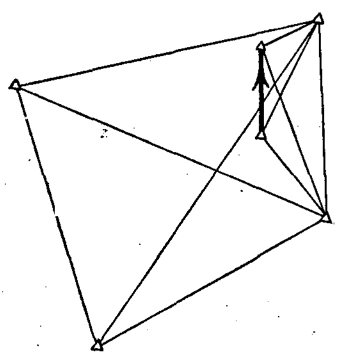
FIGURE 1.—Simple junction figure containing base and Laplace azimuth

PRELIMINARY ADJUSTMENT OF SECTIONS

After a junction figure has been adjusted, the lengths and azimuths of the lines to which the sections join are known. The sections can then be adjusted between the junction figures holding these lengths and azimuths. All of the conditions will thus be included except the conditions for closure in latitude and longitude. As a matter of expediency the latitude and longitude equations were made up in this preliminary adjustment with no closure term added to them. The "forward" solution of the normal equations could thus be made with these equations carried along as the last two equations of the set. In the preliminary "back" solution these two equations were ignored. However, after the discrepancy in closure was later determined, it was only necessary to carry the forward solution through these equations, neglecting the constant terms of the other normals except for a small correction to the constant term of the last azimuth equation of each section due to the change in the Lapace azimuth caused by the change in the longitude. In this way an additional set of corrections were derived which, added algebraically to the preliminary corrections, gave the final complete corrections. In this way an economy in the labor required was attained. However, in the application of the Bowie method, after the final positions in the junction figures have been determined (in the manner to be indicated later), a complete new adjustment of each section, including all of the conditions at one time, could be made if so desired. It would require considerable more work, however, and the results would not be any more satisfactory. Economy of calculation should have some weight in deciding which course to follow. There is no question but that the method used in the present adjustment saved a great deal of time and consequent expense.

The preliminary adjustment is no more than the usual figure adjustment including the angle equations, the side equations, the length equations to hold the lengths of the measured bases and of the fixed lines at the ends of the section, and the azimuth equations to hold any intermediate Laplace azimuths and the azimuths of the junction lines. All possible conditions are thus included except the conditions of closure in latitude and longitude.

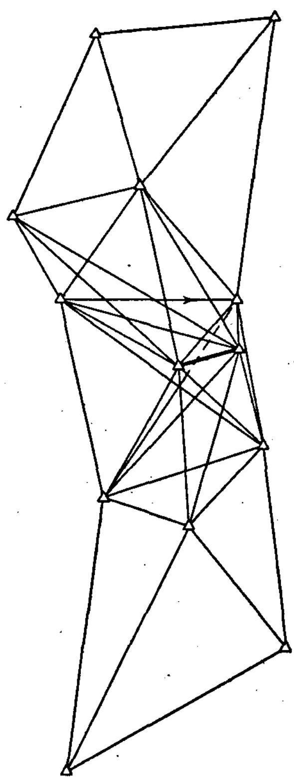

FIGURE 2.—Complicated junction figure involving several stations

COMPUTATION OF PRELIMINARY POSITIONS

After these preliminary adjustments have all been made the geographic positions can be computed, starting from the point whose latitude and longitude is to be held fixed as the fundamental datum. These preliminary positions can be computed along any of the arcs and, consequently, there will be certain junction stations at which there will be discrepancies of closure. If computations are to be carried further from such a junction, one or the other of the consistent set of data should be used. Thus the positions of all stations can be traced in unbroken line through some continuous arcs back to the fundamental station.

From these positions an assumed position is taken for some chosen point in each of the junction figures. The formation of the observation equations is simplified if these assumed positions are made to correspond in each case with the preliminary positions as computed through one of the continuous arcs. In this way the constant term in many of the equations will be zero and the work of formation of the latitude and longitude equations will be simplified thereby.

ADJUSTMENT OF LATITUDE AND LONGITUDE CLOSURES

After the positions have been assumed for the junction points, the observation equations are formed for both the latitude closures and the longitude closures. The procedure is the same in each case.

The difference between the assumed latitudes of the junction points at the two ends of a section gives the assumed value for this difference. By subtracting from this quantity the computed value for the same we get an equation for the v in latitude for that section; a similar procedure gives the v in longitude for the section. The constant term in these will be expressed in units of seconds of arc. Seconds of arc, however, represent different linear lengths depending upon the lati-

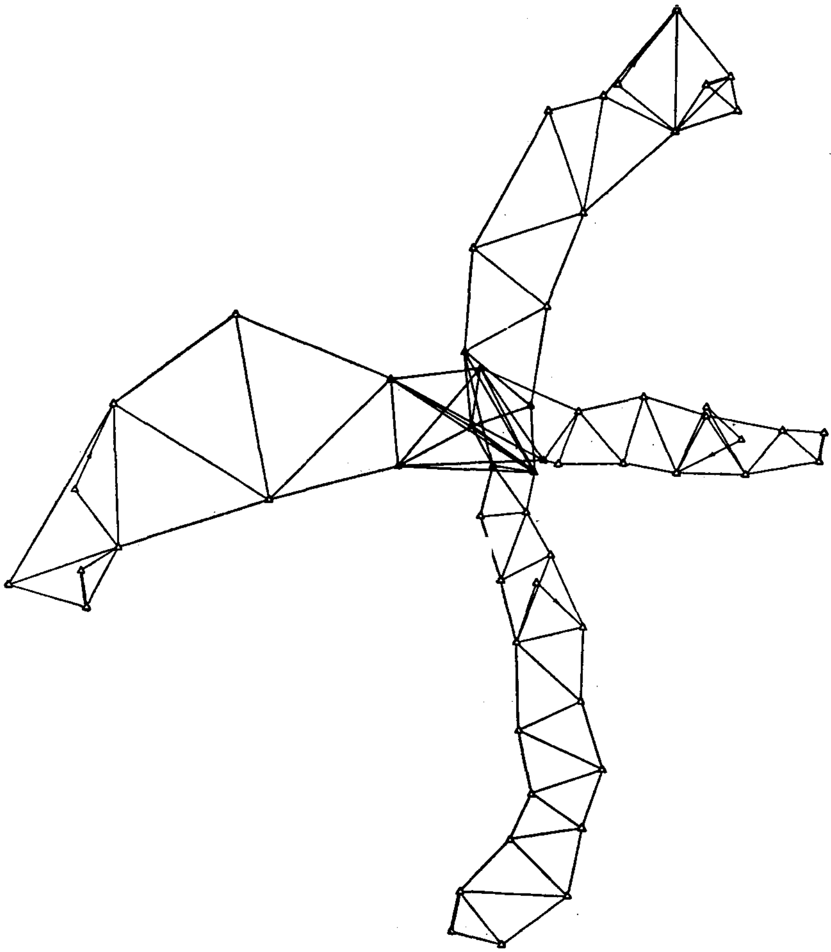

FIGURE 3.—Junction figure which contains no bases or Laplace azimuths

The lengths in the junction figure at the intersection of the arcs are controlled by the four bases indicated by the heavy lines and the azimuths by the four Laplace azimuths indicated by the arrow heads.

tude. To correlate these values in various sections of the country, the values in seconds of arc were reduced to linear measure with 10 feet as the unit. This unit gave the best balance for these quantities in the formation of the normal equations. In this manner the set of equations for latitude and the set for longitude shown in the table on page 16 were formed. The constant terms in these equations must be multiplied by 10 if it is desired to get the closures in feet.

After these two sets of observation equations have been formed, it becomes necessary to decide upon a proper weighting of the various sections. If they were all of the same length, the weight of each sec-

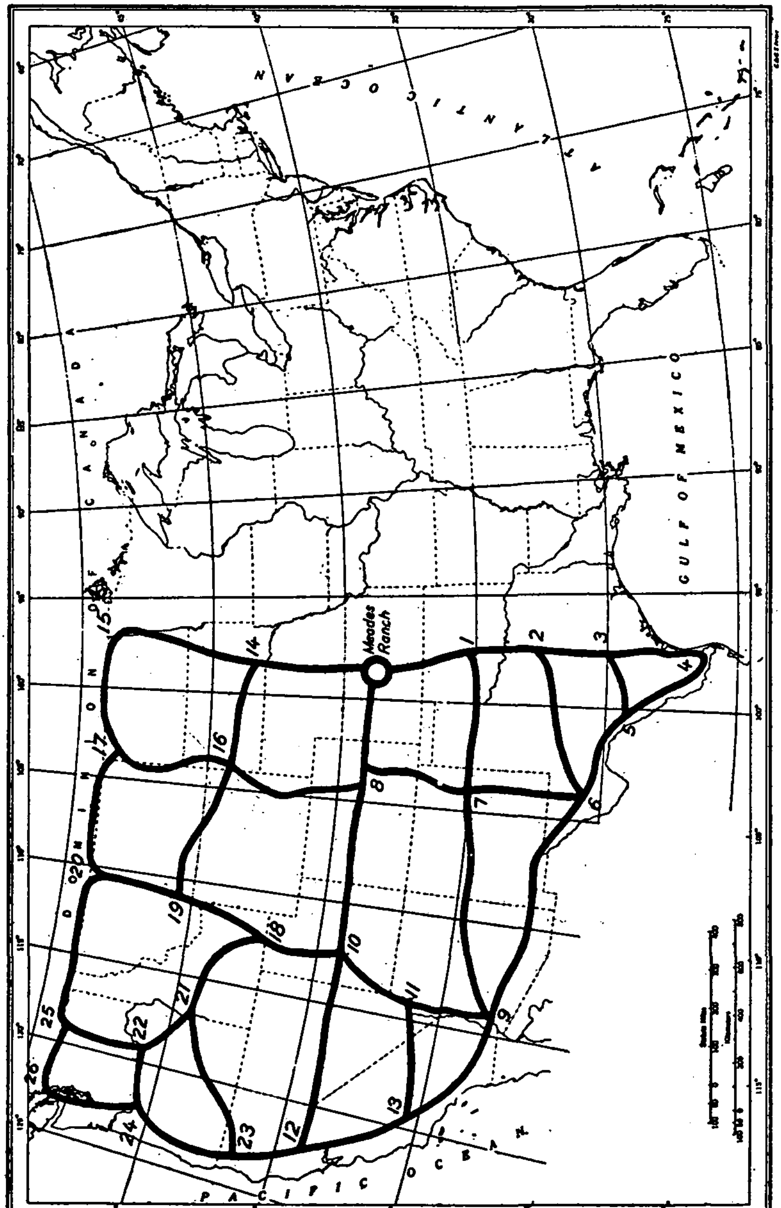

FIGURE 4.—Diagram of net showing numbers assigned to junction points

tion could be taken as unity, but such a condition would probably never be found in actual practice. In this adjustment it was decided to weight the sections inversely proportional to their length. A section of about average length was assumed as having weight unity. A section approximately one-half as long would then have the weight

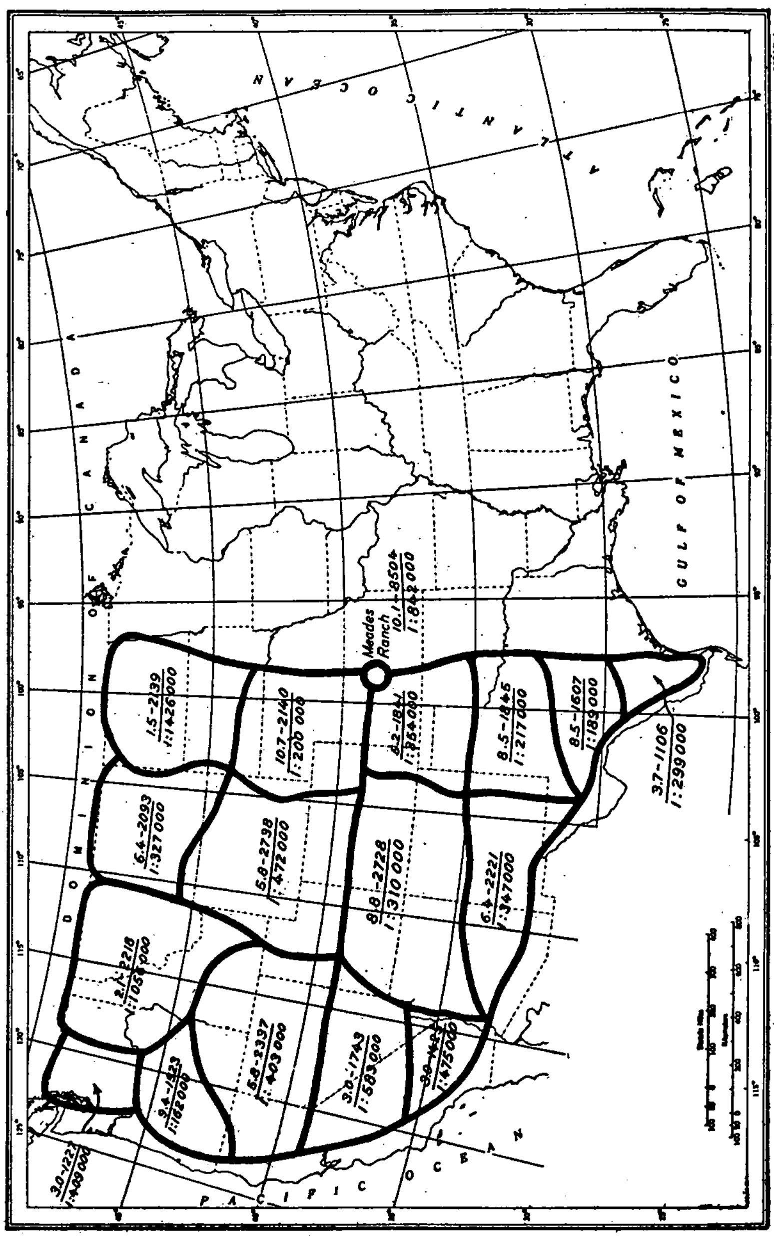

FIGURE 5.—Loop closures resulting from adjustment

The first number above the line is the total closure in meters and the second number is the approximate length of the loop in kilometers. Below the line is the approximate proportional part of the whole circuit represented by the closure.

2, while a section twice as long as the unit section would have the weight 0.5, and so on. No attempt was made to get these relative lengths with any great exactness; they were actually scaled roughly from a progress map, since a slight variation of the weight would have but little effect on the final value of the resulting v for the given section.

In this way there was formed a set of 42 observation equations for latitude and the same number for longitude, this being the number of sections in the whole net. The symbols x_1, x_2, etc., were used as corrections to the assumed latitudes of the junction points, and y_1, y_2, etc., were used as corrections to the assumed longitudes of the same points. The 42 expressions for the v's gave 26 normal equations in each case, this being the number of the junction points, omitting, of course, Meade's Ranch which was adopted as the fundamental position and which was not to change from the assumed value.

After the solution of the normal equations was made and the values of the x's and y's were determined, the expressions in units of 10 feet were reduced back to fractions of a second of latitude and longitude. When these values were added to the assumed latitudes and longitudes the position of a junction point in each junction figure was determined. A correction was then made to the Laplace azimuth or to the adopted azimuth as the case might be, due to the change in the geodetic longitude in the junction figure. After this was done all of the positions of the stations in the junction figures were computed. This gave a line at each end of the various sections that was fixed in all respects—in length, in azimuth, and in position upon the ellipsoid of reference. The sections could then be adjusted with inclusion of the equations for closure in latitude and longitude and thus the whole net could be placed on one continuous consistent datum.

FORMATION OF OBSERVATION EQUATIONS

The table of the observation equations on page 16 makes it practically self-evident as to how the equations are formed. However, to show the method in cases where there is found a failure in closure, we will give a sample of the formation of one such in the list. Between Donna and Peters (see tables on pp. 14 and 15) we have the following differences:

	In latitude	In longitude
	° ′ ″	° ′ ″
Assumed difference	3 00 52.382$-x_4+x_5$	2 33 17.613$-y_4+y_5$
Computed difference	3 00 52.154$+v_5$	2 33 17.712$+v_5$

Since, after adjustment, these values should agree, we find for latitude that $v_5 = +0.''228 - x_4 + x_5$, and for longitude that $v_5 = -0.''099 - y_4 + y_5$.

This discrepancy of closure was developed at station Peters and at this latitude 0.″228 in latitude is equal to 23.0 feet and 0.″099 in longitude is equal to 8.8 feet. For convenience in the solution, as stated on page 8, the unit was assumed as 10 feet; accordingly, the observation equations become, in latitude

$$v_5 = +2.30 - x_4 + x_5$$

and in longitude

$$v_5 = -0.88 - y_4 + y_5$$

In the same way all of the other observation equations were formed.

From the procedure indicated above, it is seen that the extent in latitude and longitude of each section is considered as if it were a quantity directly measured; the v's for each section are the corrections in latitude and in longitude that are to be applied to these quantities. The solution of the observation equations by least squares gives in each case the most probable distribution of the closures at the junction points. The later adjustment of the sections between the junction figures, thus fixed in all respects upon the spheroid (that is, fixed in length, position, and azimuth) brings about a just distribution of the position closures among the various arc sections in the whole net of triangulation.

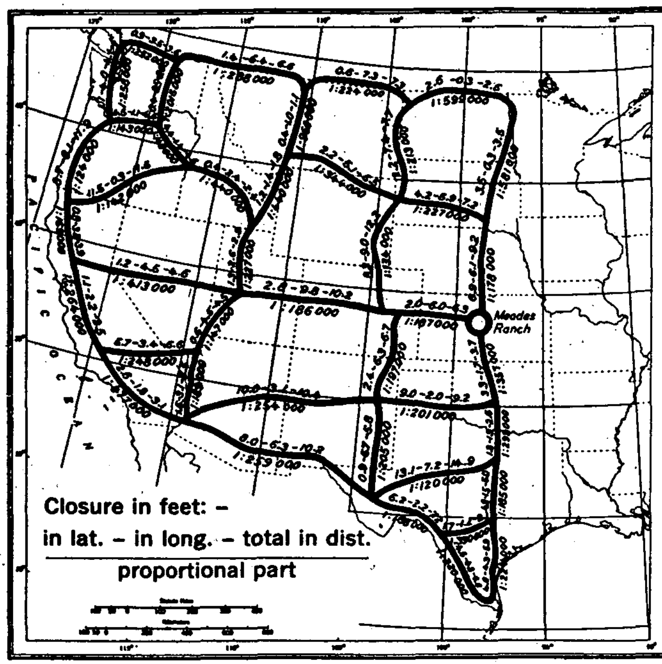

FIGURE 6.—Section closures resulting from apportionment of loop closures

The first number above the line is the closure in latitude in feet, the second number the closure in longitude in feet, and the third number the total closure in feet. On the other side of the line representing the section is the approximate proportional part of the length of the section represented by the total closure.

SOLUTION OF NORMAL EQUATIONS

The forward solution of the normal equations can be considerably shortened by carrying the solution of the latitude equations and that of the longitude equations at the same time. The coefficients of the x's are the same as the coefficients of the corresponding y's, and hence the only differences between the two solutions are in the η and Σ columns. These can both be appended to the one solution, as is done in the solution given on pages 21 to 25 and much computation is avoided in this way. The control of the solution of each set is given in its respective Σ column, and hence the single solution is just as effective as a double solution would be. As a further control of the solution, the values of $(p\eta^2)$, $(p\eta\Sigma)$ and $(p\Sigma^2)$ are appended to the normals and the resulting work is appended to the forward solution. (See p. 25.) It should be noted that the values in the columns η and Σ on page 25 for the latitude equations are derived in the regular way, and that the corresponding values in the longitude set are similarly derived just as if the latitude values were not present. In other words, the cross multiplication for the latitude set is only applied to the η and Σ column of that set and then the corresponding multiplier is used for the longitude set. After adding up the three columns occurring beyond the solution of equation No. 26, the three results

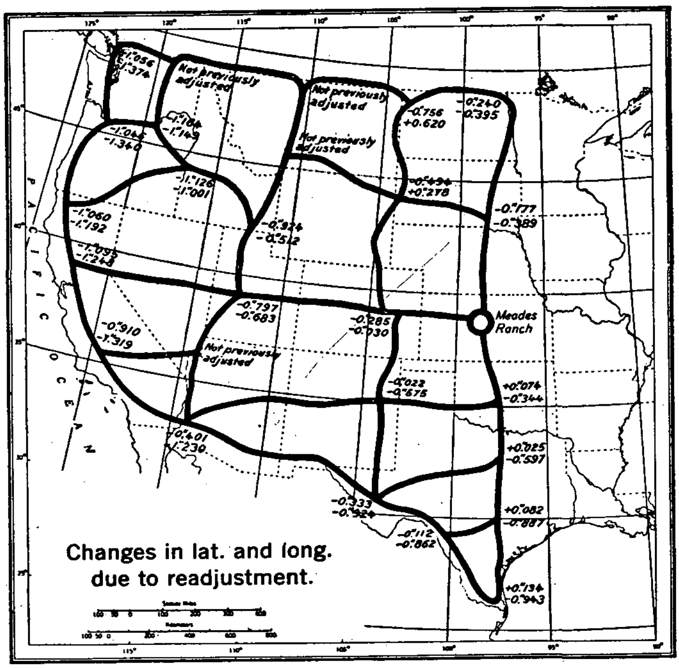

FIGURE 7.—Changes in positions of junction points, previously adjusted; caused by the readjustment

The sign indicates in which direction the position was changed. The upper number is the change in latitude and the lower number the change in longitude.

should be equal, each being the sum of the respective pv^2's. It will be seen that with the exception of the unavoidable uncertainty in the last places of decimals, the same result is obtained in both sets of normals. It will also be seen from the sum of the pv^2's given in the table on page 28 that these solution values are checked except for the uncertainty due to the dropping of decimals.

COMPUTATIONS

On the following pages are assembled in tabular form the computations involved in adjusting the latitude and longitude closures of the junction points.

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Assumed positions of junction stations

Station	Assumed latitude	Assumed longitude
	° ′ ″	° ′ ″
Carson	35 16 24.889 + x_1	97 57 32.437 + y_1
Kyle	32 49 18.024 + x_2	98 19 11.654 + y_2
Mission	29 42 52.781 + x_3	98 09 52.133 + y_3
Donna	26 09 40.491 + x_4	98 02 44.447 + y_4
Peters	29 10 32.873 + x_5	100 36 02.060 + y_5
Ingle	31 35 48.698 + x_6	103 35 24.147 + y_6
Phillips	34 59 03.367 + x_7	104 07 59.481 + y_7
Aroya	38 48 09.701 + x_8	103 10 55.504 + y_8
Whitetank	33 34 01.744 + x_9	112 33 27.214 + y_9
Mount Nebo	39 48 38.324 + x_{10}	111 45 56.284 + y_{10}
Black	35 55 51.691 + x_{11}	115 02 35.657 + y_{11}
Mount Diablo	37 52 54.246 + x_{12}	121 54 46.912 + y_{12}
Tepusquet	34 54 36.391 + x_{13}	120 11 08.121 + y_{13}
Farmer	43 42 21.280 + x_{14}	97 40 34.106 + y_{14}
States	49 00 01.298 + x_{15}	97 07 39.003 + y_{15}
Sundance	44 28 44.100 + x_{16}	104 27 02.892 + y_{16}
Noige	48 53 37.986 + x_{17}	103 47 20.841 + y_{17}
Oxford	42 16 10.863 + x_{18}	112 05 49.476 + y_{18}
Mount Ellis	45 34 38.959 + x_{19}	110 57 18.429 + y_{19}
Goldstone	48 52 55.718 + x_{20}	110 29 18.696 + y_{20}
Dry	44 10 08.208 + x_{21}	117 39 39.828 + y_{21}
Alder	45 50 59.400 + x_{22}	119 56 21.043 + y_{22}
Bally	40 36 10.730 + x_{23}	122 38 59.025 + y_{23}
Red	45 56 06.077 + x_{24}	121 49 11.048 + y_{24}
Oroville	48 53 44.253 + x_{25}	119 20 12.795 + y_{25}
Birch Point	48 56 30.780 + x_{26}	122 49 12.025 + y_{26}

Fixed position [1]

Station	Latitude	Longitude
	° ′ ″	° ′ ″
Meade's Ranch	39 13 26.686	98 32 30.506

[1] This is the fixed position on which the North American Datum depends.

Computed differences of positions of junction stations

No. of difference	Junction stations	Difference of latitude			Difference of longitude		
		°	′	″	°	′	″
1	Carson-Meade's Ranch	−3	57	01.797	−0	34	58.069
2	Kyle-Carson	−2	27	06.865	+0	21	39.217
3	Mission-Kyle	−3	06	25.243	−0	09	19.521
4	Donna-Mission	−3	33	12.290	−0	07	07.686
5	Peters-Donna	+3	00	52.154	+2	33	17.712
6	Peters-Mission	−0	32	20.068	+2	26	10.142
7	Ingle-Peters	+2	25	15.825	+2	59	22.087
8	Ingle-Kyle	−1	13	29.224	+5	16	12.826
9	Phillips-Carson	−0	17	21.648	+6	10	27.188
10	Aroya-Meade's Ranch	−0	25	16.985	+4	38	24.998
11	Phillips-Ingle	+3	23	14.669	+0	32	35.334
12	Aroya-Phillips	+3	49	06.334	−0	57	03.977
13	Whitetank-Ingle	+1	58	13.046	+8	58	03.067
14	Whitetank-Phillips	−1	25	01.810	+8	25	27.838
15	Mount Nebo-Aroya	+1	00	28.623	+8	35	00.780
16	Black-Mount Nebo	−3	52	46.633	+3	16	39.373
17	Black-Whitetank	+2	21	50.021	+2	29	08.021
18	Mount Diablo-Mount Nebo	−1	55	43.917	+10	08	50.926
19	Tepusquet-Mount Diablo	−2	58	17.855	−1	43	38.791
20	Tepusquet-Whitetank	+1	20	34.895	+7	37	40.852
21	Farmer-Meade's Ranch	+4	28	54.594	−0	51	56.400
22	States-Farmer	+5	17	40.270	−0	32	54.667
23	Sundance-Farmer	+0	46	23.031	+6	46	29.153
24	Sundance-Aroya	+5	40	34.399	+1	16	07.388
25	Norge-Sundance	+4	24	53.886	−0	39	42.051
26	Norge-States	−0	06	23.312	+6	39	41.838
27	Oxford-Mount Nebo	+2	27	32.539	+0	19	53.192
28	Mount Ellis-Oxford	+3	18	28.096	−1	08	31.047
29	Mount Ellis-Sundance	+1	05	54.703	+6	30	15.401
30	Goldstone-Mount Ellis	+3	18	16.759	−0	27	59.733
31	Goldstone-Norge	−0	00	42.422	+6	41	57.399
32	Dry-Oxford	+1	53	57.345	+5	33	50.352
33	Alder-Dry	+1	40	51.192	+2	16	41.215
34	Red-Alder	+0	05	06.677	+1	52	50.005
35	Red-Bally	+5	19	55.347	−0	49	47.977
36	Bally-Mount Diablo	+2	43	16.484	+0	44	12.113
37	Bally-Dry	−3	33	57.190	+4	59	19.331
38	Oroville-Goldstone	+0	00	48.535	+8	50	54.099
39	Oroville-Alder	+3	02	44.788	−0	36	08.211
40	Tepusquet-Black	−1	01	15.224	+5	08	32.823
41	Birch Point-Oroville	+0	02	46.660	+3	28	59.089
42	Birch Point-Red	+3	00	24.703	+1	00	00.977

Observation equations

Latitude equation	Weight, p	Longitude equation
$v_1=x_1$	1.00	$v_1=y_1$
$v_2=x_2-x_1$	1.65	$v_2=y_2-y_1$
$v_3=x_3-x_2$	1.45	$v_3=y_3-y_2$
$v_4=x_4-x_3$	1.00	$v_4=y_4-y_3$
$v_5=+2.30-x_4+x_5$	.90	$v_5=-0.88-y_4+y_5$
$v_6=+1.62-x_3+x_5$	1.65	$v_6=-1.91-y_3+y_5$
$v_7=x_6-x_5$	1.00	$v_7=y_6-y_5$
$v_8=-1.03-x_2+x_6$	.75	$v_8=-2.88-y_2+y_6$
$v_9=+1.27-x_1+x_7$	.70	$v_9=-1.20-y_1+y_7$
$v_{10}=x_8$	1.10	$v_{10}=y_8$
$v_{11}=x_7-x_6$	1.10	$v_{11}=y_7-y_6$
$v_{12}=x_8-x_7$	1.00	$v_{12}=y_8-y_7$
$v_{13}=x_9-x_6$	.50	$v_{13}=y_9-y_6$
$v_{14}=+1.89-x_7+x_9$	.50	$v_{14}=-0.89-y_7+y_9$
$v_{15}=x_{10}-x_8$	.70	$v_{15}=y_{10}-y_8$
$v_{16}=x_{11}-x_{10}$	2.00	$v_{16}=y_{11}-y_{10}$
$v_{17}=-0.75-x_9+x_{11}$	2.00	$v_{17}=+3.57-y_9+y_{11}$
$v_{18}=-1.63-x_{10}+x_{12}$	.70	$v_{18}=-2.39-y_{10}+y_{12}$
$v_{19}=x_{13}-x_{12}$	2.00	$v_{19}=y_{13}-y_{12}$
$v_{20}=-2.51-x_9+x_{13}$	.90	$v_{20}=+0.47-y_9+y_{13}$
$v_{21}=x_{14}$	.80	$v_{21}=y_{14}$
$v_{22}=-2.55-x_{14}+x_{15}$	.65	$v_{22}=-2.91-y_{14}+y_{15}$
$v_{23}=-2.14-x_{14}+x_{16}$	.80	$v_{23}=-2.70-y_{14}+y_{16}$
$v_{24}=x_{16}-x_8$	.80	$v_{24}=y_{16}-y_8$
$v_{25}=x_{17}-x_{16}$	.80	$v_{25}=y_{17}-y_{16}$
$v_{26}=x_{17}-x_{15}$	.85	$v_{26}=y_{17}-y_{15}$
$v_{27}=x_{18}-x_{10}$	2.00	$v_{27}=y_{18}-y_{10}$
$v_{28}=x_{19}-x_{18}$	1.65	$v_{28}=y_{19}-y_{18}$
$v_{29}=+1.58-x_{16}+x_{19}$	.70	$v_{29}=+0.97-y_{16}+y_{19}$
$v_{30}=x_{20}-x_{19}$	1.25	$v_{30}=y_{20}-y_{19}$
$v_{31}=+1.56-x_{17}+x_{20}$	.80	$v_{31}=+3.05-y_{17}+y_{20}$
$v_{32}=x_{21}-x_{18}$	1.25	$v_{32}=y_{21}-y_{18}$
$v_{33}=x_{22}-x_{21}$	2.00	$v_{33}=y_{22}-y_{21}$
$v_{34}=x_{24}-x_{22}$	2.00	$v_{34}=y_{24}-y_{22}$
$v_{35}=x_{24}-x_{23}$	.90	$v_{35}=y_{24}-y_{23}$
$v_{36}=x_{23}-x_{12}$	2.00	$v_{36}=y_{23}-y_{12}$
$v_{37}=-2.91-x_{21}+x_{23}$	.80	$v_{37}=-1.03-y_{21}+y_{23}$
$v_{38}=x_{25}-x_{20}$	.70	$v_{38}=y_{25}-y_{20}$
$v_{39}=+0.66-x_{22}+x_{25}$	1.20	$v_{39}=-0.25-y_{22}+y_{25}$
$v_{40}=-0.77-x_{11}+x_{13}$	.80	$v_{40}=-2.95-y_{11}+y_{13}$
$v_{41}=-1.35-x_{25}+x_{26}$	1.45	$v_{41}=+0.94-y_{25}+y_{26}$
$v_{42}=x_{26}-x_{24}$	1.25	$v_{42}=y_{26}-y_{24}$

Table for formation of latitude normal equations

v	p	η	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	pη	pΣ
1	1.00	0	+1																										0	+1.00
2	1.65	0	−1	+1																									0	0
3	1.45	0		−1	+1																								0	0
4	1.00	0			−1	+1																							0	0
5	.90	+2.30				−1	+1																						+2.0700	+2.0700
6	1.65	+1.62			−1		+1																						+2.6730	+2.6730
7	1.00	0					−1	+1																					0	0
8	.75	−1.03		−1				+1																					−.7725	−.7725
9	.70	+1.27	−1						+1																				+.8890	+.8890
10	1.10	0								+1																			0	+1.10
11	1.10	0						−1	+1																				0	0
12	1.00	0							−1	+1																			0	0
13	.50	0						−1			+1																		0	0
14	.50	+1.89							−1		+1																		+.9450	+.9450
15	.70	0								−1		+1																	0	0
16	2.00	0										−1	+1																0	0
17	2.00	−.75									−1		+1																−1.5000	−1.5000
18	.70	−1.63										−1		+1															−1.1410	−1.1410
19	2.00	0												−1	+1														0	0
20	.90	−2.51									−1				+1														−2.2590	−2.2590
21	.80	0														+1													0	+.80
22	.65	−2.55														−1	+1												−1.6575	−1.6575
23	.80	−2.14														−1		+1											−1.7120	−1.7120
24	.80	0								−1								+1											0	0
25	.80	0																−1	+1										0	0
26	.85	0															−1		+1										0	0
27	2.00	0										−1								+1									0	0
28	1.65	0																		−1	+1								0	0
29	.70	+1.58																−1			+1								+1.1060	+1.1060
30	1.25	0																			−1	+1							0	0
31	.80	+1.56																	−1			+1							+1.2480	+1.2480
32	1.25	0																		−1			+1						0	0
33	2.00	0																					−1	+1					0	0
34	2.00	0																						−1		+1			0	0
35	.90	0																							−1	+1			0	0
36	2.00	0												−1											+1				0	0
37	.80	−2.91																					−1		+1				−2.3280	−2.3280
38	.70	0																				−1					+1		0	0
39	1.20	+.66																						−1			+1		+.7920	+.7920
40	.80	−.77											−1		+1														−.6160	−.6160
41	1.45	−1.35																									−1	+1	−1.9575	−1.9575
42	1.25	0																								−1		+1	0	0

Table for formation of longitude normal equations—Continued

v	p	η	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	pη	pΣ
1	1.00	0	+1																										0	+1.00
2	1.65	0	−1	+1																									0	0
3	1.45	0		−1	+1																								0	0
4	1.00	0			−1	+1																							0	0
5	.90	−.88				−1	+1																						−.7920	−.7920
6	1.65	−1.91			−1		+1																						−3.1515	−3.1515
7	1.00	0					−1	+1																					0	0
8	.75	−2.88		−1				+1																					−2.1600	−2.1600
9	.70	−1.20	−1						+1																				−.8400	−.8400
10	1.10	0								+1																			0	+1.10
11	1.10	0						−1	+1																				0	0
12	1.00	0							−1	+1																			0	0
13	.50	0						−1			+1																		0	0
14	.50	−.89							−1		+1																		−.4450	−.4450
15	.70	0								−1		+1																	0	0
16	2.00	0										−1	+1																0	0
17	2.00	+3.57									−1		+1																+7.1400	+7.1400
18	.70	−2.39										−1		+1															−1.6730	−1.6730
19	2.00	0												−1	+1														0	0
20	.90	+.47									−1				+1														+.4230	+.4230
21	.80	0														+1													0	+.80
22	.65	−2.91														−1	+1												−1.8915	−1.8915
23	.80	−2.70														−1		+1											−2.1600	−2.1600
24	.80	0								−1								+1											0	0
25	.80	0																−1	+1										0	0
26	.85	0															−1		+1										0	0
27	2.00	0										−1								+1									0	0
28	1.65	0																		−1	+1								0	0
29	.70	+.97																−1			+1								+.6790	+.6790
30	1.25	0																			−1	+1							0	0
31	.80	+3.05																	−1			+1							+2.4400	+2.4400
32	1.25	0																		−1			+1						0	0
33	2.00	0																					−1	+1					0	0
34	2.00	0																						−1		+1			0	0
35	.90	0																							−1	+1			0	0
36	2.00	0											−1												+1				0	0
37	.80	−1.03																					−1		+1				−.8240	−.8240
38	.70	0																				−1					+1		0	0
39	1.20	−.25																						−1			+1		−.3000	−.3000
40	.80	−2.95											−1		+1														−2.3600	−2.3600
41	1.45	+.94																									−1	+1	+1.3630	+1.3630
42	1.25	0																								−1		+1	0	0

Normal equations for latitude

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	η	Σ
1	+3.35	−1.65					−0.70												−0.8890	+0.1110
2		+3.85	−1.45			−0.75													+.7725	+.7725
3			+4.10	−1.00	−1.65														−2.6730	−2.6730
4				+1.90	−.90														−2.0700	−2.0700
5					+3.55	−1.00													+4.7430	+4.7430
6						+3.35	−1.10		−0.50										−.7725	−.7725
7							+3.30	−1.00	−.50										−.0560	−.0560
8								+3.60		−0.70						−0.80			0	+1.1000
9									+3.90		−2.00		−0.90						+4.7040	+4.7040
10										+5.40	−2.00	−0.70						−2.00	+1.1410	+1.1410

	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	η	Σ
11	+4.80		−0.80														−0.8840	−0.8840
12		+4.70	−2.00										−2.00				−1.1410	−1.1410
13			+3.70														−2.8750	−2.8750
14				+2.25	−0.65	−0.80											+3.3695	+4.1695
15					+1.50		−0.85										−1.6575	−1.6575
16						+3.10	−.80		−0.70								−2.8180	−2.8180
17							+2.45			−0.80							−1.2480	−1.2480
18								+4.90	−1.65		−1.25						0	0
19									+3.60	−1.25							+1.1060	+1.1060
20										+2.75					−0.70		+1.2480	+1.2480
21											+4.05	−2.00	−.80				+2.3280	+2.3280
22												+5.20		−2.00	−1.20		−.7920	−.7920
23													+3.70	−.90			−2.3280	−2.3280
24														+4.15		−1.25	0	0
25															+3.35	−1.45	+2.7495	+2.7495
26																+2.70	−1.9575	−1.9575
																	+43.4557	+43.4557
																		+46.3557

Normal equations for longitude—Continued

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	η	Σ
1	+3.35	−1.65					−0.70												+0.8400	+1.8400
2		+3.85	−1.45			−0.75													+2.1600	+2.1600
3			+4.10	−1.00	−1.65														+3.1515	+3.1515
4				+1.90	−.90														+.7920	+.7920
5					+3.55	−1.00													−3.9435	−3.9435
6						+3.35	−1.10		−0.50										−2.1600	−2.1600
7							+3.30	−1.00	−.50										−.3950	−.3950
8								+3.60		−0.70						−0.80			0	+1.1000
9									+3.90		−2.00		−0.90						−8.0080	−8.0080
10										+5.40	−2.00	−0.70						−2.00	+1.6730	+1.6730

	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	η	Σ
11	+4.80		−0.80														+9.5000	+9.5000
12		+4.70	−2.00										−2.00				−1.6730	−1.6730
13			+3.70														−1.9370	−1.9370
14				+2.25	−0.65	−0.80											+4.0515	+4.8515
15					+1.50		−0.85										−1.8915	−1.8915
16						+3.10	−.80		−0.70								−2.8390	−2.8390
17							+2.45			−0.80							−2.4400	−2.4400
18								+4.90	−1.65		−1.25						0	0
19									+3.60	−1.25							+.6790	+.6790
20										+2.75					−0.70		+2.4400	+2.4400
21											+4.05	−2.00	−.80				+.8240	+.8240
22												+5.20		−2.00	−1.20		+.3000	+.3000
23													+3.70	−.90			−.8240	−.8240
24														+4.15		−1.25	0	0
25															+3.35	−1.45	−1.6630	−1.6630
26																+2.70	+1.3630	+1.3630
																	+72.6321	+72.6321
																		+75.5321

Solution of normal equations

1	2	3	4	5	6	7	8	9	η (Lat.)	Σ (Lat.)	η (Long.)	Σ (Long.)
+3.35	−1.65					−0.70			−0.8890	+0.1110	+0.8400	+1.8400
$x_1=$	+.49254					+.20896			+.26537	−.03313	−.25075	−.54925
	+3.85	−1.45			−0.75				+.7725	+.7725	+2.1600	+2.1600
1	−.8127					−.3448			−.4379	+.0547	+.4137	+.9063
	+3.0373	−1.45			−.75	−.3448			+.3346	+.8272 1	+2.5737	+3.0663 2
	$x_2=$	+.47740			+.24693	+.11352			−.11016	−.27231	−.84736	−1.00952
		+4.10	−1.00	−1.65					−2.6730	−2.6730	+3.1515	+3.1515
	2	−.6922			−.3580	−.1646			+.1597	+.3949	+1.2287	+1.4638
		+3.4078	−1.00	−1.65	−.3580	−.1646			−2.5133	−2.2781	+4.3802	+4.6153 4
		$x_3=$	+.29344	+.48418	+.10505	+.04830			+.73751	+.66850	−1.28535	−1.35436
			+1.90	−.90					−2.0700	−2.0700	+.7920	+.7920
		3	−.2934	−.4842	−.1050	−.0483			−.7375	−.6685	+1.2853	+1.3543
			+1.6066	−1.3842	−.1050	−.0483			−2.8075	−2.7385 4	+2.0773	+2.1463 4
			$x_4=$	+.86157	+.06536	+.03006			+1.74748	+1.70447	−1.29298	−1.33599
				+3.55	−1.00				+4.7430	+4.7430	−3.9435	−3.9435
			3	−.7989	−.1733	−.0797			−1.2169	−1.1030	+2.1208	+2.2347
			4	−1.1926	−.0905	−.0416			−2.4189	−2.3593	+1.7897	+1.8493
				+1.5585	−1.2638	−.1213			+1.1072	+1.2807 6	−.0330	+.1405 4
				$x_5=$	+.81091	+.07783			−.71043	−.82169	+.02017	−.09009
					+3.35	−1.10		−0.50	−.7725	−.7725	−2.1600	−2.1600
				2	−.1852	−.0851			+.0826	+.2042	+.6355	+.7571
				3	−.0376	−.0173			−.2640	−.2393	+.4601	+.4848
				4	−.0069	−.0032			−.1835	−.1790	+.1358	+.1403
				5	−1.0248	−.0984			+.8978	+1.0385	−.0268	+.1139
					+2.0955	−1.3040		−.50	−.2396	+.0519	−.9554	−.6639
					$x_6=$	+.62229		+.23861	+.11434	−.02477	+.45593	+.31682
						+3.30	−1.00	−.50	−.0560	−.0560	−.3950	−.3950
					1	−.1463			−.1858	+.0233	+.1755	+.3845
					2	−.0391			+.0380	+.0939	+.2922	+.3481
					3	−.0080			−.1214	−.1100	+.2116	+.2229
					4	−.0015			−.0844	−.0823	+.0624	+.0645
					5	−.0094			+.0862	+.0997	−.0026	+.0109
					6	−.8115		−.3111	−.1491	+.0323	−.5945	−.4131
						+2.2842	−1.00	−.8111	−.4725	+.0006	−.2504	+.2228 7
						$x_7=$	+.43779	+.35509	+.20686	−.00026	+.10962	−.09750

Solution of normal equations—Continued

	8	9	10	12	14	15	16	17	18	19	23	11	13	η (Lat.)	Σ (Lat.)	η (Long.)	Σ (Long.)
	+3.60		−0.70				−0.80							0	+1.10	0	+1.10
7	−.4378	−.3551												−.2069	+.0003	−.1096	+.0975
	+3.1622	−.3551	−.70				−.80							−.2069	+1.1003 2	−.1096	+1.1975
	x_8=	+.11230	+.22136				+.25299							+.06543	−.34792	+.03466	−.37869
		+3.90										−2.00	−0.90	+4.7040	+4.7040	−8.0080	−8.0080
	6	−.1193												−.0572	+.0124	−.2280	−.1584
	7	−.2880												−.1678	+.0002	−.0889	+.0791
	8	−.0399	−.0786				−.0898							−.0232	+.1236	−.0123	+.1345
		+3.4528	−.0786				−.0898					−2.00	−.90	+4.4558	+4.8402	−8.3372	−7.9528
		x_9=	+.02276				+.02601					+.57924	+.26066	−1.29049	−1.40182	+2.41462	+2.30329
			+5.40	−0.70					−2.00			−2.00		+1.1410	+1.1410	+1.6730	+1.6730
		8	−.1550				−.1771							−.0458	+.2435	−.0243	+.2651
		9	−.0018				−.0020					−.0455	−.0205	+.1014	+.1102	−.1898	−.1810
			+5.2432	−.70			−.1791		−2.00			−2.0455	−.0205	+1.1966	+1.4947	+1.4589	+1.7571 0
			x_{10}=	+.13351			+.03416		+.38145			+.39012	+.00391	−.22822	−.28507	−.27825	−.33510
				+4.70							−2.00		−2.00	−1.1410	−1.1410	−1.6730	−1.6730
			10	−.0935			−.0239		−.2670			−.2731	−.0027	+.1593	+.1990	+.1943	+.2339
				+4.6065			−.0239		−.2670		−2.00	−.2731	−2.0027	−.9817	−.9420 19	−1.4787	−1.4391 89
				x_{12}=			+.00519		+.05796		+.43417	+.05929	+.43476	+.21311	+.20447	+.32100	+.31236
					+2.25	−0.65	−.80							+3.3695	+4.1695	+4.0515	+4.8515
					x_{14}=	+.28889	+.35556							−1.49756	−1.85311	−1.80067	−2.15622
						+1.50		−0.85						−1.6575	−1.6575	−1.8915	−1.8915
					14	−.1878	−.2311							+.9734	+1.2045	+1.1704	+1.4015
						+1.3122	−.2311	−.85						−.6841	−.4530	−.7211	−.4900
						x_{15}=	+.17612	+.64777						+.52134	+.34522	+.54954	+.37342
							+3.10	−.80		−0.70				−2.8180	−2.8180	−2.8390	−2.8390
						8	−.2024							−.0523	+.2783	−.0277	+.3030
						9	−.0023					−.0520	−.0234	+.1159	+.1259	−.2169	−.2069
						10	−.0061		−.0683			−.0699	−.0007	+.0409	+.0511	+.0498	+.0600
						12	−.0001		−.0014		−.0104	−.0014	−.0104	−.0051	−.0049	−.0077	−.0075
						14	−.2844							+1.1981	+1.4825	+1.4406	+1.7250
						15	−.0407	−.1497						−.1205	−.0798	−.1270	−.0863
							+2.5640	−.9497	−.0697	−.70	−.0104	−.1233	−.3045	−1.6410	−.9649 6	−1.7279	−1.0517 5
							x_{16}=	+.37040	+.02718	+.27301	+.00406	+.04809	+.01346	+.64002	+.37621	+.67391	+.41010

Solution of normal equations—Continued

	17	18	19	20	21	22	23	11	13	24	25	η (Lat.)	Σ (Lat.)	η (Long.)	Σ (Long.)
	+2.45			−0.80								−1.2480	−1.2480	−2.4400	−2.4400
15	−.5506											−.4431	−.2934	−.4671	−.3174
16	−.3518	−0.0258	−0.2593				−0.0039	−0.0457	−0.0128			−.6078	−.3573	−.6400	−.3895
	+1.5476	−.0258	−.2593	−.80			−.0039	−.0457	−.0128			−2.2989	−1.8987 8	−3.5471	−3.1469 70
	x_{17}=	+.01667	+.16755	+.51693			+.00252	+.02953	+.00827			+1.48546	+1.22693	+2.29200	+2.03347
		+4.90	−1.65		−1.25							0	0	0	0
	10	−.7629						−.7803	−.0078			+.4564	+.5702	+.5565	+.6702
	12	−.0155					−.1159	−.0158	−.1161			−.0569	−.0546	−.0857	−.0834
	16	−.0019	−.0190				−.0003	−.0034	−.0009			−.0446	−.0262	−.0470	−.0286
	17	−.0004	−.0043	−.0133			−.0001	−.0008	−.0002			−.0383	−.0317	−.0591	−.0525
		+4.1193	−1.6733	−.0133	−1.25		−.1163	−.8003	−.1250			+.3166	+.4577	+.3647	+.5057 8
		x_{18}=	+.40621	+.00323	+.30345		+.02823	+.19428	+.03034			−.07686	−.11111	−.08853	−.12279
			+3.60	−1.25								+1.1060	+1.1060	+.6790	+.6790
		16	−.1911				−.0028	−.0337	−.0094			−.4480	−.2633	−.4717	−.2871
		17	−.0434	−.1340			−.0007	−.0077	−.0021			−.3852	−.3181	−.5943	−.5273
		18	−.6797	−.0054	−.5078		−.0472	−.3251	−.0508			+.1286	+.1859	+.1481	+.2055
			+2.6858	−1.3894	−.5078		−.0507	−.3665	−.0623			+.4014	+.7105	−.2389	+.0701 2
			x_{19}=	+.51731	+.18907		+.01888	+.13646	+.02320			−.14945	−.26454	+.08895	−.02614
				+2.75							−0.70	+1.2480	+1.2480	+2.4400	+2.4400
			17	−.4135			−.0020	−.0236	−.0066			−1.1884	−.9815	−1.8336	−1.6268
			18	0	−.0040		−.0004	−.0026	−.0004			+.0010	+.0015	+.0012	+.0016
			19	−.7188	−.2627		−.0262	−.1896	−.0322			+.2076	+.3675	−.1236	+.0363
				+1.6177	−.2667		−.0286	−.2158	−.0392		−.70	+.2682	+.6355 6	+.4840	+.8511 4
				x_{20}=	+.16486		+.01768	+.13340	+.02423		+.43271	−.16579	−.39290	−.29919	−.52630
					+4.05	−2.00	−.80					+2.3280	+2.3280	+.8240	+.8240
				18	−.3793		−.0353	−.2429	−.0379			+.0961	+.1389	+.1107	+.1535
				19	−.0960		−.0096	−.0693	−.0118			+.0759	+.1343	−.0452	+.0133
				20	−.0440		−.0047	−.0356	−.0065		−.1154	+.0442	+.1048	+.0798	+.1404
					+3.5307	−2.00	−.8496	−.3478	−.0562		−.1154	+2.5442	+2.7060 59	+.9693	+1.1312 0
					x_{21}=	+.56646	+.24063	+.09851	+.01592		+.03268	−.72059	−.76639	−.27453	−.32033
						+5.20				−2.00	−1.20	−.7920	−.7920	+.3000	+.3000
					21	−1.1329	−.4813	−.1970	−.0318		−.0654	+1.4412	+1.2328	+.5491	+.6407
						+4.0671	−.4813	−.1970	−.0318	−2.00	−1.2654	+.6492	+.7408	+.8491	+.9407
						x_{22}=	+.11834	+.04844	+.00782	+.49175	+.31113	−.15962	−.18214	−.20877	−.23130

Solution of normal equations—Continued

	23	11	13	24	25	η (Lat.)	Σ (Lat.)	η (Long.)	Σ (Long.)
	+3.70			−0.90		−2.3280	−2.3280	−0.8240	−0.8240
12	−.8683	−0.1186	−0.8695			−.4262	−.4089	−.6420	−.6247
16	0	−.0005	−.0001			−.0067	−.0039	−.0070	−.0043
17	0	−.0001				−.0058	−.0048	−.0089	−.0079
18	−.0033	−.0226	−.0035			+.0089	+.0129	+.0103	+.0143
19	−.0010	−.0069	−.0012			+.0076	+.0134	−.0045	+.0013
20	−.0005	−.0038	−.0007		−0.0124	+.0047	+.0112	+.0086	+.0151
21	−.2044	−.0837	−.0135		−.0278	+.6122	+.6511	+.2332	+.2722
22	−.0570	−.0233	−.0038	−.2367	−.1497	+.0768	+.0877	+.1005	+.1113
	+2.5655	−.2595	−.8923	−1.1367	−.1899	−2.0565	−1.9693 4	−1.1338	−1.0467
	x_{23}=	+.10115	+.34781	+.44307	+.07402	+.80160	+.76765	+.44194	+.40799
		+4.80	−.80			−.8840	−.8840	+9.5000	+9.5000
	9	−1.1585	−.5213			+2.5810	+2.8036	−4.8292	−4.6066
	10	−.7980	−.0080			+.4668	+.5831	+.5691	+.6854
	12	−.0162	−.1187			−.0582	−.0558	−.0877	−.0853
	16	−.0059	−.0017			−.0789	−.0464	−.0831	−.0506
	17	−.0013	−.0004			−.0679	−.0561	−.1047	−.0929
	18	−.1555	−.0243			+.0615	+.0889	+.0708	+.0983
	19	−.0500	−.0085			+.0548	+.0970	−.0326	+.0096
	20	−.0288	−.0052		−.0934	+.0358	+.0848	+.0646	+.1136
	21	−.0343	−.0055		−.0114	+.2506	+.2666	+.0955	+.1114
	22	−.0095	−.0015	−.0969	−.0613	+.0314	+.0359	+.0411	+.0456
	23	−.0262	−.0903	−.1150	−.0192	−.2080	−.1992	−.1147	−.1059
		+2.5158	−1.5854	−.2119	−.1853	+2.1849	+2.7181	+5.0891	+5.6226 3
		x_{11}=	+.63018	+.08423	+.07365	−.86847	−1.08041	−2.02286	−2.23480
			+3.70			−2.8750	−2.8750	−1.9370	−1.9370
		9	−.2346			+1.1614	+1.2616	−2.1732	−2.0730
		10	−.0001			+.0047	+.0058	+.0057	+.0069
		12	−.8707			−.4268	−.4095	−.6429	−.6256
		16	−.0005			−.0221	−.0130	−.0233	−.0142
		17	−.0001			−.0190	−.0157	−.0293	−.0260
		18	−.0038			+.0096	+.0139	+.0111	+.0153
		19	−.0014			+.0093	+.0165	−.0055	+.0016
		20	−.0009		−.0170	+.0065	+.0154	+.0117	+.0206
		21	−.0009		−.0018	+.0405	+.0431	+.0154	+.0180
		22	−.0002	−.0156	−.0099	+.0051	+.0058	+.0066	+.0074
		23	−.3104	−.3954	−.0660	−.7153	−.6850	−.3943	−.3641
		11	−.9991	−.1335	−.1168	+1.3769	+1.7129	+3.2070	+3.5431
			+1.2773	−.5445	−.2115	−1.4442	−.9232 29	−1.9480	−1.4270 67
			x_{13}=	+.42629	+.16558	+1.13067	+.72254	+1.52509	+1.11697

Solution of normal equations—Continued

	24	25	26	η (Lat.)	Σ (Lat.)	η (Long.)	Σ (Long.)
	+4.15	-----	−1.25	0	0	0	0
22	−.9835	−0.6223	-----	+0.3192	+0.3643	+0.4175	+0.4626
23	−.5036	−.0841	-----	−.9112	−.8726	−.5024	−.4638
11	−.0178	−.0156	-----	+.1840	+.2289	+.4287	+.4736
13	−.2321	−.0902	-----	−.6156	−.3934	−.8304	−.6082
	+2.4130	−.8122	−1.25	−1.0236	−.6728	−.4866	−.1358
	x_{24}=	+.33659	+.51803	+.42420	+.27882	+.20166	+.05628
		+3.35	−1.45	+2.7495	+2.7495	−1.6630	−1.6630
	20	−.3029	-----	+.1161	+.2750	+.2094	+.3684
	21	−.0038	-----	+.0831	+.0884	+.0317	+.0370
	22	−.3937	-----	+.2020	+.2305	+.2642	+.2927
	23	−.0141	-----	−.1522	−.1458	−.0839	−.0775
	11	−.0136	-----	+.1609	+.2002	+.3748	+.4141
	13	−.0350	-----	−.2391	−.1528	−.3225	−.2362
	24	−.2734	−.4207	−.3445	−.2265	−.1638	−.0457
		+2.3135	−1.8707	+2.5758	+3.0185 6	+1.3531	−.9102 3
		x_{25}=	+.80860	−1.11338	−1.30478	+.58487	+.39347
			+2.70	−1.9575	−1.9575	+1.3630	+1.3630
		24	−.6475	−.5303	−.3485	−.2521	−.0703
		25	−1.5126	+2.0828	+2.4408	−1.0941	−.7361
			+.5399	−.4050	+.1348 9	+.0168	+.5562 7
			x_{26}=	+.75014	−.24986	−.03112	−1.03112
				+43.4557	+43.4557	+72.6321	+72.6321
			1	−.2359	+.0295	−.2106	−.4614
			2	−.0369	−.0911	−2.1809	−2.5982
			3	−1.8536	−1.6801	−5.6301	−5.9324
			4	−4.9061	−4.7853	−2.6859	−2.7753
			5	−.7866	−.9098	−.0007	+.0028
			6	−.0274	+.0059	−.4356	−.3027
			7	−.0977	+.0001	−.0274	+.0244
			8	−.0135	+.0720	−.0038	+.0415
			9	−5.7502	−6.2462	−20.1312	−19.2030
			10	−.2731	−.3411	−.4059	−.4889
			12	−.2092	−.2007	−.4747	−.4619
			14	−5.0460	−6.2441	−7.2954	−8.7360
			15	−.3566	−.2362	−.3963	−.2693
			16	−1.0503	−.6174	−1.1644	−.7086
			17	−3.4149	−2.8206	−8.1300	−7.2129
			18	−.0243	−.0352	−.0323	−.0448
			19	−.0600	−.1062	−.0213	+.0062
			20	−.0445	−.1054	−.1448	−.2547
			21	−1.8333	−1.9498	−.2661	−.3105
			22	−.1036	−.1182	−.1773	−.1964
			23	−1.6485	−1.5787	−.5011	−.4626
			11	−1.8975	−2.3606	−10.2945	−11.3731
			13	−1.6329	−1.0435	−2.9709	−2.1758
			24	−.4342	−.2854	−.0981	−.0274
			25	−2.8678	−3.3608	−.7914	−.5324
			26	−.3038	+.1012	−.0005	−.0173
				+8.5473	+8.5480	+8.1609	+8.1614
					+46.3557	-----	+75.5321
				1	−.0037	-----	−1.0106
				2	−.2252	-----	−3.0954
				3	−1.5229	-----	−6.2509
				4	−4.6675	-----	−2.8676
				5	−1.0523	-----	−.0126
				6	−.0013	-----	−.2103
				7	0	-----	−.0217
				8	−.3828	-----	−.4535
				9	−6.7851	-----	−18.3176
				10	−.4261	-----	−.5888
				12	−.1926	-----	−.4495
				14	−7.7265	-----	−10.4609
				15	−.1564	-----	−.1830
				16	−.3629	-----	−.4312
				17	−2.3297	-----	−6.3993
				18	−.0509	-----	−.0621
				19	−.1880	-----	−.0018
				20	−.2497	-----	−.4481
				21	−2.0738	-----	−.3623
				22	−.1349	-----	−.2176
				23	−1.5118	-----	−.4270
				11	−2.9367	-----	−12.5647
				13	−.6668	-----	−1.5936
				24	−.1876	-----	−.0076
				25	−3.9386	-----	−.3582
				26	−.0337	-----	−.5740
					+8.5482	-----	+8.1622

Back solution for latitude

26	25	24	13	11	23	22	21	20
+0.75014 +.75014 +.075 +."074	−1.11338 +.60656 −.50682 −.051 −."050	+0.42420 +.38860 −.17059 +.64221 +.064 +."063	+1.13067 −.08392 +.27377 +1.32052 +.132 +."131	−0.86847 −.03733 +.05409 +.83217 −.01954 −.002 −."002	+0.80160 −.03751 +.28454 +.45929 −.00198 +1.50594 +.151 +."149	−0.15962 −.15769 +.31581 +.01033 −.00095 +.17821 +.18609 +.019 +."019	−0.72059 −.01656 +.02102 −.00192 +.36237 +.10541 −.25027 −.025 −."025	−0.16579 −.21931 +.03200 −.00261 +.02663 −.04126 −.37034 −.037 −."037

19	18	17	16	15	14	12	10	9
−0.14945 +.03064 −.00267 +.02843 −.04732 −.19158 −.33195 −.033 −."033	−0.07686 +.04006 −.00380 +.04251 −.07594 −.00120 −.13484 −.21007 −.021 −."021	+1.48546 +.01092 −.00058 +.00379 −.19144 −.05562 −.00350 +1.24903 +.125 +."124	+0.64002 +.01777 −.00094 +.00611 −.09063 −.00571 +.46264 +1.02926 +.103 +."102	+0.52134 +.80908 +.18127 +1.51169 +.151 +."149	−1.49756 +.36596 +.43671 −.69489 −.069 −."068	+0.21311 +.57411 −.00116 +.65383 −.01218 +.00534 +1.43305 +.143 +."141	−0.22822 +.00516 −.00762 −.08013 +.03516 +.19133 −.08432 −.008 −."008	−1.29049 +.34421 −.01132 +.02677 −.00192 −.93275 −.093 −."092

8	7	6	5	4	3	2	1
+0.06543 +.26039 −.01867 −.10475 +.20240 +.020 +."020	+0.20686 −.33121 +.08861 −.03574 −.004 −."004	+0.11434 −.22256 −.02224 −.13046 −.013 −."013	−0.71043 −.00278 −.10579 −.81900 −0.082 −."081	+1.74748 −.00107 −.00853 −.70563 +1.03225 +.103 +."102	+0.73751 −.00173 −.01370 −.39654 +.30290 +.62844 +.063 +."062	−0.11016 −.00406 −.03221 +.30002 +.15359 +.015 +."015	+.26537 −.00747 +.07565 +.33355 +.033 +."033

Back solution for longitude—Continued

26	25	24	13	11	23	22	21	20
−0. 03112 −. 03112 −. 003 −. ″004	+0. 58487 −. 02516 +. 55971 +. 056 +. ″084	+. 20166 −. 01612 +. 18839 +. 37393 +. 037 +. ″052	+1. 52509 +. 09268 +. 15940 +1. 77717 +. 178 +. ″214	−2. 02286 +. 04122 +. 03150 +1. 11994 −. 83020 −. 083 −. ″101	+0. 44194 +. 04143 +. 16568 +. 61812 −. 08397 +1. 18320 +. 118 +. ″153	−0. 20877 +. 17414 +. 18388 +. 01390 −. 04021 +. 14002 +. 26296 +. 026 +. ″037	−0. 27453 +. 01829 +. 02829 −. 08178 +. 28471 +. 14896 +. 12394 +. 012 +. ″016	−0. 29919 +. 24219 +. 04306 −. 11075 +. 02092 +. 02043 −. 08334 −. 008 −. ″012

19	18	17	16	15	14	12	10	9
+0. 08895 +. 04123 −. 11329 +. 02234 +. 02343 −. 04311 +. 01955 +. 002 +. ″003	−0. 08853 +. 05392 −. 16129 +. 03340 +. 03761 −. 00027 +. 00794 −. 11722 −. 012 −. ″016	+2. 29200 +. 01470 −. 02452 +. 00298 −. 04308 +. 00328 −. 00195 +2. 24341 +. 224 +. ″335	+0. 67391 +. 02392 −. 03992 +. 00480 +. 00534 −. 00319 +. 83096 +1. 49582 +. 150 +. ″207	+0. 54954 +1. 45321 +. 26344 +2. 26619 +. 227 +. ″340	−1. 80067 +. 53185 +. 65468 −. 61414 −. 061 −. ″083	+0. 32100 +. 77264 −. 04922 +. 51371 −. 00679 +. 00776 +1. 55910 +. 156 +. ″195	−0. 27825 +. 00695 −. 32388 −. 04471 +. 05110 +. 20816 −. 38063 −. 038 −. ″049	+2. 41462 +. 46324 −. 48089 +. 03891 −. 00866 +2. 42722 +. 243 +. ″287

8	7	6	5	4	3	2	1
+0. 03466 +. 37843 −. 08426 +. 27258 +. 60141 +. 060 +. ″076	+0. 10962 +. 86188 +. 26329 +1. 23479 +. 123 +. ″148	+0. 45593 +. 57916 +. 76840 +1. 80349 +. 180 +. ″208	+0. 02017 +. 09610 +1. 46247 +1. 57874 +. 158 +. ″178	−1. 29298 +. 03712 +. 11788 +1. 36020 +. 22222 +. 022 +. ″024	−1. 28535 +. 05964 +. 18946 +. 76439 +. 06521 −. 20665 −. 021 −. ″024	−0. 84736 +. 14017 +. 44534 −. 09865 −. 36050 −. 036 −. ″042	−0. 25075 +. 25802 −. 17756 −. 17029 −. 017 −. ″021

Latitude and longitude v's and total closures

Number of section	Latitude			p	Longitude			Total closure in feet	Closure as proportional part. One part in—
	v	pv	pv^2		v	pv	pv^2		
1	+0.33	+0.33	0.11	1.00	−0.17	−0.17	0.03	3.7	357,000
2	−.18	−.30	.05	1.65	−.19	−.31	.06	2.6	292,000
3	+.48	+.70	.34	1.45	+.15	+.22	.03	5.0	185,000
4	+.40	+.40	.16	1.00	+.43	+.43	.18	5.9	224,000
5	+.45	+.40	.18	.90	+.48	+.43	.21	6.6	220,000
6	+.17	+.28	.05	1.65	−.12	−.20	.02	2.1	390,000
7	+.69	+.69	.48	1.00	+.22	+.22	.05	7.2	183,000
8	−1.31	−.98	1.28	.75	−.72	−.54	.39	14.9	120,000
9	+.90	+.63	.57	.70	+.20	+.14	.03	9.2	201,000
10	+.20	+.22	.04	1.10	+.60	+.66	.40	6.3	187,000
11	+.09	+.10	.01	1.10	−.57	−.63	.36	5.8	205,000
12	+.24	+.24	.06	1.00	−.63	−.63	.40	6.7	197,000
13	−.80	−.40	.32	.50	+.63	+.32	.20	10.2	259,000
14	+1.00	+.50	.50	.50	+.31	+.16	.05	10.4	254,000
15	−.28	−.20	.06	.70	−.98	−.69	.68	10.2	186,000
16	+.06	+.12	.01	2.00	−.45	−.90	.40	4.5	147,000
17	+.16	+.32	.05	2.00	+.31	+.62	.19	3.5	189,000
18	−.12	−.08	.01	.70	−.45	−.32	.14	4.6	413,000
19	−.11	−.22	.02	2.00	+.22	+.44	.10	2.5	264,000
20	−.26	−.23	.06	.90	−.18	−.16	.03	3.1	477,000
21	−.69	−.55	.38	.80	−.61	−.49	.30	9.2	178,000
22	−.35	−.23	.08	.65	−.03	−.02	.00	3.5	581,000
23	−.42	−.34	.14	.80	−.59	−.48	.28	7.2	227,000
24	+.83	+.66	.55	.80	+.90	+.72	.65	12.2	134,000
25	+.22	+.18	.04	.80	+.74	+.59	.44	7.7	213,000
26	−.26	−.22	.06	.85	−.03	−.03	.00	2.6	599,000
27	−.13	−.26	.03	2.00	+.26	+.52	.14	2.9	227,000
28	−.12	−.20	.02	1.65	+.14	+.23	.03	1.8	440,000
29	+.22	+.15	.03	.70	−.51	−.36	.18	5.5	344,000
30	−.04	−.05	.00	1.25	−.10	−.12	.01	1.1	960,000
31	−.06	−.05	.00	.80	+.73	+.58	.42	7.3	224,000
32	−.04	−.05	.00	1.25	+.24	+.30	.07	2.4	440,000
33	+.44	+.88	.39	2.00	+.14	+.28	.04	4.6	143,000
34	+.45	+.90	.40	2.00	+.11	+.22	.02	4.6	143,000
35	−.87	−.78	.68	.90	−.81	−.73	.59	11.9	124,000
36	+.08	+.16	.01	2.00	−.38	−.76	.29	3.9	169,000
37	−1.15	−.92	1.07	.80	+.03	+.02	.00	11.5	142,000
38	−.14	−.10	.01	.70	+.64	+.45	.29	6.6	288,000
39	−.04	−.05	.00	1.20	+.05	+.06	.00	0.6	2,015,000
40	+.57	+.46	.26	.80	−.34	−.27	.09	6.6	248,000
41	−.09	−.13	.01	1.45	+.35	+.51	.18	3.6	252,000
42	+.11	+.14	.02	1.25	−.40	−.50	.20	4.1	258,000
	Sum		8.54				8.17		

Changes in positions of junction points caused by the readjustment

Number of junction	Station	Final latitude and longitude resulting from the readjustment			Latitude and longitude resulting from old adjustment			New minus old
		°	′	″	°	′	″	″
1	Carson	35	16	24.922	35	16	24.848	+0.074
		97	57	32.416	97	57	32.760	−.344
2	Kyle	32	49	18.039	32	49	18.014	+.025
		98	19	11.612	98	19	12.209	−.597
3	Mission	29	42	52.843	29	42	52.761	+.082
		98	09	52.109	98	09	52.996	−.887
4	Donna	26	09	40.593	26	09	40.459	+.134
		98	02	44.471	98	02	45.414	−.943
5	Peters	29	10	32.792	29	10	32.904	−.112
		100	36	02.238	100	36	03.100	−.862
6	Ingle	31	35	48.685	31	35	49.018	−.333
		103	35	24.355	103	35	25.284	−.929
7	Phillips	34	59	03.363	34	59	03.385	−.022
		104	07	59.629	104	07	60.204	−.575
8	Aroya	38	48	09.721	38	48	10.006	−.285
		103	10	55.580	103	10	55.610	−.030
9	Whitetank	33	34	01.652	33	34	02.053	−.401
		112	33	27.501	112	33	28.731	−1.230
10	Mount Nebo	39	48	38.316	39	48	39.113	−.797
		111	45	56.235	111	45	56.918	−.683
11	Black	35	55	51.689	Not previously adjusted.			
		115	02	35.556				
12	Mount Diablo	37	52	54.387	37	52	55.482	−1.095
		121	54	47.107	121	54	48.355	−1.248
13	Tepusquet	34	54	36.522	34	54	37.432	−.910
		120	11	08.335	120	11	09.654	−1.319
14	Farmer	43	42	21.212	43	42	21.389	−.177
		97	40	34.023	97	40	34.412	−.389
15	States	49	00	01.447	49	00	01.687	−.240
		97	07	39.343	97	07	39.738	−.395
16	Sundance	44	28	44.202	44	28	44.696	−.494
		104	27	03.099	104	27	02.821	+.278
17	Norge	48	53	38.110	48	53	38.866	−.756
		103	47	21.176	103	47	20.556	+.620
18	Oxford	42	16	10.842	42	16	11.766	−.924
		112	05	49.460	112	05	49.972	−.512
19	Mount Ellis	45	34	38.926	Not previously adjusted.			
		110	57	18.432				
20	Goldstone	48	52	55.681	Not previously adjusted.			
		110	29	18.684				
21	Dry	44	10	08.183	44	10	09.309	−1.126
		117	39	39.844	117	39	40.845	−1.001
22	Alder	45	50	59.419	45	50	60.583	−1.164
		119	56	21.080	119	56	22.229	−1.149
23	Bally	40	36	10.879	40	36	11.939	−1.060
		122	38	59.178	122	38	60.370	−1.192
24	Red	45	56	06.140	45	56	07.249	−1.109
		121	49	11.100	121	49	12.344	−1.244
25	Oroville	48	53	44.203	Not previously adjusted.			
		119	29	12.879				
26	Birch Point	48	56	30.854	48	56	31.910	−1.056
		122	49	12.021	122	49	13.395	−1.374

CLOSURES FROM THE ADJUSTMENT

The loop closures are shown by the equations on page 16, but a better understanding of the closures can be had by computing them for the several individual loops. The figures in the equations represent in many cases the closures in overlapping loops and hence are not as enlightening as the results for the simple loops. In Figure 5 these individual loop closures are shown; the first number above the line gives the closure in meters, the second number is the approximate length of the loop in kilometers, and the number below the line shows the approximate proportional part that the closure bears to the whole length of the loop. Only 2 of these closures are greater than 1 part in 200,000, the largest one being 1 part in 162,000. The mean of all of the closures is approximately 1 part in 450,000. The closure around the entire outer boundary is 10.1 meters in a distance of some 8,504 kilometers, or approximately 1 part in 842,000. The total length of the arcs in the adjustment is approximately 20,000 kilometers.

Some idea of what these closures mean may be obtained when we consider that they are the result of a combination of various types of measurements which have been made under conditions in the field that are oftentimes very unfavorable. The base measurements form one element of control, the angle measurements serve to carry the length through the arcs from base to base, and finally the astronomic measurements furnish the azimuth control of the various arcs. Some of this work was done as much as 50 years ago, but the greater part of it has been done in the past 25 years.

In the network included in this adjustment the length is controlled by 50 bases, and the orientation is maintained by 74 Laplace azimuths, both classes of control being more or less evenly distributed throughout the various sections of the net. The 50 bases have been measured within the past 50 years with various types of measuring apparatus. The older bases were measured with base bars or rods. In 1900–01 the measurement of nine bases along the ninety-eighth meridian was made with both bars and steel tapes, and in 1907 six bases were measured with both steel and invar tapes. Since 1907 all first-order bases have been measured with invar tapes. With these three types of apparatus different methods of measurement were employed. In spite of these facts the resulting loop closures demonstrate clearly that the general accord of bases is even much better than could be expected, and the fitting together of the various sections is satisfactory in every way.

In the previous adjustment of the transcontinental arc along the thirty-ninth parallel and of the arcs extending southward from this parallel, both along the ninety-eighth meridian as far as Texas and near the coast of California, the equations to hold the Laplace azimuths were not included. Because of this fact, the azimuths of the arcs became progressively bad and introduced errors in position that increased more and more as the distance from the starting point increased. On account of this swerving of the triangulation, as it is technically called, it was found very unsatisfactory to adjust the new arcs to close the loops, since all of the error of closure had to be absorbed by the new work. The introduction of Laplace azimuths to control the orientations of the arcs was an important step forward in geodetic work.

CHANGES IN POSITIONS

After the loop closures were determined and the equations given in the table on page 16 were formed, these equations were solved by the method of least squares to determine the best positions to be held in the junction figures. This solution resulted in the determination of the final geodetic position of a station in each one of the 26 junction figures. A list of these final positions is given on page 29. The positions of all of these same stations that had been previously adjusted are also given, together with the amount of the change of each brought about by the readjustment. These same changes are shown in Figure 7. It will be noted that junction No. 26 (see figs. 4 and 7) has changed $-1''.056$ in latitude and $-1''.374$ in longitude, or something like 30 meters in both latitude and longitude. This is the point that controls the data that have since been carried northward into western Canada and Alaska.

SECTION CLOSURES

Figure 6 and the table on page 28 give the amounts of latitude and longitude discrepancies that had to be absorbed by the various sections. They also give the total closure in length that had to be adjusted into each section and finally the approximate proportional part of the length of each section that this closure represents. It will be seen that no closure in position in any section is greater than 1 part in 120,000, and that the great majority of the sections have closures of 1 part in 200,000 or better. The mean of all of the closures of sections is approximately 1 part in 317,000. It is clear that these closures could be absorbed by the sections by the introduction of the latitude and longitude equations without any violence to the work in any of its parts. The whole network has therefore been fitted together in a rigid system without undue strain in any of its parts. Any short arc that may be observed in the future between sections of this framework should fit into the general scheme with comparatively small closure in position.

KIND OF FIGURES

Special attention should be called to the fact that throughout the 20,000 kilometers of arcs only complete quadrilaterals or central point figures were used except in a single case in which at the last minute it was found necessary to reject one diagonal of a quadrilateral because of bad closures in the figure. It is believed that the excellence of the results developed in the closures of loops is due in large measure to the use of such figures in carrying the arcs over such great extent of territory. It is not necessary to do more than cite the results as found to justify such an opinion. It is the policy of this bureau to observe all first-order triangulation in this way whenever it extends such control into new territory. Experience certainly justifies such precautions in geodetic work that is to serve for basic control.

CONCLUSION

The loop closures resulting from this adjustment clearly show that the methods employed in the geodetic operations in this region are fully adequate for all purposes whether practical or scientific. The instructions under which these observations were made called for an average triangle closure of about 1 second with a maximum closure of 3 seconds except in a few cases; for Laplace azimuths to be observed every 6 to 10 figures in the arcs; and for bases to be measured when the sum of the R_1's was somewhere between 80 and 100.[1] These ideal conditions were not always rigidly observed, but the exceptions are few and in all cases due to unusual circumstances. Taken all in all, the loop closures obtained speak very highly of the excellence of the field work during the fifty-odd years in which the work was in progress. These results fully justify our belief that experience has taught this bureau a method of observation that meets all demands of accuracy for high-grade geodetic work.

The results of this adjustment show unquestionably that the method of observations is fully justified and also that the Bowie method of adjustment is economically and scientifically adequate for application to such complicated conditions. Much credit is due to the field officers of this bureau for the conscientious manner in which they have endeavored to attain the ideals of accuracy called for in their instructions. Much credit is also due to the office mathematicians for their efficient handling of the complicated calculations that the adjustment demanded.

[1] The sum of the R_1's is a function of the angles that carry the length through the chain of triangles See Special Publication No. 120, p. 5.