Errata sheet for Special Publication No. 68, Fifth edition (1948 printing).

Page 5. First two entries listed as being on page 208 are actually on page 207.

Page 77. The values of x and y for latitude 45° longitude 25° should be 0.329923 and 0.780665 instead of 0.329244 and 0.779058. These two values appear in two places on this page.

The values of x and y for latitude 25° longitude 45° should be 0.707523 and 0.466582 instead of 0.706066 and 0.465622.

Page 106. The second line from the bottom of the page should read, "Mercator; and the only projection on which any great circle is represented as a straight".

Page 207. Under the center heading "The oblique conic conformal projection" the fifth line should read "America, scale 1:5,000,000".

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ELEMENTS OF MAP PROJECTION

WITH

APPLICATIONS TO MAP AND CHART CONSTRUCTION

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(1945)

BY
CHARLES H. DEETZ
AND
OSCAR S. ADAMS

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PREFACE

In this publication it has been the aim of the authors to present in simple form some of the ideas that lie at the foundation of the subject of map projections. Many people, even people of education and culture, have rather hazy notions of what is meant by a map projection, to say nothing of the knowledge of the practical construction of such a projection.

The two parts of the publication are intended to meet the needs of such people; the first part treats the theoretical side in a form that is as simple as the authors could make it; the second part attacks the subject of the practical construction of some of the most important projections, the aim of the authors being to give such detailed directions as are necessary to present the matter in a clear and simple manner.

Some ideas and principles lying at the foundation of the subject, both theoretical and practical, are from the very nature of the case somewhat complicated, and it is a difficult matter to state them in simple manner. The theory forms an important part of the differential geometry of surfaces, and it can only be fully appreciated by one familiar with the ideas of that branch of science. Fortunately, enough of the theory can be given in simple form to enable one to get a clear notion of what is meant by a map projection and enough directions for the construction can be given to aid one in the practical development of even the more complicated projections.

It is hoped that this publication may meet the needs of people along both of the lines indicated above and that it may be found of some interest to those who may already have a thorough grasp of the subject as a whole.

PREFACE TO FOURTH EDITION

In this edition there has been added a résumé of the systems of map projection which are of special interest in problems of the present day. Tables for the gnomonic projection of a map of the United States have also been added, the center of the projection or point of tangency being in latitude 40°, and the middle longitude recommended being 96° west. The intervals of the projection are given for every degree. A new table for the construction of a parabolic equal-area projection for world or sectional mapping is given. As this table is based on the authalic (or equal-area) latitude, a greater precision in results is thereby obtained. In addition to the foregoing tables there are also included the tables of two conformal projections for a map of the world, one being the development of the sphere within the area inclosed by a two-cusped epicycloid, the other, a projection by Lagrange showing the earth's surface within a circle with center on the Equator, the variations being most conspicuous in the polar regions.

PREFACE TO THE FIFTH EDITION

This edition contains a new plate showing outline of the United States in which the Lambert projection is superimposed on the Mercator projection on a scale sufficiently large to illustrate the difference in their properties and resulting delineation. Figures illustrating the nature of the Mercator projection as compared with the perspective projection upon a tangent cylinder have also been added with explanatory text.

A new equal-area projection of the hemispheres by Dr. Oscar S. Adams is brought to notice and is shown on Plate XIV.

The interesting property of isoperimetric curves (curves of true scale) which is inherent on all equal-area projections, is presented by illustration, (see Plate III), and also by figure No. 59, in its application to an Albers projection. Other features, more or less incidental, including Tissot's Indicatrix as applied to the sinusoidal projection, have been illustrated or briefly noted.

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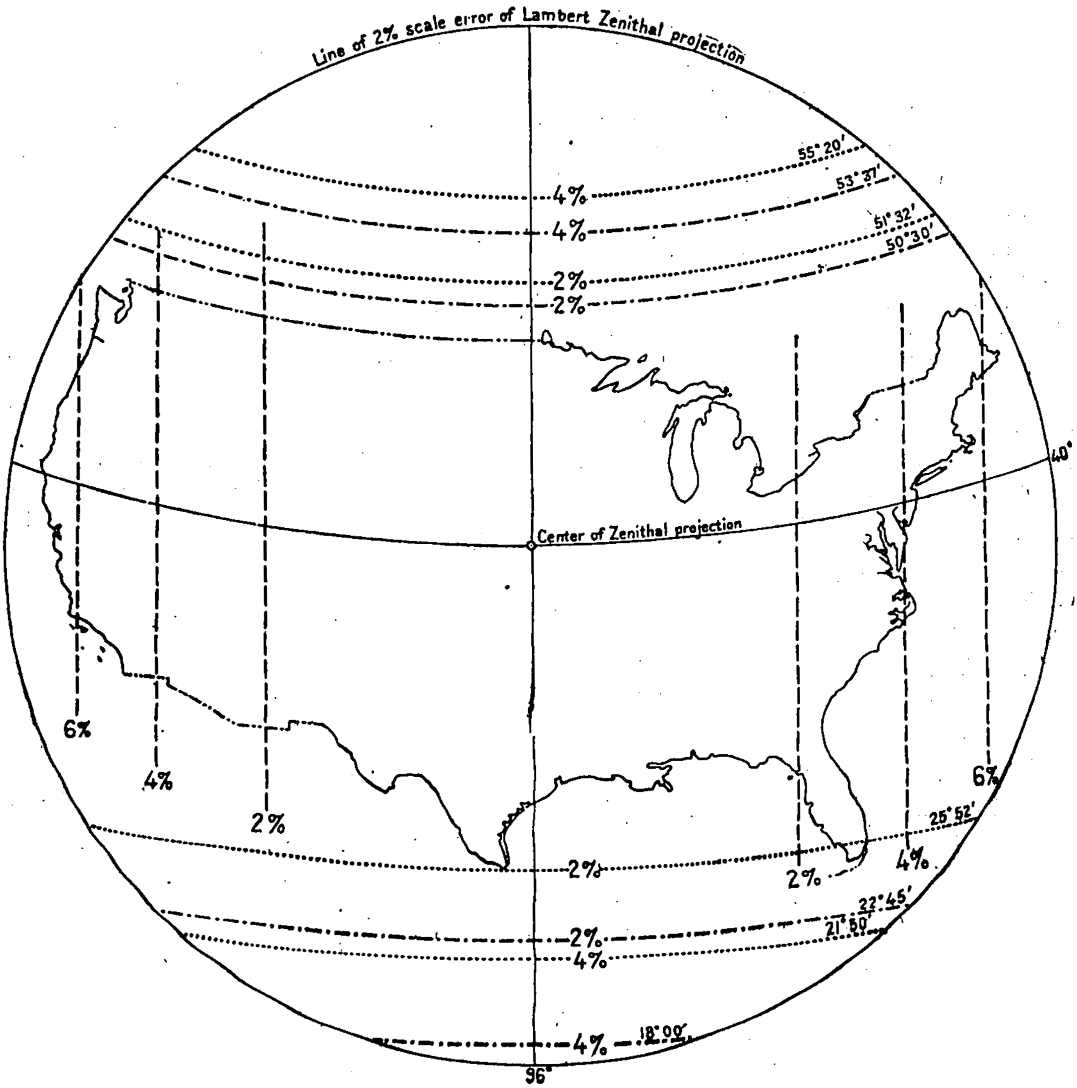

Lines of scale error or linear distortion

Polyconic projection.........2%, 4% and 6%, shown by vertical broken lines
Lambert Conformal.........2% and 4%, shown by dotted lines (east and west)
Lambert Zenithal.............2%, shown by bounding circle
Albers.................................2% and 4%, shown by dot and dash lines (east and west)

FRONTISPIECE.—Diagram showing lines of equal scale error or linear distortion in the polyconic, Lambert zenithal, Lambert conformal, and Albers projections. (See statistics on pp. 57, 58.)

ELEMENTS OF MAP PROJECTION WITH APPLICATIONS TO MAP AND CHART CONSTRUCTION

PART I

GENERAL STATEMENT[1]

Whatever may be the destiny of man in the ages to come, it is certain that for the present his sphere of activity is restricted to the outside shell of one of the smaller planets of the solar system—a system which after all is by no means the largest in the vast universe of space. By the use of the imagination and of the intellect with which he is endowed he may soar into space and investigate, with more or less certainty, domains far removed from his present habitat; but as regards his actual presence, he can not leave, except by insignificant distances, the outside crust of this small earth upon which he has been born, and which has formed in the past, and must still form, the theater upon which his activities are displayed.

The connection between man and his immediate terrestrial surroundings is therefore very intimate, and the configuration of the surface features of the earth would thus soon attract his attention. It is only reasonable to suppose that, even in the most remote ages of the history of the human race, attempts were made, however crude they may have been, to depict these in some rough manner. No doubt these first attempts at representation were scratched upon the sides of rocks and upon the walls of the cave dwellings of our primitive forefathers. It is well, then, in the light of present knowledge, to consider the structure of the framework upon which this representation is to be built. At best we can only partially succeed in any attempt at representation, but the recognition of the possibilities and the limitations will serve as valuable aids in the consideration of any specific problem.

We may reasonably assume that the earliest cartographical representations consisted of maps and plans of comparatively small areas, constructed to meet some need of the times, and it would be later on that any attempt would be made to extend the representation to more extensive regions. In these early times map making, like every other science or art, was in its infancy, and probably the first attempts of the kind were not what we should now call plans or maps at all, but rough perspective representations of districts or sketches with hills, forests, lakes, etc., all shown as they would appear to a person on the earth's surface. To represent these features in plan form, with the eye vertically over the various objects, although of very early origin, was most likely a later development; but we are now never likely to know who started the idea, since, as we have seen, it dates back far into antiquity.

[1] Paraphrased from "Maps and Map-making," by E. A. Reeves, London, 1910.

Geography is many-sided, and has numerous branches and divisions; and though it is true that map making is not the whole of geography, as it would be well for us to remind ourselves occasionally, yet it is, at any rate, a very important part of it, and it is, in fact, the foundation upon which all other branches must necessarily depend. If we wish to study the structure of any region we must have a good map of it upon which the various land forms can be shown. If we desire to represent the distribution of the races of mankind, or any other natural phenomenon, it is essential, first of all, to construct a reliable map to show their location. For navigation and for military operations, charts, plans, and maps are indispensable, as they are also for the demarcation of boundaries, land taxation, and for many other purposes. It may, therefore, be clearly seen that some knowledge of the essential qualities inherent in the various map structures or frameworks is highly desirable, and in any case the makers of maps should have a thorough grasp of the properties and limitations of the various systems of projection.

ANALYSIS OF THE BASIC ELEMENTS OF MAP PROJECTION [2]

PROBLEM TO BE SOLVED

A map is a small-scale, flat-surface representation of some portion of the surface of the earth. Nearly every person from time to time makes use of maps, and our ideas with regard to the relative areas of the various portions of the earth's surface are in general derived from this source. The shape of the land masses and their positions with respect to one another are things about which our ideas are influenced by the way these features are shown on the maps with which we become familiar.

It is fully established to-day that the shape of the earth is that of a slightly irregular spheroid, with the polar diameter about 26 miles shorter than the equatorial. The spheroid adopted for geodetic purposes is an ellipsoid of revolution formed by revolving an ellipse about its shorter axis. For the purpose of the present discussion the earth may be considered as a sphere, because the irregularities are very small compared with the great size of the earth. If the earth were represented by a spheroid with an equatorial diameter of 25 feet, the polar diameter would be approximately 24 feet 11 inches.

The problem presented in map making is the question of representing the surface of the sphere upon a plane. It requires some thought to arrive at a proper appreciation of the difficulties that have to be overcome, or rather that have to be dealt with and among which there must always be a compromise; that is, a little of one desirable property must be sacrificed to attain a little more of some other special feature.

In the first place, no portion of the surface of a sphere can be spread out in a plane without some stretching or tearing. This can be seen by attempting to flatten out a cap of orange peel or a portion of a hollow rubber ball; the outer part must be stretched or torn, or generally both, before the central part will come into the plane with the outer part. This is exactly the difficulty that has to be contended with in map making. There are some surfaces, however, that can be spread out in a plane without any stretching or tearing. Such surfaces are called developable surfaces and those

[2] Some of the elementary text in Part I and a number of the illustrations were adapted from a publication issued by G. Philip & Son (Ltd.), of London, England. The work is entitled "A Little Book on Map Projection," and its author is William Garnett writing under the pseudonym of Mary Adams. We wish to acknowledge our indebtedness to this work which sets forth in simple illustrations some of the fundamental facts in regard to map construction.

like the sphere are called nondevelopable. The cone and the cylinder are the two well-known surfaces that are developable. If a cone of revolution, or a right circular cone as it is called, is formed of thin material like paper and if it is cut from some point in the curve bounding the base to the apex, the conical surface can be spread out in a plane with no stretching or tearing. (See figs. 1 and 2.) Any curve drawn on the surface will have exactly the same length after development that it had before. In the same way, if a cylindrical surface is cut from base to base the whole surface can be rolled out in the plane, if the surface consists of thin pliable material. (See figs. 3 and 4.) In this case also there is no stretching or tearing of any part of the surface. Attention is called to the developable property of these surfaces, because use will be made of them in the later discussion of the subject of map making.

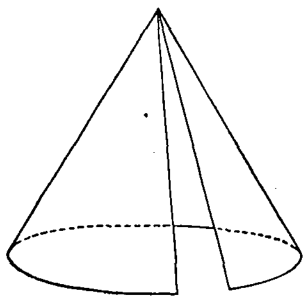

FIG. 1.—Conical surface cut from base to apex.

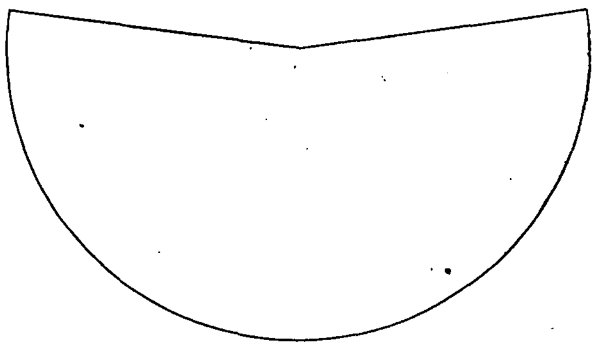

FIG. 2.—Development of the conical surface.

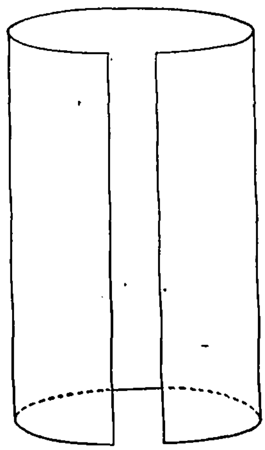

FIG. 3.—Cylindrical surface cut from base to base.

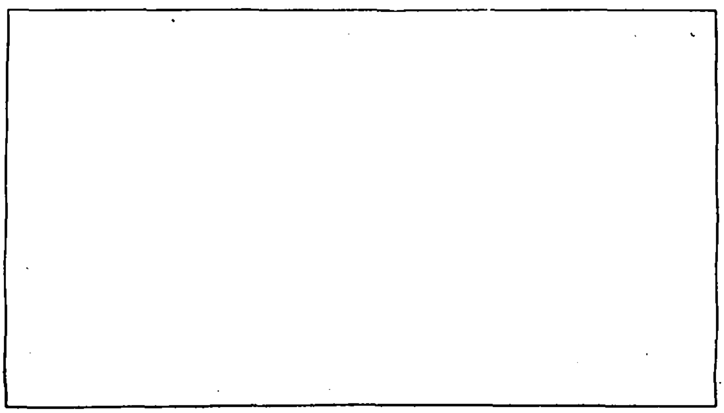

FIG. 4.—Development of the cylindrical surface.

REFERENCE POINTS ON THE SPHERE

A sphere is such that any point of it is exactly like any other point; there is neither beginning nor ending as far as differentiation of points is concerned. On the earth it is necessary to have some points or lines of reference so that other points may be located with regard to them. Places on the earth are located by latitude and longitude, and it may be well to explain how these quantities are related to the terrestrial sphere. The earth sphere rotates on its axis once a day, and this axis is therefore a definite line that is different from every other diameter. The ends of this diameter are called the poles, one the North Pole and the other the South Pole. With these as starting points, the sphere is supposed to be divided into two equal parts or hemispheres by a plane perpendicular to the axis midway between the poles. The circle formed by the intersection of this plane with the surface of the earth is called the Equator. Since this line is defined with reference to the poles, it is a definite line upon the earth. All circles upon the earth which divide it into two equal parts are called great circles, and the Equator, therefore, is a great circle. It is customary to divide the circle into four quadrants and each of these into 90 equal parts called degrees. There is no reason why the quadrant should not be divided into 100 equal parts, and in fact this division is sometimes used, each part being then called a grade. In this country the division of the quadrant into 90° is almost universally used; and accordingly the Equator is divided into 360°.

After the Equator is thus divided into 360°, there is difficulty in that there is no point at which to begin the count; that is, there is no definite point to count as zero or as the origin of reckoning. This difficulty is met by the arbitrary choice of some point the significance of which will be indicated after some preliminary explanations.

Any number of great circles can be drawn through the two poles and each one of them will cut the Equator into two equal parts. Each one of these great circles may be divided into 360°, and there will thus be 90° between the Equator and each pole on each side. These are usually numbered from 0° to 90° from the Equator to the pole, the Equator being 0° and the pole 90°. These great circles through the poles are called meridians. Let us suppose now that we take a point on one of these 30° north of the Equator. Through this point pass a plane perpendicular to the axis, and hence parallel to the plane of the Equator. This plane will intersect the surface of the earth in a small circle, which is called a parallel of latitude, this particular one being the parallel of 30° north latitude. Every point on this parallel will be in 30° north latitude. In the same way other small circles are determined to represent 20°, 40°, etc., both north and south of the Equator. It is evident that each of these small circles cuts the sphere, not into two equal parts, but into two unequal parts. These parallels are drawn for every 10°, or for any regular interval that may be selected, depending on the scale of the sphere that represents the earth. The point to bear in mind is that the Equator was drawn as the great circle midway between the poles; that the parallels were constructed with reference to the Equator; and that therefore they are definite small circles referred to the poles. Nothing is arbitrary except the way in which the parallels of latitude are numbered.

DETERMINATION OF LATITUDE

The latitude of a place is determined simply in the following way: Very nearly in the prolongation of the earth's axis to the north there happens to be a star, to which the name Polaris has been given. If one were at the North Pole, this star would appear to him to be directly overhead. Again, suppose a person to be at the Equator, then the star would appear to him to be on the horizon, level with his eye. It might be thought that it would be below his eye because it is in line with the earth's axis, 4,000 miles beneath his feet, but the distance of the star is so enormous that the radius of the earth is exceedingly small as compared with it. All lines to the star from different points on the earth appear to be parallel.

Suppose a person to be at A (see fig. 5), one-third of the distance between the Equator and the North Pole, the line BC will appear to him to be horizontal and he will see the star one-third of the way up from the horizon to the point in the heavens directly overhead. This point in the line of the vertical is called the zenith. It is now seen that the latitude of any place is the same as the height of Polaris above the horizon. Most people who have traveled have noticed that as they go south Polaris night by night appears lower in the heavens and gradually disappears, while the Southern Cross gradually comes into view.

At sea the latitude is determined every day at noon by an observation of the sun, but this is because the sun is brighter and more easily observed. Its distance from the pole, which varies throughout the year, is tabulated for each day in a book

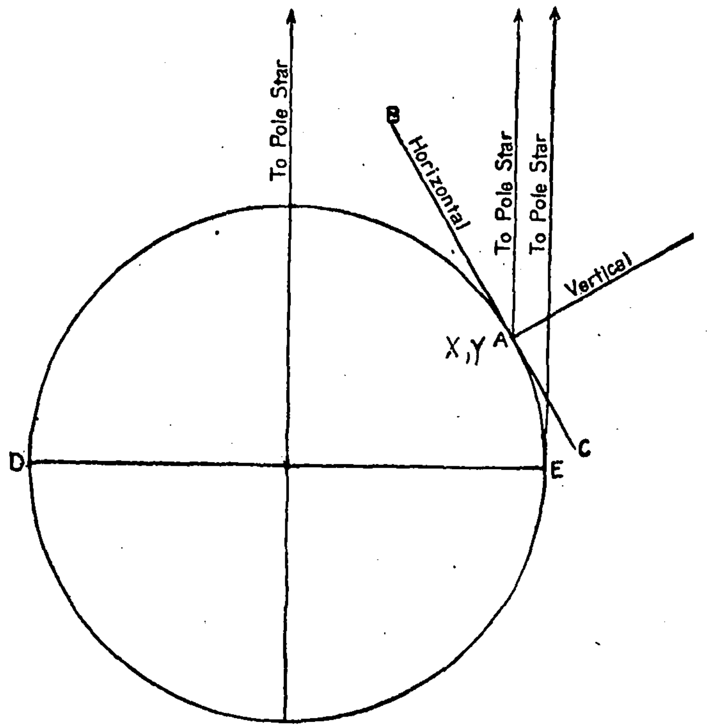

Fig. 5.—Determination of the latitude of a place.

called the Nautical Almanac. When, therefore, an observation of the sun is made, its polar distance is allowed for, and thus the latitude of the ship is determined by the height of the pole in the heavens. Even the star itself is directly observed upon from time to time. This shows that the latitude of a place is not arbitrary. If the star is one-third of the way up, measured from the horizon toward the zenith, then the point of observation is one-third of the way up from the Equator toward the pole, and nothing can alter this fact. By Polaris, in the previous discussion, is really meant the true celestial pole; that is, the point at which the prolongation of the earth's axis pierces the celestial sphere. Corrections must be made to the observations on the star to reduce them to this true pole. In the Southern Hemisphere latitudes are related in a similar way to the southern pole.

Strictly speaking, this is what is called the astronomical latitude of a place. There are other latitudes which differ slightly from that described above, partly because the earth is not a sphere and partly on account of local attractions, but the above-described latitude is not only the one adopted in all general treatises but it is also the one employed on all general maps and charts, and it is the latitude by which all navigation is conducted; and if we assume the earth to be a homogenous sphere, it is the only latitude.

DETERMINATION OF LONGITUDE

The foregoing fixes only the parallel of latitude on which a place is situated. If it be found that the latitude of one place is 10° north and that of another 20° north, then the second place is 10° north of the first, but as yet we have no means of showing whether it is east or west of it.

If at some point on the earth's surface a perpendicular pole is erected, its shadow in the morning will be on the west side of it and in the evening on the east side of it. At a certain moment during the day the shadow will lie due north and south. The moment at which this occurs is called noon, and it will be the same for all points exactly north or south of the given point. A great circle passing through the poles of the earth and through the given point is called a meridian (from *meridies*, midday), and it is therefore noon at the same moment at all points on this meridian. Let us suppose that a chronometer keeping correct time is set at noon at a given place and then carried to some other place. If noon at this latter place is observed and the time indicated by the chronometer is noted at the same moment, the difference of time will be proportional to the part of the earth's circumference to the east or west that has been passed over. Suppose that the chronometer shows 3 o'clock when it is noon at the place of arrival, then the meridian through the new point is situated one-eighth of the way around the world to the westward from the first point. This difference is a definite quantity and has nothing arbitrary about it, but it would be exceedingly inconvenient to have to work simply with differences between the various places, and all would be chaos and confusion unless some place were agreed upon as the starting point. The need of an origin of reckoning was evident as soon as longitudes began to be thought of and long before they were accurately determined. A great many places have in turn been used; but when the English people began to make charts they adopted the meridian through their principal observatory of Greenwich as the origin for reckoning longitudes and this meridian has now been adopted by many other countries. In France the meridian of Paris is most generally used. The adoption of any one meridian as a standard rather than another is purely arbitrary, but it is highly desirable that all should use the same standard.

The division of the Equator is made to begin where the standard meridian crosses it and the degrees are counted 180 east and west. The standard meridian is sometimes called the prime meridian, or the first meridian, but this nomenclature is slightly misleading, since this meridian is really the zero meridian. This great circle, therefore, which passes through the poles and through Greenwich is called the meridian of Greenwich or the meridian of 0° on one side of the globe, and the 180th meridian on the other side, it being 180° east and also 180° west of the zero meridian.

By setting a chronometer to Greenwich time and observing the hour of noon at various places their longitude can be determined, by allowing 15° of longitude to each hour of time, because the earth turns on its axis once in 24 hours and there are 360° in the entire circumference. This description of the method of determining differences of longitude is, of course, only a rough outline of the way in which they can be determined. The exact determination of a difference of longitude between two places is a work of considerable difficulty and the longitudes of the principal observatories have not even yet been determined with sufficient degree of accuracy for certain delicate observations.

PLOTTING POINTS BY LATITUDE AND LONGITUDE ON A GLOBE

If a globe has the circles of latitude and longitude drawn upon it according to the principles described above and the latitude and longitude of certain places have been determined by observation, these points can be plotted upon the globe in their proper positions and the detail can be filled in by ordinary surveying, the detail being referred to the accurately determined points. In this way a globe can be formed that is in appearance a small-scale copy of the spherical earth. This copy will be more or less accurate, depending upon the number and distribution of the accurately located points.

PLOTTING POINTS BY LATITUDE AND LONGITUDE ON A PLANE MAP

If, in the same way, lines to represent latitude and longitude be drawn on a plane sheet of paper, the places can be plotted with reference to these lines and the detail filled in by surveying as before. The art of making maps consists, in the first place, in constructing the lines to represent latitude and longitude, either as nearly like the lines on the globe as possible when transferred from a nondevelopable surface to a flat surface, or else in such a way that some one property of the lines will be retained at the expense of others. It would be practically impossible to transfer the irregular coast lines from a globe to a map; but it is comparatively easy to transfer the regular lines representing latitude and longitude. It is possible to lay down on a map the lines representing the parallels and meridians on a globe many feet in diameter. These lines of latitude and longitude may be laid down for every 10°, for every degree, or for any other regular interval either greater or smaller. In any case, the thing to be done is to lay down the lines, to plot the principal points, and then to fill in the detail by surveying. After one map is made it may be copied even on another kind of projection, care being taken that the latitude and longitude of every point are kept correct on the copy. It is evident that if the lines of latitude and longitude cannot be laid down correctly upon a plane surface, still less can the detail be laid down on such a surface without distortions.

Since the earth is such a large sphere it is clear that, if only a small portion of a country is taken, the surface included will differ but very little from a plane surface. Even two or three hundred square miles of surface could be represented upon a plane with an amount of distortion that would be negligible in practical mapping. The difficulty encountered in mapping large areas is gotten over by first making many maps of small area, generally such as to be bounded by lines of latitude and longitude. When a large number of these maps have been made it will be found that they cannot be joined together so as to lie flat. If they are carefully joined along the edges it will be found that they naturally adapt themselves to the shape of the globe. To obviate this difficulty another sheet of paper is taken and on it are laid down the lines of latitude and longitude, and the various maps are copied so as to fill the space allotted to them on this larger sheet. Sometimes this can be done by a simple reduction which does not affect the accuracy, since the accuracy of a map is independent of the scale. In most cases, however, the reduction will have to be unequal in different directions and sometimes the map has to be twisted to fit into the space allotted to it.

The work of making maps therefore consists of two separate processes. In the first place, correct maps of small areas must be made, which may be called surveying; and in the second place these small maps must be fitted into a system of lines representing the meridians and parallels. This graticule of the orderly arrangement of lines on the plane to represent the meridians and parallels of the earth is called a map projection. A discussion of the various ways in which this graticule of lines may be constructed so as to represent the meridians and parallels of the earth and at the same time so as to preserve some desired feature in the map is called a treatise on map projections.

HOW TO DRAW A STRAIGHT LINE

Few people realize how difficult it is to draw a perfectly straight line when no straightedge is available. When a straightedge is used to draw a straight line, a copy is really made of a straight line that is already in existence. A straight line is such that if any part of it is laid upon any other part so that two points of the one part coincide with two points of the other the two parts will coincide throughout. The parts must coincide when put together in any way, for an arc of a circle can be made to coincide with any other part of the same circumference if the arcs are brought together in a certain way. A carpenter solves the problem of joining two points by a straight line by stretching a chalk line between them. When the line is taut, he raises it slightly in the middle portion and suddenly releases it. Some of the powdered chalk flies off and leaves a faint mark on the line joining the points. This depends upon the principle that a stretched string tends to become as short as possible unless some other force is acting upon it than the tension in the direction of its length. This is not a very satisfactory solution, however, since the chalk makes a line of considerable width, and the line will not be perfectly straight unless extra precautions are taken.

A straightedge can be made by clamping two thin boards together and by planing the common edge. As they are planed together, the edges of the two will be alike, either both straight, in which case the task is accomplished, or they will be both convex, or both concave. They must both be alike; that is, one can not be convex and the other concave at any given point. By unclamping them it can be seen whether the planed edges fit exactly when placed together, or whether they need some more planing, due to being convex or concave or due to being convex in places and concave in other places. (See fig. 6.) By repeated trials and with sufficient patience, a straightedge can be made in this way. In practice, of course, a straightedge in process of construction is tested by one that has already been made. Machines for drawing straight lines can be constructed by linkwork, but they are seldom used in practice.

It is in any case difficult to draw straight lines of very great length. A straight line only a few hundred meters in length is not easy to construct. For very long straight lines, as in gunnery and surveying, sight lines are taken; that is, use is made of the fact that when temperature and pressure conditions are uniform, light travels through space or in air, in straight lines. If three points, *A*, *B*, and *C*, are such that *B* appears to coincide with *C* when looked at from *A*, then *A*, *B*, and *C* are in a straight line. This principle is made use of in sighting a gun and in using the telescope for astronomical measurements. In surveying, directions which are straight lines are found by looking at the distant object, the direction of which from the point of view we want to determine, through a telescope. The telescope is moved until the image of the small object seen in it coincides with a mark fixed in the telescope in the center of the field of view. When this is the case, the mark, the center of the object glass of the telescope, and the distant object are in one straight line. A graduated scale on the mounting of the telescope enables us to determine the direction of the line joining the fixed mark in the telescope and the center of the object glass. This direction is the direction of the distant object as seen by the eye, and it will be determined in terms of another direction assumed as the initial direction.

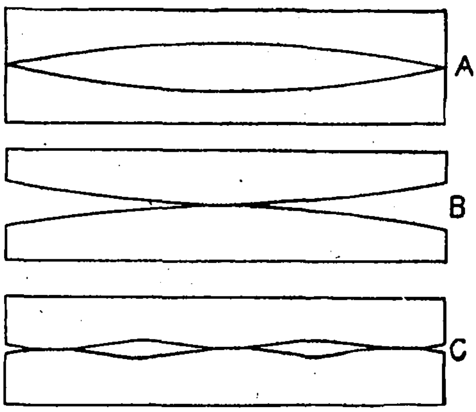

FIG. 6.—Construction of a straight edge.

HOW TO MAKE A PLANE SURFACE

While a line has length only, a surface has length and breadth. Among surfaces a plane surface is one on which a straight line can be drawn through any point in any direction. If a straightedge is applied to a plane surface, it can be turned around, and it will in every position coincide throughout its entire length with the surface. Just as a straightedge can be used to test a plane, so, equally well, can a plane surface be used to test a straightedge, and in a machine shop a plate with plane surface is used to test accurate workmanship.

The accurate construction of a plane surface is thus a problem that is of very great practical importance in engineering. A very much greater degree of accuracy is required than could be obtained by a straightedge applied to the surface in different directions. No straightedges in existence are as accurate as it is required that the planes should be. The method employed is to make three planes and to test them against one another two and two. The surfaces, having been made as truly plane as ordinary tools could render them, are scraped by hand tools and rubbed together from time to time with a little very fine red lead between them. Where they touch, the red lead is rubbed off, and then the plates are scraped again to remove the little elevations thus revealed, and the process is continued until all the projecting points have been removed. If only two planes were worked together, one might be convex (rounded) and the other concave (hollow), and if they had the same curvature they might still touch at all points and yet not be plane; but if three surfaces, *A*, *B*, and *C*, are worked together, and if *A* fits both *B* and C and *A* is concave, then *B* and *C* must be both convex, and they will not fit one another. If *B* and *C* both fit *A* and also fit one another at all points, then all three must be truly plane.

When an accurate plane-surface plate has once been made, others can be made one at a time and tested by trying them on the standard plate and moving them over the surface with a little red lead between them. When two surface plates made as truly plane as possible are placed gently on one another without any red lead between them, the upper plate will float almost without friction on a very thin layer of air, which takes a very long time to escape from between the plates, because they are everywhere so very near together.

HOW TO DRAW THE CIRCLES REPRESENTING MERIDIANS AND PARALLELS ON A SPHERE

We have seen that it is difficult to draw a straight line and also difficult to construct a plane surface with any degree of accuracy. The problem of constructing circles upon a sphere is one that requires some ingenuity if the resulting circles are to be accurately drawn. If a hemispherical cup is constructed that just fits the sphere, two points on the rim exactly opposite to one another may be determined. (See fig. 7.) To do this is not so easy as it appears, if there is nothing to mark the center of the cup. The diameter of the cup can be measured and a circle can be drawn on cardboard with the same diameter by the use of a compass. The center of this circle will be marked on the cardboard by the fixed leg of the compass and with a straight edge a diameter can be drawn through this center. This circle can then be cut out and fitted just inside the rim of the cup. The ends of the diameter drawn on the card then mark the two points required on the edge of the cup. With some suitable tool a small notch can be made at each point on the edge of the cup. Marks should then be made on the edge of the cup for equal divisions of a semicircle. If it is desired to draw the parallels for every 10° of latitude, the semicircle must be divided into 18 equal parts. This can be done by dividing the cardboard circle by means of a protractor and then by marking the corresponding points on the edge of the cup. The sphere can now be put into the cup and points on it marked corresponding to the two notches in the edge of the cup. Pins can be driven into these points and allowed to rest in the notches. If the diameter of the cup is such that the sphere just fits into it, it can be found whether the pins are exactly in the ends of a diameter by turning the sphere on the pins as an axis. If the pins are not correctly placed, the sphere will not rotate freely. The diameter determined by the pins may now be taken as the axis, one of the ends being taken as the North Pole and the other as the South Pole. With a sharp pencil or with an engraving tool circles can be drawn on the sphere at the points of division on the edge of the cup by turning the sphere on its axis while the pencil is held against the surface at the correct point. The circle midway between the poles is a great circle and will represent the Equator. The Equator is then numbered 0° and the other eight circles on either side of the Equator are numbered 10°, 20°, etc. The poles themselves correspond to 90°.

Now remove the sphere and, after removing one of the pins, insert the sphere again in such a way that the Equator lies along the edge of the cup. Marks can then be made on the Equator corresponding to the marks on the edge of the cup. In this way the divisions of the Equator corresponding to the meridians of 10° interval are determined. By replacing the sphere in its original position with the pins inserted, the meridians can be drawn along the edge of the cup through the various marks on the Equator. These will be great circles passing through the poles. One of these circles is numbered 0° and the others 10°, 20°, etc., both east and west of the zero meridian and extending to 180° in both directions. The one hundred and eightieth meridian will be the prolongation of the zero meridian through the poles and will be the same meridian for either east or west.

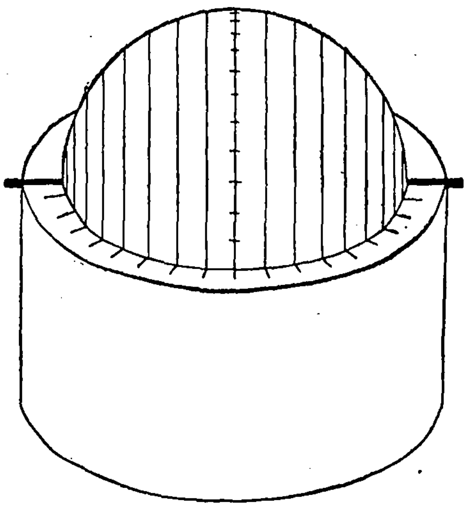

FIG. 7.—Constructing the circles of parallels and meridians on a globe.

This sphere, with its two sets of circles, the meridians and the parallels, drawn upon it may now be taken as a model of the earth on which corresponding circles are supposed to be drawn. When it is a question only of supposing the circles to be drawn, and not actually drawing them, it will cost no extra effort to suppose them drawn and numbered for every degree, or for every minute, or even for every second of arc, but no one would attempt actually to draw them on a model globe for intervals of less than 1°. On the the earth itself a second of latitude corresponds to a little more than 100 feet. For the purpose of studying the principles of map projection it is quite enough to suppose that the circles are drawn at intervals of 10°.

It was convenient in drawing the meridians and parallels by the method just described to place the polar axis horizontal, so that the sphere might rest in the cup by its own weight. Hereafter, however, we shall suppose the sphere to be turned so that its polar axis is vertical with the North Pole upward. The Equator and all the parallels of latitude will be horizontal, and the direction of rotation corresponding to the actual rotation of the earth will carry the face of the sphere at which we are looking from left to right; that is, from west to east, according to the way in which the meridians were marked. As the earth turns from west to east a person on its surface, unconscious of its movement and looking at the heavenly bodies, naturally thinks that they are moving from east to west. Thus, we say that the sun rises in the east and sets in the west.

THE TERRESTRIAL GLOBE

With the sphere thus constructed with the meridians and parallels upon it, we get a miniature representation of the earth with its imaginary meridians and parallels. On this globe the accurately determined points may be plotted and the shore line drawn in, together with the other physical features that it is desirable to show. This procedure, however, would require that each individual globe should be plotted by hand, since no reproductions could be printed. To meet this difficulty, ordinary terrestrial globes are made in the following way: It is well known that a piece of paper cannot be made to fit on a globe but a narrow strip can be made to fit fairly well by some stretching. If the strip is fastened upon the globe when it is wet, the paper will stretch enough to allow almost a perfect fit. Accordingly 12 gores are made as shown in figure 8, such that when fastened upon the globe they will reach from the parallel of 70° north to 70° south. A circular cap is then made to extend from each of these parallels to the poles. Upon these gores the projection lines and the outlines of the continents are printed. They can then be pasted upon the globe and with careful stretching they can be made to adapt themselves to the spherical surface. It is obvious that the central meridian of each gore is shorter than the bounding meridians, whereas upon the globe all of the meridians are of the same length. Hence in adapting the gores to the globe the central meridian of each gore must be slightly stretched in comparison with the side meridians. The figure 8 shows on a very small scale the series of gores and the polar caps printed for covering a globe. These gores do not constitute a map. They are as nearly as may be on a plane surface, a facsimile of the surface of the globe, and only require bending with a little stretching in certain directions or contraction in others or both to adapt themselves precisely to the spherical surface. If the reader examines the parts of the continent of Asia as shown on the separate gores which are almost a facsimile of the same portion of the globe, and tries to piece them together without bending them over the curved surface of the sphere, the problem of map projection will probably present itself to him in a new light.

It is seen that although the only way in which the surface of the earth can be represented correctly consists in making the map upon the surface of a globe, yet this is a difficult task, and, at the best, expedients have to be resorted to or the cost of construction will be prohibitive. It should be remembered, however, that the only source of true ideas regarding the mapping of large sections of the surface of the earth must of necessity be obtained from its representation on a globe. Much good would result from making the globe the basis of all elementary teaching in geography. The pupils should be warned that maps are very generally used because of their convenience. Within proper limitations they serve every purpose for which they are intended. Errors are dependent upon the system of projection used and

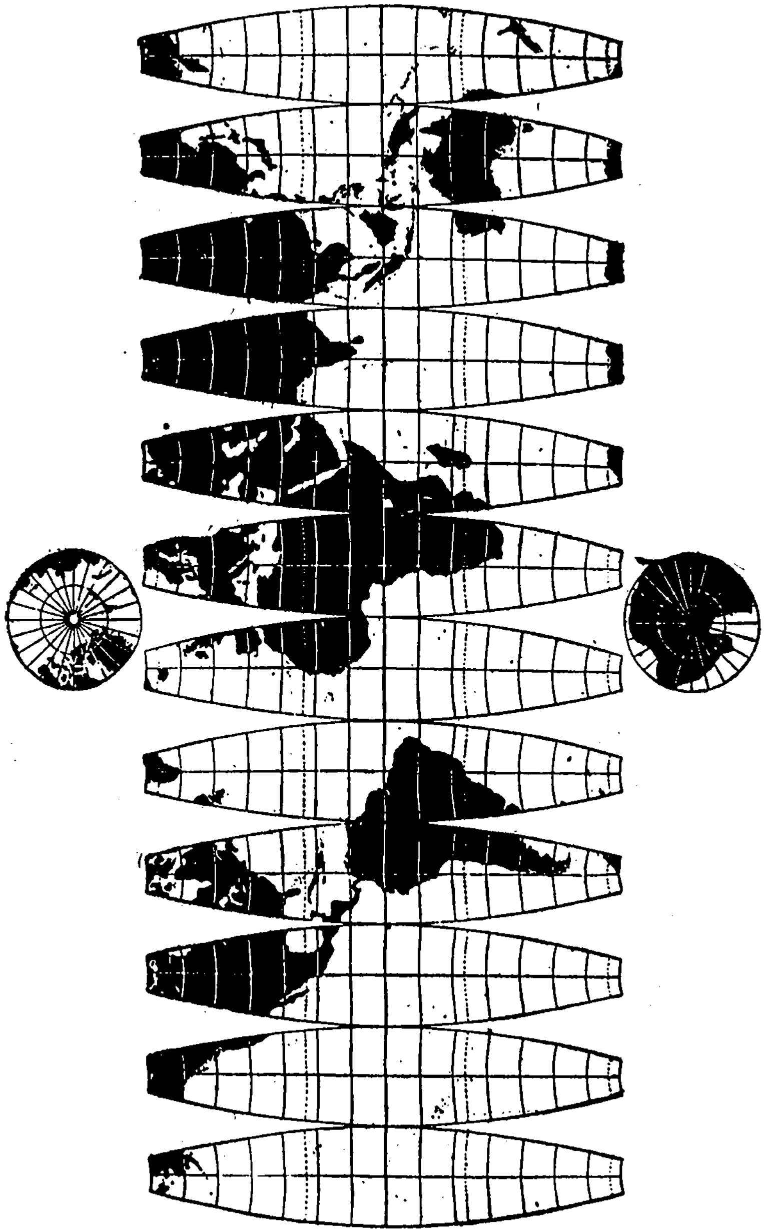

FIG. 8.—Covering for a terrestrial globe.

when map and globe [3] do not agree, the former is at fault. This would seem to be a criticism against maps in general and where large sections are involved and where unsuitable projections are used, it often is such. Despite defects which are inherent in the attempt to map a spherical surface upon a plane, maps of large areas, comprising continents, hemispheres or even the whole sphere, are employed because of their convenience both in construction and handling.

[3] Perfect globes are seldom seen on account of the expense involved in their manufacture.

Makers of globes would confer a benefit on future generations if they would make cheap globes on which is shown, not as much as possible, but essential geographic features only, omitting, perhaps, the line of the ecliptic which only leads to confusion when found upon a terrestrial globe. If the oceans were shown by a light blue tint and the continents by darker tints of another color, and if the principal great rivers and mountain chains were shown, it would be sufficient. The names of oceans and countries, and a few great cities, noted capes, etc., are all that should appear. The globe then would serve as the index to the maps of continents, which again would serve as indexes to the maps of countries.

REPRESENTATION OF THE SPHERE UPON A PLANE

THE PROBLEM OF MAP PROJECTION

It seems, then, that if we have the meridians and parallels properly drawn on any system of map projection, the outline of a continent or island can be drawn in from information given by the surveyors respecting the latitude and longitude of the principal capes, inlets, or other features, and the character of the coast between them. Copies of maps are commonly made in schools upon blank forms on which the meridians and parallels have been drawn, and these, like squared paper, give assistance to the free-hand copyist. Since the meridians and parallels can be drawn as closely together as we please, we can get as many points as we require laid down with strict accuracy. The meridians and parallels being drawn on the globe, if we have a set of lines upon a plane sheet to represent them we can then transfer detail from the globe to the map. The problem of map projection, therefore, consists in finding some method of transferring the meridians and parallels from the globe to the map.

DEFINITION OF MAP PROJECTION

The lines representing the meridians and parallels can be drawn in an arbitrary manner, but to avoid confusion we must have a one-to-one correspondence. In practice all sorts of liberties are taken with the methods of drawing the meridians and parallels in order to secure maps which best fulfill certain required conditions, provided always that the methods of drawing the meridians and parallels follow some law or system that will give the one-to-one correspondence. Hence a map projection may be defined as a systematic drawing of lines representing meridians and parallels on a plane surface, either for the whole earth or for some portion of it.

DISTORTION

In order to decide on the system of projection to be employed, we must consider the purpose for which the map is to be used and the consequent conditions which it is most important for the map to fulfill. In geometry, size and shape are the two fundamental considerations. If we want to show without exaggeration the extent of the different countries on a world map, we do not care much about the shape of the country, so long as its area is properly represented to scale. For statistical purposes, therefore, a map on which all areas are correctly represented to scale is valuable, and such a map is called an "equal-area projection." It is well known that parallelograms on the same base and between the same parallels, that is of the same height, have equal area, though one may be rectangular or upright and the other very oblique. The sloping sides of the oblique parallelograms must be very much longer than the upright sides of the other, but the areas of the figures will be the same though the shapes are so very different. The process by which the oblique parallelogram can be formed from the rectangular parallelogram is called by engineers "shearing." A pack of cards as usually placed together shows as profile a rectangular parallelogram. If a book be stood up against the ends of the cards as in figure 9 and then made to slope as in figure 10 each card will slide a little over the one below and the profile of the pack will be the oblique parallelogram shown in figure 10. The height of the parallelogram will be the same, for it is the thickness of the pack. The base will remain unchanged, for it is the long edge of the bottom card. The area will be unchanged, for it is the sum of the areas of the edges of the cards. The shape of the parallelogram is very different from its original shape. The sloping sides, it is true,

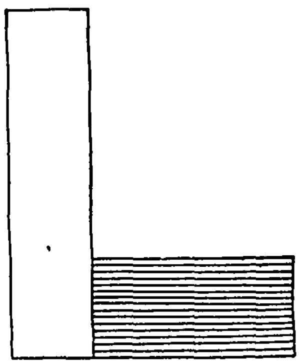

FIG. 9.—Pack of cards before "shearing."

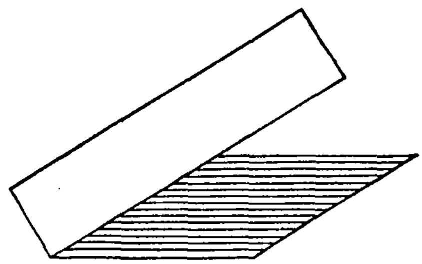

FIG. 10.—Pack of cards after "shearing."

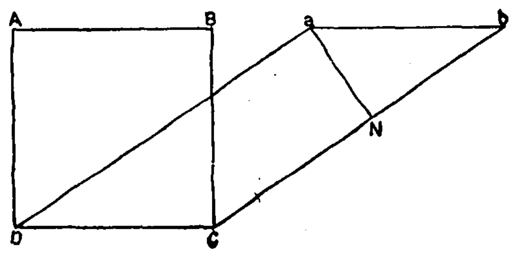

FIG. 11.—Square "sheared" into an equivalent parallelogram.

are not straight lines, but are made up of 52 little steps, but if instead of cards several hundred very thin sheets of paper or metal had been used the steps would be invisible and the sloping edges would appear to be straight lines. This sliding of layer upon layer is a "simple shear." It alters the shape without altering the area of the figure.

This shearing action is worthy of a more careful consideration in order that we may understand one very important point in map projection. Suppose the square *A B C D* (see fig. 11) to be sheared into the oblique parallelogram *a b C D*. Its base and height remain the same and its area is unchanged, but the parallelogram *a b C D* may be turned around so that *C b* is horizontal, and then *C b* is the base, and the line *a N* drawn from *a* perpendicular to *b C* is the height. Then the area is the product of *b C* and *a N*, and this is equal to the area of the original square and is constant whatever the angle of the parallelogram and the extent to which the side *B C* has been stretched. The perpendicular *a N*, therefore, varies inversely as the length of the side *b C*, and this is true however much *B C* is stretched. Therefore in an equal-area projection, if distances in one direction are increased, those measured in the direction at right angles are reduced in the corresponding ratio if the lines that they represent are perpendicular to one another upon the earth.

If lines are drawn at a point on an equal-area projection nearly at right angles to each other, it will in general be found that if the scale in the one direction is increased that in the other is diminished. If one of the lines is turned about the point, there must be some direction between the original positions of the lines in which the scale is exact. Since the line can be turned in either of two directions, there must be two directions at the point in which the scale is unvarying. This is true at every point of such a map, and consequently curves could be drawn on such a projection that would represent directions in which there is no variation in scale (isoperimetric curves).

In maps drawn on an equal-area projection, some tracts of country may be sheared so that their shape is changed past recognition, but they preserve their area unchanged. In maps covering a very large area, particularly in maps of the whole world, this generally happens to a very great extent in parts of the map which are distant from both the horizontal and the vertical lines drawn through the center of the map. (See fig. 12.)

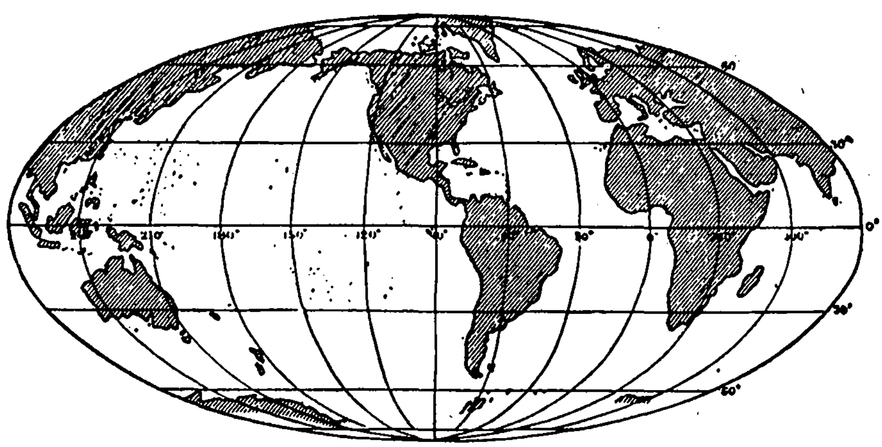

FIG. 12.—The Mollweide equal-area projection of the sphere.

It will be noticed that in the shearing process that has been described every little portion of the rectangle is sheared just like the whole rectangle. It is stretched parallel to *B C* (see fig. 11) and contracted at right angles to this direction. Hence when in an equal-area projection the shape of a tract of country is changed, it follows that the shape of every square mile and indeed of every square inch of this country will be changed, and this may involve a considerable inconvenience in the use of the map. In the case of the pack of cards the shearing was the same at all points. In the case of equal-area projections the extent of shearing or distortion varies with the position of the map and is zero at the center. It usually increases along the diagonal lines of the map. It may, however, be important for the purpose for which the map is required, that small areas should retain their shape even at the cost of the area being increased or diminished, so that different scales have to be used at different parts of the map. The projections on which this condition is secured are called "conformal" projections. If it were possible to secure equality of area and exactitude of shape at all points of the map, the whole map would be an exact counterpart of the corresponding area on the globe, and could be made to fit the globe at all points by simple bending without any stretching or contraction, which would imply alteration of scale. But a plane surface cannot be made to fit a sphere in this way. It must be stretched in some direction or contracted in others (as in the process of "raising" a dome or cup by hammering sheet metal) to fit the sphere, and this means that the scale must be altered in one direction or in the other or in both directions at once. It is therefore impossible for a map to preserve the same scale in all directions at all points; in other words a map cannot accurately represent both size and shape of the geographical features at all points of the map.

CONDITIONS FULFILLED BY A MAP PROJECTION

If, then, we endeavor to secure that the shape of a very small area, a square inch or a square mile, is preserved at all points of the map, which means that if the scale of the distance north and south is increased the scale of the distance east and west must be increased in exactly the same ratio, we must be content to have some parts of the map represented on a greater scale than others. The conformal projection, therefore, necessitates a change of scale at different parts of the map, though the scale is the same in all directions at any one point. Now, it is clear that if in a map of North America the northern part of Canada is drawn on a much larger scale than the southern States of the United States, although the shape of every little bay or headland, lake or township is preserved, the shape of the whole continent on the map must be very different from its shape on the globe. In choosing our system of map projection, therefore, we must decide whether we want—

(1) To keep the area directly comparable all over the map at the expense of correct shape (equal-area projection), or

(2) To keep the shapes of the smaller geographical features, capes, bays, lakes, etc., correct at the expense of a changing scale all over the map (conformal projection) and with the knowledge that large tracts of country will not preserve their shape, or

(3) To make a compromise between these conditions so as to minimize the errors when both shape and area are taken into account.

There is a fourth consideration which may be of great importance and which is very important to the navigator, while it will be of much greater importance to the aviator when aerial voyages of thousands of miles are undertaken, and that is that directions of places taken from the center of the map, and as far as possible when taken from other points of the map, shall be correct. The horizontal direction of an object measured from the south is known as its azimuth. Hence a map which preserves these directions correctly is called an "azimuthal projection." We may, therefore, add a fourth object, viz:

(4) To preserve the correct directions of all lines drawn from the center of the map (azimuthal projection).

Projections of this kind are sometimes called zenithal projections, because in maps of the celestial sphere the zenith point is projected into the central point of the map. This is a misnomer, however, when applied to a map of the terrestrial sphere.

We have now considered the conditions which we should like a map to fulfill, and we have found that they are inconsistent with one another. For some particular purpose we may construct a map which fulfills one condition and rejects another, or vice versa; but we shall find that the maps most commonly used are the result of compromise, so that no one condition is strictly fulfilled, nor, in most cases, is it extravagantly violated.

CLASSIFICATION OF PROJECTIONS

There is no way in which projections can be divided into classes that are mutually exclusive; that is, such that any given projection belongs in one class, and only in one. There are, however, certain class names that are made use of in practice principally as a matter of convenience, although a given projection may fall in two or more of the classes. We have already spoken of the equivalent or equal-area type and of the conformal, or, as it is sometimes called, the orthomorphic type.

The equal-area projection preserves the ratio of areas constant; that is, any given part of the map bears the same relation to the area that it represents that the whole map bears to the whole area represented. This can be brought clearly before the mind by the statement that any quadrangular-shaped section of the map formed by meridians and parallels will be equal in area to any other quadrangular area of the same map that represents an *equal* area on the earth. This means that all sections between two given parallels on any equal-area map formed by meridians that are equally spaced are equal in area upon the map just as they are equal in area on the earth. In another way, if two silver dollars are placed upon the map, one in one place and the other in any other part of the map the two areas upon the earth that are represented by the portions of the map covered by the silver dollars will be equal. Either of these tests forms a valid criterion provided that the areas selected may be situated on *any* portion of the map. There are other projections besides the equal-area ones in which the same results would be obtained on *particular* portions of the map.

A conformal projection is one in which the shape of any small section of the surface mapped is preserved on the map. The term orthomorphic, which is sometimes used in place of conformal, means right shape; but this term is somewhat misleading, since, if the area mapped is large, the shape of any continent or large country will not be preserved. The true condition for a conformal map is that the scale at any point be the same in all directions; the scale will change from point to point, but it will be independent of the azimuth at all points. The scale will be the same in all directions at a point if two directions upon the earth at right angles to one another are mapped in two directions that are also at right angles and along which the scale is the same. If, then, we have a projection in which the meridians and parallels of the earth are represented by curves that are perpendicular each to each, we need only to determine that the scale along the meridian is equal to that along the parallel. The meridians and parallels of the earth intersect at right angles, and a conformal projection preserves the angle of intersection of any two curves on the earth; therefore, the meridians of the map must intersect the parallels of the map at right angles. The one set of lines are then said to be the orthogonal trajectories of the other set. If the meridians and parallels of any map do not intersect at right angles in all parts of the map, we may at once conclude that it is not a conformal map.

Besides the equal-area and conformal projections we have already mentioned the azimuthal or, as they are sometimes called, the zenithal projections. In these the azimuth or direction of all points on the map as seen from some central point are the same as the corresponding azimuths or directions on the earth. This would be a very desirable feature of a map if it could be true for all points of the map as well as for the central point, but this could not be attained in any projection; hence the azimuthal feature is generally an incidental one unless the map is intended for some special purpose in which the directions from some one point are very important.

Besides these classes of projections there is another class called perspective projections, or, as they are sometimes called, geometric projections. The principle of these projections consists in the direct projection of the points of the earth by straight lines drawn through them from some given point. The projection is generally made upon a plane tangent to the sphere at the end of the diameter joining the point of projection and the center of the earth. If the projecting point is the center of the sphere, the point of tangency is chosen in the center of the area to be mapped. The plane upon which the map is made does not have to be tangent to the earth, but this position gives a simplification. Its position anywhere parallel to itself would only change the scale of the map and in any position not parallel to itself the same result would be obtained by changing the point of tangency with mere change of scale. Projections of this kind are generally simple, because they can in most cases be constructed by graphical methods without the aid of the analytical expressions that determine the elements of the projection.

Instead of using a plane directly upon which to lay out the projection, in many cases use is made of one of the developable surfaces as an intermediate aid. The two surfaces used for this purpose are the right circular cone and the circular cylinder. The projection is made upon one or the other of these two surfaces, and then this surface is spread out or developed in the plane. As a matter of fact, the projection is not constructed upon the cylinder or cone, but the principles are derived from a consideration of these surfaces, and then the projection is drawn upon the plane just as it would be after development. The developable surfaces, therefore, serve only as guides to us in grasping the principles of the projection. After the elements of the projections are determined, either geometrically or analytically, no further attention is paid to the cone or cylinder. A projection is called conical or cylindrical, according to which of the two developable surfaces is used in the determination of its elements. Both kinds are generally included in the one class of conical projections, for the cylinder is just a special case of the cone. In fact, even the azimuthal projections might have been included in the general class. If we have a cone tangent to the earth and then imagine the apex to recede more and more while the cone still remains tangent to the sphere, we shall have at the limit the tangent cylinder. On the other hand, if the apex approaches nearer and nearer to the earth the circle of tangency will get smaller and smaller, and in the end it will become a point and will coincide with the apex, and the cone will be flattened out into a tangent plane.

Besides these general classes there are a number of projections that are called conventional projections, since they are projections that are merely arranged arbitrarily. Of course, even these conform enough to law to permit their expression analytically, or sometimes more easily by geometric principles.

THE IDEAL MAP

There are various properties that it would be desirable to have present in a map that is to be constructed. (1) It should represent the countries with their true shape; (2) the countries represented should retain their relative size in the map; (3) the distance of every place from every other should bear a constant ratio to the true distances upon the earth; (4) great circles upon the sphere—that is, the shortest distances joining various points—should be represented by straight lines which are the shortest distances joining the points on the map; (5) the geographic latitudes and longitudes of the places should be easily found from their positions on the map, and, conversely, positions should be easily plotted on the map when we have their latitudes and longitudes. These properties could very easily be secured if the earth were a plane or one of the developable surfaces. Unfortunately for the cartographer, it is not such a surface, but is a spherical surface which cannot be developed in a plane without distortion of some kind. It becomes, then, a matter of selection from among the various desirable properties enumerated above, and even some of these can not in general be attained. It is necessary, then, to decide what purpose the map to be constructed is to fulfill, and then we can select the projection that comes nearest to giving us what we want.

PROJECTIONS CONSIDERED WITHOUT MATHEMATICS

If it is a question of making a map of a small section of the earth, it will so nearly conform to a plane surface that a projection can be made that will represent the true state to such a degree that any distortion present will be negligible. It is thus possible to consider the earth made up of a great number of plane sections of this kind, such that each of them could be mapped in this way. If the parallels and meridians are drawn each at 15° intervals and then planes are passed through the points of intersection, we should have a regular figure made up of plane quadrangular figures as in figure 13. Each of these sections could be made into a self-consistent map, but if we attempt to fit them together in one plane map, we shall find that they will not join together properly, but the effect shown in figure 13 will be observed. A section 15° square would be too large to be mapped without error, but the same principle could be applied to each square degree or to even smaller sections. This projection is called the polyhedral projection and it is in substance very similar to the method used by the United States Geological Survey in their topographic maps of the various States.

Instead of considering the earth as made up of small regular quadrangles, we might consider it made by narrow strips cut off from the bases of cones as in figure 14. The whole east-and-west extent of these strips could be mapped equally accurately as shown in figure 15. Each strip would be all right in itself, but they would not fit together, as is shown in figure 15. If we consider the strips to become very narrow while at the same time they increase in number, we get what is called the polyconic projections. These same difficulties or others of like nature are met with in every projection in which we attempt to hold the scale exact in some part. At best we can only adjust the errors in the representation, but they can never all be avoided.

Viewed from a strictly mathematical standpoint, no representation based on a system of map projection can be perfect. A map is a compromise between the various

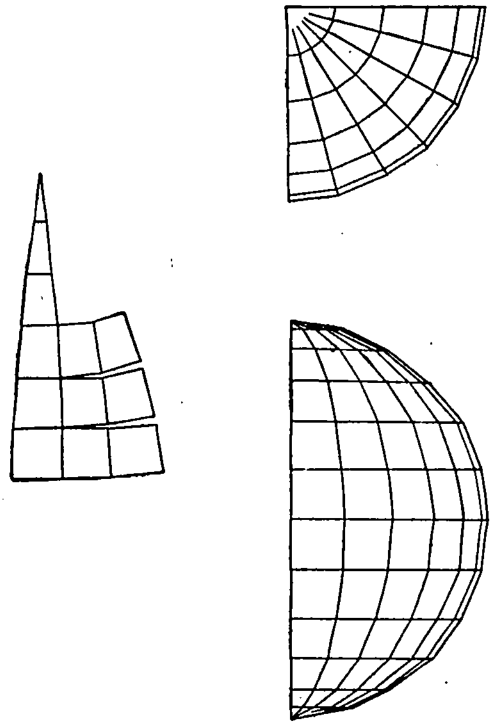

FIG. 13.—Earth considered as formed by plane quadrangles.

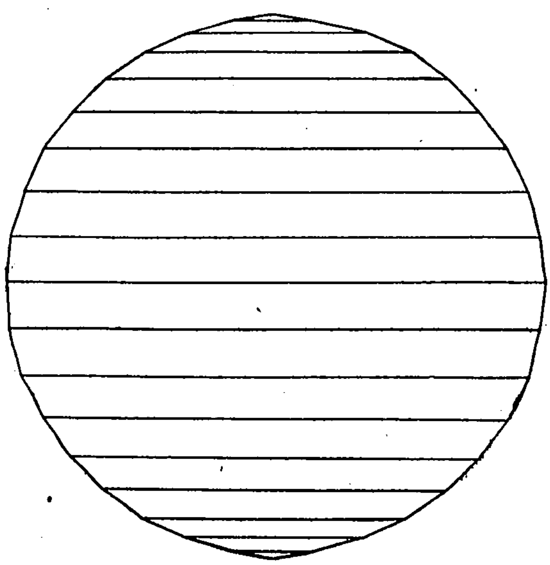

FIG. 14.—Earth considered as formed by frustrums of cones.

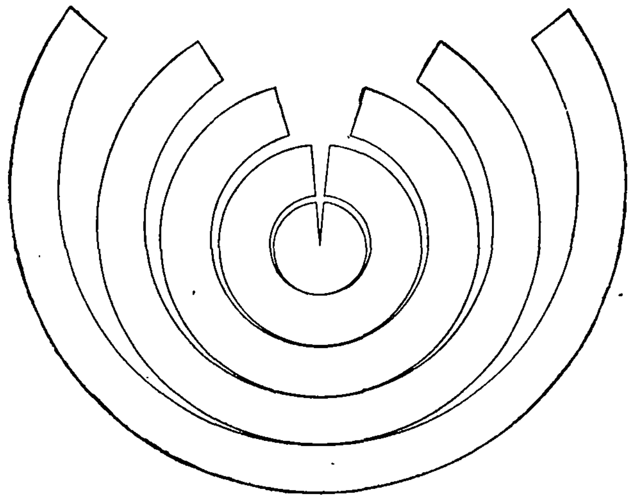

FIG. 15.—Development of the frustrum surfaces.

conditions not all of which can be satisfied, and is the best solution of the problem that is possible without encountering other difficulties that surpass those due to a varying scale and distortion of other kinds. It is possible only on a globe to represent the countries with their true relations and our general ideas should be continually corrected by reference to this source of knowledge.

In order to point out the distortion that may be found in projections, it will be well to show some of those systems that admit of easy construction. The perspective or geometrical projections can always be constructed graphically, but it is sometimes easier to make use of a computed table, even in projections of this class.

ELEMENTARY DISCUSSION OF VARIOUS FORMS OF PROJECTION

CYLINDRICAL EQUAL-AREA PROJECTION

This projection is one that is of very little use for the construction of a map of the world, although near the Equator it gives a fairly good representation. We shall use it mainly for the purpose of illustrating the modifications that can be introduced into cylindrical projections to gain certain desirable features.

In this projection a cylinder tangent to the sphere along the Equator is employed. The meridians and parallels are straight lines forming two parallel systems mutually perpendicular. The lines representing the meridians are equally spaced. These features are in general characteristic of all cylindrical projections in which the cylinder is supposed to be tangent to the sphere along the Equator. The only feature as yet undetermined is the spacing of the parallels. If planes are passed through the various parallels they will intersect the cylinder in circles that become straight lines when the cylinder is developed or rolled out in the plane. With this condition it is evident that the construction given in figure 16 will give the network of meridians and parallels for 10° intervals. The length of the map is evidently π (about $3\frac{1}{7}$) times the diameter of the circle that represents a great circle of the sphere. The semicircle is divided by means of a protractor into 18 equal arcs, and these points of division are projected by lines parallel to the line representing the Equator or perpendicular to the bounding diameter of the semicircle. This gives an equivalent or equal-area map, because, as we recede from the Equator, the distances representing differences of latitude are decreased just as great a per cent as the distances representing differences of longitude are increased. The result in a world map is the appearance of contraction toward the Equator, or, in another sense, as an east-and-west stretching of the polar regions.

CYLINDRICAL EQUAL-SPACED PROJECTION

If the equal-area property be disregarded, a better cylindrical projection can be secured by spacing the meridians and parallels equally. In this way we get rid of the very violent distortions in the polar regions, but even yet the result is very unsatisfactory. Great distortions are still present in the polar regions, but they are much less than before, as can be seen in figure 17. As a further attempt, we can throw part of the distortion into the equatorial regions by spacing the parallels equally and the meridians equally, but by making the spacing of the parallels greater than that of the meridians. In figure 18 is shown the whole world with the meridian and parallel spacings in the ratio of two to three. The result for a world map is still highly unsatisfactory even though it is slightly better than that obtained by either of the former methods.

PROJECTION FROM THE CENTER UPON A TANGENT CYLINDER

As a fourth attempt we might project the points by lines drawn from the center of the sphere upon a cylinder tangent to the Equator. This would have a tendency to stretch the polar regions north and south as well as east and west. The result of this method is shown in figure 19, in which the polar regions are shown up to 70° of latitude. The poles could not be shown, since as the projecting line approaches them indefinitely, the required intersection with the cylinder recedes indefinitely, or, in mathematical language, the pole is represented by a line at an infinite distance.

MERCATOR PROJECTION

Instead of stretching the polar regions north and south to such an extent, it is customary to limit the stretching in latitude to an equality with the stretching in longitude. (See fig. 20.) In this way we get a conformal projection in which any small area is shown with practically its true shape, but in which large areas will be distorted by the change in scale from point to point. In this projection the pole is represented by a line at infinity, so that the map is seldom extended much beyond 80° of latitude. This projection can not be obtained directly by graphical construction, but the spacings of the parallels have to be taken from a computed table. This is the most important of the cylindrical projections and is widely used for the construction of sailing charts. Its common use for world maps is very misleading, since the polar regions are represented upon a very enlarged scale.

Since a degree is one three-hundred-and-sixtieth part of a circle, the degrees of latitude are everywhere equal on a sphere, as the meridians are all equal circles. The degrees of longitude, however, vary in the same proportion as the sizes of the parallels vary at the different latitudes. The parallel of 60° latitude is just one-half of the

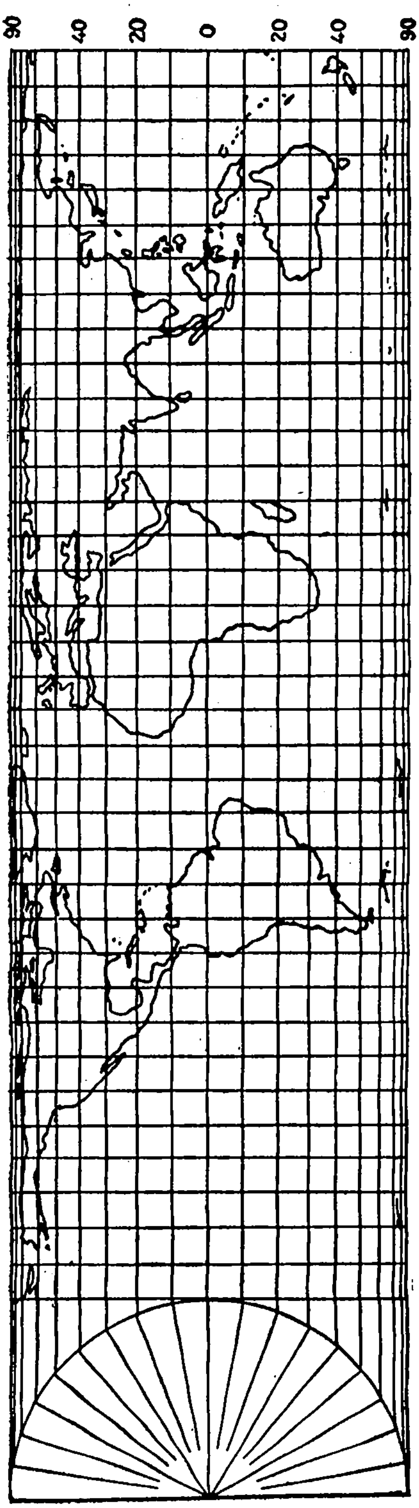

FIG. 16.—Cylindrical equal-area projection.

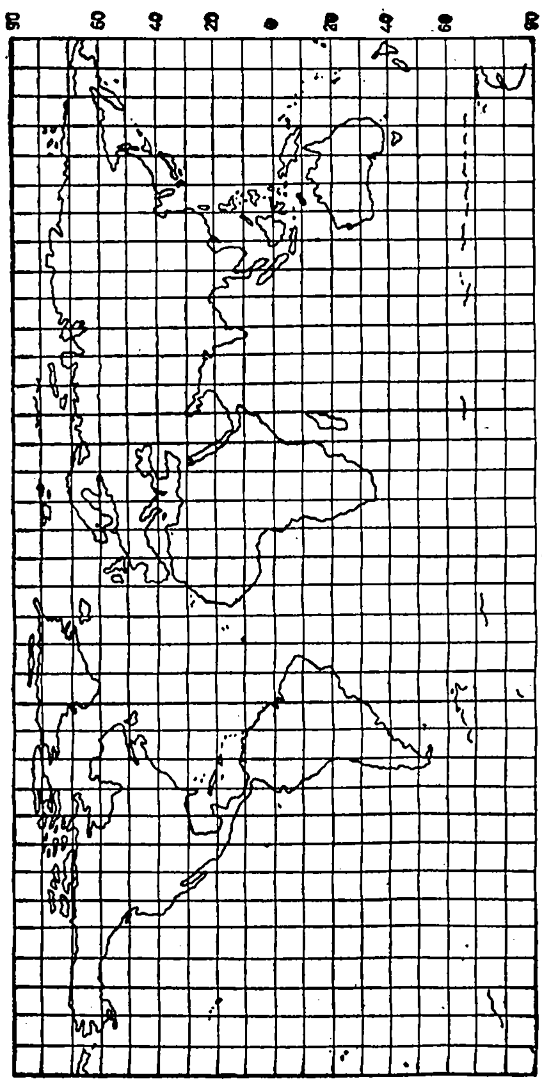

FIG. 17.—Cylindrical equal-spaced projection.

length of the Equator. A square-degree quadrangle at 60° of latitude has the same length north and south as has such a quadrangle at the Equator, but the extent east and west is just one-half as great. Its area, then, is approximately one-half the area of the one at the Equator. Now, on the Mercator projection the longitude at 60° is stretched to double its length, and hence the scale along the meridian has to be increased an equal amount. The area is therefore increased fourfold. At 80° of latitude the area is increased to 36 times its real size, and at 89° an area would be more than 3000 times as large as an equal-sized area at the Equator.

This excessive exaggeration of area is a most serious matter if the map be used

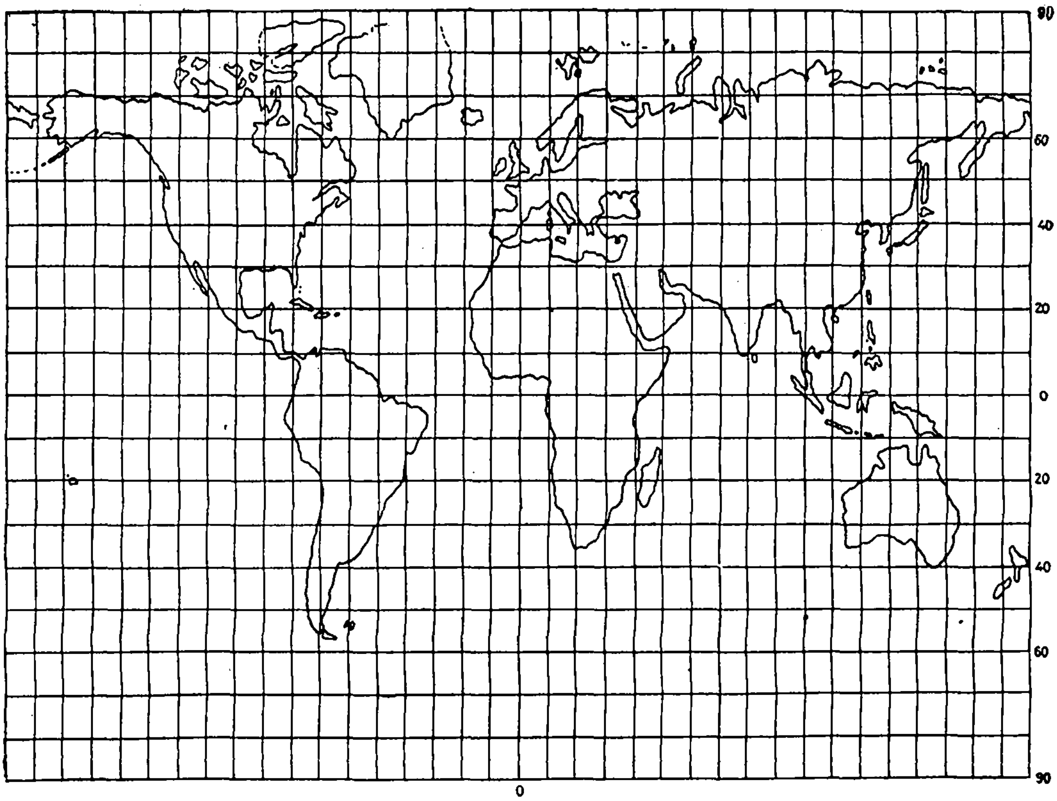

FIG. 18.—Modified cylindrical equal-spaced projection.

for general purposes, and this fact ought to be emphasized because it is undoubtedly true that in the majority of cases peoples' general ideas of geography are based on Mercator maps. On the map Greenland shows larger than South America, but in reality South America is nine times as large as Greenland. As will be shown later, this projection has many good qualities for special purposes, and for some general purposes it may be used for areas not very distant from the Equator. No suggestion is therefore made that it should be abolished, or even reduced from its position among the first-class projections, but it is most strongly urged that no one should use it without recognizing its defects, and thereby guarding against being misled by false appearances. This projection is often used because on it the whole inhabited world can be

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shown on one sheet, and, furthermore, it can be prolonged in either an east or west direction; in other words, it can be repeated so as to show part of the map twice. By this means the relative positions of two places that would be on opposite sides of the projection when confined to 360° can be indicated more definitely.

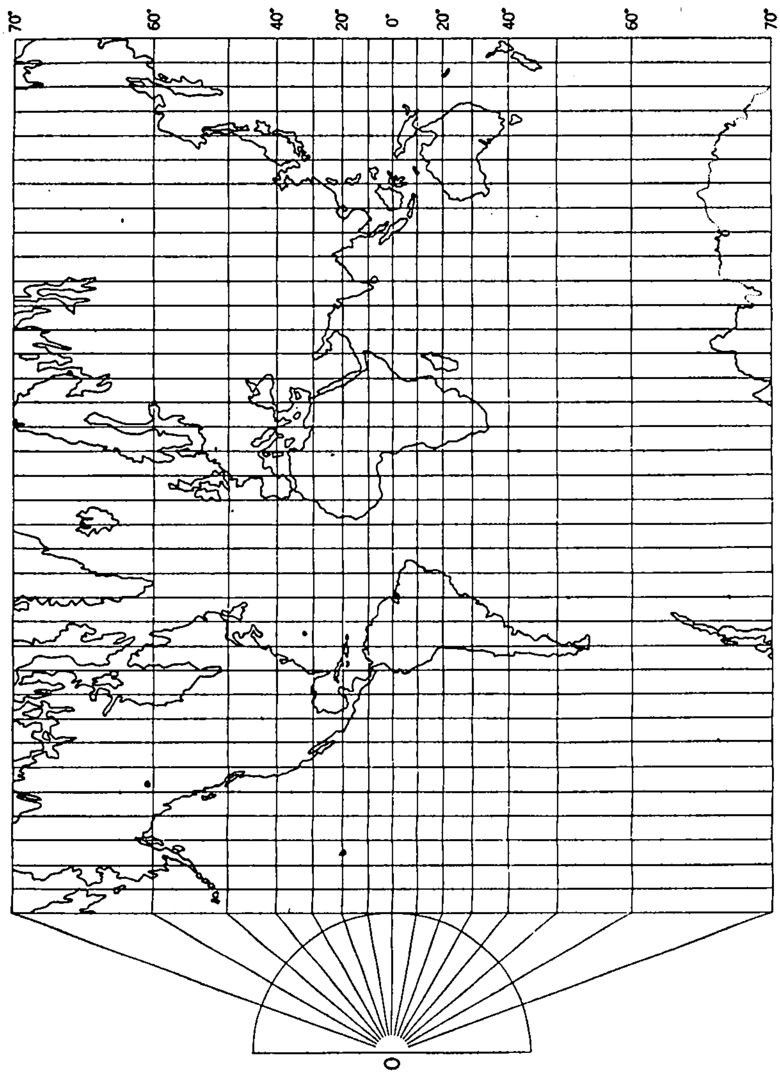

FIG. 19.—Perspective projection upon a tangent cylinder. This projection should not be interpreted as a Mercator projection, a sketch of which is given in fig. 20.

COMPARISON OF THE PERSPECTIVE PROJECTION WITH THE MERCATOR PROJECTION

In Fig. 19 we have a semicircle described from O, with radius representing the sphere. By drawing lines from the center at O through the equal divisions of the semicircle to the tangent line, we obtain the spacing of the parallels of a *perspective projection* upon a tangent cylinder.

In Fig. 20, from a semicircle with center at O′, the radius being the same as before,

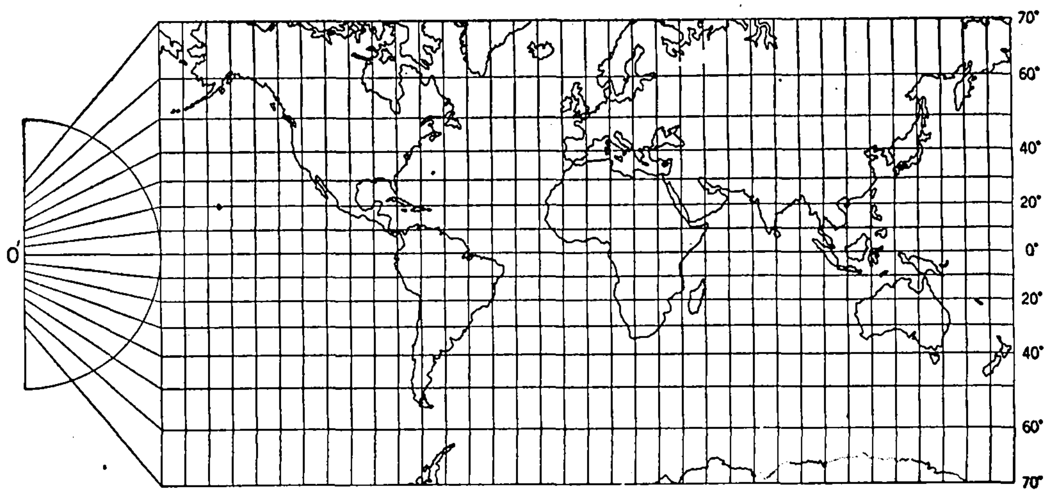

FIG. 20.—Mercator projection—a comparison.

let us plot to scale the intervals of the parallels along the tangent line *from* a Mercator projection. We find from the nature of this projection that the projecting lines passing through the equal divisions of the semicircle originate, not at the central point O′, but at increasing distances from it, as shown in the figure. While these points along the polar and equatorial axes (see also Fig. 21), can be computed and located, and the successive parallels determined, it is nevertheless difficult to see what useful purpose this would serve.

It can thus be seen that it is practically impossible to visualize the Mercator projection as a direct geometrical projection because of the variable distances between the points in the axis from which the various parallels are projected.

Mercator refers to his representation simply as "*a new proportion and a new arrangement* of the meridians with reference to the parallels"—a mathematical navigational *device* to make the *rhumb line* or loxodromic curve *a right line* on the chart, preserving the same angle of bearing with respect to the intersected meridians as does the track of a vessel under a true course.

In this mathematical transformation, Mercator did not employ a tangent cylinder nor is it ever employed in this projection. This conception did not originate with him, but is a later invention. The statements to this effect that appear in many textbooks may be dismissed as erroneous and misleading.

In confirmation of what has here been expressed as to the nature of this projection, it is interesting to note that Edward Wright, author of the tables published in "Certaine Errors in Navigation," 1599, and likewise the Admiralty Manual of Navigation, 1914, did not employ the cylinder in their principles of computation and construction of Mercator's chart.

From a certain viewpoint, Fig. 21 has a pictorial value in showing that the parallels are not projected from a common point, i. e., the center of the sphere. As a forward step in the computation and construction of a Mercator projection, it has no practical value, and only serves to illustrate the confusion that has so often arisen as to the nature of the Mercator projection.

The figure shows the spacing of the parallels of a Mercator projection on a tangent cylinder as related to the polar and equatorial axes of the sphere. From the sketch below it is obvious that the Mercator projection does not result from a geometric projection upon a cylinder tangent at the equator.

It will be seen that the projecting lines to the parallels do not originate from the center of the sphere and that their location in the polar and equatorial axes requires unnecessary computation.

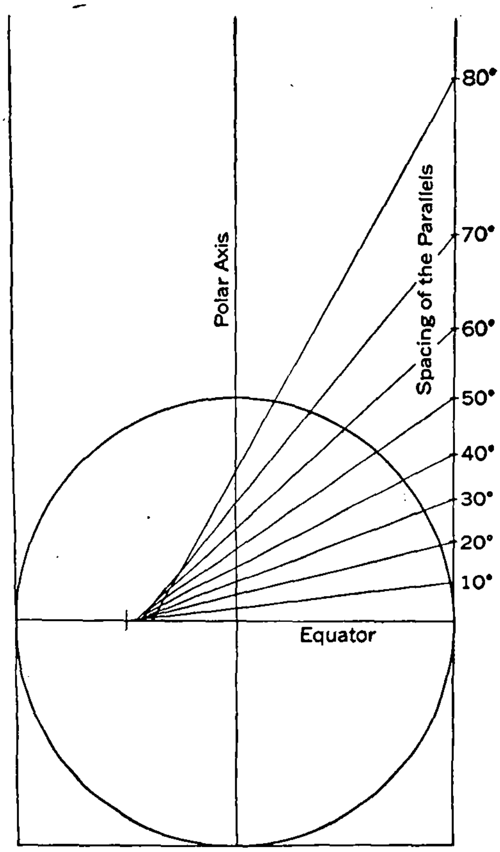

FIG. 21.—Mercator projection showing spacing of the parallels.

In some projections, such as the Mercator projection, the reference lines of latitude and longitude are in *no sense whatever projected.* In practice the Mercator projection is computed as a conformal projection upon a plane without resort to the cylinder or to the location of points in either of the axes. Many other projections are not constructed upon a cylinder or a cone, but a picture of their principles may, in a general way, be more readily referred to a consideration of geometric surfaces than is the case of a Mercator projection.

GEOMETRICAL AZIMUTHAL PROJECTIONS

Many of the projections of this class can be constructed graphically with very little trouble. This is especially true of those that have the pole at the center. The meridians are then represented by straight lines radiating from the pole and the parallels are in turn represented by concentric circles with the pole as center. The angles between the meridians are equal to the corresponding longitudes, so that they are represented by radii that are equally spaced.

STEREOGRAPHIC POLAR PROJECTION

This is a perspective conformal projection with the point of projection at the South Pole when the northern regions are to be projected. The plane upon which the projection is made is generally taken as the equatorial plane. A plane tangent at the North Pole could be used equally well, the only difference being in the scale of the projection. In figure 22 let *N E S W* be the plane of a meridian with *N* representing the North Pole. Then *N P* will be the trace of the plane tangent at the North Pole. Divide the arc *N E* into equal parts, each in the figure being for 10° of latitude. Then all points at a distance of 10° from the North Pole will lie on a circle with radius *Np*,

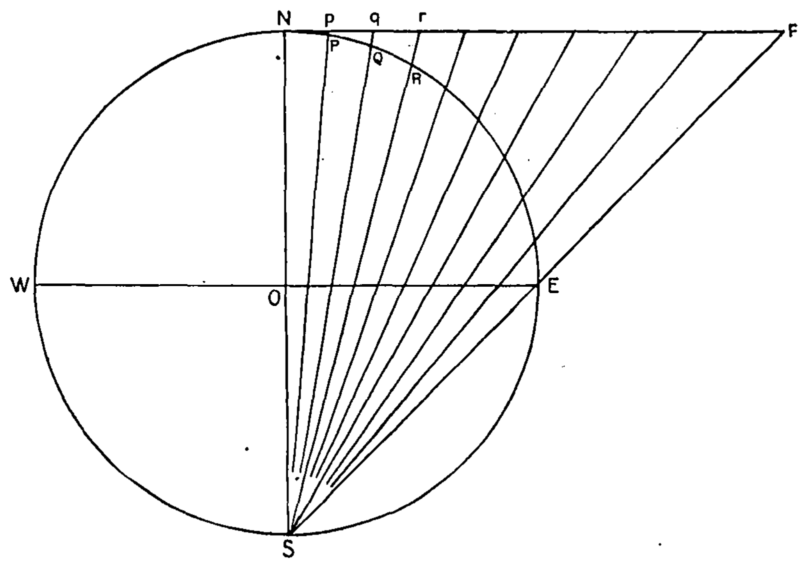

FIG. 22.—Determination of radii for stereographic polar projection.

those at 20° on a circle with radius *Nq*, etc. With these radii we can construct the map as in figure 23. On the map in this figure the lines are drawn for each 10° both in latitude and longitude; but it is clear that a larger map could be constructed on which lines could be drawn for every degree. We have seen that a practically correct map can be made for a region measuring 1° each way, because curvature in such a size is too slight to be taken into account. Suppose, then, that correct maps were made separately of all the little quadrangular portions. It would be found that by simply reducing each of them to the requisite scale it could be fitted almost exactly into the space to which it belonged. We say almost exactly, because the edge nearest the center of the map would have to be a little smaller in scale, and hence would have to be compressed a little if the outer edges were reduced the exact amount, but the compression would be so slight that it would require very careful measurement to detect it.

It would seem, then, at first sight that this projection is an ideal one, and, as a matter of fact, it is considered by most authorities as the best projection of a hemisphere for general purposes, but, of course, it has a serious defect. It has been stated that each plan has to be compressed at its inner edge, and for the same reason each plan in succession has to be reduced to a smaller average scale than the one outside of it. In other words, the *shape* of each space into which a plan has to be fitted is practically correct, but the size is less in proportion at the center than at the edges; so that if a correct plan of an area at the edge of the map has to be reduced, let us say to a scale of 500 miles to an inch to fit its allotted space, then a plan of an area at the center has to be reduced to a scale of more than 500 miles to an inch. Thus a moderate area has its true shape, and even an area as large as one of the States is not distorted to such an extent as to be visible to the ordinary observer, but to obtain this advantage relative size has to be sacrificed; that is, the property of equivalence of area has to be entirely disregarded.

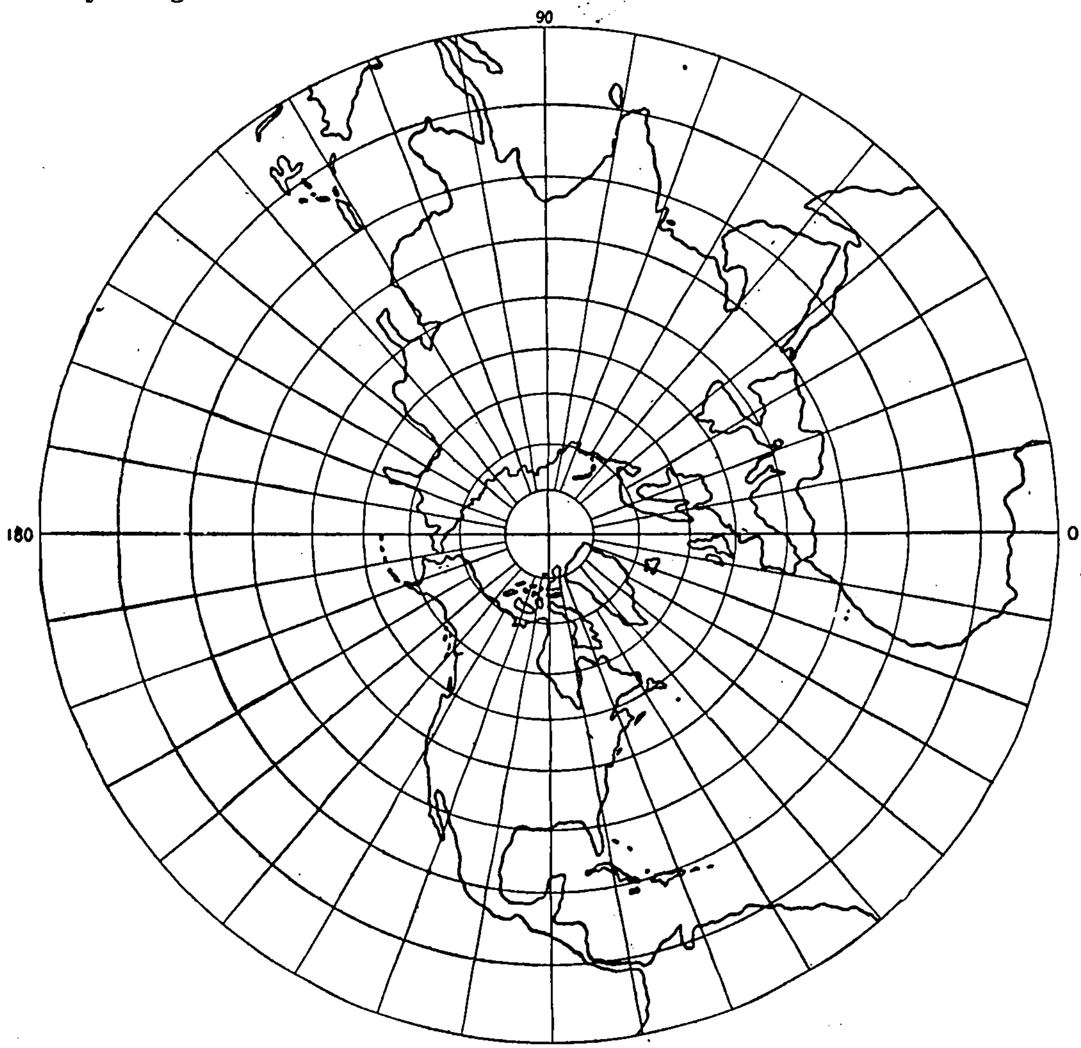

FIG. 23.—Stereographic polar projection.

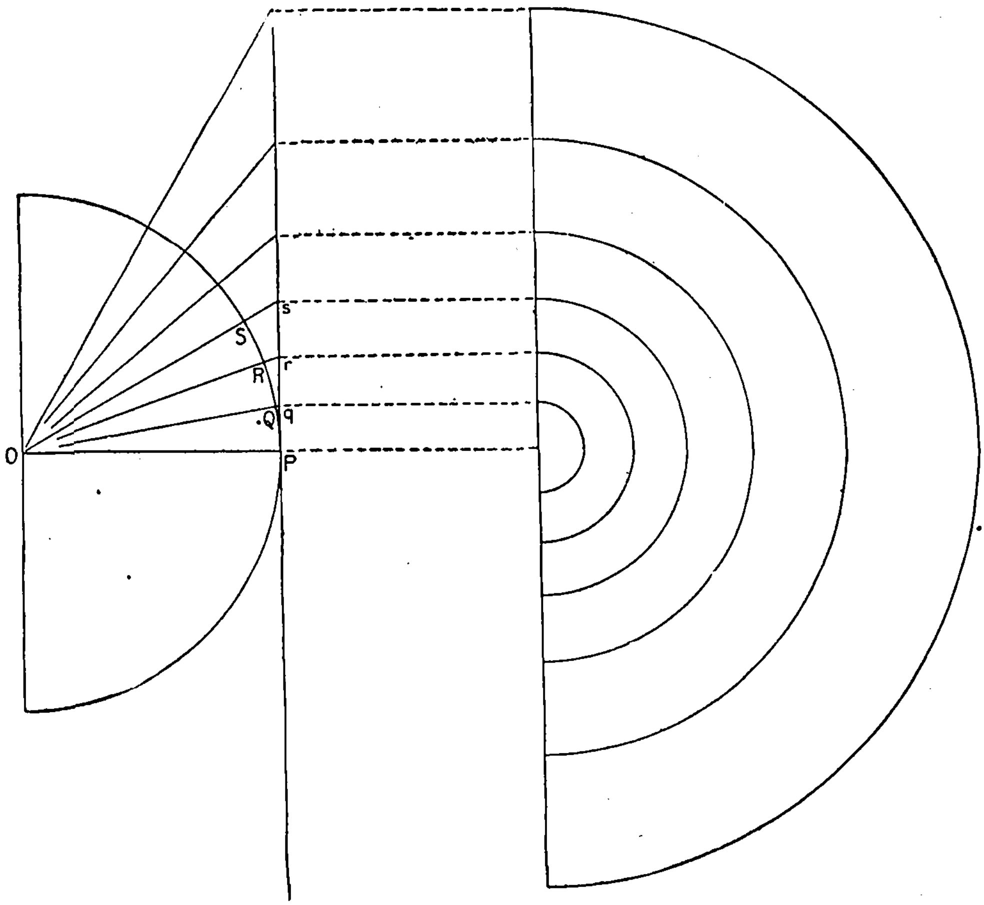

FIG. 24.—Determination of radii for gnomonic polar projection.

CENTRAL OR GNOMONIC PROJECTION

In this projection the center of the sphere is the point from which the projecting lines are drawn and the map is made upon a tangent plane. When the plane is tangent at the pole, the parallels are circles with the pole as common center and the meridians are equally spaced radii of these circles. In figure 24 it can be seen that the lengths of the various radii of the parallels are found by drawing lines from the center of a circle representing a meridian of the sphere and by prolonging them to intersect a tangent line. In the figure let P be the pole and let PQ, QR, etc., be arcs of 10°, then Pq, Pr, etc., will be the radii of the corresponding parallels. It is at once evident that a complete hemisphere can not be represented upon a plane, for the radius of 90° from the center would become infinite. The North Pole regions extending to latitude 30° is shown in figure 25.

The important property of this projection is the fact that all great circles are represented by straight lines. This is evident from the fact that the projecting lines would all lie in the plane of the circle and the circle would be represented by the intersection of this plane with the mapping plane. Since the shortest distance be-

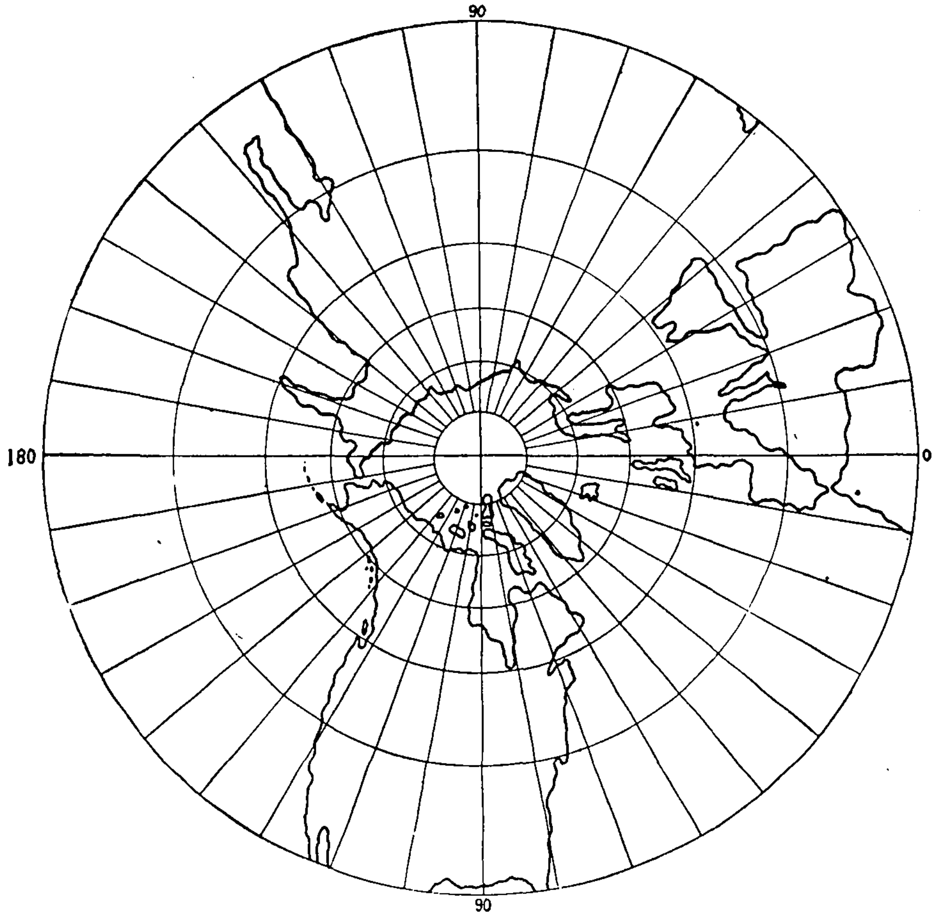

FIG. 25.—Gnomonic polar projection.

tween two given points on the sphere is an arc of a great circle, the shortest distance between the points on the sphere is represented on the map by the straight line joining the projection of the two points which, in turn, is the shortest distance joining the projections; in other words, shortest distances upon the sphere are represented by shortest distances upon the map. The change of scale in the projection is so rapid that very violent distortions are present if the map is extended any distance. A map of this kind finds its principal use in connection with the Mercator charts, as will be shown in the second part of this publication.

LAMBERT AZIMUTHAL EQUAL-AREA PROJECTION

This projection does not belong in the perspective class, but when the pole is the center it can be easily constructed graphically. The radius for the circle representing a parallel is taken as the chord distance of the parallel from the pole. In figure 26 the chords are drawn for every 10° of arc, and figure 27 shows the map of the Northern Hemisphere constructed with these radii.

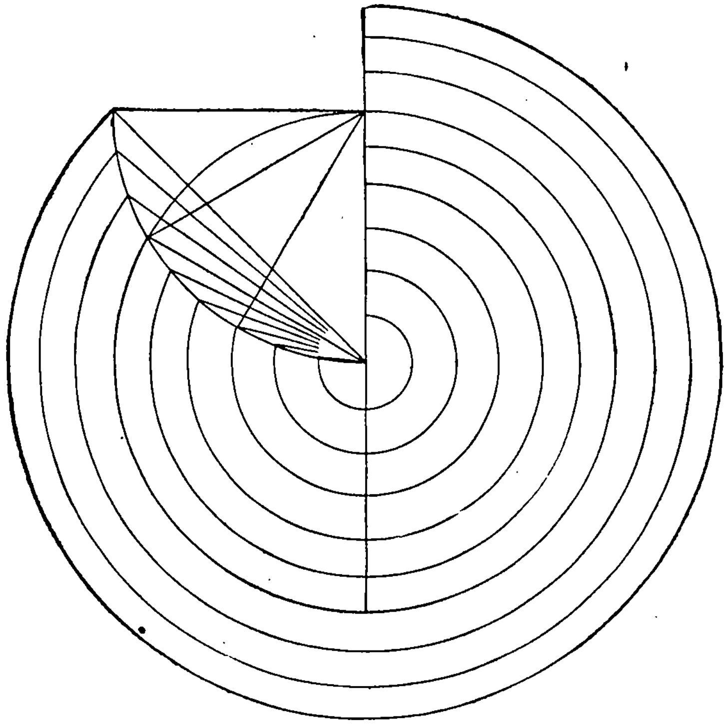

FIG. 26.—Determination of radii for Lambert equal-area polar projection.

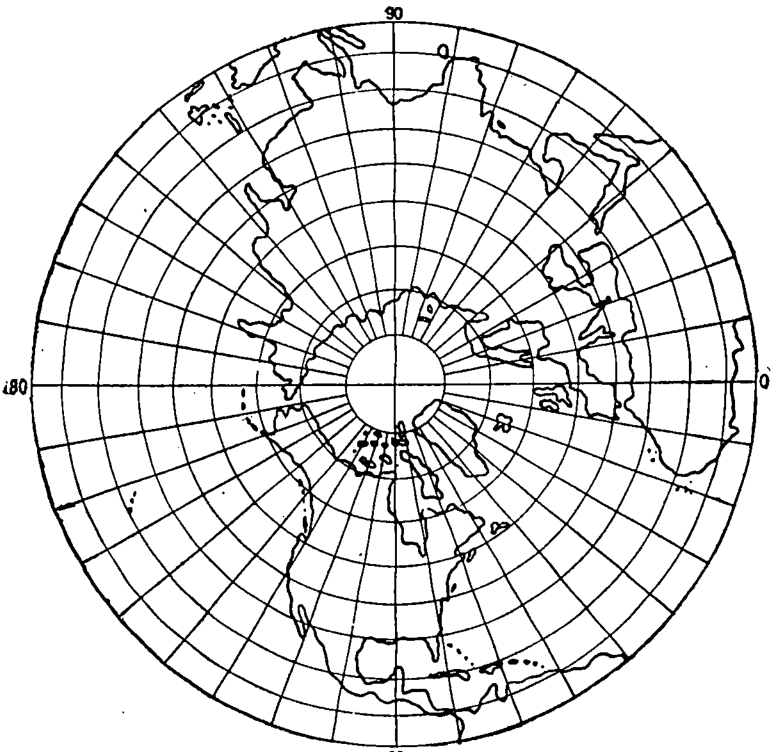

FIG. 27.—Lambert equal-area polar projection.

ORTHOGRAPHIC POLAR PROJECTION

When the pole is the center, an orthographic projection may be constructed graphically by projecting the parallels by parallel lines. It is a perspective projection in which the point of projection has receded indefinitely, or speaking mathematically, the point of projection is at infinity. Each parallel is really constructed with a radius

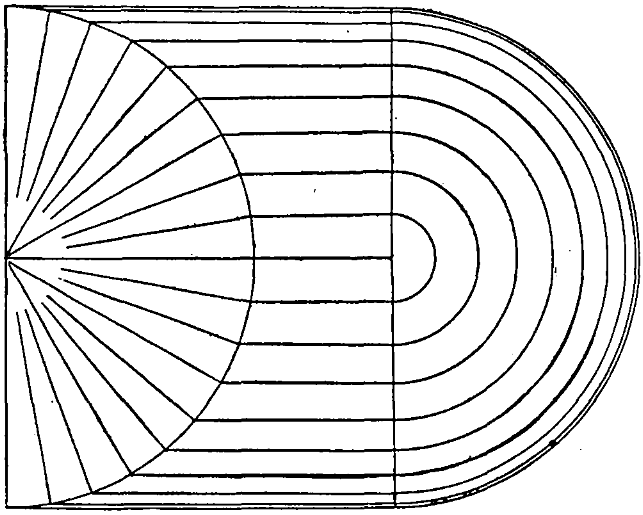

FIG. 28.—Determination of radii for orthographic polar projection.

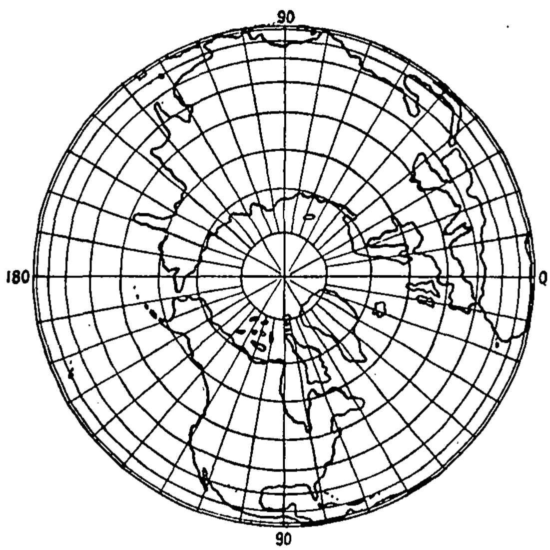

FIG. 29.—Orthographic polar projection.

proportional to its radius on the sphere. It is clear, then, that the scale along the parallels is unvarying, or, as it is called, the parallels are held true to scale. The method of construction is indicated clearly in figure 28, and figure 29 shows the Northern Hemisphere on this projection. Maps of the surface of the moon are usually constructed on this projection, since we really see the moon projected upon the celestial sphere practically as the map appears.

AZIMUTHAL EQUIDISTANT PROJECTION

In the orthographic polar projection the scale along the parallels is held constant, as we have seen. We can also have a projection in which the scale along the meridians is held unvarying. If the parallels are represented by concentric circles equally spaced, we shall obtain such a projection. The projection is very easily constructed, since we need only to draw the system of concentric, equally spaced circles with the meridians represented, as in all polar azimuthal projections, by the equally spaced radii of the system of circles. Such a map of the Northern Hemisphere is shown in figure 30. This projection has the advantage that it is somewhat a mean between the stereographic and the equal area. On the whole, it gives a fairly good representation, since it stands as a compromise between the projections that cause distortions of opposite kind in the outer regions of the maps.

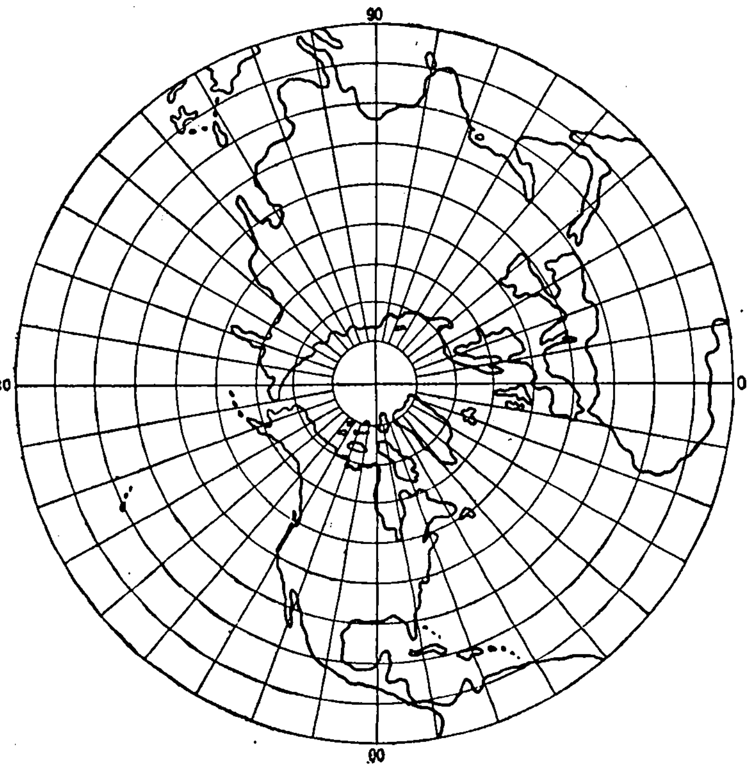

FIG. 30.—Azimuthal equidistant polar projection.

OTHER PROJECTIONS IN FREQUENT USE

In figure 31 the Western Hemisphere is shown on the stereographic projection. A projection of this nature is called a meridional projection or a projection on the plane of a meridian, because the bounding circle represents a meridian and the North and South Poles are shown at the top and the bottom of the map, respectively. The central meridian is a straight line and the Equator is represented by another straight line perpendicular to the central meridian; that is, the central meridian and the Equator are two perpendicular diameters of the circle that represents the outer meridian and that forms the boundary of the map.

FIG. 31.—Stereographic projection of the Western Hemisphere.

In figure 32 a part of the Western Hemisphere is represented on a gnomonic projection with a point on the Equator as the center.

A meridional equal-area projection of the Western Hemisphere is shown in figure 33.

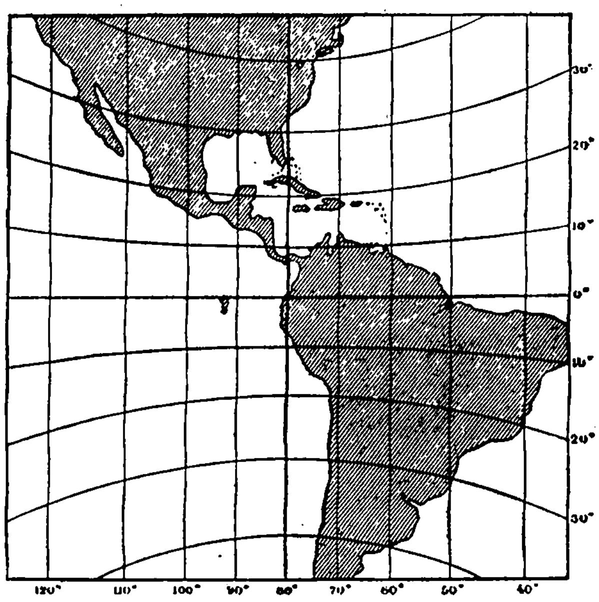

FIG. 32.—Gnomonic projection of part of the Western Hemisphere.

An orthographic projection of the same hemisphere is given in figure 34. In this the parallels become straight lines and the meridians are arcs of ellipses.

A projection that is often used in the mapping of a hemisphere is shown in figure 35. It is called the globular projection. The outer meridian and the central meridian are divided each into equal parts by the parallels which are arcs of circles. The Equator is also divided into equal parts by the meridians, which in turn are arcs of circles. Since all of the meridians pass through each of the poles, these conditions are sufficient to determine the projection. By comparing it with the stereographic it will be seen that

FIG. 33.—Lambert equal-area projection of the Western Hemisphere.

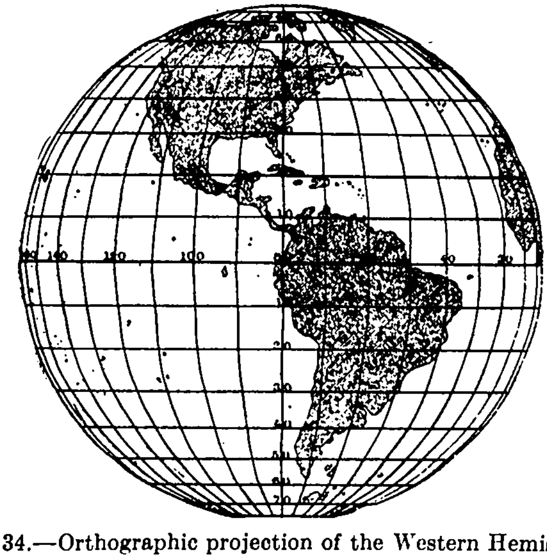

FIG. 34.—Orthographic projection of the Western Hemisphere.

the various parts are not violently sheared out of shape, and a comparison with the equal-area will show that the areas are not badly represented. Certainly such a representation is much less misleading than the Mercator which is too often employed in the school geographies for the use of young people.

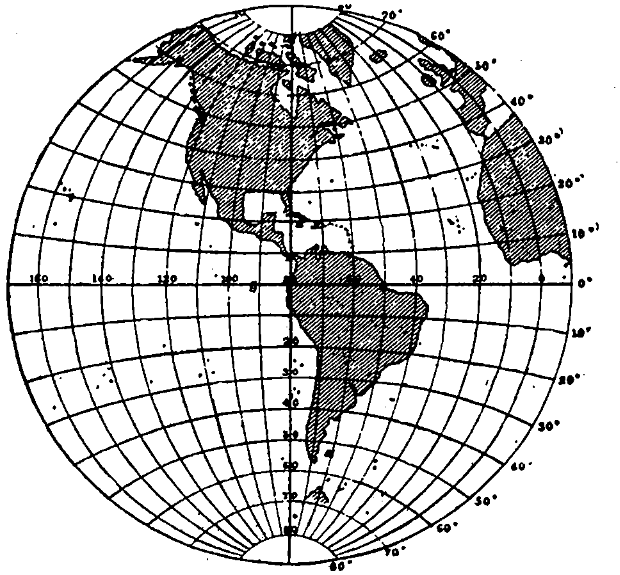

FIG. 35.—Globular projection of the Western Hemisphere.

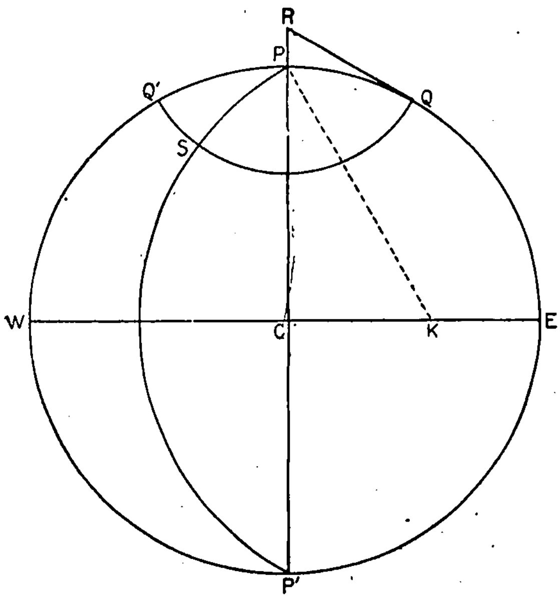

FIG. 36.—Determination of the elements of a stereographic projection on the plane of a meridian.

CONSTRUCTION OF A STEREOGRAPHIC MERIDIONAL PROJECTION

Two of the projections mentioned under the preceding heading—the stereographic and the gnomonic—lend themselves readily to graphic construction. In figure 36 let the circle *PQP′* represent the outer meridian in the stereographic projection. Take the arc *PQ*, equal to 30°; that is, *Q* will lie in latitude 60°. At *Q* construct the tangent *RQ;* with *R* as a center, and with a radius *RQ* construct the arc *QSQ′*. This arc represents the parallel of latitude 60°. Lay off OK equal to RQ; with K as a center, and with a radius KP construct the arc PSP'; then this arc represents the meridian of longitude 60° reckoned from the central meridian POP'. In the same way all the meridians and parallels can be constructed so that the construction is very simple. Hemispheres constructed on this projection are very frequently used in atlases and geographies.

CONSTRUCTION OF A GNOMONIC PROJECTION WITH POINT OF TANGENCY ON THE EQUATOR

In figure 37 let $PQP'Q'$ represent a great circle of the sphere. Draw the radii OA, OB, etc., for every 10° of arc. When these are prolonged to intersect the tangent at P, we get the points on the equator of the map where the meridians intersect it. Since the meridians of the sphere are represented by parallel straight lines perpendicular to the straight-line equator, we can draw the meridians when we know their points of intersection with the equator.

The central meridian is spaced in latitude just as the meridians are spaced on the equator. In this way we determine the points of intersection of the parallels with the central meridian. The projection is symmetrical with respect to the central meridian and also with respect to the equator. To determine the points of intersection of the parallels with any meridian, we proceed as indicated in figure 37, where the determination is made for the meridian 30° out from the central meridian. Draw CK perpendicular to OC; then CD', which equals CD, determines D', the intersection of the parallel of 10° north with the meridian of 30° in longitude east of the central meridian. In like manner $CE'=CE$, and so on. These same values can be transferred to the meridian of 30° in longitude west of the central meridian. Since the projection is symmetrical to the equator, the spacings downward on any meridian are the same as those upward on the same meridian. After the points of intersection of the parallels with the various meridians are determined, we can draw a smooth curve through those that lie on any given parallel, and this curve will represent the parallel in question. In this way the complete projection can be constructed. The distortions in this projection are very great, and the representation must always be less than a hemisphere, because the projection extends to infinity in all directions. As has already been stated, the projection is used in connection with Mercator sailing charts to aid in plotting great-circle courses.

CONICAL PROJECTIONS

In the conical projections, when the cone is spread out in the plane, the 360 degrees of longitude are mapped upon a sector of a circle. The magnitude of the angle at the center of this sector has to be determined by computation from the condition imposed upon the projection. Most of the conical projections are determined analytically; that is, the elements of the projection are expressed by mathematical formulas instead of being determined projectively. There are two classes of conical projections—one called a projection upon a tangent cone and another called a projection upon a secant cone. In the first the scale is held true along one parallel and in the second the scale is maintained true along two parallels.

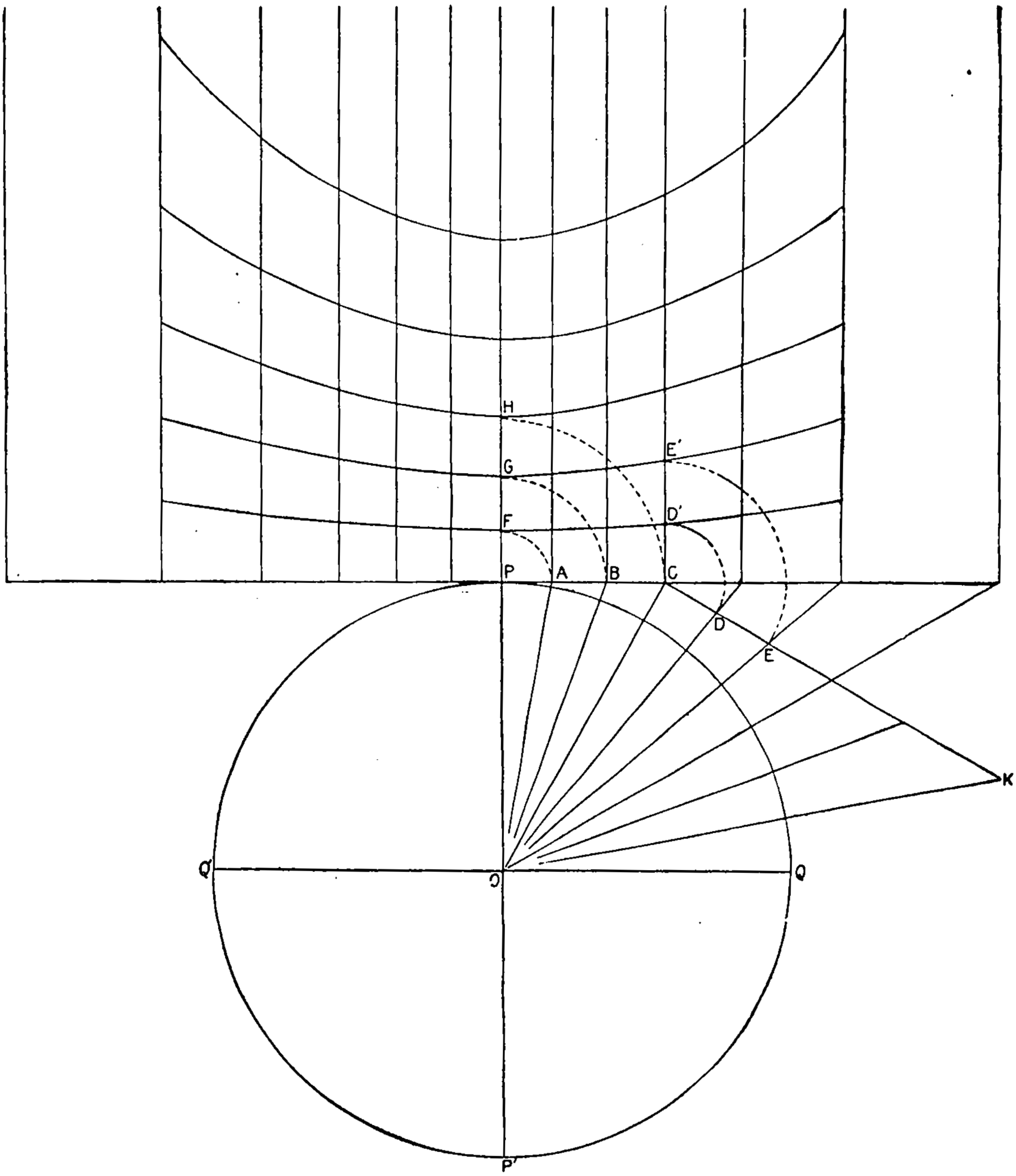

FIG. 37.—Construction of a gnomonic projection with plane tangent at the Equator.

CENTRAL PROJECTION UPON A CONE TANGENT AT LATITUDE 30°

As an illustration of conical projections we shall indicate the construction of one which is determined by projection from the center upon a cone tangent at latitude 30°. (See fig. 38). In this case the full circuit of 360° of longitude will be mapped upon a semicircle. In figure 39 let $PQP'Q'$ represent a meridian circle; draw CB tangent to the circle at latitude 30°, then CB is the radius for the parallel of 30° of latitude on the projection. CR, CS, CT, etc., are the radii for the parallels of 80°, 70°, 60°, etc., respectively. The map of the Northern Hemisphere on this projection is shown in figure 40; this is, on the whole, not a very satisfactory projection, but it serves to illustrate some of the principles of conical projection. We might determine the radii for the parallels by extending the planes of the same until they intersect the cone. This would vary the spacings of the parallels, but would not change the sector on which the projection is formed.

A cone could be made to intersect the sphere and to pass through any two chosen parallels. Upon this we could project the sphere either from the center or from any other point that we might choose. The general appearance of the projection would

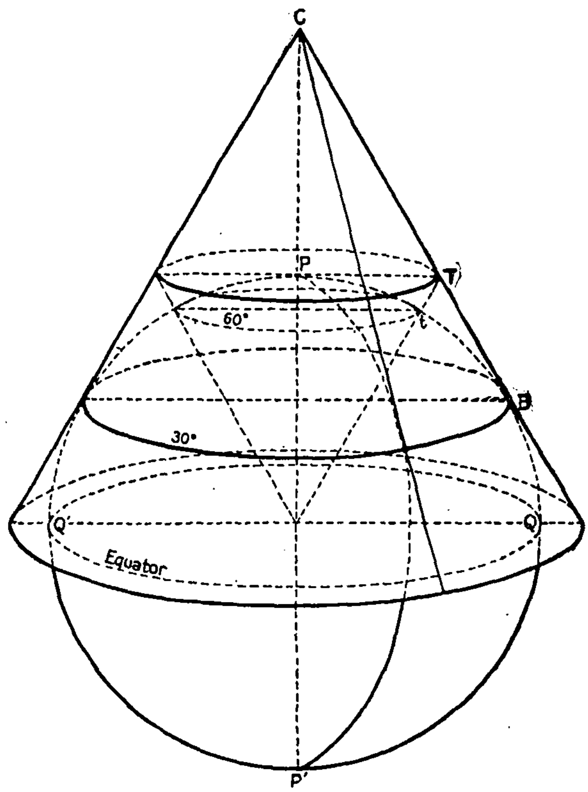

Fig. 38.—Cone tangent to the sphere at latitude 30°.

be similar to that of any conical projection, but some computation would be required for its construction. As has been stated, almost all conical projections in use have their elements determined analytically in the form of mathematical formulas. Of these the one with two standard parallels is not, in general, an intersecting cone, strictly speaking. Two separate parallels are held true to scale, but if they were held equal in length to their length on the sphere the cone could not, in general, be made to intersect the sphere so as to have the two parallels coincide with the circles that represent them. This could only be done in case the distance between the two circles on the cone was equal to the chord distance between the parallels on the sphere. This would be true

600106°—45——4

in a perspective projection, but it would ordinarily not be true in any projection determined analytically. Probably the two most important conical projections are the Lambert conformal conical projection with two standard parallels and the Albers equal-area conical projection. The latter projection has also two standard parallels.

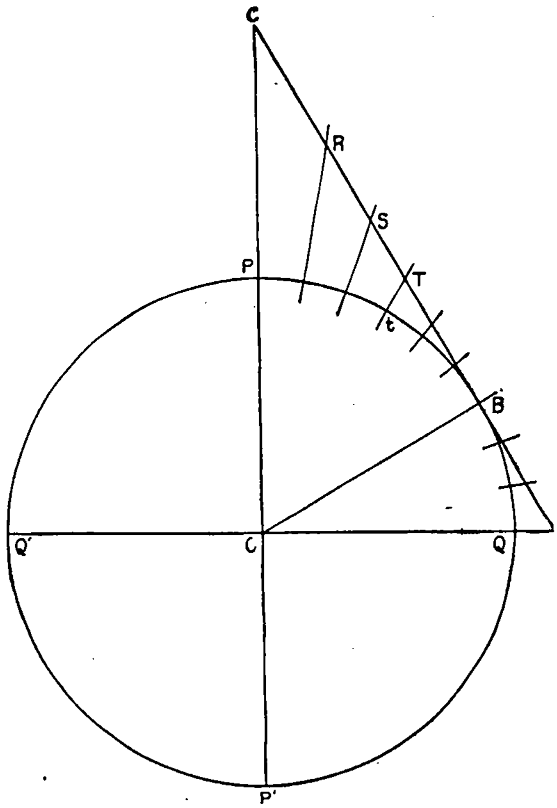

FIG. 39.—Determination of radii for conical central perspective projection.

BONNE PROJECTION

There is a modified conical equal-area projection that has been much used in map making called the Bonne projection. In general a cone tangent along the parallel in the central portion of the latitude to be mapped gives the radius for the arc representing this parallel. A system of concentric circles is then drawn to represent the other parallels with the spacings along the central meridian on the same scale as that of the standard parallel. Along the arcs of these circles the longitude distances are laid off on the same scale in both directions from the central meridian, which is a straight line. All of the meridians except the central one are curved lines concave toward the straight-line central meridian. This projection has been much used in atlases partly because it is equal-area and partly because it is comparatively easy to construct. A map of the United States is shown in figure 41 on this projection.

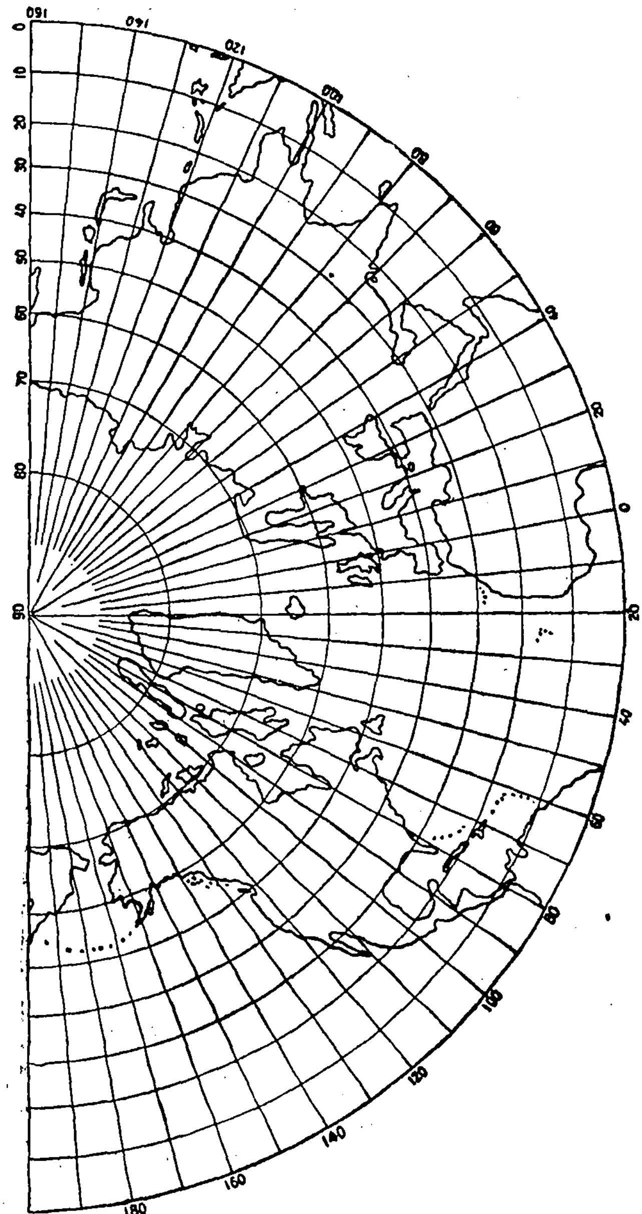

Fig. 40.—Central perspective projection on cone tangent at latitude 30°.

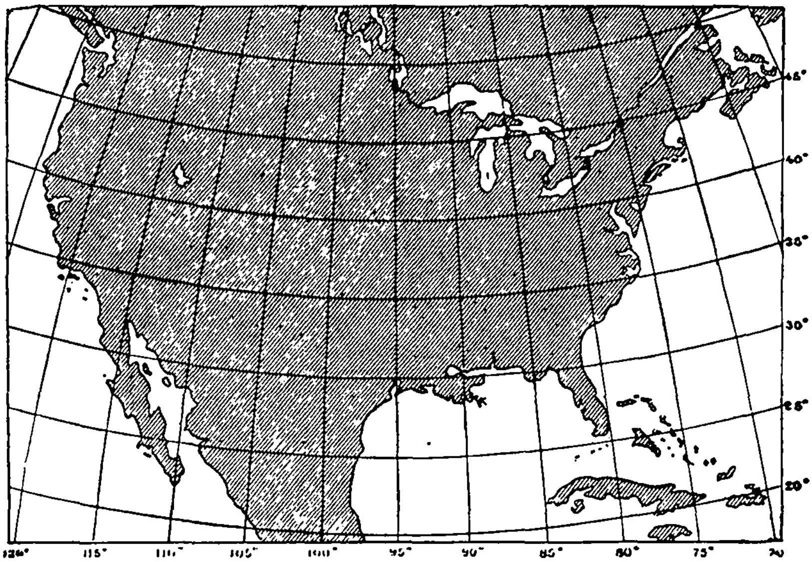

FIG. 41.—Bonne projection of the United States.

POLYCONIC PROJECTION

In the polyconic projection the central meridian is represented by a straight line and the parallels are represented by arcs of circles that are not concentric, but the centers of which all lie in the extension of the central meridian. The distances between the parallels along the central meridian are made proportional to the true distances between the parallels on the earth. The radius for each parallel is determined by an element of the cone tangent along the given parallel. When the parallels are constructed in this way, the arcs along the circles representing the parallels are laid off proportional to the true lengths along the respective parallels. Smooth curves drawn through the points so determined give the respective meridians. In figure 15 it may be seen in what manner the exaggeration of scale is introduced by this method of projection. A map of North America on this projection is shown in figure 42. The great advantage of this projection consists in the fact that a general table can be computed for use in any part of the earth.. In most other projections there are certain elements that have to be determined for the region to be mapped. When this is the case a separate table has to be computed for each region that is under consideration. With this projection, regions of narrow extent of longitude can be mapped with an accuracy such that no departure from true scale can be detected. A quadrangle of 1° on each side can be represented in such a manner, and in cases where the greatest accuracy is either not required or in which the error in scale may be taken into account, regions of much greater extent can be successfully mapped. The general table is very convenient for making topographic maps of limited extent in which it is desired to represent the region in detail. Of course, maps of neighboring regions on such a projection could not be fitted together exactly to form an extended map. This same restriction would apply to any projection on which the various regions were represented on an unvarying scale with minimum distortions.

FIG. 42.—Polyconic projection of North America.

ILLUSTRATIONS OF RELATIVE DISTORTIONS

A striking illustration of the distortion and exaggerations inherent in various systems of projection is given in figures 43–46. In figure 43 we have shown a man's head drawn with some degree of care on a globular projection of a hemisphere. The other three figures have the outline of the head plotted, maintaining the latitude and longitude the same as they are found in the globular projection. The distortions and exaggerations are due solely to those that are found in the projection in question.

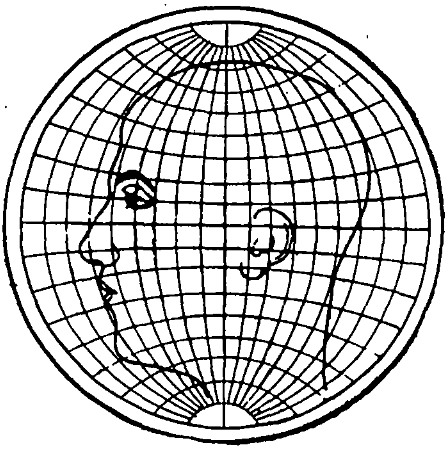

FIG. 43.—Man's head drawn on globular projection.

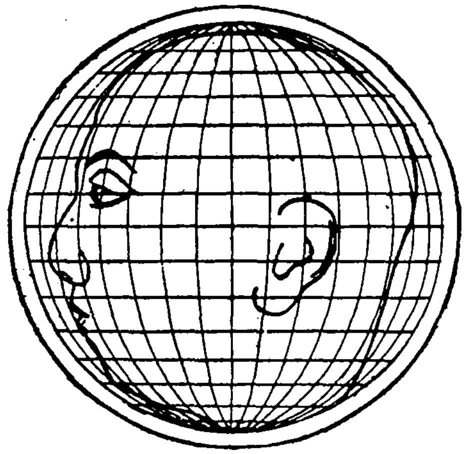

FIG. 44.—Man's head plotted on orthographic projection.

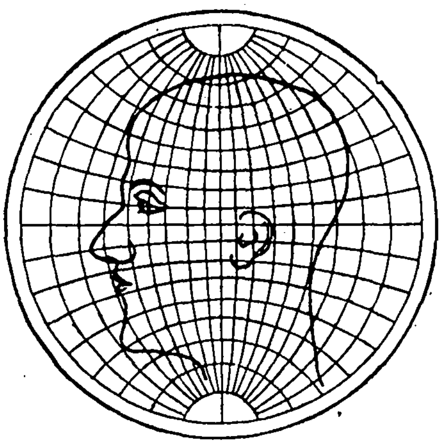

FIG. 45.—Man's head plotted on stereographic projection.

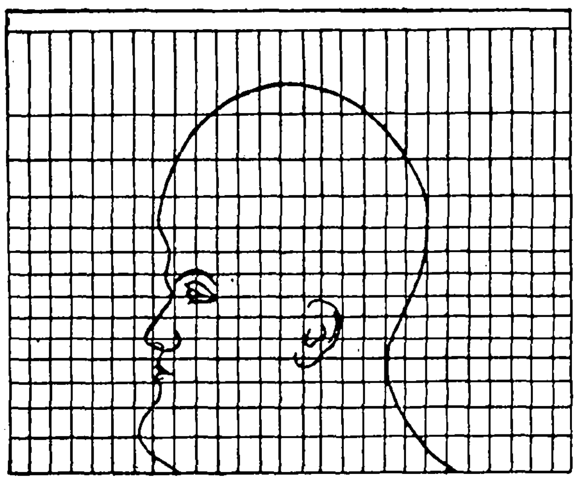

FIG. 46.—Man's head plotted on Mercator projection.

This does not mean that the globular projection is the best of the four, because the symmetrical figure might be drawn on any one of them and then plotted on the others. By this method we see shown in a striking way the relative differences in distortion of the various systems. The principle could be extended to any number of projections that might be desired, but the four figures given serve to illustrate the method.

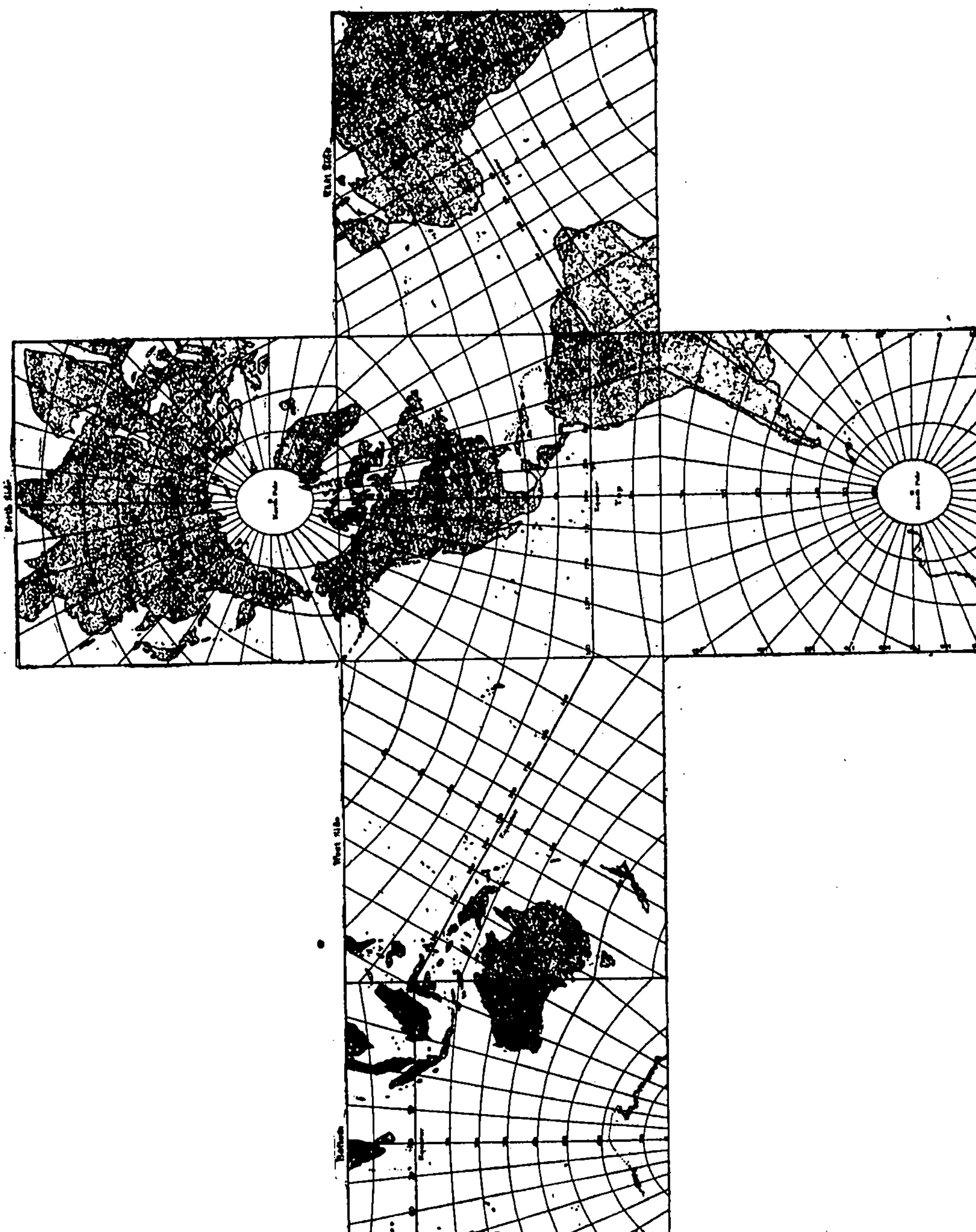

FIG. 47.—Gnomonic projection of the sphere on a circumscribed cube.

PART II

INTRODUCTION

It is the purpose in Part II of this review to give a comprehensive description of the nature, properties, and construction of the better systems of map projection in use at the present day. Many projections have been devised for map construction which are nothing more than geometric trifles, while others have attained prominence at the expense of better and ofttimes simpler types.

It is largely since the outbreak of the first World War that an increased demand for better maps has created considerable activity in mathematical cartography, and, as a consequence, a marked progress in the general theory of map projections has been in evidence.

Through military necessities and educational requirements, the science and art of cartography have demanded better draftsmanship and greater accuracy, to the extent that many of the older studies in geography are not now considered as worthy of inclusion in the present-day class.

The whole field of cartography, with its component parts of history and surveys, map projection, compilation, nomenclature and reproduction is so important to the advancement of scientific geography that the higher standard of today is due to a general development in every branch of the subject.

The selection of suitable projections is receiving far more attention than was formerly accorded to it. The exigencies of the problem at hand can generally be met by special study, and, as a rule, that system of projection can be adopted which will give the best results for the area under consideration, whether the desirable conditions be a matter of correct angles between meridians and parallels, scaling properties, equivalence of areas, rhumb lines, etc.

The favorable showing required to meet any particular mapping problem may oftentimes be retained at the expense of other less desirable properties, or a compromise may be effected. A method of projection which will answer for a country of small extent in latitude will not at all answer for another country of great length in a north-and-south direction; a projection which serves for the representation of the polar regions may not be at all applicable to countries near the Equator; a projection which is the most convenient for the purposes of the navigator is of little value to the Bureau of the Census; and so throughout the entire range of the subject, particular conditions have constantly to be satisfied and special rather than general problems to be solved. The use of a projection for a purpose to which it is not best suited is, therefore, generally unnecessary and can be avoided.

PROJECTIONS DESCRIBED IN PART II

In the description of the different projections and their properties in the following pages the mathemetical theory and development of formulas are not generally included where ready reference can be given to other manuals containing these features. In several instances, however, the mathematical development is given in somewhat closer detail than heretofore.

In the selection of projections to be presented in this discussion, the authors have, with two exceptions, confined themselves to two classes, viz, *conformal projections* and *equivalent* or *equal-area projections*. The exceptions are the polyconic and gnomonic projections—the former covering a field entirely its own in its general employment for field sheets in any part of the world and in maps of narrow longitudinal extent, the latter in its application and use to navigation.

It is within comparatively recent years that the demand for equal-area projections has been rather persistent, and there are frequent examples where the mathematical property of conformality is not of sufficient practical advantage to outweigh the useful property of equal area.

The critical needs of conformal mapping, however, were demonstrated at the commencement of the first World War, when the French adopted the Lambert conformal conic projection as a basis for their new battle maps, in place of the Bonne projection heretofore in use. By the new system, a combination of minimum of angular and scale distortion was obtained, and a precision which is unique in answering every requirement for knowledge of orientation, distances, and quadrillage (system of kilometric squares).

CONFORMAL MAPPING is not new since it is a property of the stereographic and Mercator projections. It is, however, somewhat surprising that the comprehensive study and practical application of the subject as developed by Lambert in 1772 and, from a slightly different point of view, by Lagrange in 1779, remained more or less in obscurity for many years. It is a problem in an important division of cartography which has been solved in a manner so perfect that it is impossible to add a word. This rigid analysis is due to Gauss, by whose name the Lambert conformal conic projection is sometimes known. In the representation of any surface upon any other by similarity of infinitely small areas, the credit for the advancement of the subject is due to him.

EQUAL-AREA MAPPING.—The problem of an equal-area or equivalent projection of a spheroid has been simplified by the introduction of an intermediate equal-area projection upon a sphere of equal surface, the link between the two being the authalic [4] latitude. A table of authalic latitudes for every half degree has been computed (see U. S. Coast and Geodetic Survey, Special Publication No. 67), and this can be used in the computations of any equal-area projection. The coordinates for the Albers equal-area projection of the United States were computed by use of this table.

THE CHOICE OF PROJECTION

Although the uses and limitations of the different systems of projections are given under their subject headings, a few additional observations may be of interest. (See frontispiece.)

COMPARISON OF ERRORS OF SCALE AND ERRORS OF AREA IN A MAP OF THE UNITED STATES ON FOUR DIFFERENT PROJECTIONS

Maximum Scale Error

	Per cent
Polyconic projection	7
Lambert conformal conic projection with standard parallels of latitude at 33° and 45°	2½
(Between latitudes 30½° and 47½°, only one-half per cent. Strictly speaking, in the Lambert conformal conic projection these percentages are not *scale error* but *change of scale*.)	
Lambert zenithal equal-area projection	1⅛
Albers projection with standard parallels at 29°30′ and 45°30′	1¼

[4] The term *authalic* was first employed by Tissot, in 1881, signifying *equal area*.

Maximum Error of Area

	Per cent
Polyconic	7
Lambert conformal conic	5
Lambert zenithal	0
Albers	0

Maximum Error of Azimuth

Polyconic	1° 56′
Lambert conformal conic	0° 00′
Lambert zenithal	1° 04′
Albers	0° 43′

An improper use of the polyconic projection for a map of the North Pacific Ocean during the period of the Spanish-American War resulted in distances being distorted along the Asiatic coast to double their true amount, and brought forth the query whether the distance from Shanghai to Singapore by straight line was longer than the combined distances from Shanghai to Manila and thence to Singapore.

The polyconic projection is not adapted to mapping areas of predominating longitudinal extent and should not generally be used for distances east or west of its central meridian exceeding 500 statute miles. Within these limits it is sufficiently close to other projections that are in some respects better, as not to cause any inconvenience. The extent to which the projection may be carried in latitude is not limited. On account of its tabular superiority and facility for constructing field sheets and topographical maps, it occupies a place beyond all others.[5]

Straight lines on the polyconic projection (excepting its central meridian and the Equator) are neither great circles nor rhumb lines, and hence the projection is not suited to navigation beyond certain limits.

The polyconic projection has no advantages in scale; neither is it conformal nor equal-area, but rather a compromise of various conditions which determine its choice within certain limits.

The modified polyconic projection with two standard meridians may be carried to a greater extent of longitude than the former, but for narrow zones of longitude the Bonne projection is in some respects preferable to either, as it is an equal-area representation.

For a map of the United States in a single sheet the choice rests between the *Lambert conformal conic* projection with two standard parallels and the *Albers equal-area* projection with two standard parallels. The selection of a polyconic projection for this purpose is indefensible. The longitudinal extent of the United States is too great for this system of projection and its errors are not readily accounted for. The Lambert conformal and Albers are peculiarly suited to mapping in the Northern Hemisphere, where the lines of commercial importance are generally east and west.

In Plate I about one-third of the Northern Hemisphere is mapped in an easterly and westerly extent. With similar maps on both sides of the one referred to, and with suitably selected standard parallels, we would have an interesting series of the Northern Hemisphere.

The transverse polyconic is adapted to the mapping of comparatively narrow

[5] The polyconic projection has always been employed by the Coast and Geodetic Survey for field sheets, and general tables for the construction of this projection are published by this Bureau. A projection for any small part of the world can readily be constructed by the use of these tables and the accuracy of this system within the limits specified are good reasons for its general use.

areas of considerable extent along any great circle. (See Plate II.) A *Mercator projection* can be turned into a transverse position in a similar manner and will give us conformal mapping.

The Lambert conformal and Albers projections are desirable for areas of predominating east-and-west extent, and the choice is between *conformality*, on the one hand, or *equal area*, on the other, depending on which of the two properties may be preferred. The authors would prefer Albers projection for mapping the United States. A comparison of the two indicates that their difference is very small, but the certainty of definite equal-area representation is, for general purposes, the more desirable property. When latitudinal extent increases, conformality with its preservation of shapes becomes generally more desirable than equivalence with its resultant distortion, until a limit is reached where a large extent of area has equal dimensions in both or all directions. Under the latter condition—viz, the mapping of large areas of approximately equal magnitudes in all directions approaching the dimensions of a hemisphere, combined with the condition of preserving azimuths from a central point—the Lambert zenithal equal-area projection and the stereographic projection are preferable, the former being the equal-area representation and the latter the conformal representation.

A study in the distortion of scale of four different projections is given in the frontispiece. Deformation tables giving errors in scale, area, and angular distortion in various projections are published in Tissot's Mémoire sur la Représentation des Surfaces. These elements of the Polyconic projection are given on pages 166–167, U. S. Coast and Geodetic Survey Special Publication No. 57.

The mapping of extensive areas of hemispherical proportions on any projection, whether conformal or equivalent, introduces serious scale variations or serious distortions of areas in certain parts of the map. It is better to reserve the outer areas for title space as in Plate I rather than to extend the mapping into them. The polar regions should in any event be mapped separately on a suitable polar projection. For an equatorial belt a *cylindrical conformal* or a *cylindrical equal-area* projection intersecting two parallels equidistant from the Equator may be employed.

The lack of mention of a large number of excellent map projections in Part II of this treatise should not cause one to infer that the authors deem them unworthy. It was not intended to cover the subject *in toto* at this time, but rather to caution against the misuse of certain types of projections, and bring to notice a few of the interesting features in the progress of mathematical cartography, in which the theory of functions of a complex variable plays no small part to-day. Without the elements of this subject a proper treatment of conformal mapping is impossible.

On account of its specialized nature, the mathematical element of cartography has not appealed to the amateur geographer, and the number of those who have received an adequate mathematical training in this field of research are few. A broad gulf has heretofore existed between the geodesist, on the one hand, and the cartographer, on the other. The interest of the former too frequently ceases at the point of presenting with sufficient clearness the value of his labors to the latter, with the result that many chart-producing agencies resort to such systems of map projection as are readily available rather than to those that are ideal.

It is because of this utilitarian tendency or negligence, together with the manifest aversion of the cartographer to cross the threshold of higher mathematics, that those who care more for the theory than the application of projections have not received the recognition due them, and the employment of autogonal [6] (conformal) projections has not been extensive. The labors of Lambert, Lagrange, and Gauss are now receiving full appreciation.

In this connection, the following quotation from volume IV, page 408, of the collected mathematical works of George William Hill is of interest:

> Maps being used for a great variety of purposes, many different methods of projecting them may be admitted; but when the chief end is to present to the eye a picture of what appears on the surface of the earth, we should limit ourselves to projections which are conformal. And, as the construction of the réseau of meridians and parallels is, except in maps of small regions, an important part of the labor involved, it should be composed of the most easily drawn curves. Accordingly, in a well-known memoir, Lagrange recommended circles for this purpose, in which the straight line is included as being a circle whose center is at infinity.

An attractive field for future research will be in the line in which Prof. Goode, of the University of Chicago, has contributed so substantially. Possibilities of other combinations or interruptions in the same or different systems of map projection may solve some of the other problems of world mapping. Several interesting studies given in illustration at the end of the book will, we hope, suggest ideas to the student in this particular branch.

On all recent French maps the name of the projection appears in the margin. This is excellent practice and should be followed at all times. As different projections have different distinctive properties, this feature is of no small value and may serve as a guide to an intelligible appreciation of the map.

THE POLYCONIC PROJECTION

DESCRIPTION

[See Fig. 48.]

The polyconic projection, devised by Ferdinand Hassler, the first Superintendent of the Coast and Geodetic Survey, possesses great popularity on account of mechanical ease of construction and the fact that a general table [7] for its use has been calculated for the whole spheroid.

It may be interesting to quote Prof. Hassler [8] in connection with two projections, viz, the intersecting conic projection and the polyconic projection:

> 1. *Projection on an intersecting cone.*—The projection which I intended to use was the development of a part of the earth's surface upon a cone, either a tangent to a certain latitude, or *cutting two given parallels* and two meridians, equidistant from the middle meridian, and extended on both sides of the meridian, and in latitude, only so far as to admit no deviation from the real magnitudes, sensible in the detail surveys.
>
> 2. *The polyconic projection.*—* * * This distribution of the projection, in an assemblage of sections of surfaces of successive cones, tangents to or cutting a regular succession of parallels, and upon regularly changing central meridians, appeared to me the only one applicable to the coast of the United States.
>
> Its direction, nearly diagonal through meridian and parallel, would not admit any other mode founded upon a single meridian and parallel without great deviations from the actual magnitudes and shape, which would have considerable disadvantages in use.

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[6] Page 75, Tissot's Mémoire sur la Représentation des Surfaces, Paris, 1881—"Nous appellerons *autogonales* les projectio ns qu conservent les angles, et *authaliques* celles qui conservent les aires."

[7] Tables for the polyconic projection of maps, Coast and Geodetic Survey, Special Publication No. 5.

[8] Papers on various subjects connected with the survey of the coast of the United States, by F. R. Hassler; communicated Mar. 3, 1820 (in Trans. Am. Phil. Soc., new series, vol. 2, pp. 406–408, Philadelphia, 1825).

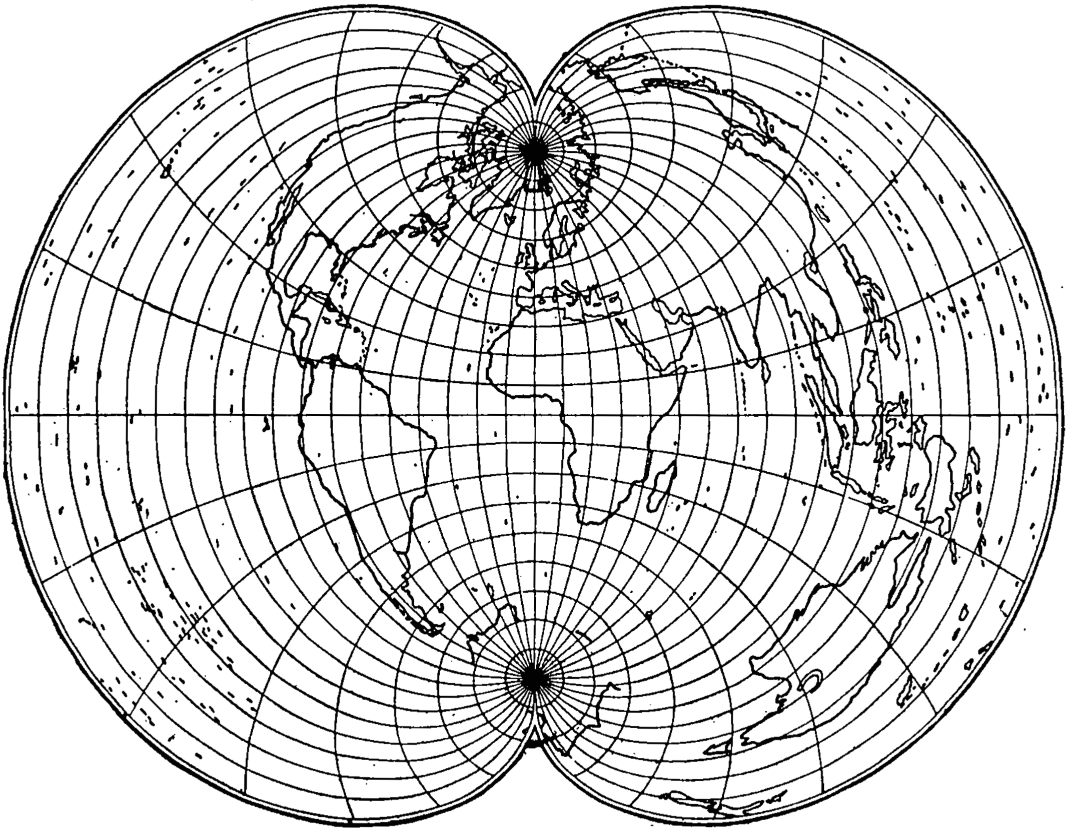

FIG. 48.—Polyconic development of the sphere.

At the left of figure 49 there are shown the centers (K, K_1, K_2, K_3) of circles on the projection that represent the corresponding parallels on the earth. At the right of the same figure there is shown the distortion at the outer meridian due to the varying radii of the circles in the polyconic development.

A central meridian is assumed upon which the intersections of the parallels are truly spaced. Each parallel is then separately developed by means of a tangent

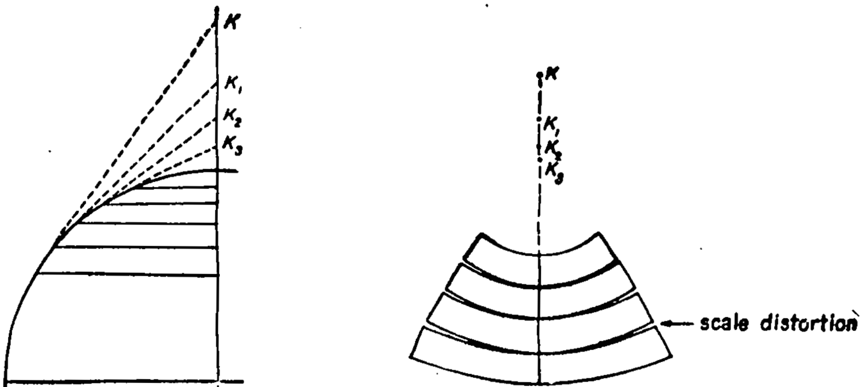

FIG. 49.—Polyconic development.

cone, the centers of the developed arcs of parallels lying in the extension of the central meridian. The arcs of the developed parallels are subdivided to true scale and the meridians drawn through the corresponding subdivisions. Since the radii for the parallels decrease as the cotangent of the latitude, the circles are not concentric, and the lengths of the arcs of latitude gradually increase as we recede from the meridian.

The central meridian is a right line; all others are curves, the curvature increasing with the longitudinal distance from the central meridian. The intersections between meridians and parallels also depart from right angles as the distance increases.

From the construction of the projection it is seen that errors in meridional distances, areas, shapes, and intersections increase with the longitudinal limits. It therefore should be restricted in its use to maps of wide latitude and narrow longitude.

The *polyconic* projection may be considered as in a measure *only compromising* various conditions impossible to be represented on any one map or chart, such as relate to—

First. Rectangular intersections [9] of parallels and meridians.

Second. Equal scale [9] over the whole extent (the error in scale not exceeding 1 percent for distances within 560 statute miles of the great circle used as its central meridian).

Third. Facilities for using great circles and azimuths within distances just mentioned.

Fourth. Proportionality of areas [9] with those on the sphere, etc.

The polyconic projection is by construction not conformal, neither do the parallels and meridians intersect at right angles, as is the case with all conical or single-cone projections, whether these latter are conformal or not.

It is sufficiently close to other types possessing in some respects better properties that its great tabular advantages should generally determine its choice within certain limits.

As stated in Hinks' Map Projections, it is a link between those projections which have some definite scientific value and those generally called conventional, but possessing properties of convenience and use.

The three projections, polyconic, Bonne, and Lambert zenithal, may be considered as practically identical within areas not distant more than 3° from a common central point, the errors from construction and distortion of the paper exceeding those due to the system of projection used.

The general theory of polyconic projections is given in Special Publication No. 57, U. S. Coast and Geodetic Survey.

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[9] The errors in meridional scale and area are expressed in percentage very closely by the formula

$$E = +\left(\frac{l^\circ \cos \phi}{8.1}\right)^2$$

in which l° = distance of point from central meridian expressed in degrees of longitude, and ϕ = latitude.

EXAMPLE.—*For latitude 39° the error for 10° 25′ 22″ (560 statute miles) departure in longitude is 1 percent for scale along the* meridian and the same amount for area.

The angular distortion is a variable quantity not easily expressed by an equation. In latitude 30° this distortion is 1° 27′ on the meridian 15° distant from the central meridian; at 30° distant it increases to 5° 36′.

The greatest angular distortion in this projection is at the Equator, decreasing to zero as we approach the pole. The distortion of azimuth is one-half of the above amounts.

CONSTRUCTION OF A POLYCONIC PROJECTION

Having the area to be covered by a projection, determine the scale and the interval of the projection lines which will be most suitable for the work in hand.

SMALL-SCALE PROJECTIONS (1–500,000 AND SMALLER)

Draw a straight line for a central meridian and a construction line (*ab* in the figure) perpendicular thereto, each to be as central to the sheet as the selected interval of latitude and longitude will permit.

On this central meridian and from its intersection with the construction line lay off the extreme intervals of latitude, north and south (mm_2 and mm_4) and subdivide the intervals for each parallel (m_1 and m_3) to be represented, all distances [11] being taken from the table (p. 7, Spec. Pub. No. 5, "Lengths of degrees of the meridian").

Through each of the points (m_1, m_2, m_3, m_4) on the central meridian draw additional construction lines (*cd*, *ef*, *gh*, *ij*) perpendicular to the central meridian, and mark off the abscissas (x, x_1, x_2, x_3, x_4, x_5) from the central meridian corresponding to the values [11] of "X" taken from the table under "Coordinates of curvature" (pp. 11 to 189 Spec. Pub. No. 5), for every meridian to be represented.

At the points (x, x_1, x_2, x_3, x_4, x_5) lay off from each of the construction lines the corresponding values [11] of "Y" [12] from the table under "Coordinates of curvature" (pp. 11 to 189, Spec. Pub. No. 5), in a direction parallel to the central meridian, above the construction lines if north of the Equator, to determine points on the meridians and parallels.

Draw curved lines through the points thus determined for the meridians and parallels of the projection.

LARGE-SCALE PROJECTIONS (1–10,000 AND LARGER)

The above method can be much simplified in constructing a projection on a large scale. Draw the central meridian and the construction line *ab*, as directed above. On the central meridian lay off the distances [13] mm_2 and mm_4 taken from the table under "Continuous sums of minutes" for the intervals in minutes between the middle parallel and the extreme parallels to be represented, and through the points m_2 and m_4 draw straight lines *cd* and *ef* parallel to the line *ab*. On the lines *ab*, *cd*, and *ef* lay off the distances [13] mx_5, m_2x_5, and m_4x_5 on both sides of the central meridian, taking the values from the table under "Arcs of the parallel in meters" corresponding to the latitude of the points m, m_2, and m_4, respectively. Draw straight lines through the points thus determined, x_5, for the extreme meridians.

---

[11] The lengths of the arcs of the meridians and parallels change when the latitude changes and *all distances* must be taken from the table opposite the latitude of the point in use.

[12] Approximate method of deriving the values of Y intermediate between those shown in the table.

The ratio of any two successive ordinates of curvature equals the ratio of the squares of the corresponding arcs.

Examples.—Latitude 60° to 61°. Given the value of Y for longitude 50′, 292.ᵐ8 (see table), to obtain the value of Y for longitude 55′.

$$\frac{(55)^2}{(50)^2}=\frac{Y}{292.8};\text{ hence } Y=354.^m3 \text{ (see table).}$$

Similarly, Y for 3° = 3795ᵐ.

$$\frac{4^2}{3^2}=\frac{Y}{3795};\text{ hence } Y \text{ for } 4° = 6747^m,$$

which differs 2ᵐ from the tabular value, a negligible quantity for the intermediate values of Y under most conditions.

[13] The lengths of the arcs of the meridians and parallels change when the latitude changes and *all distances* must be taken from the table opposite the latitude of the point in use.

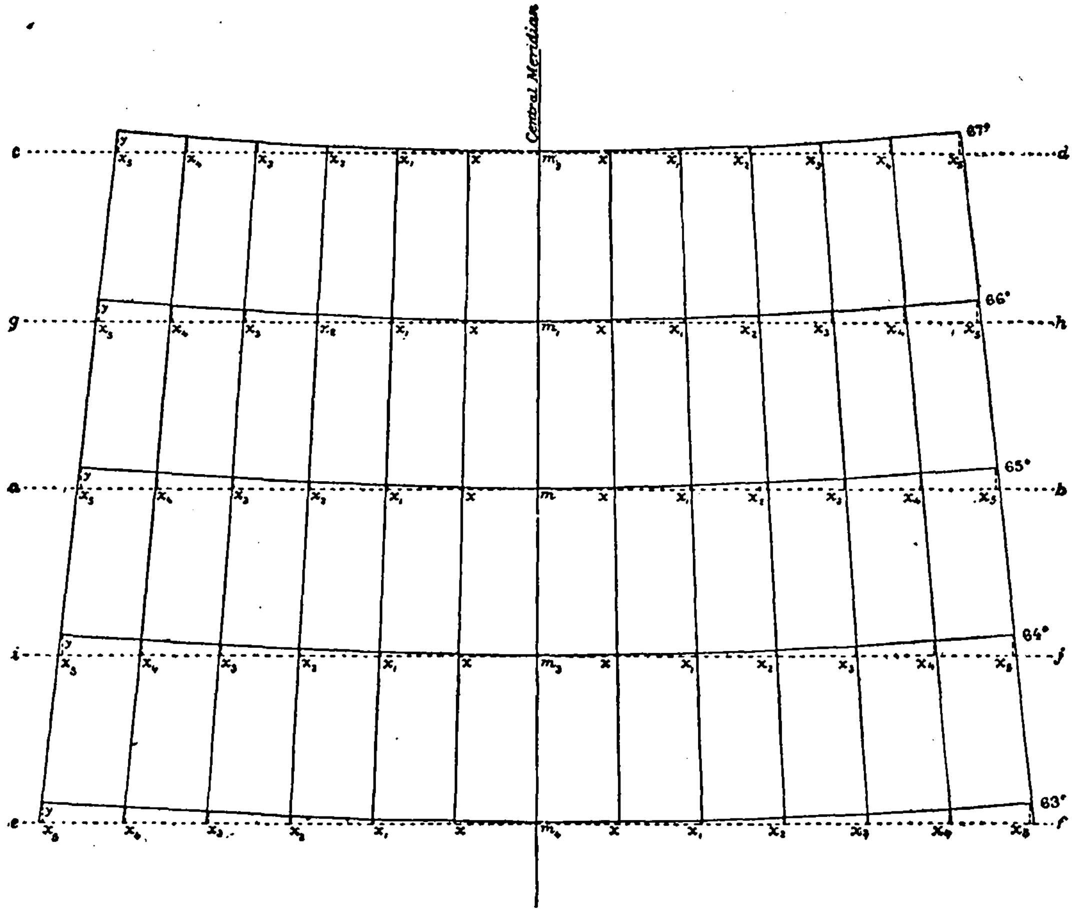

FIG. 50.—Polyconic projection—construction plate.

At the points x_5 on the line *ab* lay off the value[14] of *Y* corresponding to the interval in minutes between the central and the extreme meridians, as given in the table under "Coordinates of curvature," in a direction parallel with the central meridian and above the line, if north of the Equator, to determine points in the central parallel. Draw straight lines from these points to the point *m* for the middle parallel, and from the points of intersection with the extreme meridians lay off distances[14] on the extreme meridians, above and below, equal to the distances mm_2 and mm_4 to locate points in the extreme parallels.

Subdivide the three meridians and three parallels into parts corresponding to the projection interval and join the corresponding points of subdivision by straight lines to complete the projection.

To construct a projection on an intermediate scale, follow the rigid method for small-scale projections to the extent required to give a projection as accurate as can be constructed graphically.

In the "large-scale projection" method, the use of the table of "Arcs of the parallel" instead of X coordinates, although not theoretically correct, is sufficiently accurate for projections even up to scale 1:40,000. Due to the fact that the X coordinates are not supplied in the table for latitudes intermediate between degrees, it is convenient to use arc lengths instead of X coordinates in order to avoid

14 The lengths of the arcs of the meridians and parallels change when the latitude changes and *all distances* must be taken from the table opposite the latitude of the point in use.

interpolation. However, it becomes necessary in projections of large longitudinal extent in scales smaller than 1:40,000 to apply a check on the two sets of values, and to use the X coordinates when they become smaller than the values taken from the "Arcs of the parallel."

A more frequent use of Y coordinates is necessary as scales become smaller than 1:10,000; and on intermediate scales up to 1:40,000 it is generally sufficient to apply the Y coordinates on the central, upper, and lower parallels at their extremities, and at intermediate intervals of such frequency as will be graphically needful.

Coordinates for the projection of maps on various scales with the inch as unit, are published by the U. S. Geological Survey in Bulletin 650, Geographic Tables and Formulas, pages 34 to 107.

TRANSVERSE POLYCONIC PROJECTION

(See Plate II.)

If the map should have a predominating east-and-west dimension, the polyconic properties may still be retained, by applying the developing cones in a transverse position. A great circle at right angles to a central meridian at the middle part of the map can be made to play the part of the central meridian, the poles being transferred (in construction only) to the Equator. By transformation of coordinates a projection may be completed which will give all polyconic properties in a transverse relation. This process is, however, laborious and has seldom been resorted to.

Since the distance across the United States from north to south is less than three-fifths of that from east to west, it follows, then, by the above manipulation that the maximum distortion can be reduced from 7 to 2½ per cent.

A projection of this type (plate II) is peculiarly suited to a map covering an important section of the North Pacific Ocean. If a great circle [15] passing through San Francisco and Manila is treated in construction as a central meridian in the ordinary polyconic projection, we can cross the Pacific in a narrow belt so as to include the American and Asiatic coasts with a very small scale distortion. By transformation of coordinates the meridians and parallels can be constructed so that the projection will present the usual appearance and may be utilized for ordinary purposes.

The configuration of the two continents is such that all the prominent features of America and eastern Asia are conveniently close to this selected axis, viz Panama, Brito, San Francisco, Straits of Fuca, Unalaska, Kiska, Yokohama, Manila, Hongkong and Singapore. It is a typical case of a projection being adapted to the configuration of the locality treated. A map on a transverse polyconic projection as here suggested, while of no special navigational value, is of interest from a geographic standpoint as exhibiting in their true relations a group of important localities covering a wide expanse.

For method of constructing this modified form of polyconic projection, see Coast and Geodetic Survey, Special Publication No. 57, pages 167 to 171.

POLYCONIC PROJECTION WITH TWO STANDARD MERIDIANS, AS USED FOR THE INTERNATIONAL MAP OF THE WORLD, ON THE SCALE 1:1,000,000

The projection adopted for this map is a modified polyconic projection devised by Lallemand, and for this purpose has advantages over the ordinary polyconic projection in that the meridians are straight lines and meridional errors are lessened and distributed somewhat the same (except in an opposite direction) as in a conic projection with two standard parallels; in other words, it provides for a distribution of scale error by having two standard meridians instead of the one central meridian of the ordinary polyconic projection.

[15] A great circle tangent to parallel 45° north latitude at 160° west longitude was chosen as the axis of the projection in this plate.

The scale is slightly reduced along the central meridian, thus bringing the parallels closer together in such a way that the meridians 2° on each side of the center are made true to scale. Up to 60° of latitude the separate sheets are to include 6° of longitude and 4° of latitude. From latitude 60° to the pole the sheets are to include 12° of longitude; that is, two sheets are to be united into one. The top and bottom parallel of each sheet are constructed in the usual way; that is, they are circles constructed from centers lying on the central meridian, but not concentric. These two parallels are true to scale and truly divided. The meridians are straight lines joining the corresponding points of the top and bottom parallels. Any sheet will then join exactly along its margins with its four neighboring sheets. The correction to the length of the central meridian is very slight, amounting to only 0.01 inch at the most, and the change is almost too slight to be measured on the map.

In the resolutions of the International Map Committee, London, 1909, it is not stated how the meridians are to be divided; but, no doubt, an equal division of the central meridian was intended. Through these points, circles could be constructed with centers on the central meridian and with radii equal to $\rho_n \cot \phi$, in which ρ_n is the radius of curvature perpendicular to the meridian. In practice, however, an equal division of the straight-line meridians between the top and bottom parallels could scarcely be distinguished from the points of parallels actually constructed by means of radii or by coordinates of their intersections with the meridians. The provisions also fail to state whether in the sheets covering 12° of longitude instead of 6°, the meridians of true length shall be 4° instead of 2° on each side of the central meridian; but such was, no doubt, the intention. In any case, the sheets would not exactly join together along the parallel of 60° of latitude.

The appended tables give the corrected lengths of the central meridian from 0° to 60° of latitude and the coordinates for the construction of the 4° parallels within the same limits. Each parallel has its own origin; i. e., where the parallel in question intersects the central meridian. The central meridian is the Y axis and a perpendicular to it at the origin is the X axis; the first table, of course, gives the distance between the origins. The y values are small in every instance. In terms of the parameters these values are given by the expressions

$$x = \rho_n \cot \phi \sin (\lambda \sin \phi)$$

$$y = \rho_n \cot \phi [1 - \cos (\lambda \sin \phi)] = 2\rho_n \cot \phi \sin^2 \left(\frac{\lambda \sin \phi}{2} \right).$$

The tables as given below are all that are required for the construction of all maps up to 60° of latitude. This fact in itself shows very clearly the advantages of the use of this projection for the purpose in hand.

A discussion of the numerical properties of this map system is given by Lallemand in the Comptes Rendus, 1911, tome 153, page 559.

TABLES FOR THE PROJECTION OF THE SHEETS OF THE INTERNATIONAL MAP OF THE WORLD

[Scale: 1:1,000,000. Assumed figure of the earth: a=6378.24 km.; b=6356.56 km.]

TABLE 1.—*Corrected lengths on the central meridian, in millimeters*

Latitude	Natural length	Correction	Corrected length
° °			
From 0 to 4	442. 27	−0. 27	442. 00
4 to 8	442. 31	. 26	442. 05
8 to 12	442. 40	. 26	442. 14
12 to 16	442. 53	. 26	442. 27
16 to 20	442. 69	. 24	442. 45
20 to 24	442. 90	. 24	442. 66
24 to 28	443. 13	. 22	442. 91
28 to 32	443. 39	. 20	443. 19
32 to 36	443. 68	. 19	443. 49
36 to 40	443. 98	. 17	443. 81
40 to 44	444. 29	. 15	444. 14
44 to 48	444. 60	. 13	444. 47
48 to 52	444. 92	. 11	444. 81
52 to 56	445. 22	. 09	445. 13
56 to 60	445. 52	−0. 08	445. 44

TABLE 2.—*Coordinates of the intersections of the parallels and the meridians, in millimeters*

Latitude	Coordinates	Longitude from central meridian		
		1°	2°	3°
°				
0	x	111. 32	222. 64	333. 96
	y	0. 00	0. 00	0. 00
4	x	111. 05	222. 10	333. 16
	y	0. 07	0. 27	0. 61
8	x	110. 24	220. 49	330. 73
	y	0. 13	0. 54	1. 21
12	x	108. 90	217. 81	326. 71
	y	0. 20	0. 79	1. 78
16	x	107. 04	214. 07	321. 10
	y	0. 26	1. 03	2. 32
20	x	104. 65	209. 29	313. 93
	y	0. 31	1. 25	2. 81
24	x	101. 75	203. 50	305. 24
	y	0. 36	1. 45	3. 25
28	x	98. 36	196. 72	295. 06
	y	0. 40	1. 61	3. 63
32	x	94. 50	188. 98	283. 45
	y	0. 44	1. 75	3. 93
36	x	90. 16	180. 32	270. 46
	y	0. 46	1. 85	4. 16
40	x	85. 40	170. 78	256. 14
	y	0. 48	1. 92	4. 31
44	x	80. 21	160. 40	240. 58
	y	0. 49	1. 95	4. 38
48	x	74. 63	149. 24	223. 83
	y	0. 48	1. 94	4. 36
52	x	68. 68	137. 34	205. 98
	y	0. 47	1. 89	4. 25
56	x	62. 39	124. 77	187. 13
	y	0. 45	1. 81	4. 06
60	x	55. 80	111. 59	167. 35
	y	0. 42	1. 69	3. 80

In the debates on the international map, the ordinary polyconic projection was opposed on the ground that a number of sheets could not be fitted together on account of the curvature of both meridians and parallels. This is true from the nature of things, since it is impossible to make a map of the world in a series of flat sheets which shall fit together and at the same time be impartially representative of all meridians and parallels. Every sheet edge in the international map has an exact fit with the corresponding edges of its four adjacent sheets. (See fig 51.)

The corner sheets to complete a block of nine will not make a perfect fit along their two adjacent edges simultaneously; they will fit one or the other, but the angles of the corners are not exactly the same as the angles in which they are required to fit; and there will be in theory a slight wedge-shaped gap unfilled, as shown in the figure. It is, however, easy to calculate that the discontinuity at the points *a* or *b* in a block of nine sheets, will be no more than a tenth of an inch if the paper preserves its shape absolutely unaltered. What it will be in practice depends entirely on the paper, and a map mounter will have no difficulty in squeezing his sheets to make the junction practically perfect. If more than nine sheets are put together, the error will, of course, increase somewhat rapidly; but at the same time the sheets will become so inconveniently large that the experiment is not likely to be made very often. If the difficulty does occur, it must be considered an instructive example at once of the proposition that a spheroidal surface can not be developed on a plane without deformation, and of the more satisfying proposition that this modified projection gives a remarkably successful approximation to an unattainable ideal.

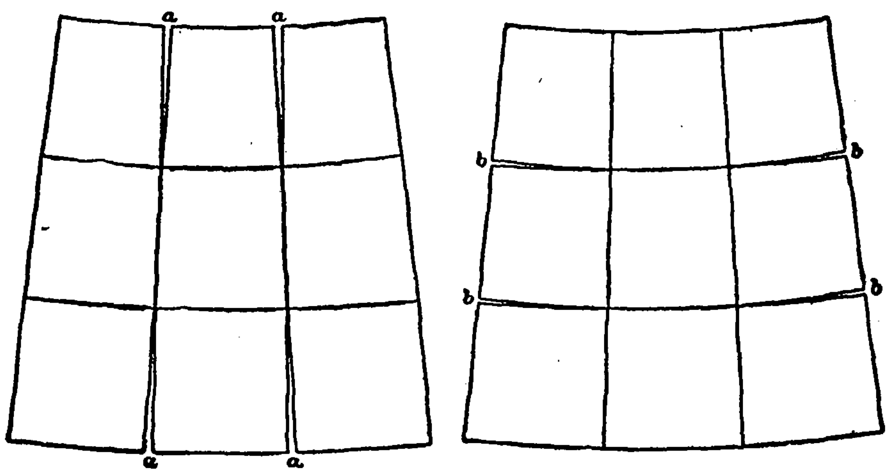

FIG. 51.—International map of the world—junction of sheets.

Concerning the modified polyconic projection for the international map, Dr. Frischauf has little to say that might be considered as favorable, partly on account of errors that appeared in the first publication of the coordinates.

The claim that the projection is not mathematically quite free from criticism and does not meet the strictest demands in the matching of sheets has some basis. The system is to some extent conventional and does not set out with any of the better scientific properties of map projections, but, within the limits of the separate sheets or of several sheets joined together, should meet all ordinary demands.

The contention that the Albers projection is better suited to the same purpose raises the problem of special scientific properties of the latter with its limitations to separate countries or countries of narrow latitudinal extent, as compared with the modified polyconic projection, which has no scientific interest, but rather a value of expediency.

In the modified polyconic projection the separate sheets are sufficiently good and can be joined any one to its four neighbors, and fairly well in groups of nine throughout the world; in the Albers projection a greater number of sheets may be joined exactly if the latitudinal limits are not so great as to necessitate new series to the north or south, as in the case of continents. The latter projection is further discussed in another chapter.

The modified polyconic projection loses the advantages of the ordinary polyconic in that the latter has the property of indefinite extension north or south, while its gain longitudinally is offset by loss of scale on the middle parallels. The system does not, therefore, permit of much extension in other maps than those for which it was designed, and a few of the observations of Prof. Rosén, of Sweden, on the limitations [16] of this projection are of interest:

The *junction of four sheets around a common point is more important than junctions* in Greek-cross arrangement, as provided for in this system.

The system does not allow a simple calculation of the degree scale, projection errors, or angular differences, the various errors of this projection being both lengthy to compute and remarkably irregular.

The length differences are unequal in similar directions from the same point, and the calculation of surface differences is especially complicated.

For simplicity in mathematical respects, Prof. Rosén favors a conformal conic projection along central parallels. By the latter system the sheets can be joined along a common meridian without a seam, but with a slight encroachment along the parallels when a northern sheet is joined to its southern neighbor. The conformal projection angles, however, being right angles, the sheets will join fully around a corner. Such a system would also serve as a better pattern in permitting wider employment in other maps.

On the other hand, the modified polyconic projection is sufficiently close, and its adaptability to small groups of sheets in any part of the world is its chief advantage. The maximum meridional error in an equatorial sheet, according to Lallemand [17] is only $\frac{1}{1300}$, *or about one-third of a millimeter in the height of a sheet; and in the direction* of the parallels $\frac{1}{1600}$, or one-fifth of a millimeter, in the width of a sheet. The error in azimuth does not exceed six minutes. Within the limits of one or several sheets these errors are negligible and inferior to those arising from drawing, printing, and hygrometric conditions.

THE BONNE PROJECTION

DESCRIPTION

[See fig. 52]

In this projection a central meridian and a standard parallel are assumed with a cone tangent along the standard parallel. The central meridian is developed along that element of the cone which is tangent to it and the cone developed on a plane.

The standard parallel falls into an arc of a circle with its center at the apex of the developing cone, and the central meridian becomes a right line which is divided to true scale. The parallels are drawn as concentric circles at their true distances apart, and all parallels are divided truly and drawn to scale.

[16] See Atti del X Congresso Internazionale di Geografia, Roma, 1913, pp. 37–42.
[17] Ibid., p. 681.

Through the points of division of the parallels the meridians are drawn. The central meridian is a straight line; all others are curves, the curvature increasing with the difference in longitude.

The scale along all meridians, excepting the central, is too great, increasing with the distance from the center, and the meridians become more inclined to the parallels, thereby increasing the distortion. The developed areas preserve a strict equality in which respect this projection is preferable to the polyconic.

Uses.—The Bonne [18] system of projection, still used to some extent in France, will be discontinued there and superseded by the Lambert system in military mapping.

BONNE PROJECTION OF HEMISPHERE

Development of cone tangent along parallel 45° N.

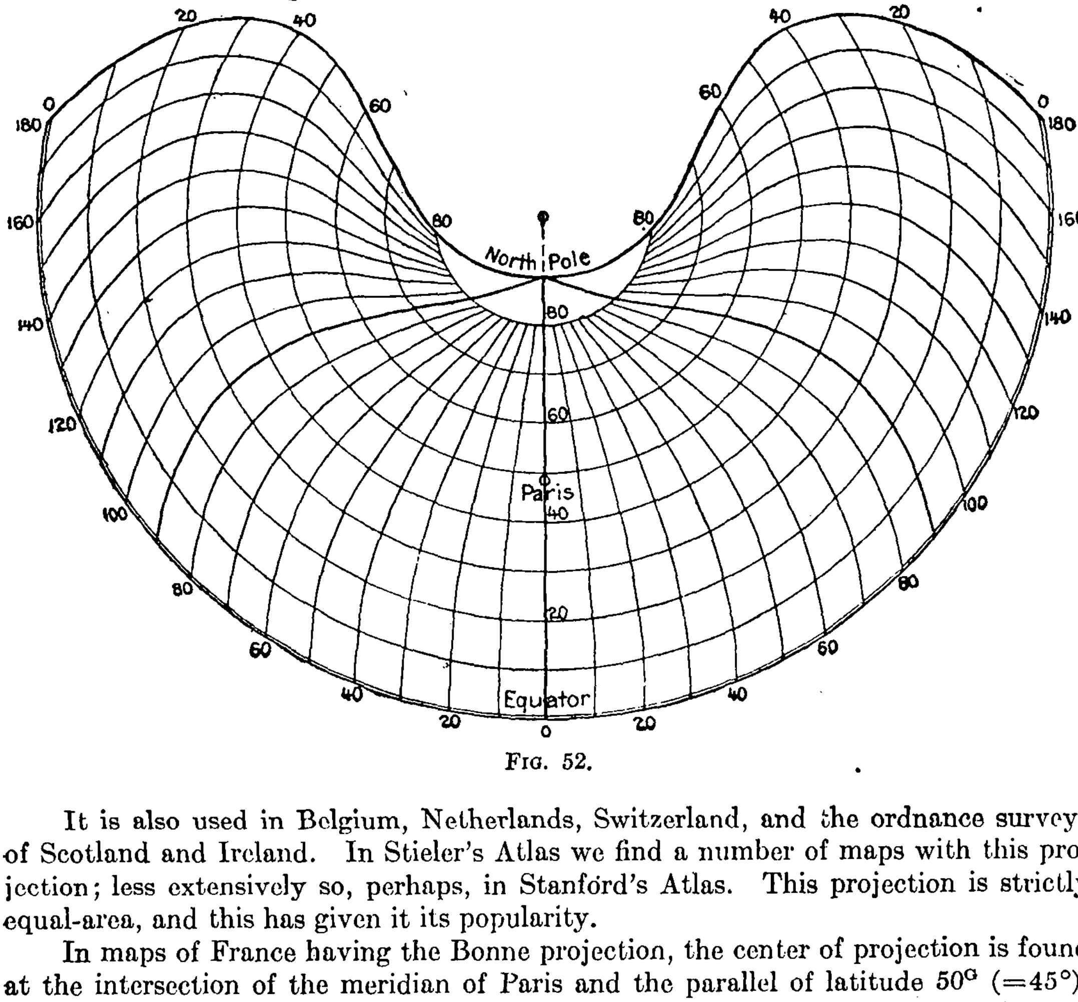

Fig. 52.

It is also used in Belgium, Netherlands, Switzerland, and the ordnance surveys of Scotland and Ireland. In Stieler's Atlas we find a number of maps with this projection; less extensively so, perhaps, in Stanford's Atlas. This projection is strictly equal-area, and this has given it its popularity.

In maps of France having the Bonne projection, the center of projection is found at the intersection of the meridian of Paris and the parallel of latitude 50^G (=45°). The border divisions and subdivisions appear in grades, minutes (centesimal), seconds, or tenths of seconds.

Limitations.—Its distortion, as the difference in longitude increases, is its chief

[18] Tables for this projection for the map of France were computed by Plessis.

defect. On the map of France the distortion at the edges reaches a value of 18′ for angles, and if extended into Alsace, or western Germany, it would have errors in distances which are inadmissible in calculations. In the rigorous tests of the military operations these errors became too serious for the purposes which the map was intended to serve.

THE SANSON-FLAMSTEED PROJECTION

In this particular case of the Bonne projection, where the Equator is chosen for the standard parallel, the projection is generally known under the name of Sanson-Flamsteed, or as the sinusoidal equal-area projection. All the parallels become straight lines parallel to the Equator and preserve the same distances as on the spheroid.

The latter projection is employed in atlases to a considerable extent in the mapping of Africa and South America, on account of its property of equal area and the comparative ease of construction. In the mapping of Africa, however, on account of its considerable longitudinal extent, the Lambert zenithal projection is preferable in that it presents less angular distortion and has decidedly less scale error. Diercke's Atlas employs the Lambert zenithal projection in the mapping of North America, Europe, Asia, Africa, and Oceania. In an equal-area mapping of South America, a Bonne projection, with center on parallel of latitude 10° or 15° south, would give somewhat better results than the Sanson-Flamsteed projection.

CONSTRUCTION OF A BONNE PROJECTION

Due to the nature of the projection, no general tables can be computed, so that for any locality special computations become necessary. The following method involves no difficult mathematical calculations:

Draw a straight line to represent the central meridian and erect a perpendicular to it at the center of the sheet. With the central meridian as Y axis, and this perpendicular as X axis, plot the points of the middle or standard parallel. The coordinates for this parallel can be taken from the polyconic tables, Special Publication No. 5. A smooth curve drawn through these plotted points will establish the standard parallel.

The radius of the circle representing the parallel can be determined as follows: The coordinates in the polyconic table are given for 30° from the central meridian.

With the x and y for 30°, we get

$$\tan\frac{\theta}{2}=\frac{y}{x}; \text{ and } r_1=\frac{x}{\sin\theta}$$

(θ being the angle at the center subtended by the arc that represents 30° of longitude). By using the largest values of x and y given in the table, the value of r_1 is better determined than it would be by using any other coordinates.

This value of r_1 can be derived rigidly in the following manner:

$$r_1=N\cot\phi$$

(N being the length of the normal to its intersection with the Y axis); but

$$N=\frac{1}{A'\sin 1''}$$

(A' being the factor tabulated in Special Publication No. 8, U. S. Coast and Geodetic Survey). Hence,

$$r_1 = \frac{\cot \phi}{A' \sin 1''}.$$

From the radius of this central parallel the radii for the other parallels can now be calculated by the addition or subtraction of the proper values taken from the table of "Lengths of degrees," U. S. Coast and Geodetic Survey Special Publication No. 5, page 7, as these values give the spacings of the parallels along the central meridian.

Let r represent the radius of a parallel determined from r_1 by the addition or subtraction of the proper value as stated above. If θ denotes the angle between the central meridian and the radius to any longitude out from the central meridian, and if P represents the arc of the parallel for 1° (see p. 6, Spec. Pub. No. 5), we obtain

$$\theta \text{ in seconds for } 1° \text{ of longitude} = \frac{P}{r \sin 1''};$$

$$\text{chord for } 1° \text{ of longitude} = 2r \sin\frac{\theta}{2}.$$

Arcs for any longitude out from the central meridian can be laid off by repeating this arc for 1°.

θ can be determined more accurately in the following way by the use of Special Publication No. 8:

λ'' = the longitude in seconds out from the central meridian; then

$$\theta \text{ in seconds} = \frac{\lambda'' \cos \phi}{rA' \sin 1''}.$$

This computation can be made for the greatest λ, and this θ can be divided proportional to the required λ.

If coordinates are desired, we get

$$x = r \sin \theta.$$

$$y = 2r \sin^2\frac{\theta}{2}.$$

The X axis for the parallel will be perpendicular to the central meridian at the point where the parallel intersects it.

If the parallel has been drawn by the use of the beam compass, the chord for the λ farthest out can be computed from the formula

$$\text{chord} = 2r \sin\frac{\theta}{2}.$$

The arc thus determined can be subdivided for the other required intersections with the meridians.

The meridians can be drawn as smooth curves through the proper intersections with the parallels. In this way all of the elements of the projection may be determined with minimum labor of computation.

THE LAMBERT ZENITHAL (OR AZIMUTHAL) EQUAL-AREA PROJECTION

DESCRIPTION

[See Frontispiece.]

This is probably the most important of the azimuthal projections and was employed by Lambert in 1772. The important property being the preservation of azimuths from a central point, the term zenithal is not so clear in meaning, being obviously derived from the fact that in making a projection of the celestial sphere the zenith is the center of the map.

In this projection the zenith of the central point of the surface to be represented appears as pole in the center of the map; the azimuth of any point within the surface, as seen from the central point, is the same as that for the corresponding points of the map; and from the same central point, in all directions, equal great-circle distances to points on the earth are represented by equal linear distances on the map.

It has the additional property that areas on the projection are proportional to the corresponding areas on the sphere; that is, any portion of the map bears the same ratio to the region represented by it that any other portion does to its corresponding region, or the ratio of area of any part is equal to the ratio of area of the whole representation.

This type of projection is well suited to the mapping of areas of considerable extent in all directions; that is, areas of approximately circular or square outline. In the frontispiece, the base of which is a Lambert zenithal projection, the line of 2 per cent scale error is represented by the bounding circle and makes a very favorable showing for a distance of 22° 44′ of arc-measure from the center of the map. Lines of other given errors of scale would therefore be shown by concentric circles (or almucantars), each one representing a small circle of the sphere parallel to the horizon.

Scale error in this projection may be determined from the scale factor of the almucantar as represented by the expression $\frac{1}{\cos \frac{1}{2}\theta}$ in which θ=actual distance in arc measure on osculating sphere from center of map to any point.

Thus we have the following percentages of scale error:

Distance in arc from center of map	Scale error
Degrees	*Percent*
5	0.1
10	0.4
20	1.2
30	3.5
40	6.4
50	10.3
60	15.5

In this projection azimuths from the center are true, as in all zenithal projections. The scale along the parallel circles (almucantars) is too large by the amounts indicated in the above table; the scale along their radii is too small in inverse proportion, for the projection is equal-area. The scale is increasingly erroneous as the distance from the center increases.

The Lambert zenithal projection is valuable for maps of considerable world areas, such as North America, Asia, and Africa, or the North Atlantic Ocean with its somewhat circular configuration. It has been employed by the Survey Department, Ministry of Finance, Egypt, for a wall map of Asia, as well as in atlases for the delineation of continents.

The projection has also been employed by the Coast and Geodetic Survey in an outline base map of the United States, scale 1:7,500,000. On account of the inclusion of the greater part of Mexico in this particular outline map, and on account of the extent of area covered and the general shape of the whole, the selection of this system of projection offered the best solution by reason of the advantages of equal-area representation combined with practically a minimum error of scale. Had the limits of the map been confined to the borders of the United States, the advantages of minimum area and scale errors would have been in favor of Albers projection, described in another chapter.

The maximum error of scale at the eastern and western limits of the United States is but 1⅛ per cent (the polyconic projection has 7 per cent), while the maximum error in azimuths is 1°04′.

Between a *Lambert zenithal projection* and a *Lambert conformal conic projection*, which is also employed for base-map purposes by the Coast and Geodetic Survey, on a scale 1:5,000,000, the choice rests largely upon the property of *equal areas* represented by the zenithal, and *conformality* as represented by the conformal conic projection. The former property is of considerable value in the practical use of the map, while the latter property is one of mathematical refinement and symmetry, the projection having two parallels of latitude of true scale, with definite scale factors available, and the advantages of straight meridians as an additional element of prime importance.

For the purposes and general requirements of a base map of the United States, disregarding scale and direction errors which are conveniently small in both projections, either of the above publications of the U. S. Coast and Geodetic Survey offers advantages over other base maps heretofore in use. However, under the subject heading of Albers projection, there is discussed another system of map projection which has advantages deserving consideration in this connection and which bids fair to supplant either of the above. (See frontispiece and table on pp. 57–58.)

Among the disadvantages of the Lambert zenithal projection should be mentioned the inconvenience of computing the coordinates and the plotting of the double system of complex curves (quartics) of the meridians and parallels; the intersection of these systems at oblique angles; and the consequent (though slight) inconvenience of plotting positions. The employment of degenerating conical projections, or rather their extension to large areas, leads to difficulties in their smooth construction and use. For this reason the Lambert zenithal projection has not been used so extensively, and other projections with greater scale and angular distortion are more frequently seen because they are more readily produced.

The center used in the frontispiece is latitude 40° and longitude 96°, corresponding closely to the geographic center [19] of the United States, which has been determined by means of this projection to be approximately in latitude 39°50′, and longitude 98°35′. Directions from this central point to any other point being true, and the

[19] "Geographic center of the United States" is here considered as a point analogous to the center of gravity of a spherical surface equally weighted (per unit area), and hence may be found by means similar to those employed to find the center of gravity.

law of radial distortion in all azimuthal directions from the central point being the same, this type of projection is admirably suited for the determination of the geographic center of the United States.

The coordinates for the following tables of the Lambert zenithal projection [20] were computed with the center on parallel of latitude 40°, on a sphere with radius equal to the geometric mean between the radius of curvature in the meridian and that perpendicular to the meridian at center. The logarithm of this mean radius in meters is 6.8044400.

THE LAMBERT EQUAL-AREA MERIDIONAL PROJECTION

This projection is also known as the Lambert central equivalent projection upon the plane of a meridian. In this case we have the projection of the parallels and meridians of the terrestrial sphere upon the plane of any meridian; the center will be upon the Equator, and the given meridional plane will cut the Equator in two points distant each 90° from the center.

It is the Lambert zenithal projection already described, but with the center on the Equator. While in the first case the bounding circle is a horizon circle, in the meridional projection the bounding circle is a meridian.

Tables for the Lambert meridional projection are given on page 77 of this publication, and also, in connection with the requisite transformation tables, in Latitude Developments Connected with Geodesy and Cartography, U. S. Coast and Geodetic Survey Special Publication No. 67.

The useful property of equivalence of area, combined with very small error of scale, makes the Lambert zenithal projection admirably suited for extensive areas having approximately equal magnitudes in all directions.

TABLE FOR THE CONSTRUCTION OF THE LAMBERT ZENITHAL EQUAL-AREA PROJECTION WITH CENTER ON PARALLEL 40°

Latitude	Longitude 0°		Longitude 5°		Longitude 10°		Longitude 15°		Longitude 20°		Longitude 25°	
	x	*y*	*x*	*y*	*x*	*y*	*x*	*y*	*x*	*y*	*x*	*y*
	Meters	*Meters*	*Meters*	*Meters*	*Meters*	*Meters*	*Meters*	*Meters*	*Meters*	*Meters*	*Meters*	*Meters*
90°	0	+5 387 885	0	+5 387 885	0	+5 387 885	0	+5 387 885	0	+5 387 885	0	+5 387 885
85°	0	+4 878 763	52 414	+4 880 599	104 453	+4 886 085	155 742	+4 895 190	205 914	+4 907 803	254 604	+4 924 009
80°	0	+4 360 354	102 679	+4 363 859	204 665	+4 374 361	305 266	+4 391 702	403 790	+4 416 058	499 587	+4 447 015
75°	0	+3 833 044	150 800	+3 838 672	300 777	+3 855 490	448 560	+3 878 743	593 609	+3 913 587	734 842	+3 958 086
70°	0	+3 299 637	196 770	+3 306 041	392 357	+3 325 225	585 579	+3 357 113	775 258	+3 401 565	960 222	+3 458 391
65°	0	+2 759 350	240 571	+2 766 994	479 775	+2 789 898	716 248	+2 827 981	948 624	+2 881 110	1 175 542	+2 949 088
60°	0	+2 213 809	282 175	+2 222 561	562 835	+2 248 789	840 467	+2 292 419	1 113 555	+2 353 321	1 380 581	+2 431 312
55°	0	+1 664 056	321 546	+1 673 787	641 463	+1 702 062	958 118	+1 751 509	1 269 876	+1 819 313	1 575 095	+1 906 212
50°	0	+1 111 133	358 645	+1 121 723	715 572	+1 153 474	1 069 062	+1 206 328	1 417 387	+1 280 187	1 758 808	+1 374 910
45°	0	+ 556 096	393 422	+ 567 424	785 065	+ 601 395	1 173 145	+ 657 961	1 555 870	+ 737 046	1 931 430	+ 838 536
40°	0	0	425 827	+ 11 951	849 835	+ 47 792	1 270 200	+ 107 490	1 685 088	+ 190 989	2 092 644	+ 298 207
35°	0	− 556 096	455 800	− 543 637	909 762	− 506 266	1 360 044	− 444 005	1 804 787	− 356 887	2 242 115	− 244 963
30°	0	−1 111 133	483 280	−1 098 277	964 722	−1 059 712	1 442 480	− 995 443	1 914 696	− 905 490	2 379 489	− 789 868
25°	0	−1 664 056	508 200	−1 650 911	1 014 578	−1 611 480	1 517 303	−1 545 757	2 014 529	−1 453 735	2 504 389	−1 335 405
20°	0	−2 213 809	530 490	−2 200 485	1 059 186	−2 160 506	1 584 288	−2 093 872	2 103 978	−2 000 539	2 616 420	−1 880 485
15°	0	−2 759 350	550 072	−2 745 953	1 098 391	−2 705 752	1 643 198	−2 638 727	2 182 718	−2 544 835	2 715 156	−2 424 020
10°	0	−3 299 637	566 863	−3 286 269	1 132 024	−3 246 157	1 693 776	−3 179 267	2 250 398	−3 085 552	2 800 148	−2 964 935
5°	0	−3 833 644	580 775	−3 820 408	1 159 907	−3 780 690	1 735 750	−3 714 453	2 300 644	−3 621 639	2 870 912	−3 502 166
0°	0	−4 360 354	591 710	−4 347 349	1 181 844	−4 308 330	1 768 820	−4 243 252	2 351 051	−4 152 000	2 926 926	−4 034 658
−5°	0	−4 878 763	599 562	−4 866 090	1 197 621	−4 828 068	1 792 661	−4 764 047	2 383 170	−4 675 776	2 967 018	−4 561 306
−10°	0	−5 387 885	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------

[20] A mathematical account of this projection is given in: Zöppritz, Prof. Dr. Karl, Leitfaden der Kartenentwurfslehre, Erster Theil, Leipzig, 1899, pp. 38-44.

TABLE FOR THE CONSTRUCTION OF THE LAMBERT ZENITHAL EQUAL-AREA PROJECTION WITH CENTER ON PARALLEL 40°—Continued

Latitude	Longitude 30°		Longitude 35°		Longitude 40°		Longitude 45°		Longitude 50°		Longitude 55°	
	x	y	x	y	x	y	x	y	x	y	x	y
	Meters	*Meters*	*Meters*	*Meters*	*Meters*	*Meters*	*Meters*	*Meters*	*Meters*	*Meters*	*Meters*	*Meters*
90°	0	+5 387 885	0	+5 387 885	0	+5 387 885	0	+5 387 885	0	+5 387 885	0	+5 387 885
85°	301 461	+4 943 517	346 141	+4 966 260	388 315	+4 992 087	427 069	+5 020 815	463 906	+5 052 256	496 749	+5 086 182
80°	591 966	+4 484 481	680 290	+4 528 232	763 928	+4 578 013	842 275	+4 633 520	914 747	+4 694 410	980 794	+4 760 300
75°	871 326	+4 012 017	1 002 146	+4 075 098	1 126 401	+4 147 003	1 243 216	+4 227 345	1 351 741	+4 315 689	1 451 160	+4 411 541
70°	1 139 309	+3 527 345	1 311 367	+3 608 121	1 475 260	+3 700 352	1 629 866	+3 803 605	1 774 088	+3 917 391	1 906 860	+4 041 146
65°	1 395 644	+3 031 666	1 607 577	+3 128 536	1 809 998	+3 239 317	2 001 571	+3 363 571	2 180 973	+3 500 777	2 346 903	+3 650 347
60°	1 640 025	+2 526 155	1 890 367	+2 637 551	2 131 174	+2 766 548	2 357 656	+2 908 476	2 571 559	+3 067 068	2 770 280	+3 240 321
55°	1 872 122	+2 011 987	2 159 301	+2 136 366	2 434 962	+2 279 018	2 697 424	+2 439 543	2 944 994	+2 617 407	3 175 970	+2 812 236
50°	2 091 574	+1 490 314	2 413 918	+1 626 160	2 724 049	+1 782 160	3 020 156	+1 957 965	3 300 406	+2 153 154	3 562 936	+2 367 231
45°	2 298 001	+ 962 283	2 653 728	+1 108 095	2 996 737	+1 275 838	3 325 112	+1 464 921	3 636 906	+1 675 294	3 930 123	+1 906 435
40°	2 490 992	+ 429 035	2 878 225	+ 583 330	3 252 512	+ 762 697	3 611 530	+ 961 575	3 953 579	+1 185 033	4 276 453	+1 430 961
35°	2 670 123	− 108 302	3 086 874	+ 53 007	3 490 384	+ 238 848	3 878 620	+ 449 076	4 249 488	+ 683 507	4 600 820	+ 941 911
30°	2 834 946	− 648 604	3 279 120	− 481 739	3 710 011	− 289 326	4 125 567	− 71 434	4 523 664	+ 171 842	4 902 092	+ 440 377
25°	2 984 985	−1 190 758	3 454 376	−1 019 784	3 910 572	− 822 474	4 351 526	− 598 827	4 [illegible]75 104	− 348 847	5 179 093	− 72 554
20°	3 119 741	−1 733 658	3 612 032	−1 560 010	4 091 331	−1 359 474	4 555 609	−1 131 980	5 002 765	− 877 448	5 430 598	− 595 799
15°	3 238 685	−2 276 209	3 751 441	−2 101 311	4 251 505	−1 899 211	4 736 897	−1 669 779	5 205 559	−1 412 861	-------	-------
10°	3 341 257	−2 817 321	3 871 917	−2 642 587	4 390 271	−2 440 579	4 894 407	−2 211 115	5 382 331	−1 953 979	-------	-------
5°	3 426 851	−3 355 917	3 972 724	−3 182 747	4 506 751	−2 982 475	5 027 097	−2 754 883	5 531 855	−2 409 703	-------	-------
0°	3 494 820	−3 890 925	4 053 078	−3 720 706	4 600 002	−3 523 808	5 133 855	−3 299 979	5 652 831	−3 048 927	-------	-------
−5°	3 544 456	−4 421 288	4 112 118	−4 255 393	4 669 003	−4 063 478	5 213 471	−3 845 293	-------	-------	-------	-------
−10°	-------	-------	4 148 912	−4 785 731	4 712 638	−4 600 397	5 264 634	−4 389 713	-------	-------	-------	-------

Latitude	Longitude 60°		Longitude 65°		Longitude 70°		Longitude 75°		Longitude 80°		Longitude 85°	
	x	y	x	y	x	y	x	y	x	y	x	y
	Meters	*Meters*	*Meters*	*Meters*	*Meters*	*Meters*	*Meters*	*Meters*	*Meters*	*Meters*	*Meters*	*Meters*
90°	0	+5 387 885	0	+5 387 885	0	+5 387 885	0	+5 387 885	0	+5 387 885	0	+5 387 885
85°	525 944	+5 122 361	551 259	+5 160 540	572 489	+5 200 446	589 457	+5 241 790	602 013	+5 284 269	610 041	+5 327 574
80°	1 039 898	+4 830 776	1 091 579	+4 905 382	1 135 398	+4 983 620	1 170 961	+5 064 964	1 197 928	+5 148 854	1 216 010	+5 234 696
75°	1 540 690	+4 514 344	1 619 593	+4 623 500	1 687 179	+4 738 343	1 742 813	+4 858 149	1 785 923	+4 982 152	1 816 003	+5 109 509
70°	2 027 143	+4 174 238	2 133 939	+4 315 959	2 226 290	+4 465 524	2 303 315	+4 622 066	2 364 175	+4 784 658	2 408 109	+4 952 257
65°	2 498 081	+3 811 608	2 633 253	+3 983 794	2 751 208	+4 166 052	2 850 778	+4 357 428	2 930 855	+4 556 864	2 990 401	+4 763 192
60°	2 952 313	+3 427 563	3 116 166	+3 628 015	3 260 368	+3 840 796	3 383 486	+4 064 920	3 484 119	+4 299 248	3 560 930	+4 542 533
55°	3 388 643	+3 023 203	3 581 299	+3 249 622	3 752 226	+3 490 627	3 899 721	+3 745 227	4 022 100	+4 012 292	4 117 713	+4 290 523
50°	3 805 858	+2 599 608	4 027 258	+2 849 595	4 225 202	+3 116 386	4 397 736	+3 399 040	4 542 899	+3 696 461	4 658 728	+4 007 376
45°	4 202 726	+2 157 841	4 452 631	+2 428 909	4 677 842	+2 719 004	4 875 760	+3 027 033	5 044 577	+3 352 226	5 181 893	+3 693 310
40°	4 577 995	+1 698 957	4 855 977	+1 988 526	5 108 094	+2 299 076	5 331 972	+2 629 879	-------	-------	-------	-------
35°	4 930 376	+1 223 997	5 235 819	+1 529 404	-------	-------	-------	-------	-------	-------	-------	-------
30°	5 258 557	+ 734 004	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------

Latitude	Longitude 90°		Longitude 95°		Longitude 100°	
	x	y	x	y	x	y
	Meters	*Meters*	*Meters*	*Meters*	*Meters*	*Meters*
90°	0	+5 387 885	0	+5 387 885	0	+5 387 885
85°	613 457	+5 371 383	-------	-------	-------	-------
80°	1 225 008	+5 321 870	1 224 673	+5 409 785	-------	-------
75°	1 832 631	+5 239 340	1 835 468	+5 370 715	-------	-------
70°	2 434 454	+5 123 768	2 442 638	+5 297 996	-------	-------
65°	3 028 467	+4 975 129	3 044 202	+5 191 278	3 036 885	+5 410 124
60°	3 612 656	+4 793 373	3 638 131	+5 050 207	3 636 304	+5 311 321
55°	4 184 969	+4 578 461	4 222 340	+4 874 439	4 228 414	+5 176 606
50°	4 743 288	+4 330 321	4 794 678	+4 663 592	4 811 080	+5 005 259
45°	5 285 429	+4 048 873	-------	-------	-------	-------
40°	-------	-------	-------	-------	-------	-------
35°	-------	-------	-------	-------	-------	-------

TABLE FOR THE CONSTRUCTION OF THE LAMBERT ZENITHAL EQUAL-AREA MERIDIONAL PROJECTION

[Coordinates in units of the earth's radius.]

Latitude	Longitude 0°		Longitude 5°		Longitude 10°		Longitude 15°		Longitude 20°		Longitude 25°	
	x	y	x	y	x	y	x	y	x	y	x	y
°												
0	0	0.000000	0.087239	0.000000	0.174311	0.000000	0.261052	0.000000	0.347296	0.000000	0.432879	0.000000
5	0	0.087239	0.086991	0.087323	0.173812	0.087571	0.260302	0.087990	0.346294	0.088582	0.431623	0.089353
10	0	0.174311	0.086241	0.174476	0.172313	0.174972	0.258051	0.175804	0.343285	0.176979	0.427851	0.178510
15	0	0.261052	0.084992	0.261297	0.169813	0.262032	0.254295	0.263265	0.338266	0.265002	0.421558	0.267277
20	0	0.347296	0.083240	0.347617	0.166306	0.348581	0.249026	0.350199	0.331226	0.352484	0.412733	0.355457
25	0	0.432879	0.080981	0.483272	0.161785	0.434451	0.242235	0.436429	0.322153	0.439222	0.401363	0.442855
30	0	0.517638	0.078211	0.518096	0.156241	0.519473	0.233908	0.521780	0.311030	0.525038	0.387426	0.529273
35	0.	0.601412	0.074923	0.601928	0.149660	0.603479	0.224026	0.606079	0.297835	0.609748	0.370897	0.614515
40	0	0.684040	0.071109	0.684605	0.142028	0.686305	0.212568	0.689152	0.282538	0.693167	0.351743	0.698379
45	0	0.765367	0.066759	0.765971	0.133325	0.767787	0.199504	0.770825	0.265103	0.775110	0.329244	0.779058
50	0	0.845237	0.061860	0.845866	0.123525	0.847760	0.184800	0.850929	0.245487	0.855389	0.305387	0.861169
55	0	0.923497	0.056398	0.924139	0.112600	0.926064	0.168412	0.929286	0.223635	0.933818	0.278071	0.939682
60	0	1.000000	0.050351	1.000635	0.100511	1.002542	0.149939	1.005727	0.199480	1.010205	0.247901	1.015991
65	0	1.074599	0.043698	1.075207	0.087211	1.077032	0.130054	1.080079	0.172940	1.084356	0.214781	1.089874
70	0	1.147153	0.036408	1.147710	0.072644	1.149380	0.108537	1.152106	0.143914	1.156072	0.178601	1.161099
75	0	1.217523	0.028444	1.218000	0.056739	1.219429	0.084733	1.221810	0.112277	1.225142	0.139220	1.229422
80	0	1.285575	0.019762	1.285937	0.039407	1.287022	0.058818	1.288828	0.077878	1.291350	0.096471	1.294579
85	0	1.351180	0.010305	1.351387	0.020542	1.352150	0.030638	1.353030	0.040529	1.354459	0.050147	1.356283
90	0	1.414214	0.000000	1.414214	0.000000	1.414214	0.000000	1.414214	0.000000	1.414214	0.000000	1.414214

Latitude	Longitude 25°		Longitude 30°		Longitude 35°		Longitude 40°		Longitude 45°		Longitude 50°	
	x	y	x	y	x	y	x	y	x	y	x	y
°												
0	0.432879	0.000000	0.517638	0.000000	0.601412	0.000000	0.684040	0.000000	0.765367	0.000000	0.845237	0.000000
5	0.431623	0.089353	0.516124	0.090310	0.599638	0.091464	0.682000	0.092826	0.763056	0.094411	0.842647	0.096237
10	0.427851	0.178510	0.511581	0.180411	0.594311	0.182701	0.675879	0.185404	0.756122	0.188550	0.834881	0.192172
15	0.421558	0.267277	0.504001	0.270093	0.585428	0.273485	0.665670	0.277488	0.744560	0.282142	0.821934	0.287499
20	0.412733	0.355457	0.493374	0.359147	0.572975	0.363589	0.651364	0.368827	0.728365	0.374912	0.803803	0.381911
25	0.401363	0.442855	0.479684	0.447861	0.556939	0.452782	0.632946	0.459168	0.706066	0.465622	0.780484	0.475097
30	0.387426	0.529273	0.462910	0.534523	0.537297	0.540832	0.610397	0.548258	0.682022	0.556868	0.751972	0.566744
35	0.370897	0.614515	0.443023	0.620417	0.514021	0.627504	0.583694	0.635835	0.651842	0.645482	0.718257	0.656527
40	0.351743	0.698379	0.419990	0.704826	0.487078	0.712559	0.552805	0.721635	0.616961	0.732126	0.679328	0.744114
45	0.329244	0.779058	0.393765	0.787531	0.456425	0.795758	0.517691	0.805385	0.577350	0.816497	0.635176	0.829164
50	0.305387	0.861169	0.364296	0.868302	0.422007	0.876829	0.478307	0.886800	0.532976	0.898275	0.585785	0.911320
55	0.278071	0.939682	0.331516	0.946908	0.383762	0.955528	0.434595	0.965586	0.483798	0.977129	0.531139	0.990210
60	0.247901	1.015991	0.295345	1.023106	0.341338	1.030750	0.386490	1.041432	0.429767	1.052708	0.471219	1.065441
65	0.214781	1.089874	0.255687	1.096644	0.295462	1.104684	0.333910	1.114008	0.370826	1.124640	0.406007	1.136597
70	0.178601	1.161099	0.212423	1.167253	0.245202	1.174540	0.276761	1.182962	0.306915	1.192524	0.334709	1.203229
75	0.139220	1.229422	0.165411	1.234646	0.190699	1.240809	0.214982	1.247906	0.237959	1.255925	0.259626	1.264857
80	0.096471	1.294579	0.114481	1.298509	0.131794	1.303128	0.148297	1.308420	0.163878	1.314370	0.178427	1.320956
85	0.050147	1.356283	0.059427	1.358496	0.068301	1.361083	0.076708	1.364033	0.084588	1.367329	0.091882	1.370953
90	0.000000	1.414214	0.000000	1.414214	0.000000	1.414214	0.000000	1.414214	0.000000	1.414214	0.000000	1.414214

TABLE FOR THE CONSTRUCTION OF THE LAMBERT ZENITHAL EQUAL-AREA MERIDIONAL PROJECTION—Continued

[Coordinates in units of the earth's radius.]

Latitude	Longitude 50°		Longitude 55°		Longitude 60°		Longitude 65°		Longitude 70°		Longitude 75°	
	x	y	x	y	x	y	x	y	x	y	x	y
°												
0	0.845237	0.000000	0.923497	0.000000	1.000000	0.000000	1.074599	0.000000	1.147153	0.000000	1.217523	0.000000
5	0.842647	0.096237	0.920622	0.098326	0.996827	0.100703	1.071145	0.103398	1.143342	0.106449	1.213365	0.109001
10	0.834881	0.192172	0.911905	0.196312	0.987311	0.201021	1.060670	0.206359	1.131919	0.212397	1.200903	0.219222
15	0.821934	0.287499	0.897621	0.293617	0.971458	0.300570	1.043276	0.308444	1.112907	0.317341	1.180179	0.327383
20	0.803803	0.381911	0 877502	0.389897	0.949282	0.398961	1.018962	0.409211	1.086352	0.420776	1.151257	0.433805
25	0.780484	0.475007	0.851641	0.484802	0.920800	0.495801	0.987761	0.508217	1.052313	0.522193	1.114235	0.537905
30	0.751972	0.566744	0.820046	0.577981	0.886036	0.590601	0.949722	0.605007	1.010871	0.621083	1.069235	0.639100
35	0.718257	0.656527	0.782723	0.669068	0.844341	0.682676	0.904904	0.699123	0.962126	0.716924	1.016411	0.736805
40	0.679328	0.744114	0.739682	0.757694	0.797784	0.772979	0.853380	0.790097	0.906201	0.809194	0.955952	0.830435
45	0.635176	0 829164	0.690934	0.843475	0.744377	0.859533	0.795240	0.877451	0.843242	0.897359	0.888073	0.919401
50	0.585785	0.911320	0.636495	0.926012	0.684853	0.942438	0.730590	0.960693	0.773421	0.980881	0.813035	1.003117
55	0.531139	0.990210	0.576381	1.004891	0.619275	1.021236	0.659555	1.039318	0.696939	1.059210	0.731128	1.080994
60	0.471219	1.065441	0.510618	1.079673	0.547723	1.095445	0.582282	1.112802	0.614031	1.131788	0.642692	1.152445
65	0.406007	1.136597	0.439234	1.149898	0.470291	1.164563	0.498947	1.180610	0.524968	1.198048	0.548109	1.216887
70	0.334709	1.203229	0.362271	1.215076	0.387095	1.228063	0.409756	1.242180	0.430061	1.257414	0.447808	1.273745
75	0.259626	1.264857	0.279782	1.274684	0.298274	1.285385	0.314953	1.296935	0.329669	1.309303	0.342275	1.322449
80	0.178427	1.320956	0.191837	1.328156	0.204003	1.335940	0.214824	1.344276	0.224204	1.353126	0.232051	1.362449
85	0.091882	1.370953	0.098534	1.374885	0.104491	1.379104	0.109706	1.383581	0.114135	1.388292	0.117736	1.393206
90	0.000000	1.414214	0.000000	1.414214	0.000000	1.414214	0.000000	1.414214	0.000000	1.414214	0.000000	1.414214

Latitude	Longitude 75°		Longitude 80°		Longitude 85°		Longitude 90°	
	x	y	x	y	x	y	x	y
°								
0	1.217523	0.000000	1.285575	0.000000	1.351180	0.000000	1.414214	0.000000
5	1.213365	0.109001	1.281044	0.113806	1.346245	0.118231	1.408832	0.123257
10	1.200903	0.219222	1.267469	0.226937	1.331607	0.235695	1.392729	0.245576
15	1.180179	0.327383	1.244912	0.338721	1.306926	0.351527	1.366025	0.366025
20	1.151257	0.433805	1.213472	0.448481	1.272775	0.465022	1.328926	0.483690
25	1.114235	0.537905	1.173287	0.555553	1.229210	0.575380	1.281713	0.597672
30	1.069235	0.639100	1.124542	0.659270	1.176491	0.681843	1.224745	0.707107
35	1.016411	0.736805	1.067459	0.758974	1.114934	0.783667	1.158456	0.811160
40	0.955952	0.830435	1.002308	0.854010	1.044910	0.880132	1.083351	0.909039
45	0.888073	0.919401	0.929400	0.943738	0.966848	0.970541	1.000000	1.000000
50	0.813035	1.003117	0.849094	1.027521	0.881231	1.054223	0.909039	1.083351
55	0.731128	1.080994	0.761799	1.104745	0.788602	1.130542	0.811160	1.158456
60	0.642692	1.152445	0.667970	1.174806	0.689552	1.198901	0.707107	1.224745
65	0.548109	1.216887	0.568115	1.237122	0.584727	1.258741	0.597673	1.281713
70	0.447808	1.273745	0.462796	1.291138	0.474823	1.300551	0.483690	1.328926
75	0.342275	1.322449	0.352628	1.336326	0.360588	1.350874	0.366025	1.366025
80	0.232051	1.362449	0.238279	1.372193	0.242811	1.382308	0.245576	1.392729
85	0.117736	1.393206	0.120476	1.398291	0.122324	1.403512	0.123257	1.408832
90	0.000000	1.414214	0.000000	1.414214	0.000000	1.414214	0.000000	1.414214

THE LAMBERT CONFORMAL CONIC PROJECTION WITH TWO STANDARD PARALLELS

DESCRIPTION

[See Plates I and XI.]

This projection, devised by Johann Heinrich Lambert, first came to notice in his Beiträge zum Gebrauche der Mathematik und deren Anwendung, volume 3, Berlin, 1772.

Although used for a map of Russia, the basin of the Mediterranean, as well as for maps of Europe and Australia in Debes' Atlas, it was not until the beginning of the first World War that its merits were fully appreciated.

The French armies, in order to meet the need of a system of mapping in which a combination of minimum angular and scale distortion might be obtained, adopted this system of projection for the battle maps which were used by the allied forces in their military operations.

Historical Outline

Lambert, Johann Heinrich (1728–1777), physicist, mathematician, and astronomer, was born at Mülhausen, Alsace. He was of humble origin, and it was entirely due to his own efforts that he obtained his education. In 1764, after some years in travel, he removed to Berlin, where he received many favors at the hand of Frederick the Great, and was elected a member of the Royal Academy of Sciences of Berlin, and in 1774 edited the Ephemeris.

He had the facility for applying mathematics to practical questions. The introduction of hyperbolic functions to trigonometry was due to him, and his discoveries in geometry are of great value, as well as his investigations in physics and astronomy. He was also the author of several remarkable theorems on conics, which bear his name.

We are indebted to A. Wangerin, in Ostwald's Klassiker, 1894, for the following tribute to Lambert's contribution to cartography:

> The importance of Lambert's work consists mainly in the fact that he was the first to make general investigations upon the subject of map projection. His predecessors limited themselves to the investigations of a single method of projection, especially the perspective, but Lambert considered the problem of the representation of a sphere upon a plane from a higher standpoint and stated certain general conditions that the representation was to fulfill, the most important of these being the preservation of angles or conformality, and equal surface or equivalence. These two properties, of course, can not be attained in the same projection.
>
> Although Lambert has not fully developed the theory of these two methods of representation, yet he was the first to express clearly the ideas regarding them. The former—conformality—has become of the greatest importance to pure mathematics as well as the natural sciences, but both of them are of great significance to the cartographer. It is no more than just, therefore, to date the beginning of a new epoch in the science of map projection from the appearance of Lambert's work. Not only is his work of importance for the generality of his ideas but he has also succeeded remarkably well in the results that he has attained.

The name Lambert occurs most frequently in this branch of geography, and, as stated by Craig, it is an unquestionable fact that he has done more for the advancement of the subject in the way of inventing ingenious and useful methods than all of those who have either preceded or followed him. The manner in which Lambert analyzes and solves his problems is very instructive. He has developed several methods of projection that are not only interesting, but are to-day in use among cartographers, the most important of these being the one discussed in this chapter.

Among the projections of unusual merit, devised by Lambert, in addition to the conformal conic, is his zenithal (or azimuthal) equivalent projection already described in this paper.

Definition of the Term "Conformality"

A conformal projection or development takes its name from the property that all small or elementary figures found or drawn upon the surface of the earth retain their original forms upon the projection.

This implies that—

All angles between intersecting lines or curves are preserved;

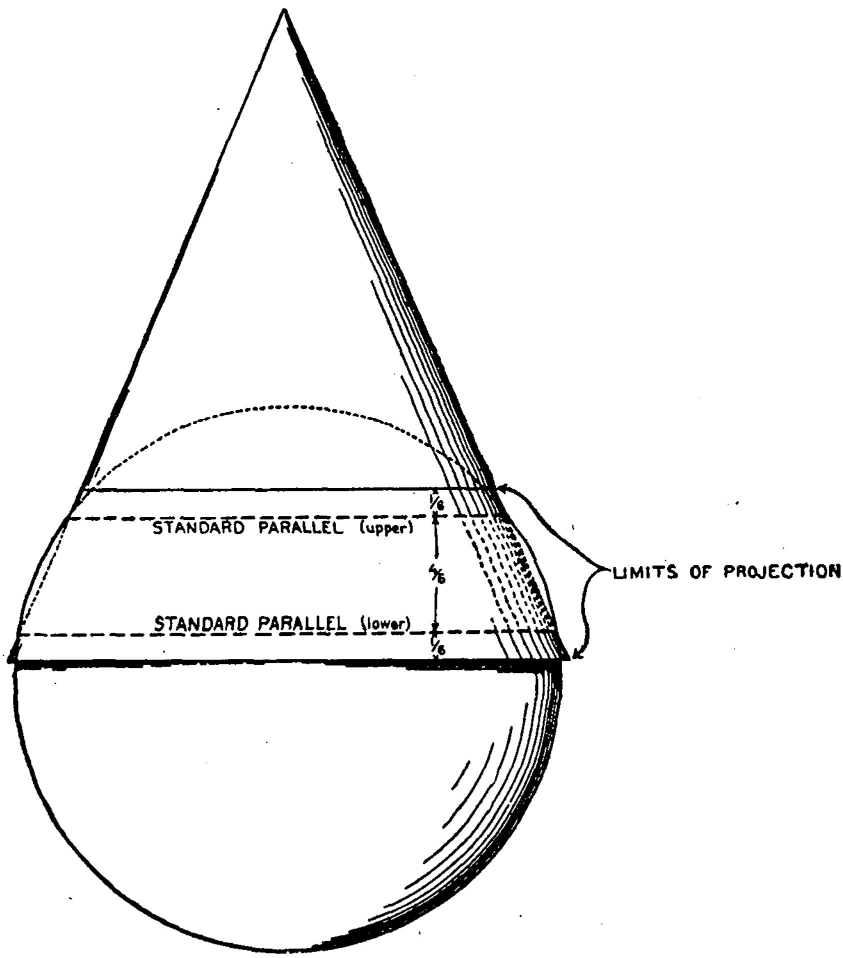

Fig. 53.—Lambert conformal conic projection.

Diagram illustrating the intersection of a cone and sphere along two standard parallels. The elements of the projection are calculated for the tangent cone and afterwards reduced in scale so as to produce the effect of a secant cone. The parallels that are true to scale do not exactly coincide with those of the earth, since they are spaced in such a way as to produce conformality.

For any given point (or restricted locality) the ratio of the length of a linear element on the earth's surface to the length of the corresponding map element is constant for all azimuths or directions in which the element may be taken.

Arthur R. Hinks, M. A., in his treatise on "Map projections," defines *orthomorphic*, which is another term for *conformal*, as follows:

> If at any point the scale along the meridian and the parallel is the same (not correct, but the same in the two directions) and the parallels and meridians of the map are at right angles to one another, then the shape of any very small area on the map is the same as the shape of the corresponding small area upon the earth. The projection is then called orthomorphic (right shape).

The Lambert Conformal Conic projection is of the simple conical type in which all meridians are straight lines that meet in a common point beyond the limits of the map, and the parallels are concentric circles whose center is at the point of intersection of the meridians. Meridians and parallels intersect at right angles and the angles formed by any two lines on the earth's surface are correctly represented on this projection.

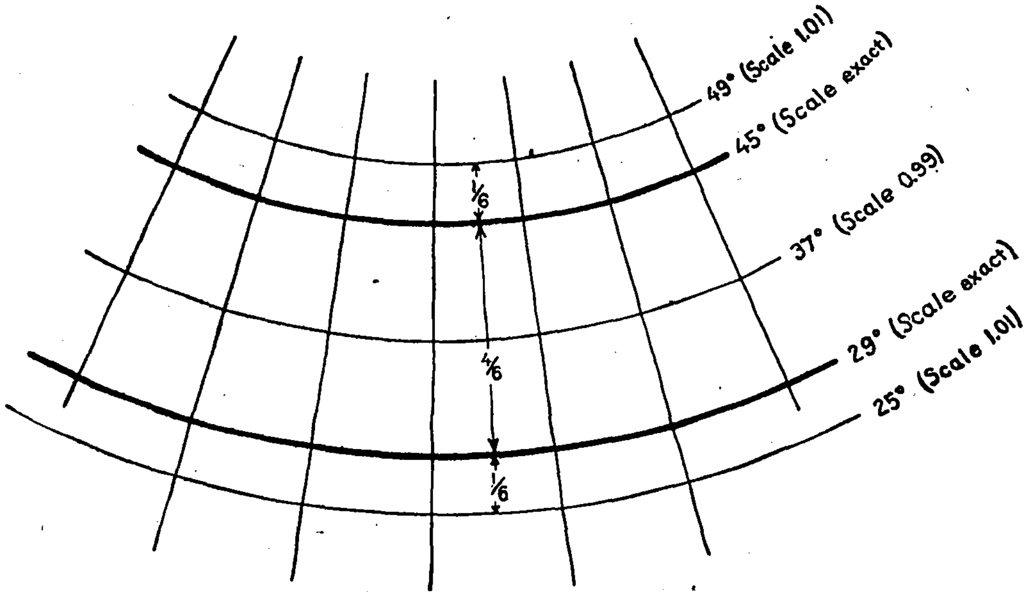

FIG. 54.—Scale distortion of the Lambert conformal conic projection with the standard parallels at 29° and 45°.

It employs a cone intersecting the spheroid at two parallels known as the standard parallels for the area to be represented. In general, for equal distribution of scale error, the standard parallels are chosen at one-sixth and five-sixths of the total length of that portion of the central meridian to be represented. It may be advisable in some localities, or for special reasons, to bring them closer together in order to have greater accuracy in the center of the map at the expense of the upper and lower border areas.

On the two selected parallels, arcs of longitude are represented in their true lengths, or to exact scale. Between these parallels the scale will be too small and beyond them too large. The projection is specially suited for maps having a pre-

609106°—45——6

dominating east-and-west dimension. For the construction of a map of the United States on this projection, see tables in U. S. Coast and Geodetic Survey Special Publication No. 52.

The chief advantage of this projection over the polyconic, as used by several Government bureaus for maps of the United States, consists in reducing the scale error from 7 per cent in the polyconic projection to 2½ or 1⅛ per cent in the Lambert projection, depending upon what parallels are chosen as standard.

The maximum scale error of 2½ per cent, noted above, applies to a base map of the United States, scale 1 : 5,000,000, in which the parallels 33° and 45° north latitude (see fig. 55) were selected as standards in order that the scale error along the central parallel of latitude might be small. As a result of this choice of standards, the maximum scale error between latitudes 30½° and 47½° is but one-half of 1 per cent, thus allowing that extensive and most important part of the United States to be favored with

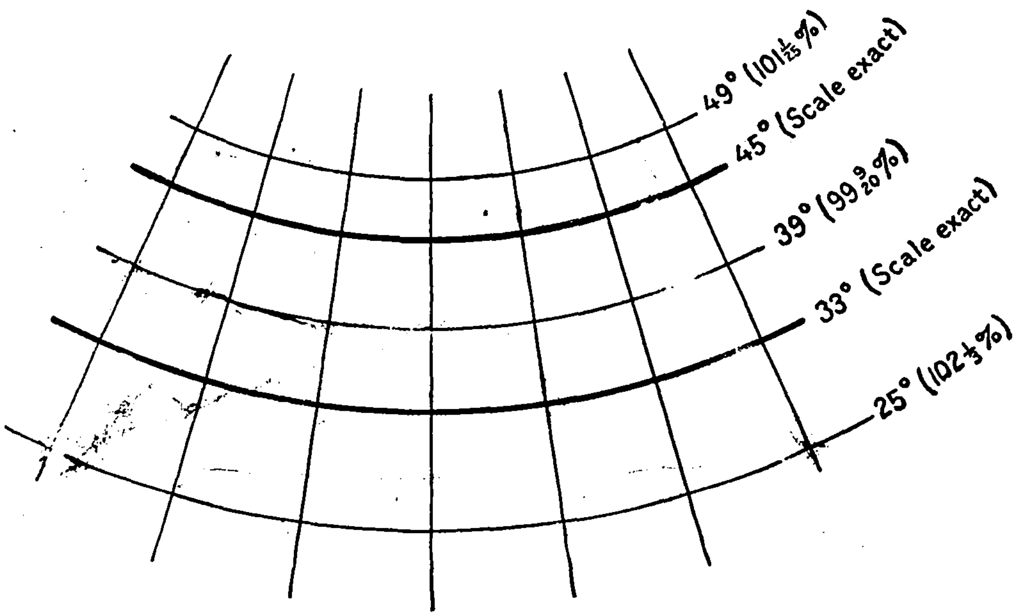

FIG. 55.—Scale distortion of the Lambert conformal conic projection with the standard parallels at 33° and 45°.

unusual scaling properties. The maximum scale error of 2½ per cent occurs in southernmost Florida. The scale error for southernmost Texas is somewhat less.

With standard parallels at 29° and 45° (see fig. 54), the maximum scale error for the United States does not exceed 1⅛ per cent, but the accuracy at the northern and southern borders is acquired at the expense of accuracy in the center of the map.

GENERAL OBSERVATIONS ON THE LAMBERT PROJECTION

In the construction of a map of France, which was extended to 7° of longitude from the middle meridian for purposes of comparison with the polyconic projection of the same area, the following results were noted:

Maximum scale error, Lambert = 0.05 per cent.
Maximum scale error, polyconic = 0.32 per cent.

Azimuthal and right line tests for orthodrome (great circle) also indicated a preference for the Lambert projection in these two vital properties, these tests indicating accuracies for the Lambert projection well within the errors of map construction and paper distortion.

In respect to areas, in a map of the United States, it should be noted that while in the polyconic projection they are misrepresented along the western margin in one dimension (that is, by meridional distortion of 7 per cent), on the Lambert projection [21] they are distorted along both the parallel and meridian as we depart from the standard parallels, with a resulting maximum error of 5 per cent.

In the Lambert projection for the map of France, employed by the allied forces in their military operations, the maximum scale errors do not exceed 1 part in 2000 and are practically negligible, while the angles measured on the map are practically equal to those on the earth. It should be remembered, however, that in the Lambert conformal conic, as well as in all other conic projections, the scale errors vary increasingly with the range of latitude north or south of the standard parallels. It follows, then, that this type of projections is not suited for maps having extensive latitudes.

AREAS.—For areas, as stated before, the Lambert projection is somewhat better than the polyconic for maps like the one of France or for the United States, where we have wide longitude and comparatively narrow latitude. On the other hand, areas are not represented as well in the Lambert projection or in the polyconic projection as they are in the Bonne or in other conical projections.

For the purpose of equivalent areas of large extent the Lambert zenithal (or azimuthal) equal-area projection offers advantages desirable for census or statistical purposes superior to other projections, excepting in areas of wide longitudes combined with narrow latitudes, where the Albers conical equal-area projection with two standard parallels is preferable.

In measuring areas on a map by the use of a planimeter, the distortion of the paper, due to the method of printing and to changes in the humidity of the air, must also be taken into consideration. It is better to disregard the scale of the map and to use the quadrilaterals formed by the latitude and longitude lines as units. The areas of quadrilaterals of the earth's surface are given for different extents of latitude and longitude in the Smithsonian Geographical Tables, 1897, Tables 25 to 29.

It follows, therefore, that for the various purposes a map may be put to, if the property of areas is slightly sacrificed and the several other properties more desirable are retained, we can still by judicious use of the planimeter or Geographical Tables overcome this one weaker property.

The idea seems to prevail among many that, while in the polyconic projection every parallel of latitude is developed upon its own cone, the multiplicity of cones so employed necessarily adds strength to the projection; but this is not true. The ordinary polyconic projection has, in fact, only one line of strength; that is, the central meridian. In this respect, then, it is no better than the Bonne.

[21] In the Lambert projection, every point has a scale factor characteristic of that point, so that the area of any restricted locality is represented by the expression

$$\text{Area} = \frac{\text{measured area on map}}{(\text{scale factor})^2}.$$

Without a knowledge of scale errors in projections that are not equivalent, erroneous results in areas are often obtained. In the table on p. 58, "Maximum error of area," only the Lambert zenithal and the Albers projections are equivalent, the polyconic and Lambert conformal being projections that have errors in area.

The Lambert projection, on the other hand, employs two lines of strength which are parallels of latitude suitably selected for the region to be mapped.

A line of strength is here used to denote a singular line characterized by the fact that the elements along it are truly represented in shape and scale.

Compensation of Scale Error

In the Lambert conformal conic projection we may supply a border scale for each parallel of latitude (see figs. 54 and 55), and in this way the scale variations may be accounted for when extreme accuracy becomes necessary.

With a knowledge of the scale factor for every parallel of latitude on a map of the United States, any sectional sheet that is a true part of the whole may have its own graphic scale applied to it. In that case the small scale error existing in the map as a whole becomes practically negligible in its sectional parts, and, although these parts have graphic scales that are slightly variant, they fit one another exactly. The system is thus truly progressive in its layout, and with its straight meridians and properties of conformality gives a precision that is unique and, within sections of 2° to 4° in extent, answers every requirement for knowledge of orientation and distances.

Caution should be exercised, however, in the use of the Lambert projection, or any conic projection, in large areas of wide latitudes, the system of projection not being suited to this purpose.

The extent to which the projection may be carried in longitude [22] is not limited, a property belonging to this general class of single-cone projections, but not found in the polyconic, where adjacent sheets have a "rolling fit" because the meridians are curved in opposite directions.

The question of choice between the Lambert and the polyconic system of projection resolves itself largely into a study of the shapes of the areas involved. The merits and defects of the Lambert and the polyconic projections may briefly be stated as being, in a general way, in opposite directions.

The Lambert conformal conic projection has unquestionably superior merits for maps of extended longitudes when the property of conformality outweighs the property of equivalence of areas. All elements retain their true forms and meridians and parallels cut at right angles, the projection belonging to the same general formula as the Mercator and stereographic, which have stood the test of time, both being likewise conformal projections.

It is an obvious advantage to the general accuracy of the scale of a map to have two standard parallels of true lengths, that is to say, two axes of strength instead of one. As an additional asset all meridians are straight lines, as they should be. Conformal projections, except in special cases, are generally of not much use in map

[22] A map (chart No. 3070, see Plate I) on the Lambert conformal conic projection of the North Atlantic Ocean, including the eastern part of the United States and the greater part of Europe, has been prepared by the Coast and Geodetic Survey. The western limits are Duluth to New Orleans; the eastern limits, Bagdad to Cairo; extending from Greenland in the north to the West Indies in the south; scale 1:10,000,000. The selected standard parallels are 36° and 54° north latitude, both these parallels being, therefore, true scale. The scale on parallel 45° (middle parallel) is but 1⅕ per cent too small; beyond the standard parallels the scale is increasingly large. This map, on certain other well-known projections covering the same area, would have distortions and scale errors so great as to render their use inadmissible. It is not intended for navigational purposes, but was constructed for the use of another department of the Government, and is designed to bring the two continents vis-à-vis in an approximately true relation and scale. The projection is based on the rigid formula of Lambert and covers a range of longitude of 165 degrees on the middle parallel. Plate I is a reduction of chart No. 3070 to approximate scale 1:25,500,000.

making unless the meridians are straight lines, this property being an almost indispensable requirement where orientation becomes a prime element.

Furthermore, the projection is readily constructed, free of complex curves and deformations, and simple in use.

It would be a better projection than the Mercator in the higher latitudes when charts have extended longitudes, and when the latter (Mercator) becomes objectionable. It can not, however, displace the latter for general sailing purposes, nor can it displace the gnomonic (or central) projection in its application and use to navigation.

Thanks to the French, it has again, after a century and a quarter, been brought to prominent notice at the expense, perhaps, of other projections that are not conformal—projections that misrepresent forms when carried beyond certain limits. Unless these latter types possess other special advantages for a subject at hand, such as the polyconic projection which, besides its special properties, has certain tabular superiority and facilities for constructing field sheets, they will sooner or later fall into disuse.

On all recent French maps the name of the projection appears in the margin. This is excellent practice and should be followed at all times. As different projections have different distinctive properties, this feature is of no small value and may serve as a guide to an intelligible appreciation of the map.

In the accompanying plate (No. I),[23] North Atlantic Ocean on a Lambert conformal conic projection, a number of great circles are plotted in red in order that their departure from a straight line on this projection may be shown.

GREAT-CIRCLE COURSES.—A great-circle course from Cape Hatteras to the English Channel, which falls within the limits of the two standard parallels, indicates a departure of only 15.6 nautical miles from a straight line on the map, in a total distance of about 3,200 nautical miles. The departure of this line on a polyconic projection is given as 40 miles in Lieut. Pillsbury's Charts and Chart Making.

DISTANCES.—The computed distance from Pittsburgh to Constantinople is 5,277 statute miles. The distance between these points as measured by the graphic scale on the map without applying the scale factor is 5,258 statute miles, a resulting error of less than four-tenths of 1 per cent in this long distance. By applying the scale factor true results may be obtained, though it is hardly worth while to work for closer results when errors of printing and paper distortion frequently exceed the above percentage.

The parallels selected as standards for the map are 36° and 54° north latitude. The coordinates for the construction of a projection with these parallels as standards are given on page 88.

CONSTRUCTION OF A LAMBERT CONFORMAL CONIC PROJECTION

FOR A MAP OF THE UNITED STATES

The mathematical development and the general theory of this projection are given in U. S. Coast and Geodetic Survey Special Publications Nos. 52 and 53. The method of construction is given on pages 20–21, and the necessary tables on pages 68 to 87 of the former publication.

[23] See footnote on p. 84.

Another simple method of construction is the following one, which involves the use of a long beam compass and is hardly applicable to scales larger than 1:2,500,000.

Draw a line for a central meridian sufficiently long to include the center of the curves of latitude and on this line lay off the spacings of the parallels, as taken from Table 1, Special Publication No. 52. With a beam compass set to the values of the radii, the parallels of latitude can be described from a common center.

(By computing chord distances for 25° of arc on the upper and lower parallels of latitude, the method of construction and subdivision of the meridians is the same as that described below under the heading, *For small-scale maps.*)

However, instead of establishing the outer meridians by chord distances on the upper and lower parallels we can determine these meridians by the following simple process:

Assume 39° of latitude as the central parallel of the map (see fig. 56), with an upper and lower parallel located at 49° and 24°. To find on parallel 24° the intersection of the meridian 25° distant from the central meridian, lay off on the central meridian the value of the y coordinate (south from the thirty-ninth parallel 1,315,273 meters, as taken from the tables, page 69, Special Pub. No. 52, second column, opposite 25°), and from this point strike an arc with the x value (2,581,184 meters, first column). The intersection with parallel 24° establishes the point of intersection of the parallel and outer meridian.

In the same manner establish the intersection of the upper parallel with the same outer meridian. The projection can then be completed by subdivision for intermediate meridians or by extension for additional ones.

The following values for radii and spacings in addition to those given in Table 1, Special Publication No. 52, may be of use for extension of the map north and south of the United States:

Latitude	Radius	Spacings from 39°
51	6 492 973	1 336 305
50	6 605 970	1 223 308
•	• • •	• • •
•	• • •	• • •
23	9 615 911	1 786 633
22	9 730 456	1 901 178

For Small-Scale Maps

In the construction of a map of the North Atlantic Ocean (see reduced copy on Plate I), scale 1:10,000,000, the process of construction is very simple.

Draw a line for a central meridian sufficiently long to include the center of the curves of latitude so that these curves may be drawn in with a beam compass set to the respective values of the radii as taken from the tables given on page 88.

To determine the meridians, a chord distance $\left(\text{chord}=2\,r\sin\frac{\theta}{2}\right)$ may be computed and described from and on each side of the central meridian on a lower parallel of latitude; preferably this chord should reach an outer meridian. The angle θ equals $l\lambda$ in which λ is the longitude out from the central meridian, and l is a factor of proportionality; r is the radius of the parallel in question. On parallel 30°, the chord of 65 degrees=6,230,277 meters.

By means of a straightedge the points of intersection of the chords at the outer ends of a lower parallel can be connected with the same center as that used in describing the parallels of latitude. This, then, will determine the outer meridians of the map. The lower parallel can then be subdivided into as many equal spaces as the meridional interval of the map may require, and the meridians can then be drawn in as straight lines to the same center as the outer ones.

If a long straightedge is not available, the spacings of the meridians on the upper parallel can be obtained from chord distance and subdivision in a similar manner to that employed on the lower parallel. Lines drawn through corresponding points on the upper and lower parallels will then determine the meridians of the map.

An adaptation of this method has been developed in connection with the construction of many aeronautical charts on this projection in recent years. Instead of the conventional rectangular coordinates ("x" and "y"), direct-plotting values ("a and "b") are used, as follows:

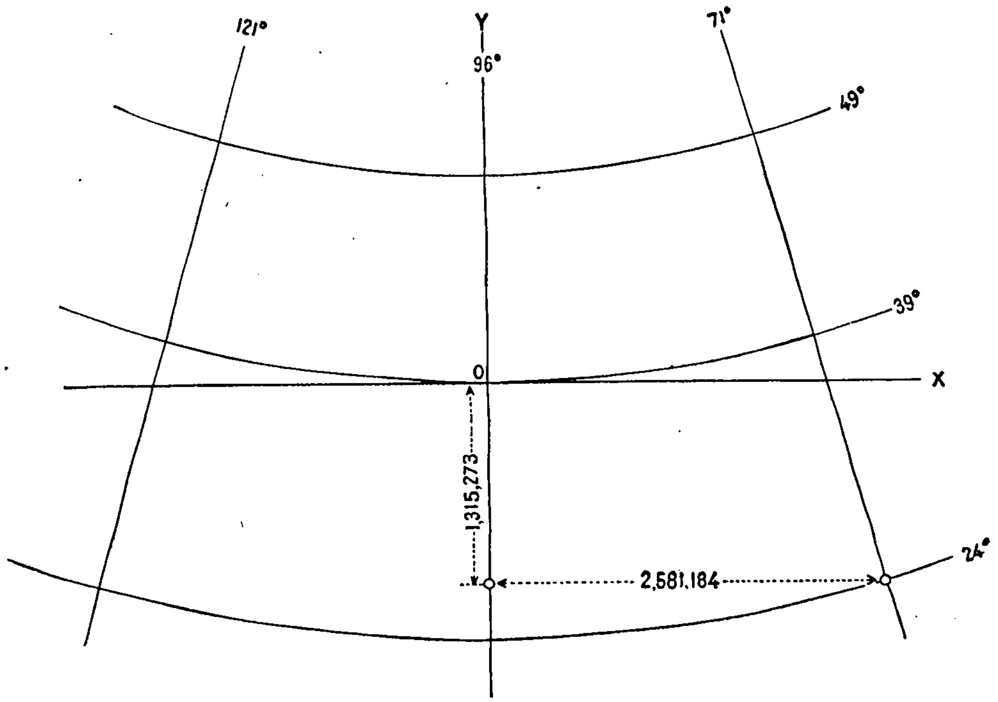

FIG. 56.—Diagram for constructing a Lambert projection of small scale.

Construction lines are drawn for the top and bottom parallels of the chart, each at right angles to the central meridian, and the "a" values for the desired differences of longitude are laid off along both. The meridians are then drawn directly, as straight lines between corresponding points on the two construction lines.

The "b" values represent the distance along each meridian from the construction line to the parallel itself, and are therefore measured along the meridians, *not* at right angles to the construction lines. The arcs of the parallels are drawn through the points so obtained on each meridian. Special tables for this method have been computed, and may be obtained from the Director, U. S. Coast and Geodetic Survey.

This method of construction for small-scale maps is far more satisfactory than the one involving rectangular coordinates.

TABLE FOR THE CONSTRUCTION OF A LAMBERT CONFORMAL CONIC PROJECTION WITH STANDARD PARALLELS AT 36° AND 54°

[This table was used in the construction of U. S. Coast and Geodetic Survey Chart No. 3070, North Atlantic Ocean, scale 1:10,000,000. See Plate I for reduced copy.]

[l=0.710105; log l=9.8513225; log K=7.0685567.]

Latitude	Radii	Spacings of parallels
°	*Meters*	*Meters*
75	2 787 926.3	3 495 899.8
70	3 430 293.7	2 853 532.4
65	4 035 253.3	2 248 572.8
60	4 615 578.7	1 668 247.4
55	5 179 778.8	1 104 052.3
50	5 734 157.3	549 068.8
45	6 283 826.1	000 000.0
40	6 833 182.5	549 356.4
35	7 386 250.0	1 102 423.9
30	7 946 910.9	1 663 084.8
25	8 519 064.7	2 235 238.6
20	9 106 795.8	2 822 969.7
15	9 714 515.9	3 430 689.8

Longitude	Coordinates of parallel 60°		Coordinates of parallel 30°		Coordinates of parallel 40°	
	x	*y*	*x*	*y*	*x*	*y*
°	*Meters*	*Meters*	*Meters*	*Meters*	*Meters*	*Meters*
5	285 837	8 859	492 142	15 253		
10	570 576	35 403	982 394	60 955		
15	853 125	79 529	1 468 876	136 930		
20	1 132 400	141 069	1 949 718	242 887		
25	1 407 327	219 785	2 423 076	378 417		
30	1 676 851	315 377	2 887 132	543 002		
35	1 939 939	427 476	3 340 105	736 010		
40	2 195 579	555 652	3 780 256	956 699		
45	2 442 790	699 415	4 205 894	1 204 222		
50	2 680 625	858 210	4 615 387	1 477 630		
55	2 908 169	1 031 430	5 007 163	1 775 872		
60	3 124 549	1 218 408	5 379 716	2 097 804		
65	3 328 933	1 418 428	5 731 616	2 442 190		
70	3 520 539	1 630 721	6 061 515	2 807 708		
75	3 698 630	1 854 473	6 368 146	3 192 953		
80	3 862 522	2 088 825			5 718 312	3 092 422
85	4 011 588	2 332 875			5 938 997	3 453 729
90	4 145 251	2 585 689			6 136 881	3 828 010

Scale along the Parallels.

Latitude—Degrees.	Scale factor.	Latitude—Degrees.	Scale factor.
20	1.079	50	0.991
30	1.021	54	1.000
36	1.000	60	1.022
40	0.992	70	1.113
45	0.988		

(To correct distances measured with graphic scale, divide by scale factor.)

TABLE FOR THE CONSTRUCTION OF A LAMBERT CONFORMAL CONIC PROJECTION WITH STANDARD PARALLELS AT 10° AND 48°40′

[This table was used in the construction of a map of the Northern and Southern Hemispheres. See Plate VII.]

[$l=\frac{1}{2}$; log K=7.1369624.]

Latitude (degrees)	Radius	Difference	Scale along the parallel
	Meters	*Meters*	
0	13 707 631	118 306	1.0746
1	13 589 325	117 319	1.0655
2	13 472 006	116 378	1.0567
3	13 355 628	115 479	1.0484
4	13 240 149	114 623	1.0404
5	13 125 526	113 807	1.0328
6	13 011 719	113 026	1.0256
7	12 898 693	112 287	1.0187
8	12 786 406	111 587	1.0121
9	12 674 819	110 920	1.0059
10	12 563 899	110 294	1.0000
11	12 453 605	109 699	0.9944
12	12 343 906	109 140	0.9891
13	12 234 766	108 618	0.9842
14	12 126 148	108 123	0.9795
15	12 018 025	107 668	0.9751
16	11 910 357	107 243	0.9711
17	11 803 114	106 850	0.9673
18	11 696 264	106 486	0.9638
19	11 580 778	106 164	0.9606
20	11 483 614	105 863	0.9576
21	11 377 751	105 598	0.9550
22	11 272 153	105 361	0.9526
23	11 166 792	105 164	0.9505
24	11 061 628	104 986	0.9487
25	10 956 642	104 847	0.9471
26	10 851 795	104 736	0.9459
27	10 747 059	104 659	0.9449
28	10 642 400	104 609	0.9442
29	10 537 791	104 594	0.9437
30	10 433 197	104 610	0.9436
31	10 328 587	104 658	0.9437
32	10 223 929	104 743	0.9442
33	10 119 186	104 852	0.9449
34	10 014 334	105 002	0.9459
35	9 909 332	105 181	0.9473
36	9 804 151	105 400	0.9489
37	9 698 751	105 651	0.9508
38	9 593 100	105 939	0.9531
39	9 487 161	106 265	0.9557
40	9 380 896		0.9586
40	9 380 896	106 629	0.9586
41	9 274 267	107 031	0.9610
42	9 167 236	107 473	0.9656
43	9 059 763	107 961	0.9696
44	8 951 802	108 491	0.9740
45	8 843 311	109 059	0.9787
46	8 734 252	109 683	0.9839
47	8 624 569	110 349	0.9896
48	8 514 220	111 072	0.9956
49	8 403 148	111 846	1.0021
50	8 291 302	112 672	1.0092
51	8 178 630	113 560	1.0167
52	8 065 070	114 510	1.0248
53	7 950 560	115 518	1.0334
54	7 835 042	116 604	1.0426
55	7 718 438	117 759	1.0525
56	7 600 679	118 993	1.0630
57	7 481 686	120 308	1.0743
58	7 361 378	121 713	1.0863
59	7 239 665	123 211	1.0992
60	7 116 454	124 812	1.1129
61	6 991 642	126 525	1.1276
62	6 865 117	128 355	1.1433
63	6 736 762	130 316	1.1601
64	6 606 446	132 418	1.1782
65	6 474 028	134 676	1.1975
66	6 339 352	137 103	1.2184
67	6 202 249	139 718	1.2408
68	6 062 531	142 545	1.2650
69	5 919 986	145 602	1.2912
70	5 774 384	148 922	1.3195
71	5 625 462	152 538	1.35
72	5 472 924	156 491	1.38
73	5 316 433	160 829	1.42
74	5 155 604	165 612	1.46
75	4 989 992	170 919	1.51
76	4 819 073	176 836	1.56
77	4 642 237	183 485	1.61
78	4 458 752	191 025	1.67
79	4 267 727	199 652	1.75
80	4 068 075	209 656	1.83
81	3 858 419	221 422	1.93
82	3 636 997		2.04
48° 30′	8 458 879	------------	0.9988

THE GRID SYSTEM OF MILITARY MAPPING

A grid system (or quadrillage) is a system of squares determined by the rectangular coordinates of the projection. This system is referred to one origin and is extended over the whole area of the original projection so that every point on the map is coordinated both with respect to its position in a given square as well as to its position in latitude and longitude.

The orientation of all sectional sheets or parts of the general map, wherever located, and on any scale, conforms to the initial meridian of the origin of coordinates. This system adapts itself to the quick computation of distances between points whose grid coordinates are given, as well as the determination of the azimuth of a line joining any two points within artillery range and, hence, is of great value to military operations.

The system was introduced by the First Army in France under the name "Quadrillage kilomètrique système Lambert," and manuals (Special Publications Nos. 47 and 49, now out of print) containing method and tables for constructing the quadrillage, were prepared by the Coast and Geodetic Survey.

As the French divide the circumference of the circle into 400 grades instead of 360°, certain essential tables were included for the conversion of degrees, minutes, and seconds into grades, as well as for miles, yards, and feet into their metric equivalents, and vice versa.

The advantage of the decimal system is obvious, and its extension to practical cartography merits consideration. The quadrant has 100 grades, and instead of 8°39′56″, we can write decimally 9.6284 grades.

GRID SYSTEM FOR PROGRESSIVE MAPS IN THE UNITED STATES

The French system (Lambert) of military mapping presented a number of features that were not only rather new to cartography but were specially adapted to the quick computation of distances and azimuths in military operations. Among these features may be mentioned: (1) A conformal system of map projection which formed the basis. Although dating back to 1772, the Lambert projection remained practically in obscurity until the outbreak of the first World War; (2) the advantage of one reference datum; (3) the grid system, or system of rectangular coordinates, already described; (4) the use of the centesimal system for graduation of the circumference of the circle, and for the expression of latitudes and longitudes in place of the sexagesimal system of usual practice.

While these departures from conventional mapping offered many advantages to an area like the French war zone, with its possible eastern extension, military mapping in the United States presented problems of its own. It was necessary to adopt a succession of zones on the polyconic projection, 7 in number, extending north and south across the United States, covering a range of 9° in longitude and having overlaps of 1° of longitude with adjacent zones east and west.

A grid system similar to the French, as already described, is projected over the whole area of each zone. The table of coordinates for one zone can be used for any other zone, as each has its own central meridian.

The overlapping area can be shown on two sets of maps, one on each grid system, thus making it possible to have progressive maps for each of the zones; or the two

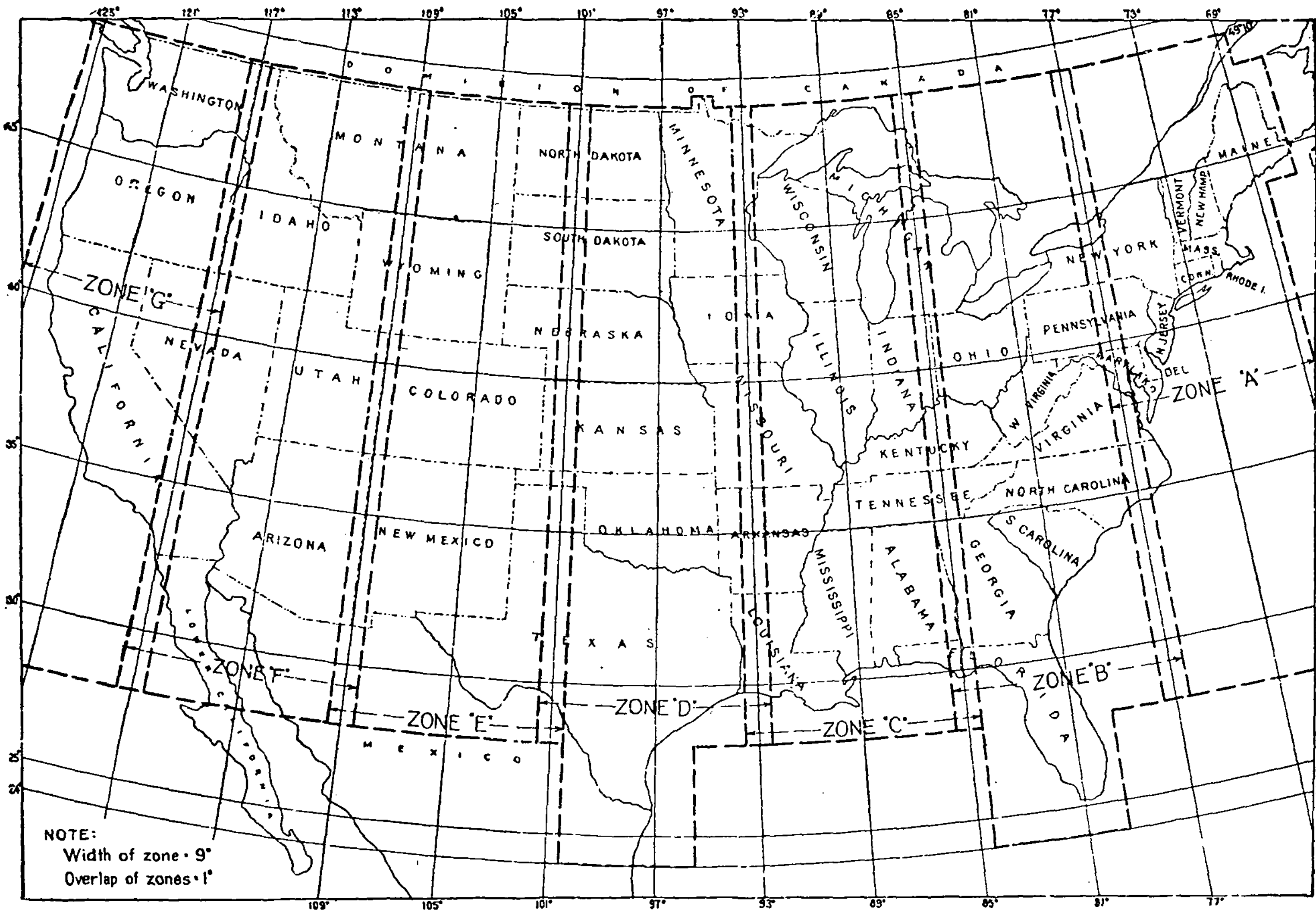

FIG. 57.—Grid zones for progressive military maps of the United States.

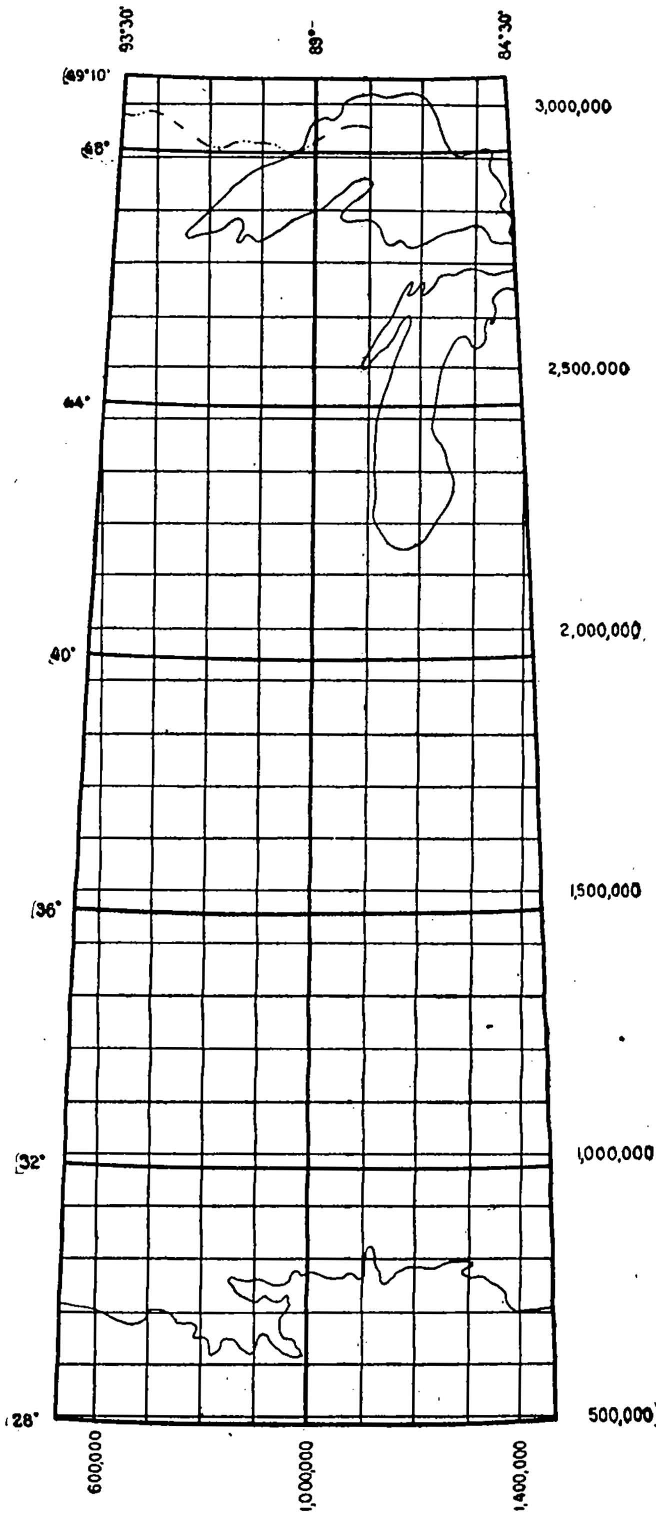

Fig. 58.—Diagram of zone C, showing grid system.

grid systems can be placed in different colors on the same overlap. (The maximum scale error within any zone will be about one part in 500.)

The system is styled progressive military mapping, but it is, in fact, an interrupted system, the overlap being the stepping-stone to a new system of coordinates. The grid system instead of being kilometric, as in the French system, is based on units of 1000 yards.

REFERENCES

Sp. Pub. 59, "Grid System for Progressive Maps in the United States" (out of print; see next paragraph).

Grid System for Military Maps of United States. 4 vols.: 0° to 7°; 7° to 28°; 24° to 50° (a reprint of Coast and Geodetic Survey Sp. Pub. 59); and 49° to 72° printed by Map Reproduction Office, U. S. Engineers.

As there are many other plane coordinate grid systems throughout the world used for military or other purposes, it would be futile under present world conditions to examine these in detail even though the necessary data were available.

THE ALBERS CONICAL EQUAL-AREA PROJECTION WITH TWO STANDARD PARALLELS

DESCRIPTION

[See Plate III]

This projection, devised by Albers[24] in 1805, possesses advantages over others now in use, which for many purposes gives it a place of special importance in cartographic work.

In mapping a country like the United States with a predominating east-and-west extent, the Albers system is peculiarly applicable on account of its many desirable properties as well as the reduction to a minimum of certain unavoidable errors.

The projection is of the conical type, in which the meridians are straight lines that meet in a common point beyond the limits of the map, and the parallels are concentric circles whose center is at the point of intersection of the meridians. Meridians and parallels intersect at right angles and the arcs of longitude along any given parallel are of equal length.

It employs a cone intersecting the spheroid at two parallels known as the standard parallels for the area to be represented. In general, for equal distribution of scale error, the standard parallels are placed within the area represented at distances from its northern and southern limits each equal to one-sixth of the total meridional distance of the map. It may be advisable in some localities, or for special reasons, to bring them closer together in order to have greater accuracy in the center of the map at the expense of the upper and lower border areas.

On the two selected parallels, arcs of longitude are represented in their true lengths. Between the selected parallels the scale along the meridians will be a trifle too large and beyond them too small.

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[24] Dr. H. C. Albers, the inventor of this projection, was a native of Lüneburg, Germany. Several articles by him on the subject of map projections appeared in Zach's *Monatliche Correspondenz* during the year 1805. Very little is known about him, not even his full name, the title "doctor" being used with his name by Germain about 1865. A book of 40 pages, entitled Unterricht im Schachsspiel (Instruction in Chess Playing) by H. C. Albers, Lüneburg, 1821, may have been the work of the inventor of this projection.

The projection is specially suited for maps having a predominating east-and-west dimension. Its chief advantage over certain other projections used for a map of the United States consists in the valuable property of equal-area representation combined with a scale error [25] that is practically the minimum attainable in any system covering this area in a single sheet.

In most conical projections, if the map is continued to the pole the latter is represented by the apex of the cone. In the Albers projection, however, owing to the fact that conditions are imposed to hold the scale exact along two parallels instead of one, as well as the property of equivalence of area, it becomes necessary to give up the requirement that the pole should be represented by the apex of the cone; this means that if the map should be continued to the pole the latter would be represented by a circle, and the series of triangular graticules surrounding the pole would be represented by quadrangular figures. This can also be interpreted by the statement that the map is projected on a truncated cone, because the part of the cone above the circle representing the pole is not used in the map.

The desirable properties obtained in mapping the United States by this system may be briefly stated as follows:

1. As stated before, it is an equal-area, or equivalent, projection. This means that any portion of the map bears the same ratio to the region represented by it that any other portion does to its corresponding region, or the ratio of any part is equal to the ratio of area of the whole representation.

2. The maximum scale error is but 1¼ per cent, which amount is about the minimum attainable in any system of projection covering the whole of the United States in a single sheet. Other projections now in use have scale errors of as much as 7 per cent.

The scale along the selected standard parallels of latitude 29½° and 45½° is true. Between these selected parallels, the meridional scale will be too great and beyond them too small. The scale along the other parallels, on account of the compensation for area, will always have an error of the opposite sign to the error in the meridional scale. It follows, then, that in addition to the two standard parallels, there are at any point two diagonal directions or curves of true-length scale approximately at right angles to each other. Curves possessing this property are termed isoperimetric curves.

On Plate III, a number of isoperimetric curves are shown in red. Due to the symmetry of the map projection, there are an infinite number of isoperimetric (true scale) curves, exact replicas of those shown.

Fig. 59 illustrates an isoperimetric curve on an Albers conical equal-area projection of the world, with standard parallels at 0° and 30° north latitude. The computation and drawing for this sketch were made by Mr. M. R. MacPhail of the Standard Oil Company of Venezuela. As a pioneer in this particular property of equal-area projections, much credit is due to him for this interesting contribution to cartography.

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[25] The standards chosen for a map of the United States on the Albers projection are parallels 29½° and 45½°, and this selection provides for a scale error slightly less than 1 per cent in the center of the map, with a maximum of 1¼ per cent along the northern and southern borders. This arrangement of the standards also places them at an even 30-minute interval.

The standards in this system of projection, as in the Lambert conformal conic projection, can be placed at will, and by not favoring the central or more important part of the United States a maximum scale error of somewhat less than 1¼ per cent might be obtained. Prof. Hartl suggests the placing of the standards so that the total length of the central meridian remain true, and this arrangement would be ideal for a country more rectangular in shape with predominating east-and-west dimensions.

With a knowledge of the scalė factors for the different parallels of latitude it would be possible to apply corrections to certain measured distances, but when we remember that the maximum scale error is practically the smallest attainable, any greater refinement in scale is seldom worth while, especially as errors due to distortion of paper, the method of printing, and to changes in the humidity of the air must also be taken into account and are frequently as much as the maximum scale error.

It therefore follows that for scaling purposes, the projection under consideration is superior to others with the exception of the Lambert conformal conic, but the latter is not equal-area. It is an obvious advantage to the general accuracy of the scale of a map to have two standard parallels of latitude of true lengths; that is to say, two axes of strength instead of one.

Caution should be exercised in the selection of standards for the use of this projection in large areas of wide latitudes, as scale errors vary increasingly with the range of latitude north or south of the standard parallels.

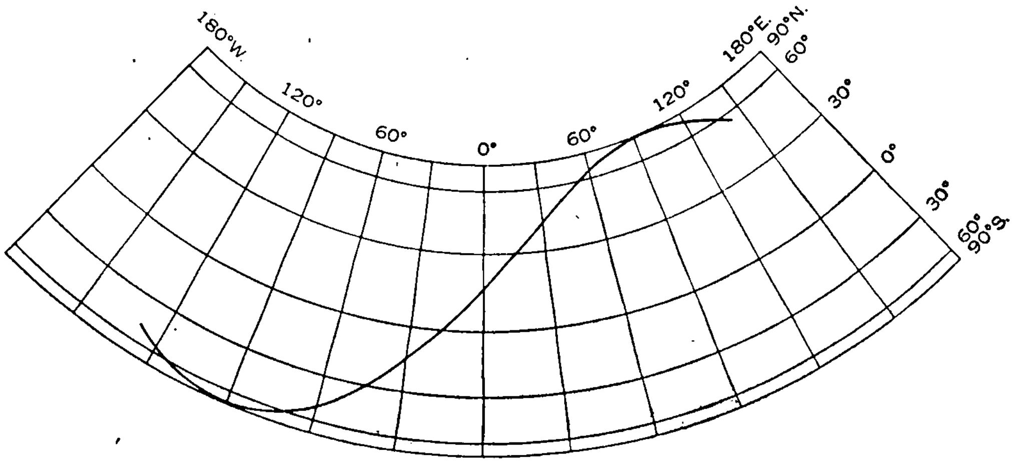


FIG. 59.—Albers conical equal-area projection of the world. With standard parallels at 0° and 30° N. latitude. Showing an isoperimetric curve.

3. The meridians are straight lines, crossing the parallels of concentric circles at right angles, thus preserving the angle of the meridians and parallels and facilitating construction. The intervals of the parallels depend upon the condition of equal-area.

The time required in the construction of this projection is but a fraction of that employed in other well-known systems that have far greater errors of scale or lack the property of equal-area.

4. The projection, besides the many other advantages, does not deteriorate as we depart from the central meridian, and by reason of straight meridians it is easy at any point to measure a direction with the protractor. In other words it is adapted to indefinite east-and-west extension, a property belonging to this general class of single-cone projections, but not found in the polyconic, where adjacent sheets con-

structed on their own central meridians have a "rolling fit," because meridians are curved in opposite directions.

Sectional maps on the Albers projection would have an exact fit on all sides, and the system is, therefore, suited to any project involving progressive equal-area mapping. The term "sectional maps" is here used in the sense of separate sheets which, as parts of the whole, are not computed independently, but with respect to the one chosen prime meridian and fixed standards. Hence the sheets of the map fit accurately together into one whole map, if desired.

The first notice of this projection appeared in Zach's Monatliche Correspondenz zur Beförderung der Erd- und Himmels-Kunde, under the title "Beschreibung einer neuen Kegelprojection von H. C. Albers," published at Gotha, November, 1805, pages 450 to 459.

A more recent development of the formulas is given in Studien über flächentreue Kegelprojectionen by Heinrich Hartl, Mittheilungen des K. u. K. Militär-Geographischen Institutes, volume 15, pages 203 to 249, Vienna, 1895–96; and in Lehrbuch der Landkartenprojectionen by Dr. Norbert Herz, page 181, Leipzig, 1885.

It was employed in a general map of Europe by Reichard at Nuremberg in 1817 and has since been adopted in the Austrian general-staff map of Central Europe; also, by reason of being peculiarly suited to a country like Russia, with its large extent of longitude, it was used in a wall map published by the Russian Geographical Society.

An interesting equal-area projection of the world by Dr. W. Behrmann appeared in Petermanns Mitteilungen, September, 1910, plate 27. In this projection equidistant standard parallels are chosen 30° north and south of the Equator, the projection being in fact a limiting form of the Albers.

In view of the various requirements a map is to fulfill and a careful study of the shapes of the areas involved, the incontestable advantages of the Albers projection for a map of the United States have been sufficiently set forth in the above description. By comparison with the Lambert conformal conic projection, we gain the practical property of equivalence of area and lose but little in conformality, the two projections being otherwise closely identical; by comparison with the Lambert zenithal we gain simplicity of construction and use, as well as the advantages of less scale error; a comparison with other familiar projections offers nothing of advantage to these latter except where their restricted special properties become a controlling factor.

MATHEMATICAL THEORY OF THE ALBERS PROJECTION

If a is the equatorial radius of the spheroid, ϵ the eccentricity, and ϕ the latitude, the radius of curvature of the meridian [26] is given in the form.

$$\rho_m=\frac{a(1-\epsilon^2)}{(1-\epsilon^2\sin^2\phi)^{3/2}},$$

and the radius of curvature perpendicular to the meridian [26] is equal to

$$\rho_n=\frac{a}{(1-\epsilon^2\sin^2\phi)^{1/2}}.$$

[26] See U. S. Coast and Geodetic Survey Special Publication No. 57, pp. 9–10.

The differential element of length of the meridian is therefore equal to the expression

$$dm=\frac{a(1-\epsilon^2)d\phi}{(1-\epsilon^2\sin^2\phi)^{3/2}},$$

and that of the parallel becomes

$$dp=\frac{a\cos\phi d\lambda}{(1-\epsilon^2\sin^2\phi)^{1/2}},$$

in which λ is the longitude.

The element of area upon the spheroid is thus expressed in the form

$$dS=dm\,dp=\frac{a^2(1-\epsilon^2)\cos\phi\,d\phi\,d\lambda}{(1-\epsilon^2\sin^2\phi)^2}.$$

We wish now to determine an equal-area projection of the spheroid in the plane.

If ρ is the radius vector in the plane, and θ is the angle which this radius vector makes with some initial line, the element of area in the plane is given by the form

$$dS'=\rho\,d\rho\,d\theta.$$

ρ and θ must be expressed as functions of ϕ and λ, and therefore

$$d\rho=\frac{\partial\rho}{\partial\phi}d\phi+\frac{\partial\rho}{\partial\lambda}d\lambda$$

and

$$d\theta=\frac{\partial\theta}{\partial\phi}d\phi+\frac{\partial\theta}{\partial\lambda}d\lambda.$$

We will now introduce the condition that the parallels shall be represented by concentric circles; ρ will therefore be a function of ϕ alone.

or

$$d\rho=\frac{\partial\rho}{\partial\phi}d\phi.$$

As a second condition, we require that the meridians be represented by straight lines, the radii of the system of concentric circles. This requires that θ should be independent of ϕ,

or

$$d\theta=\frac{\partial\theta}{\partial\lambda}d\lambda.$$

Furthermore, if θ and λ are to vanish at the same time and if equal differences of longitude are to be represented at all points by equal arcs on the parallels, θ must be equal to some constant times λ,

or

$$\theta=n\lambda,$$

in which n is the required constant.

609100°—45——7

This gives us

$$d\theta = n\,d\lambda.$$

By substituting these values in the expression for dS', we get

$$dS' = \rho \frac{\partial \rho}{\partial \phi} n\,d\phi\,d\lambda.$$

Since the projection is to be equal-area, dS' must equal $-dS$, or

$$\rho \frac{\partial \rho}{\partial \phi} n\,d\phi\,d\lambda = -\frac{a^2(1-\epsilon^2)\cos\phi\,d\phi\,d\lambda}{(1-\epsilon^2\sin^2\phi)^2}.$$

The minus sign is explained by the fact that ρ decreases as ϕ increases. By omitting the $d\lambda$, we find that ρ is determined by the integral

$$\int_0^\phi \rho \frac{\partial \rho}{\partial \phi} d\phi = -\frac{a^2(1-\epsilon^2)}{n}\int_0^\phi \frac{\cos\phi\,d\phi}{(1-\epsilon^2\sin^2\phi)^2}.$$

If R represents the radius for $\phi=0$, this becomes

$$\rho^2 - R^2 = -\frac{2a^2(1-\epsilon^2)}{n}\int_0^\phi \frac{\cos\phi\,d\phi}{(1-\epsilon^2\sin^2\phi)^2}.$$

If β is the latitude on a sphere of radius c, the right-hand member would be represented by the integral

$$u = -\frac{2c^2}{n}\int_0^\beta \cos\beta\,d\beta = -\frac{2c^2}{n}\sin\beta.$$

We may define β by setting this quantity equal to the above right-hand member, or

$$c^2\sin\beta = a^2(1-\epsilon^2)\int_0^\phi \frac{\cos\phi\,d\phi}{(1-\epsilon^2\sin^2\phi)^2}$$

$$= a^2(1-\epsilon^2)\int_0^\phi (\cos\phi + 2\epsilon^2\sin^2\phi\cos\phi + 3\epsilon^4\sin^4\phi\cos\phi + 4\epsilon^6\sin^6\phi\cos\phi + \ldots\ldots)d\phi.$$

Therefore,

$$c^2\sin\beta = a^2(1-\epsilon^2)\left(\sin\phi + \frac{2\epsilon^2}{3}\sin^3\phi + \frac{3\epsilon^4}{5}\sin^5\phi + \frac{4\epsilon^6}{7}\sin^7\phi + \ldots\ldots\right).$$

As yet c is an undetermined constant. We may determine it by introducing the condition that,

$$\text{when } \phi = \frac{\pi}{2},\ \beta \text{ shall also equal } \frac{\pi}{2}.$$

This gives

$$c^2 = a^2(1-\epsilon^2)\left(1 + \frac{2\epsilon^2}{3} + \frac{3\epsilon^4}{5} + \frac{4\epsilon^6}{7} + \ldots\ldots\right).$$

The latitude on the sphere is thus defined in the form

$$\sin \beta = \sin \phi \left(\frac{1+\frac{2\epsilon^2}{3}\sin^2\phi+\frac{3\epsilon^4}{5}\sin^4\phi+\frac{4\epsilon^6}{7}\sin^6\phi+\ldots\ldots}{1+\frac{2\epsilon^2}{3}+\frac{3\epsilon^4}{5}+\frac{4\epsilon^6}{7}+\ldots\ldots} \right).$$

This latitude on the sphere has been called the authalic latitude, the term authalic meaning equivalent or equal-area. A table of these latitudes for every half degree of geodetic latitude is given in U. S. Coast and Geodetic Survey Special Publication No. 67.

With this latitude the expression for ρ becomes

$$\rho^2 = R^2 - \frac{2c^2}{n}\sin \beta.$$

The two constants n and R are as yet undetermined.

Let us introduce the condition that the scale shall be exact along two given parallels. On the spheroid the length of the parallel for a given longitude difference λ is equal to the expression

$$P = \frac{a\lambda \cos \phi}{(1-\epsilon^2 \sin^2 \phi)^{1/2}}.$$

On the map this arc is represented by

$$\rho\theta = \rho n \lambda$$

On the two parallels along which the scale is to be exact, if we denote them by subscripts, we have

$$\rho_1 n \lambda = \frac{a\lambda \cos \phi_1}{(1-\epsilon^2 \sin^2 \phi_1)^{1/2}},$$

or, on omitting λ, we have

$$\rho_1 = \frac{a \cos \phi_1}{n(1-\epsilon^2 \sin^2 \phi_1)^{1/2}},$$

and

$$\rho_2 = \frac{a \cos \phi_2}{n(1-\epsilon^2 \sin^2 \phi_2)^{1/2}}.$$

Substituting these values in turn in the general equation for ρ, we get

$$R^2 - \frac{2c^2}{n}\sin \beta_1 = \frac{a^2 \cos^2 \phi_1}{n^2(1-\epsilon^2 \sin^2 \phi_1)}$$

and

$$R^2 - \frac{2c^2}{n}\sin \beta_2 = \frac{a^2 \cos^2 \phi_2}{n^2(1-\epsilon^2 \sin^2 \phi_2)}.$$

In U. S. Coast and Geodetic Survey Special Publication No. 8 a quantity called A' is defined as

$$A' = \frac{(1-\epsilon^2 \sin^2 \phi')^{1/2}}{a \sin 1''};$$

and is there tabulated for every minute of latitude.

Hence

$$\frac{a^2}{(1-\epsilon^2 \sin^2 \phi_1)}=\frac{1}{A_1{}^2 \sin^2 1''}.$$

(The prime on A is here omitted for convenience.)

The equations for determining R and n, therefore, become

$$R^2-\frac{2c^2}{n} \sin \beta_1=\frac{\cos^2 \phi_1}{A_1{}^2 n^2 \sin^2 1''}$$

and

$$R^2-\frac{2c^2}{n} \sin \beta_2=\frac{\cos^2 \phi_2}{A_2{}^2 n^2 \sin^2 1''}.$$

By subtracting these equations and reducing, we get

$$n=\frac{\dfrac{\cos^2 \phi_1}{A_1{}^2 \sin^2 1''}-\dfrac{\cos^2 \phi_2}{A_2{}^2 \sin^2 1''}}{2c^2(\sin \beta_2-\sin \beta_1)}$$

$$=\frac{\dfrac{\cos^2 \phi_1}{A_1{}^2 \sin^2 1''}-\dfrac{\cos^2 \phi_2}{A_2{}^2 \sin^2 1''}}{4c^2 \sin \tfrac{1}{2}(\beta_2-\beta_1) \cos \tfrac{1}{2}(\beta_2+\beta_1)}=\frac{r_1{}^2-r_2{}^2}{4c^2 \sin \tfrac{1}{2}(\beta_2-\beta_1) \cos \tfrac{1}{2}(\beta_2+\beta_1)},$$

r_1 and r_2 being the radii of the respective parallels upon the spheroid.

By substituting the value of n in the above equations, we could determine R, but we are only interested in canceling this quantity from the general equation for ρ.

Since n is determined, we have for the determination of ρ_1

$$\rho_1=\frac{a \cos \phi_1}{n(1-\epsilon^2 \sin^2 \phi_1)^{\frac{1}{2}}}=\frac{\cos \phi_1}{nA_1 \sin 1''}=\frac{r_1}{n}.$$

But

$$\rho_1{}^2=R^2-\frac{2c^2}{n} \sin \beta_1.$$

By subtracting this equation from the general equation for the determination of ρ we get

$$\rho^2-\rho_1{}^2=\frac{2c^2}{n}(\sin \beta_1-\sin \beta)$$

or

$$\rho^2=\rho_1{}^2+\frac{4c^2}{n} \sin \tfrac{1}{2}(\beta_1-\beta) \cos \tfrac{1}{2}(\beta_1+\beta).$$

In a similar manner we have

$$\rho_2=\frac{a \cos \phi_2}{n(1-\epsilon^2 \sin^2 \phi_2)^{\frac{1}{2}}}=\frac{\cos \phi_2}{nA_2 \sin 1''}=\frac{r_2}{n}$$

and

$$\rho^2=\rho_2{}^2+\frac{4c^2}{n} \sin \tfrac{1}{2}(\beta_2-\beta) \cos \tfrac{1}{2}(\beta_2+\beta).$$

The radius c is the radius of a sphere having a surface equivalent to that of the spheroid. For the Clarke spheroid of 1866 (c in meters)

$$\log c = 6.80420742$$

To obviate the difficulty of taking out large numbers corresponding to logarithms, it is convenient to use the form

$$\frac{\rho^2}{c^2} = \frac{\rho_1^{\,2}}{c^2} + \frac{4}{n} \sin \tfrac{1}{2}(\beta_1 - \beta) \cos \tfrac{1}{2}(\beta_1 + \beta),$$

until after the addition is performed in the right-hand member, and then ρ can be found without much difficulty.

For the authalic latitudes use the table in U. S. Coast and Geodetic Survey Special Publication No. 67.

Now, if λ is reckoned as longitude out from the central meridian, which becomes the Y axis, we get

$$\theta = n\lambda,$$

$$x = \rho \sin \theta,$$

$$y = -\rho \cos \theta.$$

In this case the origin is the center of the system of concentric circles, the central meridian is the Y axis, and a line perpendicular to this central meridian through the origin is the X axis. The y coordinate is negative because it is measured downward.

If it is desired to refer the coordinates to the center of the map as a single system of coordinates, the values become

$$x = \rho \sin \theta,$$

$$y = \rho_0 - \rho \cos \theta,$$

in which ρ_0 is the radius of the parallel passing through the center of the map.

The coordinates of points on each parallel may be referred to a separate origin, the point in which the parallel intersects the central meridian. In this case the coordinates become

$$x = \rho \sin \theta,$$

$$y = \rho - \rho \cos \theta = 2\rho \sin^2 \tfrac{1}{2}\theta.$$

If the map to be constructed is of such a scale that the parallels can be constructed by the use of a beam compass, it is more expeditious to proceed in the following manner:

If λ' is the λ of the meridian farthest out from the central meridian on the map, we get

$$\theta' = n\lambda'.$$

We then determine the chord on the circle representing the lowest parallel of the map, from its intersection with the central meridian to its intersection with the meridian represented by λ',

$$\text{chord} = 2\rho \sin \tfrac{1}{2}\theta'.$$

With this value set off on the beam compass, and with the intersection of the parallel with the central meridian as center, strike an arc intersecting the parallel at the point where the meridian of λ' intersects it. The arc on the parallel represents λ' degrees of longitude, and it can be divided proportionately for the other intersections.

Proceed in the same manner for the upper parallel of the map. Then straight lines drawn through corresponding points on these two parallels will determine all of the meridians.

The scale along the parallels, k_p, is given by the expression

$$k_p = \frac{n\rho_s}{r_s},$$

in which ρ_s is the radius of the circle representing the parallel of ϕ_s, and r_s is the radius of the same parallel on the spheroid;
hence

$$r_s = \frac{\cos\phi_s}{A'_s \sin 1''}.$$

The scale along the meridians is equal to the reciprocal of the expression for the scale along the parallels, or

$$k_m = \frac{r_s}{n\rho_s}.$$

CONSTRUCTION OF AN ALBERS PROJECTION

This projection affords a remarkable facility for graphical construction, requiring practically only the use of a scale, straightedge, and beam compass. In a map for the United States the central or ninety-sixth meridian can be extended far enough to include the center of the curves of latitude, and these curves can be drawn in with a beam compass set to the respective values of the radii taken from the tables.

To determine the meridians, a chord of 25° of longitude (as given in the tables) is laid off from and on each side of the central meridian, on the lower or 25° parallel of latitude. By means of a straightedge the points of intersection of the chords with parallel 25° can be connected with the same center as that used in drawing the parallels of latitude. This, then, will determine the two meridians distant 25° from the center of the map. The lower parallel can then be subdivided into as many equal spaces as may be required, and the remaining meridians drawn in similarly to the outer ones.

If a long straightedge is not available, the spacings of the meridians on parallel 45° can be obtained from chord distance and subdivision of the arc in a similar manner to that employed on parallel 25°. Lines drawn through corresponding points on parallels 25° and 45° will then determine the meridians of the map.

This method of construction is far more satisfactory than the one involving rectangular coordinates, though the length of a beam compass required for the construction of a map of the United States on a scale larger than 1:5,000,000 is rather unusual.

In equal-area projections it is a problem of some difficulty to make allowance for the ellipticity of the earth, a difficulty which is most readily obviated by an intermediate equal-area projection of the spheroid upon a sphere of equal surface. This amounts to the determination of a correction to be applied to the astronomic latitudes in order to obtain the corresponding latitudes upon the sphere. The sphere can then be projected equivalently upon the plane and the problem is solved.

The name of authalic latitudes has been applied to the latitudes of the sphere of equal surface. A table [27] of these latitudes has been computed for every half degree and can be used in the computations of any equal-area projection. This table was employed in the computations of the following coordinates for the construction of a map of the United States.

TABLE FOR THE CONSTRUCTION OF A MAP OF THE UNITED STATES ON ALBERS EQUAL-AREA PROJECTION WITH TWO STANDARD PARALLELS

Latitude ϕ	Radius of parallel ρ	Spacings of parallels Δ
	Meters	*Meters*
20°	10 253 177	107 598
21	10 145 579	108 039
22	10 037 540	108 460
23	9 929 080	108 862
24	9 820 218	109 249
25	9 710 969	109 608
26	9 601 361	109 952
27	9 491 409	110 270
28	9 381 139	110 563
29	9 270 576	110 838
29° 30′	9 215 188	
30	9 159 738	111 090
31	9 048 648	111 311
32	8 937 337	111 510
33	8 825 827	111 677
34	8 714 150	111 822
35	8 602 328	111 936
36	8 490 392	112 015
37	8 378 877	112 065
38	8 266 312	112 084
39	8 154 228	112 065
40	8 042 163	112 011
41	7 930 152	111 921
42	7 818 231	111 787
43	7 706 444	111 616
44	7 594 828	111 402
45	7 483 426	111 138
45° 30′	7 427 822	
46	7 372 288	110 829
47	7 261 459	110 472
48	7 150 987	110 062
49	7 040 925	109 592
50	6 931 333	109 069
51	6 822 264	108 484
52	6 713 780	

Additional data.

$$\frac{\rho_2^2}{c^2}=1.3592771$$
$$\text{colog } n=0.2197522$$
$$\log c=6.8042075$$

Longitude from central meridian	Chords on latitude 25°	Chords on latitude 45°
	Meters	*Meters*
1°	102 184.68	78 745.13
5°	510 866.82	393 682.00
25°	2 547 270	1 962 966
30°	----------	2 352 568

Latitude	Scale factor along the parallel	Scale factor along the meridian
49° 00′	1.0126	0.9876
45° 30′	1.0000	1.0000
37° 30′	.9904	1.0097
29° 30′	1.0000	1.0000
25° 00′	1.0124	.9878
20° 00′	1.0310	.9699

THE MERCATOR PROJECTION

DESCRIPTION

[See Plate XII, Figs. 20, 21, pp. 35, 36, Figs. 60, 61, p. 105, Fig. 62, p. 106, and Fig. 70, p. 155.]

The projection takes its name from the Latin surname of Gerhard Krämer, the inventor, who was born in Flanders in 1512, and was a graduate of the University of Louvain. With an exceptional talent he devoted his life to the betterment of maps, improving and formulating a new device in their mathematical framework, and putting in order the accumulating stores of geographic knowledge. The first known map bearing his name is a map of the world in 1538 on an equal-area projection.

[27] Latitude Developments Connected with Geodesy and Cartography, U. S. Coast and Geodetic Survey Special Publication No. 67.

In 1569, Mercator published his nautical projection in a map of the world on a new system which bears his name and is known as the Mercator projection. For the sake of historical accuracy it might be stated that his results were derived by approximate formula. By the use of a smaller initial element, a more accurate method of computation, and tables for the construction of a projection were made known 30 years later by Edward Wright of Cambridge in a publication entitled, "Certaine Errors in Navigation."

Mercator announced his results simply as "a new *arrangement of* the meridians with reference to the parallels." In consideration of the mathematical resources of the time, his "arrangement" proved to be of sufficient precision to exercise a powerful influence on the progress of navigation. Accurate values of *meridional parts*, however, did not become available until more than a century later when the calculus was invented and better values determined for the figure of the earth.

This map, or nautical chart, of 1569 was[28] the greatest achievement in cartographic history, and on account of its excellence, it was reproduced in 1931 by the International Hydrographic Bureau, Monaco, consisting of 18 sheets in all. As an original creation it made Mercator famous, transmitting his name to all time. It was the first projection in which the meridians are straight and equally spaced parallel lines, and at right angles to them the parallels are straight parallel lines their distances apart increasing toward each pole in proportion to the lengthening of the parallels with reference to the equator. The great importance of this to the mariner lies in the fact that the chart based on this new projection is the prototype of the modern marine chart.

For nautical purposes this system of projection is now universally employed and will probably be so as long as ships follow the loxodrome, i. e., base their courses on rhumb lines. It may be stated that Mercator's nautical chart stands alone in map history, isolated from his many other works as a radical departure and improvement over methods existing before his time. In contemporary judgment he was styled as "In cosmographia longe primus," which, translated, means: *In cosmography by far the first.*

Not including the large issue of pilot charts, communication charts, etc., the annual issue of Mercator nautical charts at the present day throughout the world is several million copies, the Coast and Geodetic Survey alone contributing over a million.

For ordinary geographic purposes, however, the Mercator nautical projection has no value and was not intended to be used as such by the inventor. In the mapping of political divisions and continents, he held to equal-area representations of his own or his contemporaries.

Let none dare to attribute the shame
Of misuse of projections to Mercator's name;
But smother quite, and let infamy light
Upon those who do misuse,
Publish or recite.

It is interesting to note that the word "Atlas" as used today was borrowed from the Greek mythology and introduced by Mercator as a geographic term.

[28] Mercator's original chart of the world was found in Breslau in 1889 (with two other large ones, also by Mercator). This treasury of cartographic art is polychromatic, being profusely decorated and ornamental. The map measures 82.7 inches by 51.2 inches.

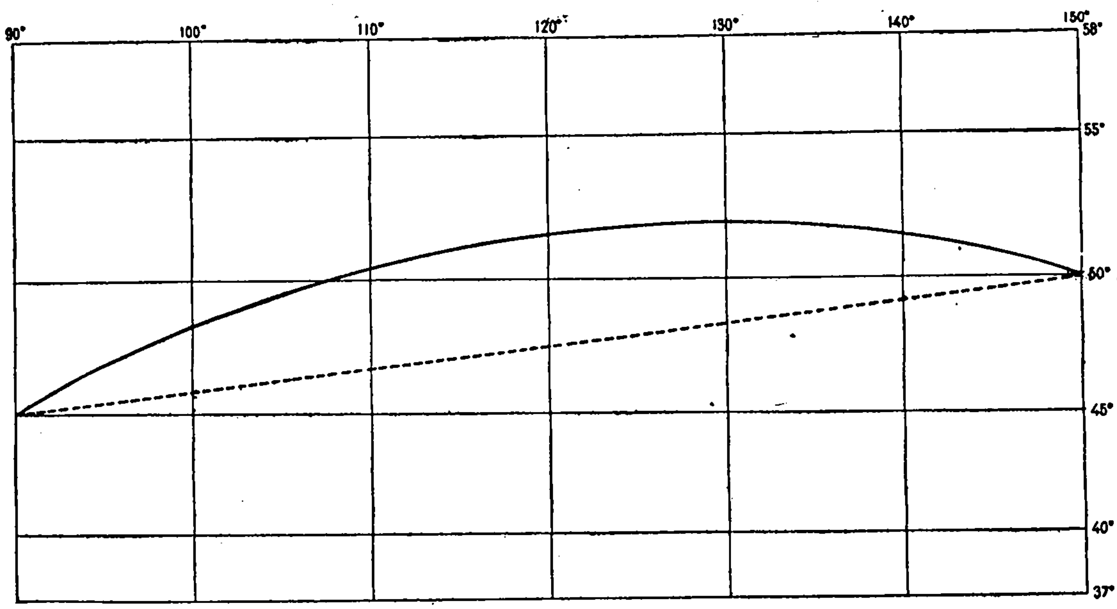


Fig. 60.—Part of a Mercator chart showing a rhumb line and a great circle.

The dotted line shows the rhumb line which is a straight line on this projection. The curve shown by a full line is the great circle track which lies on the polar side of the rhumb line. Any great circle or straight line drawn between two given points on the gnomonic projection may be plotted on the Mercator projection by noting the latitudes of the points where the track crosses the various meridians.

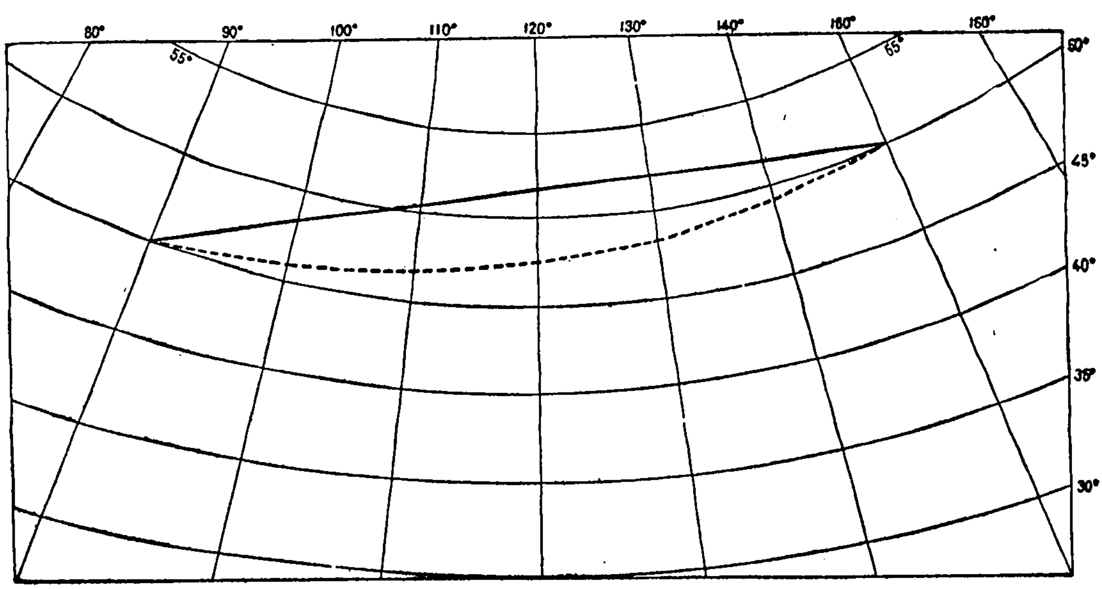


Fig. 61.—Part of a gnomonic chart showing a great circle and a rhumb line.

The full line shows the great circle track. The curve shown by a dotted line is the rhumb line. It lies on the equatorial side of the great circle track.

GREAT CIRCLES AND RHUMB LINES

The shortest line between any two given points on the surface of a sphere is the arc of the great circle that joins them; but, as the earth is a spheroid, the shortest or minimum line that can be drawn on its ellipsoidal surface between any two points is termed a geodetic line. In connection with the study of shortest distances, however, it is customary to consider the earth as a sphere and for ordinary purposes this approximation is sufficiently accurate.

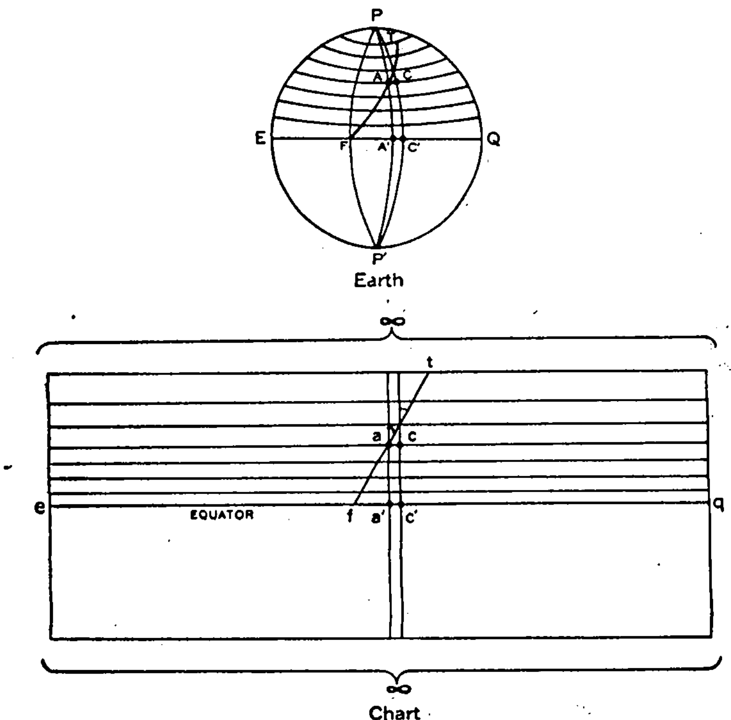


FIG. 62.—The Earth and the Mercator Projection.

The rhumb line (curve) F—T on the Earth is represented by the straight line f—t on the chart. All rhumb lines on the earth are curved lines excepting the Equator and the meridians.
The lengths of the parallels A—C, A'—C' are represented by a—c, a'—c' on the chart. The above figure is adapted from Admiralty Manual of Navigation, 1914.

A rhumb line, or loxodromic curve, is a line which crosses the successive meridians at a constant angle. A ship following always the same oblique course would continuously approach nearer and nearer to the pole without ever theoretically arriving at it. In "sailing a rhumb," a ship is, therefore, on one course continuously following a rhumb line, and will, theoretically at least, pass all points along that line exactly as they are charted.[29]

The only projection on which such a line is represented as a straight line is the Mercator; and the only projection on which the great circle is represented as a straight line is the gnomonic; but as any oblique great circle cuts the meridians of the latter at

[29] When a long route is flown in one mean magnetic course, however, a considerable departure from the intended track may result, the departure from this cause being greatest, of course, where the greatest differences in magnetic variation occur. Such an error is unrelated to the projection and is due solely to assuming a mean magnetic correction for the entire distance.

different angles, to follow such a line would necessitate constant alterations in the direction of the ship's head, an operation that would be impracticable. The choice is then between a rhumb line, which is longer than the arc of a great circle and at every point of which the direction is the same, or the arc of a great circle which is shorter than the rhumb line, but at every point of which the direction is different.

The solution of the problem thus resolves itself into the selection of points at convenient course-distances apart along the great-circle track, so that the ship may be steered from one to the other along the rhumb lines joining them; the closer the points selected to one another,—that is, the shorter the rhumb lines sailed—the more nearly will the track of the ship coincide with the great circle, or shortest sailing route. On other projections, a navigator, assuming a straight line as his course, will not track the line exactly because he is consciously or unconsciously sailing a rhumb.

For this purpose the Mercator projection, except in high latitudes, has attained an importance in surface navigation beyond all others, in that the great circle can be plotted thereon from a gnomonic chart, or it may be determined by calculation, and these arcs can then be subdivided into convenient sailing rhumbs, so that, if the courses are carefully followed, the port bound for will in due time be reached by the shortest practicable route.

It suffices for the mariner to measure by means of a protractor the angle which his course makes with any meridian. With this course corrected for magnetic variation and deviation his compass route will be established.

It may be stated here that the U. S. Coast and Geodetic Survey and the Hydrographic Office, U. S. Navy, have prepared a series of charts on the gnomonic projection which are most useful in laying off great-circle courses. As any straight line on these charts represents a great circle, by taking from them the latitudes and longitudes of a number of points along the line, the great-circle arcs may be transferred to the Mercator system where bearings are obtainable.

It should be borne in mind, moreover, that in practice the shortest course is not always necessarily the shortest passage that can be made. Alterations become necessary on account of the irregular distribution of land and water, the presence of rocks and shoals, the effect of set and drift of currents, and of the direction and strength of the wind. It is necessary, therefore, in determining a course to find out if the rhumb line (or lines) to the destination is interrupted or impracticable, and, if so, to determine intermediate points between which the rhumb lines are uninterrupted. The resolution of the problem at the start, however, must set out with the great circle, or a number of great circles, drawn from one objective point to the next. In the interests of economy, a series of courses, or composite sailing, will frequently be the solution.

ADVANTAGES OF THE MERCATOR PROJECTION

The projection commends itself to map makers because of ready tables, its ease of construction, its rectilinearity, and its use.

Another advantage of the Mercator projection is its simplicity, in that meridians, or north and south lines, are up and down, always constant throughout the chart, pointing the same way and parallel with the east and west borders of the chart—just where one expects them to be.

The essential need of straightforward knowledge of the four principal directions, North, South, East, and West, in conformity with the borders and the system of reference lines of latitude and longitude, is best provided for in the Mercator projection.

The mariner's aversion to curved lines has always been in evidence, and in connection with curved meridians, Mercator, in a legend on his nautical chart of 1569, expresses himself in the following statement translated from Latin: "Indeed, the forms of the meridians, as used till now by geographers, on account of their curvature and their convergence to each other, are not utilizable for navigation."

A course can be laid off from starting point and measured at any meridian along the line, and the border scale of a Mercator chart can be divided as minutely as desirable, leaving the body of the chart free from further subdivisions, such as are frequently necessary where one or both systems of reference lines are curved.

The latitude and longitude of any place are readily found from its position on the map, and the convenience of plotting points or positions by straightedge across the map from corresponding border divisions prevents errors, especially in navigation where two systems of projection cover the same locality. This cannot be done on any other projection in common use.

A compass course may be carried by parallel ruler from a compass rose to any part of the chart without error, and the side borders furnish a distance scale [30] convenient to all parts of the chart, as described in the chapter on "Construction of a Mercator Projection," page 117.

From the nature of the projection any sufficiently narrow belt of latitude in any part of the world, except polar regions only, enlarged or reduced to a desired scale, represents true form for the ready use of the locality it represents. All Mercator charts being similar mathematically, when brought to similar scales on a common latitude, will fit in their relationship to one another, and adjacent charts of uniform latitude scale will join exactly and will remain oriented when joined.

In a series of charts it may be convenient to have them all on such scales that the overlaps will agree, and this can best be attained by adopting the same longitude scale for the series.

In connection with latitude bands and longitude tiers covering the large commercial area of the world on the smaller scales, a uniform longitude scale may offer an advantage in the use of the Mercator system as based on the equator. Adjoining charts, north or south, will differ in scale only, but continuity of conformality will be maintained.

The Mercator projection with its nautical literature, tables, and devices, provides a complete and satisfactory system of navigation and, through centuries of use, has attained a status and popularity that is well established. The projection provides for longitudinal repetition so that continuous commercial routes, east or west, around the world may be completely shown on one map, and it avoids the discontinuities or breach of orderly arrangement seen in many other world maps, noticeably the so-called interrupted projections.

Finally, for a nautical chart, especially in ocean navigation as contrasted with coastwise navigation, the Mercator projection has stood the test of time and has acquired the momentum of success, largely due to the convenience of having one system in use by all nations.

In view of the frequent misunderstanding of the properties of this projection, a few words as to its true merits may be appropriate. It is by no means an equal-area repre-

[30] The border latitude scale will measure the correct distance in the corresponding latitude. If sufficiently important on the smaller scale charts, a diagrammatic scale can be placed on the charts, giving the scale for various latitudes.

sentation, and the mental adjustment to meet this idea in a map of the world has caused unnecessary abuse in ascribing to it properties that are peculiarly absent. But there is this distinction between it and others which give greater accuracy in the relative size and outline of countries—that, while the latter may serve as a basis for geographic and engineering purposes, the Mercator projection, and it alone, in its special delineation, has the invaluable property that a ship's track under a constant course from any point is a straight line on the chart; it can be laid off with accuracy and ease, and position can be readily ascertained. It is, therefore, a convenient working base—a projection that meets certain requirements of navigation and has a world-wide use.

The Mercator projection with scale continuously increasing, when extended beyond 60° of latitude, or even 40° of latitude, shows distances and areas seriously exaggerated, and critics invariably cite Greenland and Alaska. The remark has been made that it places "Alaska too far north." This observation after all is not so far-fetched when we consider that at 60°, the scale of proportion is increased 100%, and the corresponding distances and areas proportionally increased. Granting that the latter are enlarged relatively when sections of the map differing in latitude are compared, an intelligent use of the marginal scale will define these quantities for any limited section. In many other projections the scale is not even uniform about a given point, the scale at the point depending upon the azimuth of a line.

The increments in meridional parts must not be considered as an error of the system of projection but as entirely due to an orderly change of scale, and when in long lines the apex or several points of a great circle are transferred from a gnomonic to a Mercator projection, the measurement of distances must be made along the path of the great circle instead of the rhumb line, the measuring unit changing with the latitude. These measuring or "stepping units" can better be obtained from a horizontal diagrammatic scale based on the different latitudes.

When the great circle becomes a curved line on the Mercator projection, it may seem paradoxical that the measured distance between two points should be shorter along the curved (great-circle) line than along the straight (rhumb) line. The fact is that, in the higher latitudes, where the great circle falls above the rhumb line, the stepping units along the curve become greater than those along the straight line, and the measured distance is thereby decreased.

In many other projections when extended too far, forms and positions of regions may be distorted so much, on account of the oblique incidence of meridians and parallels, that true relations cannot be maintained or measured.

MERCATOR PROJECTION IN HIGH LATITUDES

In latitudes above 60°, where the meridional parts of a Mercator projection increase rather rapidly, charts covering considerable area may be constructed advantageously on a Lambert conformal projection, if the locality has a predominating east-and-west extent; and on a polyconic projection, or a transverse Mercator, if the locality has predominating north-and-south dimensions. In regard to suitable projections for polar regions, see page 155.

Difficulties in navigation in the higher latitudes, often ascribed to the use of the Mercator projection, have in some instances been traced to unreliable positions of landmarks due to inadequate surveys and in other instances to the application of corrections for variation and deviation in the wrong direction.

For purposes of navigation in the great commercial area of the world the Mercator projection has the indorsement of all nautical textbooks and nautical schools, and its employment by maritime nations is universal. It is estimated that of the 15,000 or more different nautical charts published by the various countries not more than 1 per cent are constructed on a system of projection that is noticeably different from Mercator charts.

The advantages of the Mercator system over other systems of projection are evident in nautical charts of small scale covering extensive areas,[31] but the larger the scale the less important these differences become. In harbor and coast charts of the United States of scales varying from 1:10,000 to 1:80,000 the difference of the various types of projection is almost inappreciable.

This being the case, there is a great practical advantage to the mariner in having one uniform system of projection for all scales and in avoiding a sharp break that would require successive charts to be constructed or handled on different principles at a point where there is no definite distinction.

The use of the Mercator projection for nautical charts by the U. S. Coast and Geodetic Survey is, therefore, not due to the habit of continuing an old system, but to the desirability of meeting the special requirements of the navigator. It was adopted by this Bureau after much deliberation, and remodeled charts appeared in 1911, superseding the polyconic projection formerly employed.

The middle latitudes employed by the U. S. Coast and Geodetic Survey in the construction of charts on the Mercator system, are as follows:

Coast and harbor charts, scales 1:80,000 and larger, are constructed to the scale of the middle latitude of each chart. This series includes 86 coast charts of the Atlantic and Gulf coasts, each on the scale 1:80,000. The use of these charts in series is probably less important than their individual local use, and the slight break in scale between adjoining charts will probably cause less inconvenience than would the variation in the scale of the series from 1:69,000 to 1:88,000 if constructed to the scale of the middle latitude of the series.

General charts of the Atlantic coast, scale 1:400,000, are constructed to the scale of latitude 40°. The scales of the different charts of the series are therefore variant, but the adjoining charts join exactly. This applies likewise to the following three groups:

General charts of the Pacific coast, San Diego to Strait of Juan de Fuca, are constructed to the scale of 1:200,000 in latitude 41°.

General charts of the Alaska coast, Dixon Entrance to Unalaska Island, are constructed to the scale of 1:200,000 in latitude 60°.

General sailing charts of the Pacific coast, San Diego to the western limit of the Aleutian Islands, are constructed to the scale of 1:1,200,000 in latitude 49°.

Some of the older charts still issued on the polyconic projection will be changed to the Mercator system as soon as practicable. Information as to the construction of nautical charts in this Bureau is given in Rules and Practice, U. S. Coast and Geodetic Survey, Special Publication No. 66.

It has frequently been stated in treatises on geography and elsewhere that the Mercator projection is a projection upon a cylinder tangent at the equator. This has caused many to think of it as a perspective or geometric projection upon the cylin-

[31] On small scale charts in the middle or higher latitudes, the difference between the Mercator and Lambert or polyconic projection is obvious to the eye and affects the method of using the charts. Latitude can not be carried across perpendicular to the east or west border in any of the conic projections.

der with all the projecting lines radiating from the center of the sphere. They do not. In the Mercator projection of the sphere the scale increases as the secant of the latitude; in the cylindrical perspective projection it increases as the square of the secant of the latitude. The distance of any point of the map in the latter projection is given by $s=a \tan \phi$; hence by the calculus,

$$\frac{ds}{a\,d\phi}=\sec^2 \phi$$

In the Mercator the lengths of all arcs of the parallels are kept equal and the same as on the equator; hence the scale along the parallels is $\sec \phi$ of the respective parallels; and since the scale is constant at a point, the scale along the meridian at the point is also equal to the secant of the latitude of the point, and not $\sec^2 \phi$ as in the perspective projection.

It is thus misleading to speak of the tangent cylinder in connection with the Mercator projection, and it is better to discard all mention of its relation to a cylinder and to view it entirely as a conformal projection upon a plane. As stated before, it is derived by mathematical analysis. The distances of the various parallels depend upon an integral, and the required values are not obtained by any simple formula.

FURTHER OBSERVATIONS ON THE MERCATOR PROJECTION

The two errors, to one or both of which all map projections are liable, are changes of area and distortion as applying to portions of the earth's surface. The former error is well illustrated in a world map on the Mercator projection where a unit of area at the Equator is represented by an area approximating infinity as we approach the pole. Errors of distortion imply deviation from right shape in the graticules or network of meridians and parallels of the map, involving deformation of angles, curvature of meridians, and errors of distance, bearing, or area.

In the Mercator projection, however, as well as in the Lambert conformal conic projection, the changes in scale and area cannot truly be considered as distortion or as errors. A mere alteration of scale or size in the same ratio in all directions is not considered distortion or error. These projections being conformal, both scale and area are correct in any restricted locality when referred to the scale of that locality, but as the scale varies with the latitude large areas are not relatively correct in representation.

There is one important feature in particular, however, in which the Mercator projection does not respond directly, and that is in the plotting of radio bearings to or from a distant point. As the path of radio signals is a great circle, it becomes necessary to convert true bearings to mercatorial bearings. This may be done by means of conversion tables, such as are given in the Coast Pilots of the U. S. Coast and Geodetic Survey, and in Radio Aids to Navigation, U. S. Navy Department, Hydrographic Office, 1939, Vol. 1, pp. 9–15.

MERCATORIAL CORRECTIONS

It is evidently necessary to determine carefully the plus or minus corrections to directional bearings on a Mercator chart, and, in view of the increasing speed of airplanes, the question arises whether time is ordinarily available for the application of mercatorial corrections in air navigation. The arguments used in the conversion table are the middle latitude and the difference of longitude between the plane's dead-reckoning position and that of the radio station. The sign of the correction is additive or subtractive, depending upon the plane's position, westward or eastward of the radio station, and other varying conditions.

The possibility of applying the correction on the wrong side calls for methods making the benefits of great-circle navigation readily available at a critical time.

It is evident from numerous experiments and years of practical use that the automatic or self-contained safety properties embodied in the Lambert projection avoid confusion and are sufficiently accurate for air-navigation purposes, and that the utility of the Mercator projection in many instances is lessened comparatively thereby.

Other comparisons of the Mercator projection with the Lambert projection for aeronautical charts appear in the Résumé, pp.193, 200.

One of the main differences between the Mercator system and that of the Lambert or other conic projections consists in laying off courses. In the former system, it is customary to base the course upon the meridian of origin; in the other systems, a true course angle is measured with the meridian nearest half-way along the course. The two methods are equally good, but they do not begin the same way.

The use of the gnomonic projection as an intermediate device in plotting radio bearings, does not ordinarily simplify the solution. On account of the angular distortion in a gnomonic projection, a specially computed rose (except at the point of tangency) is necessary in order to obtain a true bearing or azimuth from a station. The problem may be simplified either by including on the gnomonic chart compass roses computed for important stations, or by supplying for the stations on any particular chart the *Gnomonic azimuth tables*, such as are given in Special Publication No. 75, Radio-Compass Bearings, U. S. Coast and Geodetic Survey.

DERIVATION OF THE FORMULAS FOR THE COORDINATES OF THE MERCATOR PROJECTION[32]

The Mercator projection is a conformal projection which must be derived by mathematical analysis. The Equator is represented by a straight line true to scale and the meridians are represented by straight lines perpendicular to the line representing the Equator, being equally spaced in proportion to their actual distances apart upon the Equator. The parallels are represented by a second system of parallel lines perpendicular to the family of lines representing the meridians; or, in other words, they are straight lines parallel to the line representing the Equator. The only thing not yet determined is the spacings between the lines representing the parallels; or, what amounts to the same thing, the distances of these lines from the Equator.

Since the projection is conformal, the scale at any point must be the same in all directions. When the parallels and meridians are represented by lines or curves that are mutually perpendicular, the scale will be equal in all directions at a point, if the scale is the same along the parallel and meridian at that point. In the Mercator projection the lines representing the parallels are perpendicular to the lines representing the meridians. In order, then, to determine the projection, we need only to introduce the condition that the scale along the meridians shall be equal to the scale along the parallels.

[32] This projection should not be interpreted as a perspective projection from the center of the spheroid upon a tangent cylinder as is erroneously stated in certain treatises of geography. The distances of the various parallels from the Equator depend upon an integral and the required value is not obtained from any simple formula.

An element of length along a parallel is equal to the expression

$$dp=\frac{a\cos\phi\,d\lambda}{(1-\epsilon^2\sin^2\phi)^{1/2}},$$

in which a is the equatorial radius, ϕ the latitude, λ the longitude, and ϵ the eccentricity.

For the purpose before us we may consider that the meridians are spaced equal to their actual distances apart upon the earth at the Equator. In that case the element of length dp along the parallel will be represented upon the map by $a\,d\lambda$, or the scale along the parallel will be given in the form

$$\frac{dp}{a\,d\lambda}=\frac{\cos\phi}{(1-\epsilon^2\sin^2\phi)^{1/2}}.$$

An element of length along the meridian is given in the form

$$dm=\frac{a\,(1-\epsilon^2)\,d\phi}{(1-\epsilon^2\sin^2\phi)^{3/2}}.$$

Now, if ds is the element of length upon the projection that is to represent this element of length along the meridian, we must have the ratio of dm to ds equal to the scale along the parallel, if the projection is to be conformal.

Accordingly, we must have

$$\frac{dm}{ds}=\frac{a\,(1-\epsilon^2)\,d\phi}{ds\,(1-\epsilon^2\sin^2\phi)^{3/2}}=\frac{\cos\phi}{(1-\epsilon^2\sin^2\phi)^{1/2}},$$

or,

$$ds=\frac{a\,(1-\epsilon^2)\,d\phi}{(1-\epsilon^2\sin^2\phi)\cos\phi}.$$

The distance of the parallel of latitude ϕ from the Equator must be equal to the integral

$$s=\int_0^\phi\frac{a\,(1-\epsilon^2)\,d\phi}{(1-\epsilon^2\sin^2\phi)\cos\phi}$$

$$=a\int_0^\phi\frac{d\phi}{\cos\phi}+\frac{a\epsilon}{2}\int_0^\phi\frac{-\epsilon\cos\phi\,d\phi}{1-\epsilon\sin\phi}-\frac{a\epsilon}{2}\int_0^\phi\frac{\epsilon\cos\phi\,d\phi}{1+\epsilon\sin\phi}$$

$$=a\int_0^\phi\frac{d\phi}{\sin\left(\frac{\pi}{2}+\phi\right)}+\frac{a\epsilon}{2}\int_0^\phi\frac{-\epsilon\cos\phi\,d\phi}{1-\epsilon\sin\phi}+\frac{a\epsilon}{2}\int_0^\phi\frac{\epsilon\cos\phi\,d\phi}{1+\epsilon\sin\phi}$$

$$=a\int_0^\phi\frac{\cos\left(\frac{\pi}{4}+\frac{\phi}{2}\right)}{\sin\left(\frac{\pi}{4}+\frac{\phi}{2}\right)}\frac{d\phi}{2}-a\int_0^\phi\frac{-\sin\left(\frac{\pi}{4}+\frac{\phi}{2}\right)}{\cos\left(\frac{\pi}{4}+\frac{\phi}{2}\right)}\frac{d\phi}{2}+\frac{a\epsilon}{2}\int_0^\phi\frac{-\epsilon\cos\phi\,d\phi}{1-\epsilon\sin\phi}-\frac{a\epsilon}{2}\int_0^\phi\frac{\epsilon\cos\phi\,d\phi}{1+\epsilon\sin\phi}.$$

On integration this becomes

$$s=a \log_e \sin\left(\frac{\pi}{4}+\frac{\phi}{2}\right)-a \log_e \cos\left(\frac{\pi}{4}+\frac{\phi}{2}\right)+\frac{a\epsilon}{2}\log_e(1-\epsilon \sin\phi)-\frac{a\epsilon}{2}\log_e(1+\epsilon\sin\phi)$$

$$=a \log_e \tan\left(\frac{\pi}{4}+\frac{\phi}{2}\right)+\frac{a\epsilon}{2}\log_e\left(\frac{1-\epsilon\sin\phi}{1+\epsilon\sin\phi}\right)$$

$$=a \log_e\left[\tan\left(\frac{\pi}{4}+\frac{\phi}{2}\right)\cdot\left(\frac{1-\epsilon\sin\phi}{1+\epsilon\sin\phi}\right)^{\epsilon/2}\right].$$

The distance of the meridian λ from the central meridian is given by the integral

$$s'=a\int_0^\lambda d\lambda$$
$$=a\lambda.$$

The coordinates of the projection referred to the intersection of the central meridian and the Equator as origin are, therefore, given in the form

$$x=a\lambda,$$
$$y=a \log_e\left[\tan\left(\frac{\pi}{4}+\frac{\phi}{2}\right)\cdot\left(\frac{1-\epsilon\sin\phi}{1+\epsilon\sin\phi}\right)^{\epsilon/2}\right].$$

In U. S. Coast and Geodetic Survey Special Publication No. 67, the isometric or conformal latitude is defined by the expression

$$\tan\left(\frac{\pi}{4}+\frac{\chi}{2}\right)=\tan\left(\frac{\pi}{4}+\frac{\phi}{2}\right)\cdot\left(\frac{1-\epsilon\sin\phi}{1+\epsilon\sin\phi}\right)^{\epsilon/2},$$

or, if

$$\chi=\frac{\pi}{2}-z \text{ and } \phi=\frac{\pi}{2}-p,$$

$$\tan\frac{z}{2}=\tan\frac{p}{2}\cdot\left(\frac{1+\epsilon\cos p}{1-\epsilon\cos p}\right)^{\epsilon/2}.$$

With this value we get

$$y=a \log_e \cot\frac{z}{2},$$

or, expressed in common logarithms,

$$y=\frac{a}{M}\log\cot\frac{z}{2},$$

in which M is the modulus of common logarithms.

$$M=0.4342944819,$$

$$\log M=9.6377843113-10.$$

A table for the isometric colatitudes for every half degree of geodetic latitude is given in U. S. Coast and Geodetic Survey Special Publication No. 67.

The radius a is usually expressed in units of minutes on the Equator, or

$$a=\frac{10800}{\pi},$$

$$\log a=3.5362738828,$$

$$\log\left(\frac{a}{M}\right)=3.8984895715.$$

$$\log y=3.8984895715+\log\left(\log\cot\frac{z}{2}\right),$$

or,

$$y=7915'.704468\log\cot\frac{z}{2}.$$

The value of x now becomes

$$x=\frac{10800}{\pi}\lambda,$$

with λ expressed in radians;
or,

$$x=\lambda,$$

with λ expressed in minutes of arc.

The table of isometric latitudes given in U. S. Coast and Geodetic Survey Special Publication No. 67 was computed for the Clarke spheroid of 1866. If it is desired to compute values of y for any other spheroid, the expansion of y in series must be used. In this case

$$y=7915'.704468\log\tan\left(\frac{\pi}{4}+\frac{\phi}{2}\right)$$
$$-3437'.747\left(\epsilon^2\sin\phi+\frac{\epsilon^4}{3}\sin^3\phi+\frac{\epsilon^6}{5}\sin^5\phi+\frac{\epsilon^8}{7}\sin^7\phi+\ldots\right),$$

or, in more convenient form,

$$y=7915'.704468\log\tan\left(\frac{\pi}{4}+\frac{\phi}{2}\right)-3437'.747\left[\left(\epsilon^2+\frac{\epsilon^4}{4}+\frac{\epsilon^6}{8}+\frac{5\epsilon^8}{64}+\ldots\right)\sin\phi\right.$$
$$\left.-\left(\frac{\epsilon^4}{12}+\frac{\epsilon^6}{16}+\frac{3\epsilon^8}{64}+\ldots\right)\sin 3\phi+\left(\frac{\epsilon^6}{80}+\frac{\epsilon^8}{64}+\ldots\right)\sin 5\phi-\left(\frac{\epsilon^8}{448}+\ldots\right)\sin 7\phi\ldots\right].$$

If the given spheroid is defined by the flattening, ϵ^2 may be computed from the formula

$$\epsilon^2=2f-f^2,$$

in which f is the flattening.

The series for y in the sines of the multiple arcs can be written with coefficients in closed form, as follows:

$$y=7915'.704468\log\tan\left(\frac{\pi}{4}+\frac{\phi}{2}\right)-3437'.747\left(2f\sin\phi-\frac{2f^3}{3\epsilon^2}\sin 3\phi\right.$$
$$\left.+\frac{2f^5}{5\epsilon^4}\sin 5\phi-\frac{2f^7}{7\epsilon^6}\sin 7\phi+\ldots\right),$$

in which f denotes the flattening and ϵ the eccentricity of the spheroid.

DEVELOPMENT OF THE FORMULAS FOR THE TRANSVERSE MERCATOR PROJECTION

The expressions for the coordinates of the transverse Mercator projection can be determined by a transformation performed upon the sphere. If, p is the great-circle radial distance, and ω is the azimuth reckoned from a given initial, the transverse Mercator projection in terms of these elements is expressed in the form

$$x = a\,\omega,$$

$$y = a \log_e \cot \frac{p}{2}.$$

But, from the transformation triangle (Fig. 69 on page 151), we have

$$\cos p = \sin \alpha \sin \phi + \cos \alpha \cos \phi \cos \lambda,$$

$$\tan \omega = \frac{\cos \alpha \sin \phi - \sin \alpha \cos \phi \cos \lambda}{\sin \lambda \cos \phi},$$

in which α is the latitude of the point that becomes the pole in the transverse projection.

By substituting these values in the equations above, we get

$$x = a \tan^{-1}\left(\frac{\cos \alpha \sin \phi - \sin \alpha \cos \phi \cos \lambda}{\sin \lambda \cos \phi}\right)$$

and

$$y = a \log_e \cot \frac{p}{2} = \frac{a}{2} \log_e\left(\frac{1+\cos p}{1-\cos p}\right)$$

$$= \frac{a}{2} \log_e\left(\frac{1+\sin \alpha \sin \phi + \cos \alpha \cos \phi \cos \lambda}{1-\sin \alpha \sin \phi - \cos \alpha \cos \phi \cos \lambda}\right).$$

If we wish the formulas to yield the usual values when α converges to $\frac{\pi}{2}$, we must replace λ by $\lambda - \frac{\pi}{2}$ or, in other words, we must change the meridian from which λ is reckoned by $\frac{\pi}{2}$. With this change the expressions for the coordinates become

$$x = a \tan^{-1}\left(\frac{\sin \alpha \cos \phi \sin \lambda - \cos \alpha \sin \phi}{\cos \phi \cos \lambda}\right)$$

$$y = \frac{a}{2} \log_e\left(\frac{1+\sin \alpha \sin \phi + \cos \alpha \cos \phi \sin \lambda}{1-\sin \alpha \sin \phi - \cos \alpha \cos \phi \sin \lambda}\right).$$

With common logarithms the y coordinate becomes

$$y = \frac{a}{2M} \log\left(\frac{1+\sin \alpha \sin \phi + \cos \alpha \cos \phi \sin \lambda}{1-\sin \alpha \sin \phi - \cos \alpha \cos \phi \sin \lambda}\right),$$

in which M is the modulus of common logarithms.

A study of the transverse Mercator projections was made by A. Lindenkohl, U. S. Coast and Geodetic Survey, some years ago, but no charts in the modified form have ever been issued by this office.

In a transverse position the projection loses the property of straight meridians and parallels, and the loxodrome or rhumb line is no longer a straight line. Since the projection is conformal, the representation of the rhumb line must intersect the meridians on the map at a constant angle, but as the meridians become curved lines the rhumb line must also become a curved line. The transverse projection, therefore, loses this valuable property of the ordinary Mercator projection.

The distortion, or change of scale, increases with the distance from the great circle which plays the part of the Equator in the ordinary Mercator projection, but, considering the shapes and geographic location of certain areas to be charted, a transverse position would in some instances give advantageous results in the property of conformal mapping.

CONSTRUCTION OF A MERCATOR PROJECTION

On the Mercator projection, meridians are represented by parallel and equidistant straight lines, and the parallels of latitude are represented by a system of straight lines at right angles to the former, the spacings between them conforming to the condition that at every point the angle between any two curvilinear elements upon the sphere is represented upon the chart by an equal angle between the representatives of these elements.

In order to retain the correct shape and comparative size of objects as far as possible, it becomes necessary, therefore, in constructing a Mercator chart, to increase every degree of latitude toward the pole in precisely the same proportion as the degrees of longitude have been lengthened by projection.

TABLES

The table beginning on page 125 is that appearing in Traité d'Hydrographie by A. Germain, 1882, Table XIII. Attention is called to the paragraph on page 124 in regard to a more recent table published by the International Hydrographic Bureau.

The outer columns of *minutes* give the notation of minutes of latitude from the Equator to 80°.

The column of meridional distances gives the total distance of any parallel of latitude from the Equator in terms of a minute or unit of longitude on the Equator.

The column of *differences* gives the value of 1 minute of latitude in terms of a minute or unit of longitude on the Equator; thus, the length of any minute of latitude on the map is obtained by multiplying the length of a minute of longitude by the value given in the column of differences between adjacent minutes.

The first important step in the use of Mercator tables is to note the fact that a minute of longitude on the Equator is the unit of measurement and is used as an expression for the ratio of any one minute of latitude to any other. The method of construction is simple, but, on account of different types of scales employed by different chart-producing establishments, it is desirable to present two methods: (1) The diagonal metric scale method; (2) the method similar to that given in Bowditch's American Practical Navigator.

Diagonal Metric Scale Method as Used in the U. S. Coast and Geodetic Survey

Draw a straight line for a central meridian and a construction line perpendicular thereto, each to be as central to the sheet as the selected interval of latitude and longitude will permit. To insure greater accuracy on large sheets, the longer line of the two should be drawn first, and the shorter line erected perpendicular to it.

Example: Required a Mercator projection, Portsmouth, N.H., to Biddeford, Maine, extending from latitude 43° 00′ to 43° 30′; longitude 70° 00′ to 71° 00′, scale on middle parallel 1:400,000, projection interval 5 minutes.

The middle latitude being 43° 15′, we take as the unit of measurement the true value of a minute of longitude. This is given in U. S. Coast and Geodetic Survey Special Publication No. 5 entitled Tables for a Polyconic Projection of Maps and Lengths of Terrestrial Arcs of Meridian and Parallels (general spherical coordinates not being given in the Germain tables). Entering the proper column on page 96, we find the length of a minute of longitude to be 1353.5 meters.

As metric diagonal scales of 1:400,000 are neither available nor convenient, we ordinarily use a scale 1:10,000; this latter scale, being 40 times the former, the length of a unit of measurement on it will be one-fortieth of 1353.5, or 33.84.

Lines representing 5-minute intervals of longitude can now be drawn in on either side of the central meridian and parallel thereto at intervals of 5×33.84 or 169.2 apart on the 1:10,000 scale. (In practice it is advisable to determine the outer meridians first, 30 minutes of longitude being represented by 6×169.2, or 1015.2; and the 5-minute intervals by 169.2, successively.)

The Parallels of Latitude

The distance between the bottom parallel of the chart 43° 00′ and the next 5-minute parallel—that is, 43° 05′—will be ascertained from the Mercator tables by taking the difference between the values opposite these parallels and multiplying this difference by the unit of measurement. Thus:

Latitude.	Meridional distance.
° ′	
43 05	2853.987
43 00	2847.171
	6.816

6.816 multiplied by 33.84=230.6, which is the spacing from the bottom parallel to 43°05′.

The spacings of the other 5-minute intervals obtained in the same way are as follows:

Latitude.	Spacings.
° ′	
43 30	232.3
43 25	232.0
43 20	231.6
43 15	231.3
43 10	231.0
43 05	230.6
43 00	
Total height of chart	1388.8

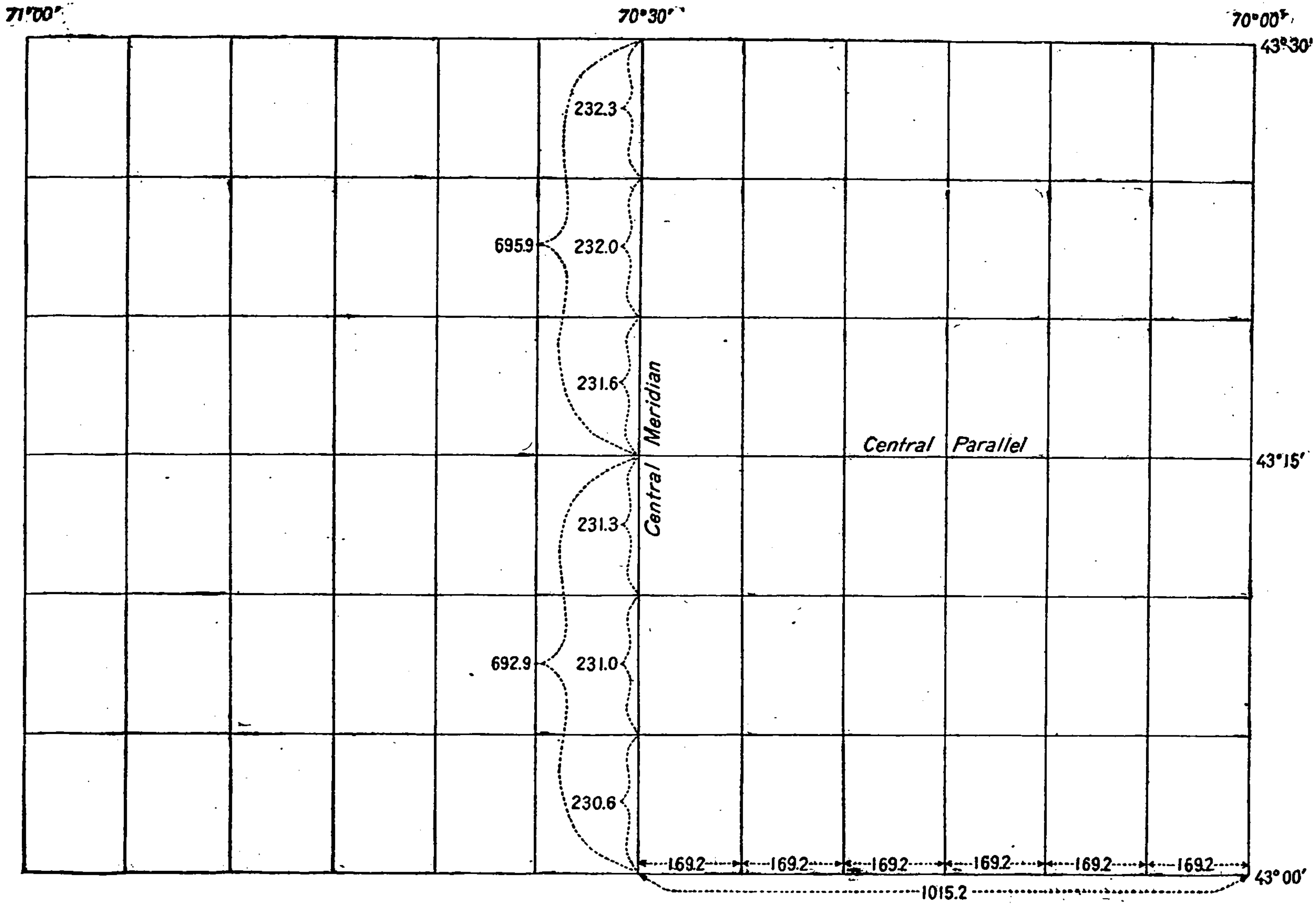


Fig. 63.—Mercator projection—construction plate.

From the central parallel, or 43°15′, the other parallels can now be stepped off and drawn in as straight lines and the projection completed. Draw then the outer neat lines of the chart at a convenient distance outside of the inner neat lines and extend to them the meridians and parallels already constructed. Between the inner and outer neat lines of the chart subdivide the degrees of latitude and longitude as minutely as the scale of the chart will permit, the subdivisions of the degrees of longitude being found by dividing the degrees into equal parts; and the subdivisions of the degrees of latitude being accurately found in the same manner as the 5-minute intervals of latitude already described, though it will generally be sufficiently exact on large-scale charts to make even subdivisions of these intervals of latitude, as in the case of the longitude.

In northern latitudes, where the meridional increments are quite noticeable, care should be taken so as to have the latitude intervals or subdivisions computed with sufficient closeness, so that their distances apart will increase progressively.

The subdivisions along the eastern, as well as those along the western neat line, will serve for measuring or estimating terrestrial distances. Distances between points bearing north and south of each other may be ascertained by referring them to the subdivisions between their latitudes. Distances represented by lines (rhumb or loxodromic) at an angle to the meridians may be measured by taking between the dividers a small number of the subdivisions near the middle latitude of the line to be measured, and stepping them off on that line. If, for instance, the terrestrial length of a line running at an angle to the meridians, between the parallels of latitude 24°00′ and 29°00′ be required, the distance shown on the neat space between 26°15′ and 26°45′ (=30 nautical miles)[33] may be taken between the dividers and stepped off on that line. An oblique line of considerable length may well be divided into parts and each part referred to its middle latitude for a unit of measurement.

To Construct a Mercator Projection by a Method Similar to that Given in Bowditch's American Practical Navigator

If the chart includes the Equator, the values found in the tables will serve directly as factors for any properly divided diagonal scale of yards, feet, meters, or miles; these factors to be reduced proportionally to the scale adopted for the chart.

If the chart does not include the Equator then the parallels of latitude should be referred to a *principal parallel*, preferably the central or the lowest parallel to be drawn upon the chart. The distance of any other parallel of latitude from the principal parallel is the difference of the values of the two taken from the tables and reduced to the scale of the chart.

If, for example, it be required to construct a chart on a scale of one-fourth of an inch to 5 minutes of arc on the Equator, the minute or unit of measurement will be $\frac{1}{5}$ of $\frac{1}{4}$ inch, or $\frac{1}{20}$ of an inch, and 10 minutes of longitude on the Equator (or 10 meridional parts) will be represented by $\frac{10}{20}$ or 0.5 inch; likewise 10 minutes of latitude north or south of the Equator will be represented by $\frac{1}{20} \times 9.932$ or 0.4966 inch. The value 9.932 is the difference between the *meridional distances* as given opposite latitudes 0° 00′ and 0° 10′.

[33] Strictly speaking, a minute of latitude is equal to a nautical mile in latitude 48° 15′ only. The length of a minute of latitude varies from 1842.8 meters at the Equator to 1861.7 meters at the pole.

If the chart does not include the Equator, and if the middle parallel is latitude 40°, and the scale of this parallel is to be one-fourth of an inch to 5 minutes, then the measurement for 10 minutes on this parallel will be the same as before, but the measurement of the interval between 40° 00′ and 40° 10′ will be $\frac{1}{20} \times 13.018$, or 0.6509 inch. The value 13.018 is the difference of the *meridional distances* as given opposite these latitudes, i. e., the difference between 2620.701 and 2607.683.

(It may often be expedient to construct a diagonal scale of inches on the drawing to facilitate the construction of a projection on the required scale.)

Sometimes it is desirable to adapt the scale of a chart to a certain allotment of paper.

Example: Let a projection be required for a chart of 14° extent in longitude between the parallels of latitude 20° 30′ and 30° 25′, and let the space allowable on the paper between these parallels be 10 inches.

Draw in the center of the sheet a straight line for the central meridian of the chart. Construct carefully two lines perpendicular to the central meridian and 10 inches apart, one near the lower border of the sheet for parallel of latitude 20° 30′ and an upper one for parallel of latitude 30° 25′.

Entering the tables in the column *meridional distance* we find for latitude 20° 30′ the value 1248.945, and for latitude 30° 25′ the value 1905.488. The difference, or 1905.488 − 1248.945 = 656.543, is the value of the meridional arc between these latitudes, for which 1 minute of arc of the Equator is taken as a unit. On the projection, therefore, 1 minute of arc of longitude will measure $\frac{10 \text{ in.}}{656.543} = 0.0152$ inch, which will be the unit of measurement. By this quantity all the values derived from the table must be multiplied before they can be used on a diagonal scale of inches for this chart.

As the chart covers 14° of longitude, the 7° on either side of the central meridian will be represented by $0.0152 \times 60 \times 7$, or 6.38 inches. These distances can be laid off from the central meridian east and west on the upper and lower parallel. Through the points thus obtained draw lines parallel to the central meridian, and these will be the eastern and western neat lines of the chart.

In order to obtain the spacing, or interval, between the parallel of latitude 21° 00′ and the bottom parallel of 20° 30′, we find the difference between their meridional distances and multiply this difference by the unit of measurement, which is 0.0152.

Thus:

$$(1280.835 - 1248.945) \times 0.0152$$
$$\text{or } 31.890 \times 0.0152 = 0.485 \text{ inch.}$$

On the three meridians already constructed lay off this distance from the bottom parallel, and through the points thus obtained draw a straight line which will be the parallel 21° 00′.

Proceed in the same manner to lay down all the parallels answering to full degrees of latitude; the distances for 22°, 23°, and 24° from the bottom parallel will be, respectively:

$$0.0152 \times (1344.945 - 1248.945) = 1.459 \text{ inches}$$
$$0.0152 \times (1409.513 - 1248.945) = 2.441 \text{ inches}$$
$$0.0152 \times (1474.566 - 1248.945) = 3.429 \text{ inches, etc.}$$

Finally, lay down in the same way the parallel 30° 25′, which will be the northern inner neat line of the chart.

A degree of longitude will measure on this chart $0.0152 \times 60 = 0.912$ inch. Lay off, therefore, on the lowest parallel of latitude, on the middle one, and on the highest parallel, measuring from the central meridian toward either side, the distances 0.912 inch, 1.824 inches, 2.736 inches, 3.648 inches, etc., in order to determine the points where meridians answering to full degrees cross the parallels drawn on the chart. Through the points thus found draw the straight lines representing the meridians.

If it occurs that a Mercator projection is to be constructed on a piece of paper where the size is controlled by the limits of longitude, the case may be similarly treated.

CONSTRUCTION OF A TRANSVERSE MERCATOR PROJECTION FOR THE SPHERE

The Anti-Gudermannian table given on pages 309 to 318 in "Smithsonian Mathematical Tables—Hyperbolic Functions" is really a table of meridional distances for the sphere. By use of this table an ordinary Mercator projection can be constructed for the sphere. Upon this graticule the transverse Mercator can be plotted by use of the table, "Transformation from geographical to azimuthal coordinates—Center on the Equator" given in U. S. Coast and Geodetic Survey Special Publication No. 67, "Latitude Developments Connected with Geodesy and Cartography, with Tables, Including a Table for Lambert Equal-Area Meridional Projection."

Figure 64 shows such a transverse Mercator projection for a hemisphere; the pole is the origin and the horizontal meridian is the central meridian. The dotted lines are the lines of the original Mercator projection. Since the projection is turned 90° in azimuth, the original meridians are horizontal lines and the parallels are vertical lines, the vertical meridian of the transverse projection being the Equator of the original projection. The numbers of the meridians in the transverse projection are the complements of the numbers of the parallels in the original projection. The same thing is true in regard to the parallels in the transverse projection and the meridians in the original projection. That is, where the number 20 is given for the transverse projection, we must read 70 in the original projection.

The table in Special Publication No. 67 consists of two parts, the first part giving the values of the azimuths reckoned from the north and the second part giving the great-circle central distances. From this table we get for the intersection of latitude 10° with longitude 10°,

	°	′	″
azimuth =	44	33	41.2
radial distance =	14	06	21.6

To the nearest minute these become

$$\alpha = 44° \, 34'$$
$$\zeta = 14 \quad 06$$

The azimuth becomes longitude in the original projection and is laid off upward from the origin, or the point marked "pole" in the figure. The radial distance is the complement of the latitude on the original projection; hence the chosen intersection lies in longitude 44° 34′ and latitude 75° 54′ on the original projection. It can be seen from the figure that there are three other points symmetrically situated with respect to this point, one in each of the other three quadrants. If the intersections in one quadrant are actually plotted, the other quadrants may be copied from this construction. Another hemisphere added either above or below will complete the sphere, with the exception, of course, of the part that passes off to infinity.

In practice the original projection need not be drawn, or, if it is drawn, the lines should be light pencil lines used for guidance only. If longitude 44° 34′ is laid off upward along a vertical line from an origin, and the meridional distance for 75° 54′ is laid off to the right, the intersection of the meridian of 10° with the parallel of 10° is located upon the map. In a similar manner, by the use of the table in Special

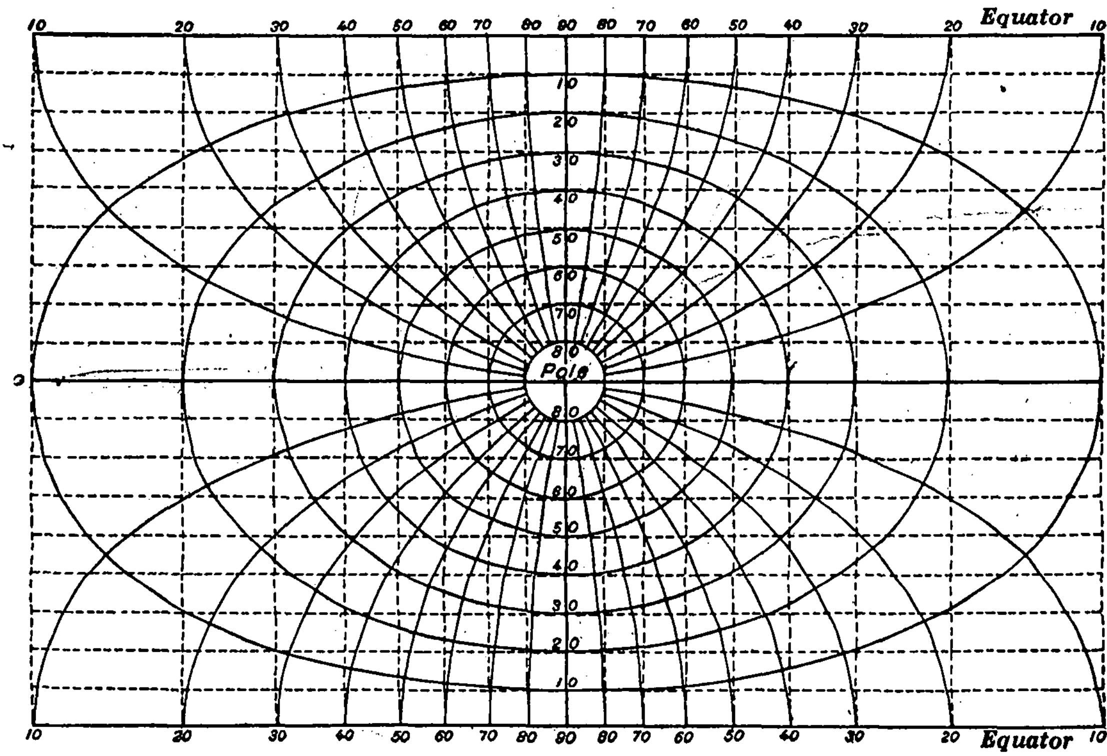


FIG. 64.—Transverse Mercator projection, construction plate.

Publication No. 67, the other intersections of the parallel of 10° can be located; then a smooth curve drawn through these points so determined will be the parallel of 10°. Also the other intersections of the meridian of 10° can be located, and a smooth curve drawn through these points will represent the meridian of 10°.

The table in Special Publication No. 67 gives the intersections for 5° intervals in both latitude and longitude for one-fourth of a hemsiphere. This is sufficient for the construction of one quadrant of the hemisphere on the map. As stated above, the remaining quadrants can either be copied from this construction, or the values may be plotted from the consideration of symmetry. In any case figure 64 will serve as a guide in the process of construction.

In the various problems of *conformal* and *equal-area* mapping, any solution that will satisfy the shapes or extents of the areas involved in the former system has generally a counterpart or natural complement in the latter system. Thus, where we map a given locality on the Lambert conformal conic projection for purposes of conformality, we may on the other hand employ the Albers projection for equal-area representation of the same region; likewise, in mapping a hemisphere, the stereographic meridional projection may be contrasted with the Lambert meridional projection, the stereographic horizon projection with the Lambert zenithal; and so, with a fair degree of accuracy, the process above described will give us conformal representation of the sphere suited to a zone predominating meridional dimensions as a counterpart of the Bonne system of equal-area mapping of the same zone.

MERCATOR PROJECTION TABLE

[Reprinted from Traité d'Hydrographie, A. Germain, Ingénieur Hydrographe de la Marine, Paris, MDCCCLXXXII, to latitude 80° only.]

Note

It is observed in this table that the meridional differences are irregular and that second differences frequently vary from plus to minus. The tables might well have been computed to one more place in decimals to insure the smooth construction of a projection.

In the use of any meridional distance below latitude 50°00′ the following process will eliminate irregularities in the construction of large scale maps and is within scaling accuracy:

To any meridional distance add the one above and the one below and take the mean, thus:

Latitude.		Meridional distances.
°	′	
28	35	1779.745
28	36	1780.877
28	37	1782.011
		5342.633

The mean to be used for latitude 28° 36′ is 1780.8777

In the original publication of these tables the author stated that they were computed for an ellipsoid of a compression $\frac{1}{294}$. As a matter of fact they were computed with the value of $e^2=0.006785$.

This makes the compression slightly less than $\frac{1}{294}$.

Although the tables of meridional distances, here given to three places of decimals, will serve for the construction of Mercator projections with sufficient accuracy for all practical purposes, it is recommended that the following publication be used to avoid the necessity of occasional interpolation: International Hydrographic Bureau, Special Publication No. 21, Monaco, 1928. These tables give the *Meridional Parts* to five places of decimals for every sexagesimal minute calculated for compression $=\frac{1}{297}$ (Ellipsoïde International, Madrid, 1924); also tables of corrections for conversion to Bessel's Ellipsoid, as well as to Clarke's Ellipsoid, 1880. In practice, however, it is found that the fifth place of decimals is unnecessary.

MERCATOR PROJECTION TABLE.

[Meridional distances for the spheroid. Compression $\frac{1}{294}$.]

Minutes.	0° Meridional distance.	0° Difference.	1° Meridional distance.	1° Difference.	2° Meridional distance.	2° Difference.	3° Meridional distance.	3° Difference.	Minutes.
	′	′	′	′	′	′	′	′	
0	0.000	0.993	59.596	0.994	119.210	0.994	178.862	0.994	0
1	0.993	993	60.590	993	120.204	994	179.856	995	1
2	1.986	994	61.583	993	121.198	994	180.851	994	2
3	2.980	993	62.576	994	122.192	994	181.845	995	3
4	3.973	993	63.570	993	123.186	994	182.840	994	4
5	4.966	993	64.563	993	124.180	994	183.834	995	5
6	5.959	993	65.556	994	125.174	994	184.829	995	6
7	6.952	994	66.550	993	126.168	994	185.824	994	7
8	7.946	993	67.543	994	127.162	993	186.818	995	8
9	8.939	993	68.537	993	128.155	994	187.813	995	9
10	9.932	0.993	69.530	0.993	129.149	0.994	188.808	0.994	10
11	10.925	993	70.523	994	130.143	994	189.802	995	11
12	11.918	994	71.517	993	131.137	994	190.797	995	12
13	12.912	993	72.510	994	132.131	994	191.792	995	13
14	13.905	993	73.504	993	133.125	994	192.787	995	14
15	14.898	993	74.497	994	134.119	994	193.782	995	15
16	15.891	993	75.491	993	135.113	994	194.777	995	16
17	16.884	994	76.484	993	136.107	994	195.772	995	17
18	17.878	993	77.477	994	137.101	994	196.767	995	18
19	18.871	993	78.471	993	138.095	994	197.762	995	19
20	19.864	0.993	79.464	0.994	139.089	0.994	198.757	0.995	20
21	20.857	994	80.458	993	140.083	994	199.752	995	21
22	21.851	993	81.451	994	141.077	995	200.747	995	22
23	22.844	993	82.445	993	142.072	994	201.742	995	23
24	23.837	994	83.438	994	143.066	994	202.737	995	24
25	24.831	993	84.432	993	144.060	994	203.732	995	25
26	25.824	993	85.425	994	145.054	994	204.727	995	26
27	26.817	993	86.419	994	146.048	994	205.722	995	27
28	27.810	994	87.413	993	147.042	994	206.717	995	28
29	28.804	993	88.406	994	148.036	994	207.712	995	29
30	29.797	0.993	89.400	0.993	149.030	0.994	208.707	0.995	30
31	30.790	993	90.393	994	150.024	995	209.702	995	31
32	31.783	994	91.387	993	151.019	994	210.697	995	32
33	32.777	993	92.380	994	152.013	994	211.692	995	33
34	33.770	993	93.374	994	153.007	994	212.687	995	34
35	34.763	994	94.368	993	154.001	995	213.682	995	35
36	35.757	993	95.361	994	154.996	994	214.677	996	36
37	36.750	993	96.355	993	155.990	994	215.673	995	37
38	37.743	993	97.348	994	156.984	994	216.668	995	38
39	38.736	994	98.342	994	157.978	995	217.663	995	39
40	39.730	0.993	99.336	0.993	158.973	0.994	218.658	0.996	40
41	40.723	993	100.329	994	159.967	994	219.654	995	41
42	41.716	994	101.323	993	160.961	995	220.649	995	42
43	42.710	993	102.316	994	161.956	994	221.644	996	43
44	43.703	993	103.310	994	162.950	994	222.640	995	44
45	44.696	993	104.304	994	163.944	995	223.635	996	45
46	45.689	994	105.298	993	164.939	994	224.631	995	46
47	46.683	993	106.291	994	165.933	995	225.626	996	47
48	47.676	993	107.285	994	166.928	994	226.622	995	48
49	48.669	994	108.279	994	167.922	995	227.617	996	49
50	49.663	0.993	109.273	0.993	168.917	0.994	228.613	0.995	50
51	50.656	993	110.266	994	169.911	994	229.608	995	51
52	51.649	994	111.260	994	170.905	995	230.603	996	52
53	52.643	993	112.254	993	171.900	994	231.599	995	53
54	53.636	993	113.247	994	172.894	995	232.594	996	54
55	54.629	994	114.241	994	173.889	994	233.590	995	55
56	55.623	993	115.235	994	174.883	995	234.585	996	56
57	56.616	993	116.229	994	175.878	994	235.581	996	57
58	57.609	994	117.223	993	176.872	995	236.577	995	58
59	58.603	0.993	118.216	0.994	177.867	0.995	237.572	0.996	59
60	59.596		119.210		178.862		238.568		60

MERCATOR PROJECTION TABLE—Continued.

[Meridional distances for the spheroid. Compression $\frac{1}{294}$.]

Minutes.	4° Meridional distance.	4° Difference.	5° Meridional distance.	5° Difference.	6° Meridional distance.	6° Difference.	7° Meridional distance.	7° Difference.	Minutes.
	′	′	′	′	′	′	′	′	
0	238.568	0.996	298.348	0.997	358.222	0.998	418.206	1.001	0
1	239.564	995	299.345	997	359.220	999	419.207	001	1
2	240.559	996	300.342	998	360.219	999	420.208	001	2
3	241.555	996	301.340	997	361.218	999	421.209	000	3
4	242.551	996	302.337	997	362.217	999	422.209	001	4
5	243.547	996	303.334	997	363.216	999	423.210	001	5
6	244.543	995	304.331	997	364.215	998	424.211	001	6
7	245.538	996	305.328	998	365.213	999	425.212	001	7
8	246.534	996	306.326	997	366.212	0.999	426.213	001	8
9	247.530	996	307.323	997	367.211	1.000	427.214	002	9
10	248.526	0.996	308.320	0.998	368.211	0.999	428.216	1.001	10
11	249.522	996	309.318	997	369.210	999	429.217	001	11
12	250.518	996	310.315	997	370.209	999	430.218	001	12
13	251.514	996	311.312	998	371.208	999	431.219	001	13
14	252.510	996	312.310	997	372.207	0.999	432.220	002	14
15	253.506	996	313.307	998	373.206	1.000	433.222	001	15
16	254.502	996	314.305	997	374.206	0.999	434.223	001	16
17	255.498	996	315.302	998	375.205	0.999	435.224	002	17
18	256.494	996	316.300	998	376.204	1.000	436.226	001	18
19	257.490	996	317.298	997	377.204	0.999	437.227	002	19
20	258.486	0.996	318.295	0.998	378.203	1.000	438.229	1.001	20
21	259.482	996	319.293	998	379.203	0.999	439.230	002	21
22	260.478	996	320.291	997	380.202	1.000	440.232	002	22
23	261.474	996	321.288	998	381.202	0.999	441.234	001	23
24	262.470	997	322.286	998	382.201	1.000	442.235	002	24
25	263.467	996	323.284	997	383.201	0.999	443.237	002	25
26	264.463	996	324.281	998	384.200	1.000	444.239	002	26
27	265.459	996	325.279	998	385.200	1.000	445.241	001	27
28	266.455	996	326.277	998	386.200	0.999	446.242	002	28
29	267.451	997	327.275	998	387.199	0.999	447.244	002	29
30	268.448	0.996	328.273	0.997	388.198	1.000	448.246	1.002	30
31	269.444	996	329.270	998	389.198	000	449.248	002	31
32	270.440	997	330.268	998	390.198	000	450.250	002	32
33	271.437	996	331.266	998	391.198	000	451.252	002	33
34	272.433	997	332.264	998	392.198	000	452.254	002	34
35	273.430	996	333.262	998	393.198	000	453.256	002	35
36	274.426	997	334.260	998	394.198	000	454.258	002	36
37	275.423	996	335.258	998	395.198	000	455.260	002	37
38	276.419	997	336.256	998	396.198	000	456.262	002	38
39	277.416	996	337.254	999	397.198	000	457.264	003	39
40	278.412	0.997	338.253	0.998	398.198	1.000	458.267	1.002	40
41	279.409	997	339.251	998	399.198	000	459.269	003	41
42	280.406	996	340.249	998	400.198	000	460.272	002	42
43	281.402	997	341.247	998	401.198	000	461.274	003	43
44	282.399	997	342.245	999	402.198	000	462.277	002	44
45	283.396	996	343.244	998	403.198	001	463.279	003	45
46	284.392	997	344.242	998	404.199	000	464.282	002	46
47	285.389	997	345.240	999	405.199	000	465.284	003	47
48	286.386	997	346.239	998	406.199	001	466.287	002	48
49	287.383	997	347.237	999	407.200	000	467.289	003	49
50	288.380	0.996	348.236	0.998	408.200	1.001	468.292	1.003	50
51	289.376	997	349.234	999	409.201	000	469.295	002	51
52	290.373	997	350.233	998	410.201	001	470.297	003	52
53	291.370	997	351.231	999	411.202	000	471.300	003	53
54	292.367	996	352.230	998	412.202	001	472.303	003	54
55	293.363	997	353.228	999	413.203	000	473.306	003	55
56	294.360	997	354.227	999	414.203	001	474.309	003	56
57	295.357	997	355.226	998	415.204	001	475.312	002	57
58	296.354	997	356.224	999	416.205	001	476.314	003	58
59	297.351	0.997	357.223	0.999	417.206	1.000	477.317	1.004	59
60	298.348		358.222		418.206		478.321		60

MERCATOR PROJECTION TABLE—Continued.

[Meridional distances for the spheroid. Compression $\frac{1}{294}$.]

Minutes.	8° Meridional distance.	8° Difference.	9° Meridional distance.	9° Difference.	10° Meridional distance.	10° Difference.	11° Meridional distance.	11° Difference.	Minutes.
	′	′	′	′	′	′	′	′	
0	478.321	1.003	538.585	1.006	599.019	1.009	659.641	1.012	0
1	479.324	003	539.591	006	600.028	009	660.653	012	1
2	480.327	003	540.597	006	601.037	009	661.665	013	2
3	481.330	003	541.603	006	602.046	008	662.678	012	3
4	482.333	004	542.609	006	603.054	009	663.690	012	4
5	483.337	003	543.615	006	604.063	009	664.702	013	5
6	484.340	003	544.621	006	605.072	009	665.715	012	6
7	485.343	004	545.627	006	606.081	010	666.727	013	7
8	486.347	003	546.633	006	607.091	009	667.740	012	8
9	487.350	004	547.639	007	608.100	009	668.752	013	9
10	488.354	1.003	548.646	1.006	609.109	1.009	669.765	1.013	10
11	489.357	004	549.652	006	610.118	010	670.778	012	11
12	490.361	004	550.658	006	611.128	009	671.790	013	12
13	491.365	004	551.664	007	612.137	009	672.803	013	13
14	492.369	003	552.671	006	613.146	010	673.816	013	14
15	493.372	004	553.677	007	614.156	010	674.829	013	15
16	494.376	004	554.684	006	615.166	009	675.842	013	16
17	495.380	004	555.690	007	616.175	010	676.855	013	17
18	496.384	004	556.697	006	617.185	010	677.868	013	18
19	497.388	004	557.703	007	618.195	009	678.881	013	19
20	498.392	1.004	558.710	1.007	619.204	1.010	679.894	1.013	20
21	499.396	004	559.717	007	620.214	010	680.907	013	21
22	500.400	004	560.724	007	621.224	010	681.920	014	22
23	501.404	004	561.731	006	622.234	010	682.934	013	23
24	502.408	004	562.737	007	623.244	010	683.947	014	24
25	503.412	004	563.744	007	624.254	010	684.961	013	25
26	504.416	004	564.751	007	625.264	011	685.974	014	26
27	505.420	004	565.758	008	626.275	010	686.988	014	27
28	506.424	005	566.766	007	627.285	010	688.002	013	28
29	507.429	004	567.773	007	628.295	010	689.015	014	29
30	508.433	1.004	568.780	1.007	629.305	1.011	690.029	1.014	30
31	509.437	005	569.787	008	630.316	010	691.043	014	31
32	510.442	004	570.795	007	631.326	011	692.057	014	32
33	511.446	005	571.802	007	632.337	010	693.071	014	33
34	512.451	004	572.809	008	633.347	011	694.085	014	34
35	513.455	005	573.817	007	634.358	011	695.099	014	35
36	514.460	005	574.824	008	635.369	010	696.113	015	36
37	515.465	004	575.832	007	636.379	011	697.128	014	37
38	516.469	005	576.839	008	637.390	011	698.142	014	38
39	517.474	005	577.847	008	638.401	011	699.156	015	39
40	518.479	1.005	578.855	1.007	639.412	1.011	700.171	1.014	40
41	519.484	005	579.862	008	640.423	011	701.185	015	41
42	520.489	005	580.870	008	641.434	011	702.200	015	42
43	521.494	005	581.878	008	642.445	011	703.215	014	43
44	522.499	005	582.886	008	643.456	011	704.229	015	44
45	523.504	005	583.894	008	644.467	011	705.244	015	45
46	524.509	005	584.902	008	645.478	011	706.259	015	46
47	525.514	005	585.910	008	646.489	011	707.274	015	47
48	526.519	006	586.918	008	647.500	012	708.289	015	48
49	527.525	005	587.926	008	648.512	011	709.304	015	49
50	528.530	1.005	588.934	1.008	649.523	1.012	710.319	1.015	50
51	529.535	005	589.942	009	650.535	011	711.334	015	51
52	530.540	006	590.951	008	651.546	012	712.349	015	52
53	531.546	005	591.959	009	652.558	012	713.364	015	53
54	532.551	006	592.968	008	653.570	011	714.379	016	54
55	533.557	006	593.976	009	654.581	012	715.395	015	55
56	534.563	005	594.985	008	655.593	012	716.410	015	56
57	535.568	006	595.993	009	656.605	012	717.425	016	57
58	536.574	006	597.002	008	657.617	012	718.441	016	58
59	537.580	1.005	598.010	1.009	658.629	1.012	719.457	1.015	59
60	538.585		599.019		659.641		720.472		60

MERCATOR PROJECTION TABLE—Continued.

[Meridional distances for the spheroid. Compression $\frac{1}{294}$.]

Minutes.	12° Meridional distance.	12° Difference.	13° Meridional distance.	13° Difference.	14° Meridional distance.	14° Difference.	15° Meridional distance.	15° Difference.	Minutes.
	′	′	′	′	′	′	′	′	
0	720.472	1.016	781.532	1.020	842.842	1.024	904.422	1.029	0
1	721.488	016	782.552	020	843.866	024	905.451	029	1
2	722.504	016	783.572	020	844.890	025	906.480	029	2
3	723.520	015	784.592	020	845.915	024	907.509	029	3
4	724.535	016	785.612	020	846.939	024	908.538	029	4
5	725.551	016	786.632	020	847.963	025	909.567	029	5
6	726.567	017	787.652	020	848.988	024	910.596	030	6
7	727.584	016	788.672	020	850.012	025	911.626	029	7
8	728.600	016	789.692	020	851.037	024	912.655	029	8
9	729.616	016	790.712	021	852.061	025	913.684	030	9
10	730.632	1.017	791.733	1.020	853.086	1.025	914.714	1.029	10
11	731.649	016	792.753	020	854.111	025	915.743	030	11
12	732.665	017	793.773	021	855.136	025	916.773	030	12
13	733.682	016	794.794	020	856.161	025	917.803	029	13
14	734.698	017	795.814	021	857.186	025	918.832	030	14
15	735.715	017	796.835	021	858.211	025	919.862	030	15
16	736.732	017	797.856	021	859.236	026	920.892	030	16
17	737.749	016	798.877	021	860.262	025	921.922	031	17
18	738.765	017	799.898	021	861.287	025	922.953	030	18
19	739.782	017	800.919	021	862.312	025	923.983	030	19
20	740.799	1.017	801.940	1.021	863.337	1.026	925.013	1.031	20
21	741.816	017	802.961	021	864.363	026	926.044	030	21
22	742.833	017	803.982	021	865.389	026	927.074	031	22
23	743.850	018	805.003	022	866.415	025	928.105	030	23
24	744.868	017	806.025	021	867.440	026	929.135	031	24
25	745.885	017	807.046	022	868.466	026	930.166	031	25
26	746.902	017	808.068	021	869.492	026	931.197	031	26
27	747.919	018	809.089	022	870.518	026	932.228	031	27
28	748.937	017	810.111	022	871.544	027	933.259	031	28
29	749.954	018	811.133	022	872.571	026	934.290	031	29
30	750.972	1.018	812.155	1.022	873.597	1.026	935.321	1.031	30
31	751.990	017	813.177	022	874.623	026	936.352	032	31
32	753.007	018	814.199	022	875.649	027	937.384	031	32
33	754.025	018	815.221	022	876.676	026	938.415	032	33
34	755.043	018	816.243	022	877.702	027	939.447	031	34
35	756.061	018	817.265	022	878.729	027	940.478	032	35
36	757.079	018	818.287	022	879.756	026	941.510	032	36
37	758.097	018	819.309	023	880.782	027	942.542	031	37
38	759.115	019	820.332	022	881.809	027	943.573	032	38
39	760.134	018	821.354	023	882.836	027	944.605	032	39
40	761.152	1.018	822.377	1.022	883.863	1.028	945.637	1.032	40
41	762.170	019	823.399	023	884.891	027	946.669	033	41
42	763.189	018	824.422	022	885.918	028	947.702	032	42
43	764.207	019	825.444	023	886.946	027	948.734	032	43
44	765.226	018	826.467	023	887.973	028	949.766	033	44
45	766.244	019	827.490	023	889.001	027	950.799	033	45
46	767.263	019	828.513	023	890.028	028	951.832	032	46
47	768.282	019	829.536	023	891.056	028	952.864	032	47
48	769.301	019	830.559	023	892.084	028	953.896	033	48
49	770.320	019	831.582	023	893.112	028	954.929	033	49
50	771.339	1.019	832.605	1.024	894.140	1.028	955.962	1.033	50
51	772.358	019	833.629	023	895.168	028	956.995	033	51
52	773.377	019	834.652	024	896.196	028	958.028	033	52
53	774.396	019	835.676	023	897.224	028	959.061	034	53
54	775.415	019	836.699	024	898.252	028	960.095	033	54
55	776.434	020	837.723	024	899.280	028	961.128	033	55
56	777.454	019	838.747	024	900.308	029	962.161	034	56
57	778.473	020	839.771	023	901.337	028	963.195	033	57
58	779.493	020	840.794	024	902.365	029	964.228	034	58
59	780.513	1.019	841.818	1.024	903.394	1.028	965.262	1.034	59
60	781.532		842.842		904.422		966.296		60

MERCATOR PROJECTION TABLE—Continued.

[Meridional distances for the spheroid. Compression $\frac{1}{294}$]

Minutes.	16° Meridional distance.	16° Difference.	17° Meridional distance.	17° Difference.	18° Meridional distance.	18° Difference.	19° Meridional distance.	19° Difference.	Minutes.
	′	′	′	′	′	′	′	′	
0	966.296	1.034	1028.483	1.039	1091.007	1.045	1153.891	1.052	0
1	967.330	034	29.522	039	92.052	046	54.943	051	1
2	968.364	034	30.561	039	93.098	045	55.994	052	2
3	969.398	034	31.600	040	94.143	045	57.046	051	3
4	970.432	034	32.640	040	95.188	046	58.097	052	4
5	971.466	034	33.680	039	96.234	045	59.149	052	5
6	972.500	034	34.719	040	97.279	046	60.201	052	6
7	973.534	034	35.759	040	98.325	045	61.253	052	7
8	974.568	035	36.799	040	1099.370	046	62.305	052	8
9	975.603	035	37.839	040	1100.416	046	63.357	052	9
10	976.638	1.035	1038.879	1.041	1101.462	1.046	1164.409	1.052	10
11	977.673	034	39.920	040	02.508	046	65.461	053	11
12	978.707	035	40.960	040	03.554	047	66.514	052	12
13	979.742	035	42.000	041	04.601	046	67.566	053	13
14	980.777	035	43.041	041	05.647	046	68.619	053	14
15	981.812	035	44.082	040	06.693	047	69.672	052	15
16	982.847	035	45.122	041	07.740	047	70.724	053	16
17	983.882	036	46.163	041	08.787	046	71.777	053	17
18	984.918	035	47.204	041	09.833	047	72.830	054	18
19	985.953	035	48.245	041	10.880	047	73.884	053	19
20	986.988	1.036	1049.286	1.041	1111.927	1.047	1174.937	1.053	20
21	988.024	036	50.327	041	12.974	047	75.990	054	21
22	989.060	035	51.368	041	14.021	048	77.044	053	22
23	990.095	036	52.409	042	15.069	047	78.097	054	23
24	991.131	036	53.451	042	16.116	047	79.151	054	24
25	992.167	036	54.493	041	17.163	048	80.205	054	25
26	993.203	036	55.534	042	18.211	048	81.259	054	26
27	994.239	037	56.576	042	19.259	048	82.313	054	27
28	995.276	036	57.618	042	20.307	047	83.367	054	28
29	996.312	036	58.660	042	21.354	048	84.421	055	29
30	997.348	1.037	1059.702	1.042	1122.402	1.049	1185.476	1.054	30
31	998.385	036	60.744	042	23.451	048	86.530	055	31
32	999.421	037	61.786	042	24.499	048	87.585	055	32
33	1000.458	037	62.828	042	25.547	048	88.640	055	33
34	01.495	037	63.870	043	26.595	049	89.695	055	34
35	02.532	037	64.913	043	27.644	049	90.750	055	35
36	03.569	037	65.956	042	28.693	048	91.805	055	36
37	04.606	037	66.998	043	29.741	049	92.860	055	37
38	05.643	037	68.041	043	30.790	049	93.915	056	38
39	06.680	038	69.084	043	31.839	049	94.971	055	39
40	1007.718	1.037	1070.127	1.043	1132.888	1.049	1196.026	1.056	40
41	08.755	038	71.170	043	33.937	050	97.082	055	41
42	09.793	037	72.213	044	34.987	049	98.137	056	42
43	10.830	038	73.257	043	36.036	050	1199.193	056	43
44	11.878	038	74.300	043	37.086	049	1200.249	056	44
45	12.906	037	75.343	044	38.135	050	01.305	056	45
46	13.943	038	76.387	044	39.185	050	02.361	056	46
47	14.981	038	77.431	044	40.235	050	03.417	057	47
48	16.019	039	78.475	043	41.285	050	04.474	056	48
49	17.058	038	79.518	044	42.335	050	05.530	057	49
50	1018.096	1.038	1080.562	1.045	1143.385	1.050	1206.587	1.056	50
51	19.134	038	81.607	044	44.435	050	07.643	057	51
52	20.172	038	82.651	044	45.485	051	08.700	057	52
53	21.210	039	83.695	044	46.536	050	09.757	057	53
54	22.249	039	84.739	045	47.586	051	10.814	057	54
55	23.288	039	85.784	044	48.637	051	11.871	058	55
56	24.327	039	86.828	045	49.688	050	12.929	057	56
57	25.366	039	87.873	045	50.738	051	13.986	058	57
58	26.405	039	88.918	045	51.789	051	15.044	057	58
59	27.444	1.039	89.963	1.044	52.840	1.051	16.101	1.058	59
60	1028.483		1091.007		1153.891		1217.159		60

MERCATOR PROJECTION TABLE—Continued.

[Meridional distances for the spheroid. Compression $\frac{1}{294}$.]

Minutes.	20° Meridional distance.	20° Difference.	21° Meridional distance.	21° Difference.	22° Meridional distance.	22° Difference.	23° Meridional distance.	23° Difference.	Minutes.
	′	′	′	′	′	′	′	′	
0	1217. 159	1. 058	1280. 835	1. 065	1344. 945	1. 072	1409. 513	1. 080	**0**
1	18. 217	058	81. 900	065	46. 017	072	10. 593	080	**1**
2	19. 275	058	82. 965	065	47. 089	073	11. 673	081	**2**
3	20. 333	059	84. 030	065	48. 162	073	12. 754	080	**3**
4	21. 392	058	85. 095	066	49. 235	072	13. 834	081	**4**
5	22. 450	059	86. 161	065	50. 307	073	14. 915	081	**5**
6	23. 509	058	87. 226	066	51. 380	073	15. 996	081	**6**
7	24. 567	059	88. 292	065	52. 453	073	17. 077	081	**7**
8	25. 626	059	89. 357	066	53. 526	074	18. 158	081	**8**
9	26. 685	059	90. 423	066	54. 600	073	19. 239	082	**9**
10	1227. 744	1. 059	1291. 489	1. 066	1355. 673	1. 074	1420. 321	1. 081	**10**
11	28. 803	059	92. 555	066	56. 747	073	21. 402	082	**11**
12	29. 862	059	93. 621	067	57. 820	074	22. 484	082	**12**
13	30. 921	059	94. 688	066	58. 894	074	23. 566	081	**13**
14	31. 980	060	95. 754	067	59. 968	074	24. 647	082	**14**
15	33. 040	059	96. 821	066	61. 042	074	25. 729	083	**15**
16	34. 099	060	97. 887	067	62. 116	075	26. 812	082	**16**
17	35. 159	059	1298. 954	067	63. 191	074	27. 894	082	**17**
18	36. 218	060	1300. 021	067	64. 265	075	28. 976	083	**18**
19	37. 278	060	01. 088	067	65. 340	075	30. 059	083	**19**
20	1238. 340	1. 061	1302. 155	1. 068	1366. 415	1. 074	1431. 142	1. 083	**20**
21	39. 399	060	03. 223	067	67. 489	075	32. 225	083	**21**
22	40. 459	060	04. 290	068	68. 564	076	33. 308	083	**22**
23	41. 519	061	05. 358	067	69. 640	075	34. 391	083	**23**
24	42. 580	060	06. 425	068	70. 715	075	35. 474	083	**24**
25	43. 640	061	07. 493	068	71. 790	076	36. 557	084	**25**
26	44. 701	061	08. 561	068	72. 866	076	37. 641	084	**26**
27	45. 762	061	09. 629	068	73. 942	075	38. 725	084	**27**
28	46. 823	061	10. 697	068	75. 017	076	39. 809	084	**28**
29	47. 884	061	11. 765	069	76. 093	076	40. 893	084	**29**
30	1248. 945	1. 061	1312. 834	1. 068	1377. 169	1. 076	1441. 977	1. 084	**30**
31	50. 006	062	13. 902	069	78. 245	077	43. 061	085	**31**
32	51. 068	061	14. 971	069	79. 322	076	44. 146	084	**32**
33	52. 129	062	16. 040	069	80. 398	077	45. 230	085	**33**
34	53. 191	061	17. 109	069	81. 475	076	46. 315	085	**34**
35	54. 252	062	18. 178	069	82. 551	077	47. 400	085	**35**
36	55. 314	062	19. 247	069	83. 628	077	48. 485	085	**36**
37	56. 376	062	20. 316	070	84. 705	077	49. 570	085	**37**
38	57. 438	063	21. 386	069	85. 782	078	50. 655	086	**38**
39	58. 501	062	22. 455	070	86. 860	077	51. 741	085	**39**
40	1259. 563	1. 063	1323. 525	1. 070	1387. 937	1. 077	1452. 826	1. 086	**40**
41	60. 626	062	24. 595	070	89. 014	078	53. 912	086	**41**
42	61. 688	063	25. 665	070	90. 092	078	54. 998	086	**42**
43	62. 751	063	26. 735	070	91. 170	078	56. 084	086	**43**
44	63. 814	063	27. 805	070	92. 248	078	57. 170	086	**44**
45	64. 877	063	28. 875	070	93. 326	078	58. 256	087	**45**
46	65. 940	063	29. 945	071	94. 404	078	59. 343	086	**46**
47	67. 003	064	31. 016	070	95. 482	079	60. 429	087	**47**
48	68. 067	063	32. 086	071	96. 561	078	61. 516	087	**48**
49	69. 130	064	33. 157	071	97. 639	079	62. 603	087	**49**
50	1270. 194	1. 063	1334. 228	1. 071	1398. 718	1. 079	1463. 690	1. 086	**50**
51	71. 257	064	35. 299	071	1399. 797	079	64. 776	088	**51**
52	72. 321	064	36. 370	072	1400. 876	079	65. 864	087	**52**
53	73. 385	064	37. 442	071	01. 955	079	66. 951	087	**53**
54	74. 449	064	38. 513	072	03. 034	080	68. 038	088	**54**
55	75. 513	064	39. 585	072	04. 114	079	69. 126	088	**55**
56	76. 577	065	40. 657	071	05. 193	080	70. 214	088	**56**
57	77. 642	064	41. 728	072	06. 273	080	71. 302	088	**57**
58	78. 706	065	42. 800	072	07. 353	080	72. 390	088	**58**
59	79. 771	1. 064	43. 872	1. 073	08. 433	1. 080	73. 478	1. 088	**59**
60	1280. 835		1344. 945		1409. 513		1474. 566		**60**

MERCATOR PROJECTION TABLE—Continued.

[Meridional distances for the spheroid. Compression $\frac{1}{294}$.]

Minutes.	24° Meridional distance.	24° Difference.	25° Meridional distance.	25° Difference.	26° Meridional distance.	26° Difference.	27° Meridional distance.	27° Difference.	Minutes.
	′	′	′	′	′	′	′	′	
0	1474.566	1.089	1540.134	1.097	1606.243	1.106	1672.923	1.117	0
1	75.655	088	41.231	097	07.349	107	74.040	116	1
2	76.743	089	42.328	098	08.456	107	75.156	117	2
3	77.832	089	43.426	098	09.563	107	76.273	117	3
4	78.921	089	44.524	098	10.670	107	77.390	117	4
5	80.010	089	45.622	098	11.777	107	78.507	117	5
6	81.099	090	46.720	098	12.884	108	79.624	117	6
7	82.189	089	47.818	098	13.992	107	80.741	118	7
8	83.278	090	48.916	099	15.099	108	81.859	117	8
9	84.368	090	50.015	098	16.207	108	82.976	118	9
10	1485.458	1.090	1551.113	1.099	1617.315	1.108	1684.094	1.118	10
11	86.548	090	52.212	099	18.423	100	85.212	119	11
12	87.638	090	53.311	099	19.532	108	86.331	118	12
13	88.728	091	54.410	099	20.640	109	87.449	118	13
14	89.819	090	55.509	100	21.749	109	88.567	119	14
15	90.909	091	56.609	099	22.858	109	89.686	119	15
16	92.000	091	57.708	100	23.967	109	90.805	119	16
17	93.091	091	58.808	100	25.076	109	91.924	119	17
18	94.182	091	59.908	100	26.185	110	93.043	120	18
19	95.273	091	61.008	100	27.295	109	94.163	119	19
20	1496.364	1.091	1562.108	1.101	1628.404	1.110	1695.282	1.120	20
21	97.455	092	63.209	100	29.514	110	96.402	120	21
22	98.547	092	64.309	101	30.624	110	97.522	120	22
23	1499.639	091	65.410	101	31.734	110	98.642	120	23
24	1500.730	092	66.511	101	32.844	111	1699.762	121	24
25	01.822	092	67.612	101	33.955	110	1700.883	120	25
26	02.914	093	68.713	101	35.065	111	02.003	121	26
27	04.007	092	69.814	101	36.176	111	03.124	121	27
28	05.099	093	70.915	102	37.287	111	04.245	121	28
29	06.192	092	72.017	102	38.398	111	05.366	121	29
30	1507.284	1.093	1573.119	1.102	1639.509	1.112	1706.487	1.122	30
31	08.377	093	74.221	102	40.621	112	07.609	121	31
32	09.470	093	75.323	102	41.733	111	08.730	122	32
33	10.563	093	76.425	102	42.844	112	09.852	122	33
34	11.656	094	77.527	102	43.956	112	10.974	122	34
35	12.750	093	78.629	103	45.068	113	12.096	123	35
36	13.843	094	79.732	103	46.181	112	13.219	122	36
37	14.937	094	80.835	103	47.293	113	14.341	123	37
38	16.031	094	81.938	103	48.406	112	15.464	122	38
39	17.125	094	83.041	103	49.518	113	16.586	123	39
40	1518.219	1.094	1584.144	1.104	1650.631	1.113	1717.709	1.124	40
41	19.313	095	85.248	103	51.744	113	18.833	123	41
42	20.408	094	86.351	104	52.857	114	19.956	124	42
43	21.502	095	87.455	104	53.971	113	21.080	123	43
44	22.597	095	88.559	104	55.084	114	22.203	124	44
45	23.692	095	89.663	104	56.198	114	23.327	124	45
46	24.787	095	90.767	104	57.312	114	24.451	124	46
47	25.882	096	91.871	105	58.426	114	25.575	125	47
48	26.978	095	92.976	105	59.540	114	26.700	124	48
49	28.073	096	94.081	105	60.654	115	27.824	125	49
50	1529.169	1.096	1595.186	1.105	1661.769	1.115	1728.949	1.125	50
51	30.265	096	96.291	105	62.884	114	30.074	125	51
52	31.361	096	97.396	105	63.998	115	31.199	125	52
53	32.457	096	98.501	106	65.113	116	32.324	126	53
54	33.553	097	1599.607	105	66.229	115	33.450	125	54
55	34.650	096	1600.712	106	67.344	115	34.575	126	55
56	35.746	097	01.818	106	68.459	116	35.701	126	56
57	36.843	097	02.924	106	69.575	116	36.827	126	57
58	37.940	097	04.030	106	70.691	116	37.953	127	58
59	39.037	1.097	05.136	1.107	71.807	1.116	39.080	1.126	59
60	1540.134		1606.243		1672.923		1740.206		60

MERCATOR PROJECTION TABLE—Continued.

[Meridional distances for the spheroid. Compression $\frac{1}{294}$.]

Minutes.	28° Meridional distance.	28° Difference.	29° Meridional distance.	29° Difference.	30° Meridional distance.	30° Difference.	31° Meridional distance.	31° Difference.	Minutes.
	′	′	′	′	′	′	′	′	
0	1740.206	1.127	1808.122	1.138	1876.706	1.149	1945.992	1.161	0
1	41.333	127	09.260	138	77.855	149	47.153	161	1
2	42.460	127	10.398	137	79.004	149	48.314	162	2
3	43.587	127	11.535	138	80.153	150	49.476	161	3
4	44.714	127	12.673	139	81.303	150	50.637	162	4
5	45.841	128	13.812	138	82.453	150	51.799	162	5
6	46.969	127	14.950	139	83.603	150	52.961	162	6
7	48.096	128	16.089	139	84.753	150	54.123	162	7
8	49.224	128	17.228	139	85.903	150	55.285	163	8
9	50.352	129	18.367	139	87.053	151	56.448	163	9
10	1751.481	1.128	1819.506	1.139	1888.204	1.151	1957.611	1.163	10
11	52.609	129	20.645	140	89.355	151	58.774	163	11
12	53.738	128	21.785	139	90.506	151	59.937	163	12
13	54.866	129	22.924	140	91.657	152	61.100	163	13
14	55.995	129	24.064	140	92.809	151	62.263	164	14
15	57.124	130	25.204	141	93.960	152	63.427	164	15
16	58.254	129	26.345	140	95.112	152	64.591	165	16
17	59.383	130	27.485	141	96.264	152	65.756	164	17
18	60.513	130	28.626	141	97.416	153	66.920	165	18
19	61.643	130	29.767	141	98.569	152	68.085	164	19
20	1762.773	1.130	1830.908	1.141	1899.721	1.153	1969.249	1.165	20
21	63.903	130	32.049	141	1900.874	153	70.414	166	21
22	65.033	131	33.190	142	02.027	154	71.580	165	22
23	66.164	131	34.332	142	03.181	153	72.745	166	23
24	67.295	131	35.474	142	04.334	154	73.911	166	24
25	68.426	131	36.616	142	05.488	154	75.077	166	25
26	69.557	131	37.758	142	06.642	154	76.243	166	26
27	70.688	132	38.900	143	07.796	154	77.409	166	27
28	71.820	131	40.043	143	08.950	155	78.575	167	28
29	72.951	132	41.186	143	10.105	154	79.742	167	29
30	1774.083	1.132	1842.329	1.143	1911.259	1.155	1980.909	1.167	30
31	75.215	132	43.472	143	12.414	155	82.076	168	31
32	76.347	132	44.615	144	13.569	155	83.244	167	32
33	77.479	133	45.759	143	14.724	156	84.411	168	33
34	78.612	133	46.902	144	15.880	155	85.579	168	34
35	79.745	132	48.046	144	17.035	156	86.747	168	35
36	80.877	134	49.190	145	18.191	156	87.915	169	36
37	82.011	133	50.335	144	19.347	156	89.084	168	37
38	83.144	133	51.479	145	20.503	157	90.252	169	38
39	84.277	134	52.624	145	21.660	156	91.421	169	39
40	1785.411	1.134	1853.769	1.145	1922.816	1.157	1992.590	1.169	40
41	86.545	134	54.914	145	23.973	157	93.759	170	41
42	87.679	134	56.059	145	25.130	157	94.929	169	42
43	88.813	135	57.204	146	26.287	158	96.098	170	43
44	89.948	134	58.350	146	27.445	158	97.268	170	44
45	91.082	135	59.496	146	28.603	157	98.438	171	45
46	92.217	135	60.642	146	29.760	158	1999.609	170	46
47	93.352	135	61.788	146	30.918	159	2000.779	171	47
48	94.487	135	62.934	147	32.077	158	01.950	171	48
49	95.622	136	64.081	147	33.235	159	03.121	171	49
50	1796.758	1.135	1865.228	1.147	1934.394	1.159	2004.292	1.171	50
51	97.893	136	66.375	147	35.553	159	05.463	172	51
52	1799.029	136	67.522	147	36.712	159	06.635	172	52
53	1800.165	136	68.669	148	37.871	160	07.807	172	53
54	01.301	137	69.817	147	39.031	160	08.979	172	54
55	02.438	136	70.964	148	40.191	160	10.151	172	55
56	03.574	137	72.112	148	41.351	160	11.323	173	56
57	04.711	137	73.260	149	42.511	160	12.496	173	57
58	05.848	137	74.409	148	43.671	161	13.669	173	58
59	06.985	1.137	75.557	1.149	44.832	1.160	14.842	1.173	59
60	1808.122		1876.706		1945.992		2016.015		60

MERCATOR PROJECTION TABLE—Continued.

[Meridional distances for the spheroid. Compression $\frac{1}{294}$.]

Minutes.	32° Meridional distance.	32° Difference.	33° Meridional distance.	33° Difference.	34° Meridional distance.	34° Difference.	35° Meridional distance.	35° Difference.	Minutes.
	′	′	′	′	′	′	′	′	
0	2016.015	1.174	2086.814	1.187	2158.428	1.201	2230.898	1.215	0
1	17.189	174	88.001	187	59.629	201	32.113	216	1
2	18.363	174	89.188	188	60.830	201	33.329	216	2
3	19.537	174	90.376	187	62.031	201	34.545	216	3
4	20.711	174	91.563	188	63.232	202	35.761	216	4
5	21.885	175	92.751	188	64.434	202	36.977	217	5
6	23.060	175	93.939	188	65.636	202	38.194	217	6
7	24.235	175	95.127	188	66.838	203	39.411	217	7
8	25.410	175	96.315	189	68.041	202	40.628	217	8
9	26.585	176	97.504	189	69.243	203	41.845	218	9
10	2027.761	1.175	2098.693	1.189	2170.446	1.203	2243.063	1.218	10
11	28.936	176	2099.882	189	71.649	204	44.281	218	11
12	30.112	176	2101.071	189	72.853	203	45.499	218	12
13	31.288	176	02.260	190	74.056	204	46.717	219	13
14	32.464	177	03.450	190	75.260	204	47.936	219	14
15	33.641	177	04.640	190	76.464	204	49.155	219	15
16	34.818	177	05.830	191	77.668	205	50.374	219	16
17	35.995	177	07.021	190	78.873	204	51.593	220	17
18	37.172	177	08.211	191	80.077	205	52.813	220	18
19	38.349	178	09.402	191	81.282	206	54.033	220	19
20	2039.527	1.178	2110.593	1.192	2182.488	1.205	2255.253	1.220	20
21	40.705	178	11.785	191	83.693	206	56.473	220	21
22	41.883	178	12.976	192	84.899	206	57.693	221	22
23	43.061	178	14.168	192	86.105	206	58.914	221	23
24	44.239	179	15.360	192	87.311	207	60.135	222	24
25	45.418	179	16.552	193	88.518	206	61.357	221	25
26	46.597	179	17.745	192	89.724	207	62.578	222	26
27	47.776	179	18.937	193	90.931	207	63.800	222	27
28	48.955	179	20.130	193	92.138	208	65.022	223	28
29	50.134	180	21.323	194	93.346	208	66.245	222	29
30	2051.314	1.181	2122.517	1.194	2194.554	1.208	2267.467	1.223	30
31	52.495	180	23.711	193	95.762	208	68.690	223	31
32	53.675	181	24.904	194	96.970	208	69.913	224	32
33	54.856	180	26.098	195	98.178	208	71.137	224	33
34	56.036	181	27.293	194	2199.386	209	72.361	224	34
35	57.217	182	28.487	195	2200.595	209	73.585	224	35
36	58.399	181	29.682	195	01.804	210	74.809	224	36
37	59.580	182	30.877	195	03.014	209	76.033	225	37
38	60.762	182	32.072	196	04.223	210	77.258	225	38
39	61.944	182	33.268	196	05.433	210	78.483	225	39
40	2063.126	1.182	2134.464	1.196	2206.643	1.211	2279.708	1.226	40
41	64.308	183	35.660	196	07.854	211	80.934	225	41
42	65.491	183	36.856	196	09.065	211	82.159	226	42
43	66.674	183	38.052	197	10.276	211	83.385	227	43
44	67.857	183	39.249	197	11.487	211	84.612	226	44
45	69.040	183	40.446	197	12.698	212	85.838	227	45
46	70.223	184	41.643	198	13.910	212	87.065	227	46
47	71.407	184	42.841	197	15.122	212	88.292	227	47
48	72.591	184	44.038	198	16.334	212	89.519	228	48
49	73.775	184	45.236	198	17.546	213	90.747	228	49
50	2074.959	1.185	2146.434	1.199	2218.759	1.213	2291.975	1.228	50
51	76.144	184	47.633	198	19.972	213	93.203	228	51
52	77.328	185	48.831	199	21.185	213	94.431	229	52
53	78.513	185	50.030	199	22.398	213	95.660	229	53
54	79.698	186	51.229	199	23.611	214	96.889	229	54
55	80.884	185	52.428	1.199	24.825	214	98.118	229	55
56	82.069	186	53.627	1.200	26.039	214	2299.347	230	56
57	83.255	186	54.827	200	27.253	215	2300.577	230	57
58	84.441	187	56.027	200	28.468	215	01.807	230	58
59	85.628	1.186	57.227	1.201	29.683	1.215	03.037	1.230	59
60	2086.814		2158.428		2230.898		2304.267		60

MERCATOR PROJECTION TABLE—Continued.

[Meridional distances for the spheroid. Compression $\frac{1}{294}$.]

Minutes.	36° Meridional distance.	36° Difference.	37° Meridional distance.	37° Difference.	38° Meridional distance.	38° Difference.	39° Meridional distance.	39° Difference.	Minutes.
	′	′	′	′	′	′	′	′	
0	2304.267	1.231	2378.581	1.247	2453.888	1.264	2530.238	1.281	0
1	05.498	231	79.828	247	55.152	264	31.519	282	1
2	06.729	231	81.075	248	56.416	264	32.801	282	2
3	07.960	232	82.323	247	57.680	265	34.083	283	3
4	09.192	231	83.570	248	58.945	265	35.366	283	4
5	10.423	232	84.818	248	60.210	265	36.649	283	5
6	11.655	232	86.066	249	61.475	266	37.932	283	6
7	12.887	233	87.315	249	62.741	266	39.215	284	7
8	14.120	233	88.564	249	64.007	266	40.499	284	8
9	15.353	233	89.813	249	65.273	266	41.783	285	9
10	2316.586	1.233	2391.062	1.250	2466.539	1.267	2543.068	1.284	10
11	17.819	234	92.312	250	67.806	267	44.352	285	11
12	19.053	234	93.562	250	69.073	267	45.637	285	12
13	20.287	234	94.812	250	70.340	268	46.922	286	13
14	21.521	234	96.062	251	71.608	268	48.208	286	14
15	22.755	235	97.313	251	72.876	268	49.494	287	15
16	23.990	235	98.564	252	74.144	269	50.781	286	16
17	25.225	235	2399.816	251	75.413	268	52.067	287	17
18	26.460	235	2401.067	252	76.681	269	53.354	287	18
19	27.695	236	02.319	252	77.950	270	54.641	288	19
20	2328.931	1.236	2403.571	1.253	2479.220	1.269	2555.929	1.287	20
21	30.167	237	04.824	252	80.489	270	57.216	288	21
22	31.404	236	06.076	253	81.759	271	58.504	289	22
23	32.640	237	07.329	253	83.030	270	59.793	288	23
24	33.877	237	08.582	254	84.300	271	61.081	289	24
25	35.114	237	09.836	254	85.571	271	62.370	290	25
26	36.351	238	11.090	254	86.842	272	63.660	289	26
27	37.589	238	12.344	254	88.114	271	64.949	290	27
28	38.827	238	13.598	255	89.385	272	66.239	290	28
29	40.065	238	14.853	255	90.657	273	67.529	291	29
30	2341.303	1.239	2416.108	1.255	2491.930	1.272	2568.820	1.291	30
31	42.542	239	17.363	255	93.202	273	70.111	291	31
32	43.781	239	18.618	256	94.475	273	71.402	292	32
33	45.020	240	19.874	256	95.748	274	72.694	292	33
34	46.260	240	21.130	256	97.022	274	73.986	292	34
35	47.500	240	22.386	257	98.296	274	75.278	292	35
36	48.740	240	23.643	257	2499.570	274	76.570	293	36
37	49.980	241	24.900	257	2500.844	275	77.863	293	37
38	51.221	241	26.157	258	02.119	275	79.156	293	38
39	52.462	241	27.415	257	03.394	275	80.449	294	39
40	2353.703	1.241	2428.672	1.258	2504.669	1.276	2581.743	1.294	40
41	54.944	241	29.930	259	05.945	276	83.037	294	41
42	56.185	242	31.189	259	07.221	276	84.331	295	42
43	57.427	242	32.448	259	08.497	276	85.626	295	43
44	58.669	243	33.707	259	09.773	277	86.921	295	44
45	59.912	242	34.966	259	11.050	277	88.216	295	45
46	61.154	243	36.225	260	12.327	277	89.511	296	46
47	62.397	244	37.485	260	13.604	278	90.807	296	47
48	63.641	243	38.745	261	14.882	278	92.103	297	48
49	64.884	244	40.006	260	16.160	278	93.400	297	49
50	2366.128	1.244	2441.266	1.261	2517.438	1.279	2594.697	1.297	50
51	67.372	244	42.527	261	18.717	279	95.994	298	51
52	68.616	245	43.788	262	19.996	279	97.292	298	52
53	69.861	245	45.050	261	21.275	279	98.590	298	53
54	71.106	245	46.311	262	22.554	280	2599.888	298	54
55	72.351	246	47.573	263	23.834	280	2601.186	299	55
56	73.597	245	48.836	262	25.114	281	02.485	1.299	56
57	74.842	246	50.098	263	26.395	280	03.784	1.300	57
58	76.088	247	51.361	263	27.675	281	05.084	1.299	58
59	77.335	1.246	52.624	1.264	28.956	1.282	06.383	1.300	59
60	2378.581		2453.888		2530.238		2607.683		60

MERCATOR PROJECTION TABLE—Continued.

[Meridional distances for the spheroid. Compression $\frac{1}{294}$.]

Minutes.	40° Meridional distance.	40° Difference.	41° Meridional distance.	41° Difference.	42° Meridional distance.	42° Difference.	43° Meridional distance.	43° Difference.	Minutes.
	′	′	′	′	′	′	′	′	
0	2607.683	1.301	2686.280	1.320	2766.089	1.341	2847.171	1.362	0
1	08.984	300	87.600	320	67.430	341	48.533	363	1
2	10.284	301	88.920	321	68.771	341	49.896	364	2
3	11.585	301	90.241	321	70.112	342	51.260	363	3
4	12.886	302	91.562	322	71.454	342	52.623	364	4
5	14.188	302	92.884	322	72.796	342	53.987	365	5
6	15.490	302	94.206	322	74.138	343	55.352	364	6
7	16.792	303	95.528	322	75.481	343	56.716	365	7
8	18.095	303	96.850	323	76.824	344	58.081	366	8
9	19.398	303	98.173	323	78.168	344	59.447	366	9
10	2620.701	1.303	2699.496	1.324	2779.512	1.344	2860.813	1.366	10
11	22.004	304	2700.820	323	80.856	345	62.179	367	11
12	23.308	304	02.143	324	82.201	345	63.546	367	12
13	24.612	305	03.467	325	83.546	345	64.913	367	13
14	25.917	305	04.792	325	84.891	346	66.280	368	14
15	27.222	305	06.117	325	86.237	346	67.648	368	15
16	28.527	306	07.442	325	87.583	347	69.016	368	16
17	29.833	306	08.767	326	88.930	347	70.384	369	17
18	31.139	306	10.093	326	90.277	347	71.753	370	18
19	32.445	306	11.419	327	91.624	347	73.123	369	19
20	2633.751	1.307	2712.746	1.327	2792.971	1.348	2874.492	1.370	20
21	35.058	307	14.073	327	94.319	348	75.862	371	21
22	36.365	307	15.400	327	95.667	349	77.233	371	22
23	37.672	308	16.727	328	97.016	349	78.604	371	23
24	38.980	308	18.055	328	98.365	349	79.975	372	24
25	40.288	309	19.383	329	2799.714	350	81.347	372	25
26	41.597	309	20.712	329	2801.064	350	82.719	372	26
27	42.906	309	22.041	329	02.414	350	84.091	373	27
28	44.215	309	23.370	330	03.764	351	85.464	373	28
29	45.524	310	24.700	330	05.115	351	86.837	374	29
30	2646.834	1.310	2726.030	1.330	2806.466	1.352	2888.211	1.374	30
31	48.144	310	27.360	330	07.818	352	89.585	374	31
32	49.454	311	28.690	331	09.170	352	90.959	374	32
33	50.765	311	30.021	331	10.522	353	92.333	375	33
34	52.076	312	31.352	332	11.875	353	93.708	376	34
35	53.388	312	32.684	332	13.228	353	95.084	376	35
36	54.700	312	34.016	332	14.581	354	96.460	376	36
37	56.012	312	35.348	333	15.935	354	97.836	376	37
38	57.324	313	36.681	333	17.289	354	2899.212	377	38
39	58.637	313	38.014	333	18.643	355	2900.589	377	39
40	2659.950	1.313	2739.347	1.334	2819.998	1.355	2901.966	1.378	40
41	61.263	314	40.681	334	21.353	356	03.344	378	41
42	62.577	314	42.015	335	22.709	356	04.722	378	42
43	63.891	314	43.350	334	24.065	356	06.100	379	43
44	65.205	315	44.684	335	25.421	356	07.479	379	44
45	66.520	315	46.019	336	26.777	357	08.858	380	45
46	67.835	315	47.355	336	28.134	358	10.238	380	46
47	69.150	316	48.691	336	29.492	358	11.618	380	47
48	70.466	316	50.027	336	30.850	358	12.998	381	48
49	71.782	317	51.363	337	32.208	358	14.379	381	49
50	2673.099	1.316	2752.700	1.338	2833.566	1.359	2915.760	1.382	50
51	74.415	317	54.038	337	34.925	359	17.142	382	51
52	75.732	317	55.375	338	36.284	359	18.524	382	52
53	77.049	318	56.713	339	37.643	360	19.906	383	53
54	78.367	318	58.052	338	39.003	361	21.289	383	54
55	79.685	318	59.390	339	40.364	360	22.672	384	55
56	81.003	319	60.729	340	41.724	361	24.056	384	56
57	82.322	319	62.069	340	43.085	362	25.440	384	57
58	83.641	319	63.409	340	44.447	362	26.824	385	58
59	84.960	1.320	64.749	1.340	45.809	1.362	28.209	1.385	59
60	2686.280		2766.089		2847.171		2929.594		60

MERCATOR PROJECTION TABLE—Continued.

[Meridional distances for the spheroid. Compression $\frac{1}{294}$.]

Minutes.	44°		45°		46°		47°		Minutes.
	Meridional distance.	Difference.	Meridional distance.	Difference.	Meridional distance.	Difference.	Meridional distance.	Difference.	
	′	′	′	′	′	′	′	′	
0	2929.594	1.385	3013.427	1.410	3098.747	1.435	3185.634	1.462	0
1	30.979	386	14.837	410	3100.182	435	87.096	462	1
2	32.365	386	16.247	410	01.617	436	88.558	463	2
3	33.751	387	17.657	411	03.053	437	90.021	463	3
4	35.138	387	19.068	411	04.490	437	91.484	464	4
5	36.525	388	20.479	412	05.927	437	92.948	464	5
6	37.913	387	21.891	412	07.364	438	94.412	464	6
7	39.300	388	23.303	413	08.802	438	95.876	465	7
8	40.688	389	24.716	413	10.240	438	97.341	466	8
9	42.077	389	26.129	413	11.678	439	3198.807	466	9
10	2943.466	1.389	3027.542	1.414	3113.117	1.440	3200.273	1.466	10
11	44.855	390	28.956	414	14.557	440	01.739	467	11
12	46.245	390	30.370	414	15.997	440	03.206	468	12
13	47.635	391	31.784	415	17.437	441	04.674	468	13
14	49.026	391	33.199	416	18.878	441	06.142	468	14
15	50.417	391	34.615	416	20.319	442	07.610	469	15
16	51.808	392	36.031	416	21.761	442	09.079	469	16
17	53.200	392	37.447	416	23.203	442	10.548	470	17
18	54.592	393	38.863	417	24.645	443	12.018	470	18
19	55.985	393	40.280	418	26.088	443	13.488	471	19
20	2957.378	1.393	3041.698	1.418	3127.531	1.444	3214.959	1.471	20
21	58.771	394	43.116	418	28.975	444	16.430	472	21
22	60.165	394	44.534	419	30.419	445	17.902	472	22
23	61.559	394	45.953	420	31.864	445	19.374	472	23
24	62.953	395	47.373	419	33.309	446	20.846	473	24
25	64.348	396	48.792	420	34.755	446	22.319	474	25
26	65.744	396	50.212	421	36.201	446	23.793	474	26
27	67.140	396	51.633	421	37.647	447	25.267	474	27
28	68.536	396	53.054	421	39.094	447	26.741	475	28
29	69.932	397	54.475	422	40.541	448	28.216	475	29
30	2971.329	1.398	3055.897	1.422	3141.989	1.449	3229.691	1.476	30
31	72.727	397	57.319	422	43.438	448	31.167	476	31
32	74.124	398	58.741	423	44.886	449	32.643	477	32
33	75.522	399	60.164	424	46.335	450	34.120	477	33
34	76.921	399	61.588	424	47.785	450	35.597	478	34
35	78.320	399	63.012	424	49.235	451	37.075	478	35
36	79.719	400	64.436	424	50.686	451	38.553	479	36
37	81.119	400	65.860	426	52.137	451	40.032	479	37
38	82.519	401	67.286	425	53.588	452	41.511	480	38
39	83.920	401	68.711	426	55.040	452	42.991	480	39
40	2985.321	1.401	3070.137	1.427	3156.492	1.453	3244.471	1.480	40
41	86.722	402	71.564	427	57.945	453	45.951	481	41
42	88.124	403	72.991	427	59.398	454	47.432	482	42
43	89.527	402	74.418	428	60.852	454	48.914	482	43
44	90.929	403	75.846	428	62.306	455	50.396	482	44
45	92.332	404	77.274	428	63.761	455	51.878	483	45
46	93.736	404	78.702	429	65.216	455	53.361	483	46
47	95.140	404	80.131	430	66.671	456	54.844	484	47
48	96.544	405	81.561	430	68.127	457	56.328	485	48
49	97.949	405	82.991	430	69.584	457	57.813	485	49
50	2999.354	1.405	3084.421	1.431	3171.041	1.457	3259.298	1.485	50
51	3000.759	406	85.852	431	72.498	458	60.783	486	51
52	02.165	407	87.283	431	73.956	458	62.269	486	52
53	03.572	406	88.714	432	75.414	459	63.755	487	53
54	04.978	407	90.146	432	76.873	459	65.242	487	54
55	06.385	408	91.578	433	78.332	459	66.729	488	55
56	07.793	408	93.011	433	79.791	460	68.217	488	56
57	09.201	408	94.444	434	81.251	461	69.705	489	57
58	10.609	409	95.878	434	82.712	461	71.194	489	58
59	12.018	1.409	97.312	1.435	84.173	1.461	72.683	1.490	59
60	3013.427		3098.747		3185.634		3274.173		60

MERCATOR PROJECTION TABLE—Continued.

[Meridional distances for the spheroid. Compression $\frac{1}{294}$.]

Minutes.	48° Meridional distance.	48° Difference.	49° Meridional distance.	49° Difference.	50° Meridional distance.	50° Difference.	51° Meridional distance.	51° Difference.	Minutes.
	′	′	′	′	′	′	′	′	
0	3274.173	1.490	3364.456	1.520	3456.581	1.551	3550.654	1.585	0
1	75.663	491	65.976	521	58.132	552	52.239	585	1
2	77.154	491	67.497	521	59.684	553	53.824	586	2
3	78.645	491	69.018	521	61.237	553	55.410	587	3
4	80.136	493	70.539	522	62.790	554	56.997	587	4
5	81.629	492	72.061	523	64.344	555	58.584	588	5
6	83.121	493	73.584	523	65.899	555	60.172	589	6
7	84.614	494	75.107	524	67.454	555	61.761	589	7
8	86.108	494	76.631	524	69.009	556	63.350	589	8
9	87.602	494	78.155	525	70.565	557	64.939	590	9
10	3289.096	1.495	3379.680	1.525	3472.122	1.557	3566.529	1.591	10
11	90.591	496	81.205	526	73.679	557	68.120	592	11
12	92.087	496	82.731	526	75.236	558	69.712	592	12
13	93.583	496	84.257	526	76.794	559	71.304	592	13
14	95.079	497	85.783	527	78.353	559	72.896	593	14
15	96.576	498	87.310	528	79.912	560	74.489	594	15
16	98.074	498	88.838	529	81.472	561	76.083	594	16
17	3299.572	498	90.367	529	83.033	561	77.677	595	17
18	3301.070	499	91.896	529	84.594	561	79.272	596	18
19	02.569	500	93.425	530	86.155	562	80.868	596	19
20	3304.069	1.500	3394.955	1.530	3487.717	1.563	3582.464	1.596	20
21	05.569	500	96.485	531	89.280	563	84.060	597	21
22	07.069	501	98.016	531	90.843	563	85.657	598	22
23	08.570	501	3399.547	532	92.406	564	87.255	598	23
24	10.071	502	3401.079	533	93.970	565	88.853	599	24
25	11.573	502	02.612	533	95.535	565	90.452	600	25
26	13.075	503	04.145	533	97.100	566	92.052	600	26
27	14.578	504	05.678	534	3498.666	567	93.652	600	27
28	16.082	504	07.212	535	3500.233	567	95.252	601	28
29	17.586	504	08.747	535	01.800	567	96.853	602	29
30	3319.090	1.505	3410.282	1.535	3503.367	1.568	3598.455	1.603	30
31	20.595	505	11.817	536	04.935	569	3600.058	603	31
32	22.100	506	13.353	537	06.504	569	01.661	604	32
33	23.606	507	14.890	537	08.073	570	03.265	604	33
34	25.113	507	16.427	538	09.643	570	04.869	605	34
35	26.620	507	17.965	538	11.213	571	06.474	605	35
36	28.127	508	19.503	539	12.784	571	08.079	606	36
37	29.635	508	21.042	539	14.355	572	09.685	607	37
38	31.143	509	22.581	540	15.927	573	11.292	607	38
39	32.652	510	24.121	540	17.500	573	12.899	607	39
40	3334.162	1.510	3425.661	1.541	3519.073	1.574	3614.506	1.609	40
41	35.672	510	27.202	542	20.647	574	16.115	609	41
42	37.182	511	28.744	542	22.221	575	17.724	610	42
43	38.693	511	30.286	542	23.796	575	19.334	610	43
44	40.204	512	31.828	543	25.371	576	20.944	611	44
45	41.716	512	33.371	544	26.947	577	22.555	611	45
46	43.228	513	34.915	544	28.524	577	24.166	612	46
47	44.741	514	36.459	545	30.101	577	25.778	612	47
48	46.255	514	38.004	545	31.678	578	27.390	613	48
49	47.769	514	39.549	546	33.256	579	29.003	614	49
50	3349.283	1.515	3441.095	1.546	3534.835	1.580	3630.617	1.614	50
51	50.798	516	42.641	547	36.415	580	32.231	615	51
52	52.314	516	44.188	547	37.995	580	33.846	616	52
53	53.830	516	45.735	548	39.575	581	35.462	616	53
54	55.346	517	47.283	548	41.156	581	37.078	617	54
55	56.863	518	48.831	549	42.737	582	38.695	617	55
56	58.381	518	50.380	549	44.319	583	40.312	618	56
57	59.899	518	51.929	550	45.902	583	41.930	618	57
58	61.417	519	53.479	551	47.485	584	43.548	619	58
59	62.936	1.520	55.030	1.551	49.069	1.585	45.167	1.620	59
60	3364.456		3456.581		3550.654		3646.787		60

MERCATOR PROJECTION TABLE—Continued.

[Meridional distances for the spheroid. Compression $\frac{1}{294}$.]

Minutes.	52° Meridional distance.	52° Difference.	53° Meridional distance.	53° Difference.	54° Meridional distance.	54° Difference.	55° Meridional distance.	55° Difference.	Minutes.
	′	′	′	′	′	′	′	′	
0	3646.787	1.621	3745.105	1.658	3845.738	1.698	3948.830	1.740	0
1	48.408	621	46.763	658	47.436	698	50.570	741	1
2	50.029	621	48.421	659	49.134	699	52.311	741	2
3	51.650	622	50.080	660	50.833	700	54.052	742	3
4	53.272	623	51.740	661	52.533	701	55.794	743	4
5	54.895	624	53.401	661	54.234	701	57.537	744	5
6	56.519	624	55.062	662	55.935	702	59.281	744	6
7	58.143	624	56.724	662	57.637	702	61.025	745	7
8	59.767	626	58.386	663	59.339	703	62.770	746	8
9	61.393	626	60.049	664	61.042	704	64.516	746	9
10	3663.019	1.626	3761.713	1.664	3862.746	1.704	3966.262	1.747	10
11	64.645	627	63.377	665	64.450	705	68.009	748	11
12	66.272	628	65.042	666	66.155	706	69.757	749	12
13	67.900	628	66.708	666	67.861	707	71.506	749	13
14	69.528	629	68.374	667	69.568	707	73.255	750	14
15	71.157	630	70.041	668	71.275	708	75.005	751	15
16	72.787	630	71.709	668	72.983	708	76.756	752	16
17	74.417	631	73.377	669	74.691	709	78.508	752	17
18	76.048	631	75.046	669	76.400	710	80.260	753	18
19	77.679	632	76.715	670	78.110	711	82.013	754	19
20	3679.311	1.633	3778.385	1.671	3879.821	1.712	3983.767	1.755	20
21	80.944	633	80.056	672	81.533	712	85.522	755	21
22	82.577	634	81.728	672	83.245	713	87.277	756	22
23	84.211	634	83.400	673	84.958	714	89.033	757	23
24	85.845	635	85.073	673	86.672	714	90.790	758	24
25	87.480	636	86.746	674	88.386	715	92.548	758	25
26	89.116	636	88.420	675	90.101	715	94.306	759	26
27	90.752	637	90.095	676	91.816	717	96.065	760	27
28	92.389	638	91.771	676	93.533	717	97.825	761	28
29	94.027	638	93.447	677	95.250	717	3999.586	761	29
30	3695.665	1.639	3795.124	1.677	3896.967	1.719	4001.347	1.762	30
31	97.304	639	96.801	678	3898.686	719	03.109	763	31
32	3698.943	640	3798.479	679	3900.405	720	04.872	763	32
33	3700.583	641	3800.158	679	02.125	720	06.635	764	33
34	02.224	642	01.837	680	03.845	721	08.399	765	34
35	03.866	642	03.517	681	05.566	722	10.164	766	35
36	05.508	642	05.198	681	07.288	723	11.930	767	36
37	07.150	643	06.879	682	09.011	723	13.697	767	37
38	08.793	644	08.561	683	10.734	724	15.464	768	38
39	10.437	645	10.244	684	12.458	725	17.232	769	39
40	3712.082	1.645	3811.928	1.684	3914.183	1.726	4019.001	1.769	40
41	13.727	646	13.612	685	15.909	726	20.770	771	41
42	15.373	646	15.297	685	17.635	727	22.541	771	42
43	17.019	647	16.982	686	19.362	728	24.312	772	43
44	18.666	648	18.668	687	21.090	728	26.084	772	44
45	20.314	648	20.355	688	22.818	729	27.856	774	45
46	21.962	649	22.043	688	24.547	730	29.630	774	46
47	23.611	650	23.731	689	26.277	731	31.404	775	47
48	25.261	650	25.420	689	28.008	731	33.179	776	48
49	26.911	651	27.109	690	29.739	732	34.955	776	49
50	3728.562	1.651	3828.799	1.691	3931.471	1.732	4036.731	1.777	50
51	30.213	652	30.490	692	33.203	734	38.508	778	51
52	31.865	653	32.182	692	34.937	734	40.286	779	52
53	33.518	653	33.874	693	36.671	735	42.065	779	53
54	35.171	654	35.567	694	38.406	736	43.844	780	54
55	36.825	655	37.261	694	40.142	736	45.624	781	55
56	38.480	655	38.955	695	41.878	737	47.405	782	56
57	40.135	656	40.650	695	43.615	738	49.187	783	57
58	41.791	656	42.345	696	45.353	738	50.970	783	58
59	43.447	1.658	44.041	1.697	47.091	1.739	52.753	1.784	59
60	3745.105		3845.738		3948.830		4054.537		60

MERCATOR PROJECTION TABLE—Continued.

[Meridional distances for the spheroid. Compression $\frac{1}{294}$.]

Minutes.	56°		57°		58°		59°		Minutes.
	Meridional distance.	Difference.	Meridional distance.	Difference.	Meridional distance.	Difference.	Meridional distance.	Difference.	
	'	'	'	'	'	'	'	'	
0	4054.537	1.784	4163.027	1.833	4274.485	1.884	4389.113	1.939	0
1	56.321	785	64.860	833	76.369	885	91.052	939	1
2	58.106	786	66.693	834	78.254	885	92.991	941	2
3	59.892	787	68.527	836	80.139	887	94.932	941	3
4	61.679	788	70.363	836	82.026	887	96.873	942	4
5	63.467	788	72.199	837	83.913	888	4398.815	944	5
6	65.255	789	74.036	837	85.801	890	4400.759	944	6
7	67.044	790	75.873	839	87.691	890	02.703	945	7
8	68.834	791	77.712	839	89.581	891	04.648	946	8
9	70.625	792	79.551	840	91.472	892	06.594	947	9
10	4072.417	1.793	4181.391	1.841	4293.364	1.892	4408.541	1.948	10
11	74.210	794	83.232	842	95.256	894	10.489	949	11
12	76.004	795	85.074	843	97.150	895	12.438	950	12
13	77.799	795	86.917	844	4299.045	895	14.388	951	13
14	79.594	796	88.761	844	4300.940	896	16.339	952	14
15	81.390	797	90.605	846	02.836	898	18.291	953	15
16	83.187	797	92.451	846	04.734	898	20.244	953	16
17	84.984	799	94.297	847	06.632	899	22.197	955	17
18	86.783	799	96.144	848	08.531	900	24.152	956	18
19	88.582	800	97.992	848	10.431	901	26.108	956	19
20	4090.382	1.800	4199.840	1.850	4312.332	1.901	4428.064	1.958	20
21	92.182	801	4201.690	850	14.233	903	30.022	959	21
22	93.983	802	03.540	851	16.136	904	31.981	959	22
23	95.785	803	05.391	852	18.040	904	33.940	961	23
24	97.588	804	07.243	852	19.944	905	35.901	961	24
25	4099.392	805	09.095	854	21.849	906	37.862	963	25
26	4101.197	805	10.949	854	23.755	908	39.825	963	26
27	03.002	806	12.804	855	25.663	908	41.788	965	27
28	04.808	807	14.659	856	27.571	909	43.753	965	28
29	06.615	808	16.515	857	29.480	909	45.718	966	29
30	4108.423	1.808	4218.372	1.858	4331.389	1.911	4447.684	1.968	30
31	10.231	809	20.230	859	33.300	912	49.652	968	31
32	12.040	810	22.089	860	35.212	913	51.620	969	32
33	13.850	811	23.949	860	37.125	913	53.589	971	33
34	15.661	812	25.809	862	39.038	915	55.560	971	34
35	17.473	812	27.671	862	40.953	915	57.531	972	35
36	19.285	813	29.533	863	42.868	916	59.503	973	36
37	21.098	814	31.396	864	44.784	917	61.476	975	37
38	22.912	815	33.260	865	46.701	918	63.451	975	38
39	24.727	816	35.125	866	48.619	919	65.426	976	39
40	4126.543	1.817	4236.991	1.866	4350.538	1.920	4467.402	1.977	40
41	28.360	817	38.857	867	52.458	921	69.379	978	41
42	30.177	818	40.724	868	54.379	922	71.357	979	42
43	31.995	819	42.592	869	56.301	923	73.336	981	43
44	33.814	820	44.461	870	58.224	924	75.317	981	44
45	35.634	820	46.331	871	60.148	924	77.298	982	45
46	37.454	821	48.202	872	62.072	925	79.280	983	46
47	39.275	822	50.074	872	63.997	927	81.263	984	47
48	41.097	823	51.946	873	65.924	927	83.247	985	48
49	42.920	824	53.819	875	67.851	928	85.232	986	49
50	4144.744	1.825	4255.694	1.875	4369.779	1.930	4487.218	1.987	50
51	46.569	825	57.569	876	71.709	930	89.205	988	51
52	48.394	826	59.445	877	73.639	931	91.193	989	52
53	50.220	827	61.322	878	75.570	932	93.182	990	53
54	52.047	828	63.200	879	77.502	932	95.172	991	54
55	53.875	829	65.079	879	79.434	934	97.163	992	55
56	55.704	830	66.958	881	81.368	935	4499.155	993	56
57	57.534	830	68.839	881	83.303	936	4501.148	994	57
58	59.364	831	70.720	882	85.239	936	03.142	995	58
59	61.195	1.832	72.602	1.883	87.175	1.938	05.137	1.996	59
60	4163.027		4274.485		4389.113		4507.133		60

MERCATOR PROJECTION TABLE—Continued.

[Meridional distances for the spheroid. Compression $\frac{1}{294}$.]

Minutes.	60°		61°		62°		63°		Minutes.
	Meridional distance.	Difference.	Meridional distance.	Difference.	Meridional distance.	Difference.	Meridional distance.	Difference.	
	′	′	′	′	′	′	′	′	
0	4507.133	1.997	4628.789	2.060	4754.350	2.128	4884.117	2.200	0
1	09.130	998	30.849	061	56.478	129	86.317	201	1
2	11.128	1.999	32.910	062	58.607	129	88.518	203	2
3	13.127	2.001	34.972	063	60.736	131	90.721	204	3
4	15.128	001	37.035	064	62.867	133	92.925	205	4
5	17.129	002	39.099	066	65.000	133	95.130	206	5
6	19.131	003	41.165	066	67.133	135	97.336	208	6
7	21.134	005	43.231	068	69.268	135	4899.544	209	7
8	23.139	005	45.299	069	71.403	137	4901.753	211	8
9	25.144	006	47.368	069	73.540	138	03.964	211	9
10	4527.150	2.007	4649.437	2.071	4775.678	2.139	4906.175	2.213	10
11	29.157	009	51.508	072	77.817	141	08.388	215	11
12	31.166	009	53.580	073	79.958	141	10.603	215	12
13	33.175	010	55.653	074	82.099	143	12.818	217	13
14	35.185	012	57.727	075	84.242	144	15.035	218	14
15	37.197	012	59.802	077	86.386	145	17.253	219	15
16	39.209	013	61.879	077	88.531	146	19.472	221	16
17	41.222	015	63.956	079	90.677	148	21.693	222	17
18	43.237	015	66.035	079	92.825	148	23.915	223	18
19	45.252	017	68.114	081	94.973	150	26.138	224	19
20	4547.269	2.017	4670.195	2.082	4797.123	2.151	4928.362	2.226	20
21	49.286	019	72.277	083	4799.274	153	30.588	227	21
22	51.305	019	74.360	084	4801.427	153	32.815	228	22
23	53.324	021	76.444	085	03.580	155	35.043	230	23
24	55.345	022	78.529	086	05.735	156	37.273	231	24
25	57.367	022	80.615	088	07.891	157	39.504	232	25
26	59.389	024	82.703	088	10.048	158	41.736	234	26
27	61.143	025	84.791	090	12.206	160	43.970	234	27
28	63.438	026	86.881	091	14.366	160	46.204	237	28
29	65.464	027	88.972	092	16.526	162	48.441	237	29
30	4567.491	2.028	4691.064	2.093	4818.688	2.163	4950.678	2.239	30
31	69.519	028	93.157	094	20.851	165	52.917	240	31
32	71.547	030	95.251	095	23.016	165	55.157	241	32
33	73.577	032	97.346	097	25.181	167	57.398	243	33
34	75.609	032	4699.443	097	27.348	168	59.641	244	34
35	77.641	033	4701.540	099	29.516	169	61.885	245	35
36	79.674	034	03.639	100	31.685	171	64.130	247	36
37	81.708	035	05.739	101	33.856	171	66.377	248	37
38	83.743	037	07.840	102	36.027	173	68.625	249	38
39	85.780	037	09.942	103	38.200	174	70.874	251	39
40	4587.817	2.039	4712.045	2.104	4840.374	2.176	4973.125	2.252	40
41	89.856	039	14.149	106	42.550	176	75.377	253	41
42	91.895	041	16.255	106	44.726	178	77.630	255	42
43	93.936	042	18.361	108	46.904	179	79.885	256	43
44	95.978	042	20.469	109	49.083	180	82.141	257	44
45	4598.020	044	22.578	110	51.263	182	84.398	259	45
46	4600.064	045	24.688	111	53.445	183	86.657	260	46
47	02.109	046	26.799	113	55.628	184	88.917	261	47
48	04.155	047	28.912	113	57.812	185	91.178	263	48
49	06.202	048	31.025	115	59.997	186	93.441	263	49
50	4608.250	2.049	4733.140	2.116	4862.183	2.188	4995.704	2.266	50
51	10.299	050	35.256	117	64.371	189	4997.970	266	51
52	12.349	051	37.373	118	66.560	190	5000.236	268	52
53	14.400	052	39.491	119	68.750	192	02.504	270	53
54	16.452	054	41.610	121	70.942	192	04.774	271	54
55	18.506	054	43.731	121	73.134	194	07.045	272	55
56	20.560	056	45.852	123	75.328	196	09.317	273	56
57	22.616	056	47.975	124	77.524	196	11.590	275	57
58	24.672	058	50.099	125	79.720	198	13.865	276	58
59	26.730	2.059	52.224	2.126	81.918	2.199	16.141	2.278	59
60	4628.789		4754.350		4884.117		5018.419		60

MERCATOR PROJECTION TABLE—Continued.

[Meridional distances for the spheroid. Compression $\frac{1}{294}$.]

Minutes.	64° Meridional distance.	64° Difference.	65° Meridional distance.	65° Difference.	66° Meridional distance.	66° Difference.	67° Meridional distance.	67° Difference.	Minutes.
	′	′	′	′	′	′	′	′	
0	5018.419	2.279	5157.629	2.364	5302.164	2.457	5452.493	2.558	0
1	20.698	280	59.993	366	04.621	458	55.051	559	1
2	22.978	281	62.359	367	07.079	459	57.610	561	2
3	25.259	283	64.726	368	09.538	461	60.171	563	3
4	27.542	285	67.094	370	11.999	464	62.734	564	4
5	29.827	286	69.464	371	14.463	465	65.298	567	5
6	32.113	287	71.835	373	16.928	466	67.865	568	6
7	34.400	288	74.208	375	19.394	468	70.433	570	7
8	36.688	290	76.583	376	21.862	469	73.003	571	8
9	38.978	291	78.959	378	24.331	471	75.574	574	9
10	5041.269	2.293	5181.337	2.379	5326.802	2.473	5478.148	2.576	10
11	43.562	294	83.716	380	29.275	475	80.724	577	11
12	45.856	295	86.096	382	31.750	476	83.301	579	12
13	48.151	296	88.478	383	34.226	478	85.880	581	13
14	50.447	298	90.861	385	36.704	479	88.461	582	14
15	52.745	300	93.246	386	39.183	481	91.043	584	15
16	55.045	301	95.632	388	41.664	483	93.627	586	16
17	57.346	302	5198.020	390	44.147	484	96.213	588	17
18	59.648	304	5200.410	391	46.631	486	5498.801	589	18
19	61.952	305	02.801	393	49.117	488	5501.390	591	19
20	5064.257	2.306	5205.194	2.394	5351.605	2.489	5503.981	2.592	20
21	66.563	308	07.588	395	54.094	491	06.573	593	21
22	68.871	309	09.983	397	56.585	493	09.166	595	22
23	71.180	311	12.380	399	59.078	494	11.761	597	23
24	73.491	312	14.779	400	61.572	496	14.358	599	24
25	75.803	314	17.179	402	64.068	497	16.957	602	25
26	78.117	315	19.581	403	66.565	499	19.559	603	26
27	80.432	316	21.984	405	69.064	501	22.162	605	27
28	82.748	318	24.389	406	71.565	503	24.767	608	28
29	85.066	320	26.795	408	74.068	504	27.375	610	29
30	5087.386	2.320	5229.203	2.409	5376.572	2.506	5529.985	2.612	30
31	89.706	322	31.612	411	79.078	508	32.597	615	31
32	92.028	323	34.023	412	81.586	509	35.212	617	32
33	94.351	325	36.435	414	84.095	512	37.829	618	33
34	96.676	326	38.849	416	86.607	512	40.447	620	34
35	5099.002	328	41.265	417	89.119	515	43.067	621	35
36	5101.330	329	43.682	419	91.634	516	45.688	624	36
37	03.659	330	46.101	420	94.150	518	48.312	625	37
38	05.989	332	48.521	421	96.668	519	50.937	627	38
39	08.321	334	50.942	424	5399.187	522	53.564	628	39
40	5110.655	2.335	5253.366	2.425	5401.709	2.522	5556.192	2.630	40
41	12.990	336	55.791	426	04.231	525	58.822	632	41
42	15.326	338	58.217	428	06.756	526	61.454	634	42
43	17.664	339	60.645	429	09.282	528	64.088	635	43
44	20.003	341	63.074	431	11.810	530	66.723	637	44
45	22.344	342	65.506	433	14.340	531	69.360	640	45
46	24.686	343	67.938	435	16.871	533	72.000	641	46
47	27.029	345	70.373	436	19.404	535	74.641	643	47
48	29.374	347	72.809	437	21.939	537	77.284	645	48
49	31.721	348	75.246	440	24.476	538	79.929	647	49
50	5134.069	2.350	5277.686	2.440	5427.014	2.540	5582.576	2.649	50
51	36.419	351	80.126	442	29.554	542	85.225	650	51
52	38.770	352	82.568	444	32.096	544	87.875	653	52
53	41.122	354	85.012	445	34.640	545	90.528	654	53
54	43.476	355	87.457	448	37.185	547	93.182	657	54
55	45.831	357	89.905	449	39.732	549	95.839	658	55
56	48.188	358	92.354	449	42.281	551	5598.497	660	56
57	50.546	359	94.803	452	44.832	552	5601.157	662	57
58	52.905	361	97.255	454	47.384	554	03.819	664	58
59	55.266	2.363	5299.709	2.455	49.938	2.555	06.483	2.666	59
60	5157.629		5302.164		5452.493		5609.149		60

MERCATOR PROJECTION TABLE—Continued.

[Meridional distances for the spheroid. Compression $\frac{1}{294}$.]

Minutes.	68° Meridional distance.	68° Difference.	69° Meridional distance.	69° Difference.	70° Meridional distance.	70° Difference.	71° Meridional distance.	71° Difference.	Minutes
	′	′	′	′	′	′	′	′	
0	5609.149	2.668	5772.739	2.789	5943.955	2.923	6123.602	3.071	0
1	11.817	670	75.528	791	46.878	925	26.673	073	1
2	14.487	672	78.319	793	49.803	927	29.746	076	2
3	17.159	673	81.112	795	52.730	929	32.822	078	3
4	19.832	676	83.907	798	55.659	932	35.900	081	4
5	22.508	678	86.705	800	58.591	935	38.981	084	5
6	25.186	679	89.505	801	61.526	937	42.065	086	6
7	27.865	682	92.306	804	64.463	939	45.151	089	7
8	30.547	683	95.110	807	67.402	941	48.240	092	8
9	33.230	685	5797.917	808	70.343	944	51.332	094	9
10	5635.915	2.687	5800.725	2.810	5973.287	2.947	6154.426	3.097	10
11	38.602	690	03.535	813	76.234	948	57.523	099	11
12	41.292	691	06.348	814	79.182	951	60.622	102	12
13	43.983	693	09.162	817	82.133	954	63.724	105	13
14	46.676	695	11.979	819	85.087	956	66.829	108	14
15	49.371	697	14.798	822	88.043	958	69.937	110	15
16	52.068	699	17.620	823	91.001	960	73.047	113	16
17	54.767	701	20.443	826	93.961	964	76.160	115	17
18	57.468	703	23.269	827	96.925	965	79.275	119	18
19	60.171	705	26.096	830	5999.890	968	82.394	120	19
20	5662.876	2.707	5828.926	2.832	6002.858	2.970	6185.514	3.124	20
21	65.583	709	31.758	835	05.828	973	88.638	126	21
22	68.292	711	34.593	836	08.801	975	91.764	129	22
23	71.003	713	37.429	838	11.776	977	94.893	132	23
24	73.716	715	40.267	841	14.753	980	6198.025	134	24
25	76.431	717	43.108	843	17.733	983	6201.159	137	25
26	79.148	719	45.951	846	20.716	985	04.296	140	26
27	81.867	721	48.797	847	23.701	987	07.436	143	27
28	84.588	723	51.644	850	26.688	990	10.579	145	28
29	87.311	725	54.494	852	29.678	992	13.724	148	29
30	5690.036	2.727	5857.346	2.854	6032.670	2.995	6216.872	3.151	30
31	92.763	729	60.200	857	35.665	997	20.023	153	31
32	95.492	731	63.057	858	38.662	2.999	23.176	156	32
33	5698.223	733	65.915	861	41.661	3.003	26.332	159	33
34	5700.956	735	68.776	863	44.664	004	29.491	162	34
35	03.691	738	71.639	866	47.668	007	32.653	165	35
36	06.429	739	74.595	867	50.675	010	35.818	167	36
37	09.168	741	77.372	870	53.685	012	38.985	170	37
38	11.909	743	80.242	872	56.697	015	42.155	173	38
39	14.652	746	83.114	875	59.712	017	45.328	175	39
40	5717.398	2.747	5885.989	2.876	6062.729	3.019	6248.503	3.179	40
41	20.145	749	88.865	879	65.748	022	51.682	181	41
42	22.894	752	91.744	881	68.770	024	54.863	184	42
43	25.646	753	94.625	883	71.794	027	58.047	187	43
44	28.399	756	5897.508	886	74.821	030	61.234	190	44
45	31.155	758	5900.394	888	77.851	032	64.424	192	45
46	33.913	759	03.282	890	80.883	035	67.616	195	46
47	36.672	762	06.172	893	83.918	037	70.811	199	47
48	39.434	764	09.065	895	86.955	040	74.010	201	48
49	42.198	766	11.960	897	89.995	043	77.211	203	49
50	5744.964	2.768	5914.857	2.899	6093.038	3.045	6280.414	3.207	50
51	47.732	770	17.756	902	96.083	047	83.621	210	51
52	50.502	772	20.658	904	6099.130	050	86.831	212	52
53	53.274	775	23.562	906	6102.180	052	90.043	215	53
54	56.049	776	26.468	909	05.232	055	93.258	218	54
55	58.825	779	29.377	911	08.287	058	96.476	221	55
56	61.604	780	32.288	913	11.345	061	6299.697	224	56
57	64.384	783	35.201	916	14.406	063	6302.921	227	57
58	67.167	785	38.117	918	17.469	065	06.148	230	58
59	69.952	2.787	41.035	2.920	20.534	3.068	09.378	3.232	59
60	5772.739		5943.955		6123.602		6312.610		60

MERCATOR PROJECTION TABLE—Continued.

[Meridional distances for the spheroid. Compression $\frac{1}{294}$.]

Minutes.	72° Meridional distance.	72° Difference.	73° Meridional distance.	73° Difference.	74° Meridional distance.	74° Difference.	75° Meridional distance.	75° Difference.	Minutes.
	′	′	′	′	′	′	′	′	
0	6312.610	3.235	6512.071	3.420	6723.275	3.628	6947.761	3.864	0
1	15.845	238	15.491	423	26.903	631	51.625	868	1
2	19.083	242	18.914	426	30.534	635	55.493	872	2
3	22.325	244	22.340	430	34.169	639	59.365	877	3
4	25.569	247	25.770	433	37.808	643	63.242	881	4
5	28.816	250	29.203	437	41.451	646	67.123	885	5
6	32.066	253	32.640	439	45.097	650	71.008	890	6
7	35.319	256	36.079	443	48.747	654	74.898	894	7
8	38.575	259	39.522	446	52.401	658	78.792	898	8
9	41.834	262	42.968	450	56.059	662	82.690	902	9
10	6345.096	3.264	6546.418	3.453	6759.721	3.665	6986.592	3.906	10
11	48.360	267	49.871	456	63.386	669	90.498	911	11
12	51.627	271	53.327	459	67.055	673	94.409	915	12
13	54.898	273	56.786	463	70.728	676	6998.324	919	13
14	58.171	277	60.249	466	74.404	680	7002.243	924	14
15	61.448	279	63.715	470	78.084	684	06.167	928	15
16	64.727	283	67.185	473	81.768	688	10.095	933	16
17	68.010	286	70.658	476	85.456	692	14.028	937	17
18	71.296	288	74.134	480	89.148	696	17.965	941	18
19	74.584	292	77.614	483	92.844	699	21.906	946	19
20	6377.876	3.295	6581.097	3.486	6796.543	3.703	7025.852	3.949	20
21	81.171	297	84.583	490	6800.246	707	29.801	954	21
22	84.468	300	88.073	493	03.953	711	33.755	959	22
23	87.768	304	91.566	497	07.663	715	37.714	963	23
24	91.072	307	95.063	500	11.378	718	41.677	968	24
25	94.379	310	6598.563	504	15.096	722	45.645	972	25
26	6397.689	313	6602.067	507	18.818	727	49.617	977	26
27	6401.002	315	05.574	510	22.545	730	53.594	981	27
28	04.317	319	09.084	514	26.275	734	57.575	985	28
29	07.636	322	12.598	518	30.009	738	61.561	990	29
30	6410.958	3.325	6616.116	3.520	6833.747	3.742	7065.551	3.994	30
31	14.283	328	19.636	524	37.489	747	69.545	3.999	31
32	17.611	331	23.160	528	41.236	750	73.544	4.003	32
33	20.942	334	26.688	531	44.986	754	77.547	008	33
34	24.276	337	30.219	535	48.740	758	81.555	013	34
35	27.613	341	33.754	538	52.498	762	85.568	017	35
36	30.954	344	37.292	541	56.260	766	89.585	022	36
37	34.298	347	40.833	545	60.027	770	93.607	026	37
38	37.645	350	44.378	549	63.797	774	7097.633	031	38
39	40.995	353	47.927	552	67.571	778	7101.664	035	39
40	6444.348	3.356	6651.479	3.556	6871.349	3.782	7105.699	4.040	40
41	47.704	359	55.035	559	75.131	785	09.739	045	41
42	51.063	362	58.594	563	78.916	790	13.784	049	42
43	54.425	365	62.157	566	82.706	794	17.833	054	43
44	57.790	369	65.723	570	86.500	798	21.887	059	44
45	61.159	372	69.293	573	90.298	802	25.946	063	45
46	64.531	375	72.866	577	94.100	806	30.009	068	46
47	67.906	378	76.443	581	6897.906	810	34.077	072	47
48	71.284	381	80.024	585	6901.716	815	38.149	077	48
49	74.665	385	83.609	588	05.531	819	42.226	082	49
50	6478.050	3.387	6687.197	3.591	6909.350	3.822	7146.308	4.086	50
51	81.437	391	90.788	595	13.172	826	50.394	091	51
52	84.828	394	94.383	599	16.998	831	54.485	096	52
53	88.222	397	6697.982	602	20.829	835	58.581	101	53
54	91.619	401	6701.584	606	24.664	839	62.682	105	54
55	95.020	404	05.190	610	28.503	843	66.787	110	55
56	6498.424	407	08.800	613	32.346	847	70.897	115	56
57	6501.831	410	12.413	617	36.193	852	75.012	120	57
58	05.241	413	16.030	621	40.045	856	79.132	125	58
59	08.654	3.417	19.651	3.625	43.901	3.860	83.257	4.130	59
60	6512.071		6723.275		6947.761		7187.387		60

MERCATOR PROJECTION TABLE—Continued.

[Meridional distances for the spheriod. Compression $\frac{1}{294}$.]

Minutes.	76° Meridional distance.	76° Difference.	77° Meridional distance.	77° Difference.	78° Meridional distance.	78° Difference.	79° Meridional distance.	79° Difference.	Minutes.
	′	′	′	′	′	′	′	′	
0	7187. 387	4. 134	7444. 428	4. 447	7721. 700	4. 811	8022. 758	5. 243	0
1	91. 521	139	48. 875	452	26. 511	818	28. 001	251	1
2	95. 660	144	53. 327	458	31. 329	825	33. 252	259	2
3	7199. 804	149	57. 785	463	36. 154	831	38. 511	267	3
4	7203. 953	154	62. 248	469	40. 985	838	43. 778	275	4
5	08. 107	159	66. 717	475	45. 823	845	49. 053	283	5
6	12. 266	163	71. 192	481	50. 668	852	54. 336	292	6
7	16. 429	169	75. 673	487	55. 520	858	59. 628	299	7
8	20. 598	174	80. 160	492	60. 378	865	64. 927	307	8
9	24. 772	178	84. 652	498	65. 243	872	70. 234	316	9
10	7228. 950	4. 183	7489. 150	4. 504	7770. 115	4. 878	8075. 550	5. 323	10
11	33. 133	188	93. 654	509	74. 993	885	80. 873	330	11
12	37. 321	193	7498. 163	515	79. 878	892	86. 203	339	12
13	41. 514	198	7502. 678	521	84. 770	899	91. 542	348	13
14	45. 712	203	07. 199	527	89. 669	906	8096. 890	356	14
15	49. 915	208	11. 726	532	94. 575	912	8102. 246	364	15
16	54. 123	213	16. 258	539	7799. 487	920	07. 610	373	16
17	58. 336	219	20. 797	544	7804. 407	927	12. 983	381	17
18	62. 555	223	25. 341	550	09. 334	933	18. 364	389	18
19	66. 778	229	29. 891	556	14. 267	941	23. 753	397	19
20	7271. 007	4. 233	7534. 447	4. 561	7819. 208	4. 947	8129. 150	5. 405	20
21	75. 240	238	39. 008	567	24. 155	954	34. 555	414	21
22	79. 478	243	43. 575	574	29. 109	961	39. 969	422	22
23	83. 721	249	48. 149	579	34. 070	968	45. 391	430	23
24	87. 970	254	52. 728	585	39. 038	975	50. 821	439	24
25	92. 224	258	57. 313	592	44. 013	983	56. 260	448	25
26	7296. 482	265	61. 905	597	48. 996	990	61. 708	457	26
27	7300. 747	269	66. 502	604	53. 986	4. 997	67. 165	465	27
28	05. 016	274	71. 106	610	58. 983	5. 004	72. 630	474	28
29	09. 290	280	75. 716	616	63. 987	011	78. 104	482	29
30	7313. 570	4. 284	7580. 332	4. 621	7868. 998	5. 018	8183. 586	5. 490	30
31	17. 854	290	84. 953	628	74. 016	025	89. 076	499	31
32	22. 144	295	89. 581	634	79. 041	032	8194. 575	507	32
33	26. 439	300	94. 215	640	84. 073	040	8200. 082	516	33
34	30. 739	306	7598. 855	647	89. 113	047	05. 598	525	34
35	35. 045	311	7603. 502	652	94. 160	054	11. 123	534	35
36	39. 356	316	08. 154	659	7899. 214	062	16. 657	543	36
37	43. 672	322	12. 813	665	7904. 276	069	22. 200	552	37
38	47. 994	327	17. 478	671	09. 345	076	27. 752	561	38
39	52. 321	332	22. 149	678	14. 421	084	33. 313	570	39
40	7356. 653	4. 337	7626. 827	4. 683	7919. 505	5. 091	8238. 883	5. 578	40
41	60. 990	342	31. 510	689	24. 596	098	44. 461	586	41
42	65. 332	348	36. 199	696	29. 694	105	50. 047	595	42
43	69. 680	353	40. 895	702	34. 799	113	55. 642	605	43
44	74. 033	359	45. 597	708	39. 912	121	61. 247	614	44
45	78. 392	364	50. 305	715	45. 033	128	66. 861	623	45
46	82. 756	370	55. 020	721	50. 161	136	72. 484	633	46
47	87. 126	375	59. 741	728	55. 297	144	78. 117	642	47
48	91. 501	381	64. 469	734	60. 441	151	83. 759	650	48
49	7395. 882	386	69. 203	740	65. 592	159	89. 409	660	49
50	7400. 268	4. 391	7673. 943	4. 746	7970. 751	5. 166	8295. 069	5. 668	50
51	04. 659	396	78. 689	753	75. 917	173	8300. 737	677	51
52	09. 055	402	83. 442	759	81. 090	181	06. 414	687	52
53	13. 457	408	88. 201	766	86. 271	189	12. 101	697	53
54	17. 865	413	92. 967	773	91. 460	196	17. 798	706	54
55	22. 278	419	7697. 740	779	7996. 656	205	23. 504	715	55
56	26. 697	424	7702. 519	785	8001. 861	213	29. 219	725	56
57	31. 121	430	07. 304	792	07. 074	220	34. 944	734	57
58	35. 551	436	12. 096	799	12. 294	228	40. 678	744	58
59	39. 987	4. 441	16. 895	4. 805	17. 522	5. 236	46. 422	5. 754	59
60	7444. 428		7721. 700		8022. 758		8352. 176		60

FIXING POSITION BY WIRELESS DIRECTIONAL BEARINGS[34]

A very close approximation for plotting on a Mercator chart the position of a ship receiving wireless bearings is given in Admiralty Notice to Mariners, No. 952, June 19, 1920, as follows:

I.—GENERAL

Fixing position by directional wireless is very similar to fixing by cross bearings from visible objects, the principal difference being that, when using a chart on the Mercator projection allowance has to be made for the curvature of the earth, the wireless stations being generally at very much greater distances than the objects used in an ordinary cross bearing fix.

Although fixing position by wireless directional bearings is dependent for its accuracy upon the degree of precision with which it is at present possible to determine the direction of wireless waves, confirmation of the course and distance made good by the receipt of additional bearings, would afford confidence to those responsible in the vessel as the land is approached under weather conditions that preclude the employment of other methods.

At the present time, from shore stations with practiced operators and instruments in good adjustment, the maximum error in direction should not exceed 2° for day working, but it is to be noted that errors at night may be larger, although sufficient data on this point are not at present available.

II.—TRACK OF WIRELESS WAVE

The track of a wireless wave being a great circle is represented on a chart on the Mercator projection by a flat curve, concave toward the Equator; this flat curve is most curved when it runs in an east and west direction and flattens out as the bearing changes toward north and south. When exactly north and south it is quite flat and is then a straight line, the meridian. The true bearing of a ship from a wireless telegraph (W/T) station, or vice versa, is the angle contained by the great circle passing through either position and its respective meridian.

III.—CONVERGENCY

Meridians on the earth's surface not being parallel but converging at the poles, it follows that a great circle will intersect meridians as it crosses them at a varying angle unless the great circle itself passes through the poles and becomes a meridian. The difference in the angles formed by the intersection of a great circle with two meridians (that is, convergency) depends on the angle the great circle makes with the meridian, the middle latitude between the meridians, and the difference of longitude between the meridians.

This difference is known as the convergency and can be approximately calculated from the formula—

$$\text{Convergency in minutes} = \text{diff. long. in minutes} \times \sin \text{mid. lat.}$$

Convergency may be readily found from the convergency scale (see fig. 65), or it may be found by traverse table entering the diff. long. as distance and mid. lat. as course; the resulting departure being the convergency in minutes.

[34] A valuable contribution to this subject by G. W. Littlehales, appeared in the Journal of the American Society of Naval Engineers, February, 1920 under the title: "The Prospective Utilization of Vessel-to-Shore Radiocompass Bearings in Aerial and Transoceanic Navigation."

Our attention has been called to a diagram on Pilot Chart No. 1400, February, 1921, entitled "Position Plotting by Radio Bearings" by Elmer B. Collins, nautical expert, U. S. Hydrographic Office. On this diagram there is given a method of fixing the position of a vessel on a Mercator chart both by plotting and by computation.

The Admiralty uses dead-reckoning position for preliminary fix whereas by the Hydrographic Office method the preliminary fix is obtained by laying the radiocompass bearings on the Mercator chart. The Hydrographic Office also gives a method of computation wherein the radiocompass bearings are used in a manner very similar to Sumner lines.

See also the paragraph *wireless directional bearings* under the chapter *Gnomonic Projection*, p. 149.

IV.—TRUE AND MERCATORIAL BEARINGS

Meridians on a Mercator chart being represented by parallel lines, it follows that the *true bearing* of the ship from the station, or vice versa, can not be represented by a straight line joining the two positions, the straight line joining them being the *mean mercatorial bearing*, which differs from the true bearing by ±½ the convergency. As it is this mean mercatorial bearing which we require, all that is necessary when the true bearing is obtained from a W/T station is to add to or subtract from it ½ the convergency and lay off this bearing from the station.

NOTE.—Charts on the gnomonic projection which facilitate the plotting of true bearings are now in course of preparation by the Admiralty and the U. S. Hydrographic Office.

V.—SIGN OF THE ½ CONVERGENCY

Provided the bearings are always measured in degrees north 0° to 360° (clockwise) the sign of this ½ convergency can be simply determined as follows:

N. lat ---------- ½ convergency is + to the bearing given by the W/T station when ship is E. of station.

N. lat ---------- ½ convergency is − to the bearing given by the W/T station when ship is W. of station.

S. lat ---------- The opposite.

When the W/T station and the ship are on opposite sides of the Equator, the factor sin mid. lat. is necessarily very small and the convergency is then negligible. All great circles in the neighborhood of the Equator appear on the chart as straight lines and the convergency correction as described above is immaterial and unnecessary.

VI.—EXAMPLE

A ship is by D. R.[35] in lat. 48° 45′ N., long. 25° 30′ W., and obtains wireless bearings from Sea View 244¾° and from Ushant 277½°. What is her position?

Sea View	Lat. 55° 22′ N.	Long. 7° 19½′ W.
D. R.	Lat. 48° 45′ N.	Long. 25° 30′ W.
Mid. lat	52° 03′ N.	Diff. long. 1090.′5

Convergency = 1090.5 × sin 52° = 859′,
or ½ convergency = 7° 09′

The true bearing signaled by Sea View was 244¾°; as ship is west of the station (north lat., see Par. V) the ½ convergency will be "minus" to the true bearing signaled.

Therefore the mercatorial bearing will be 237½° nearly.

Similarly with Ushant.

Lat. D. R.	48° 45′ N.	Long. 25° 30′ W.
Lat. Ushant	48° 26½′ N.	Long. 5° 05½′ W.
Mid. lat	48° 36′ N.	Diff. long. 1224.′5 W.

Convergency = 1224.5 × sin 48° 36′ = 919′,
or ½ convergency = 7° 40′

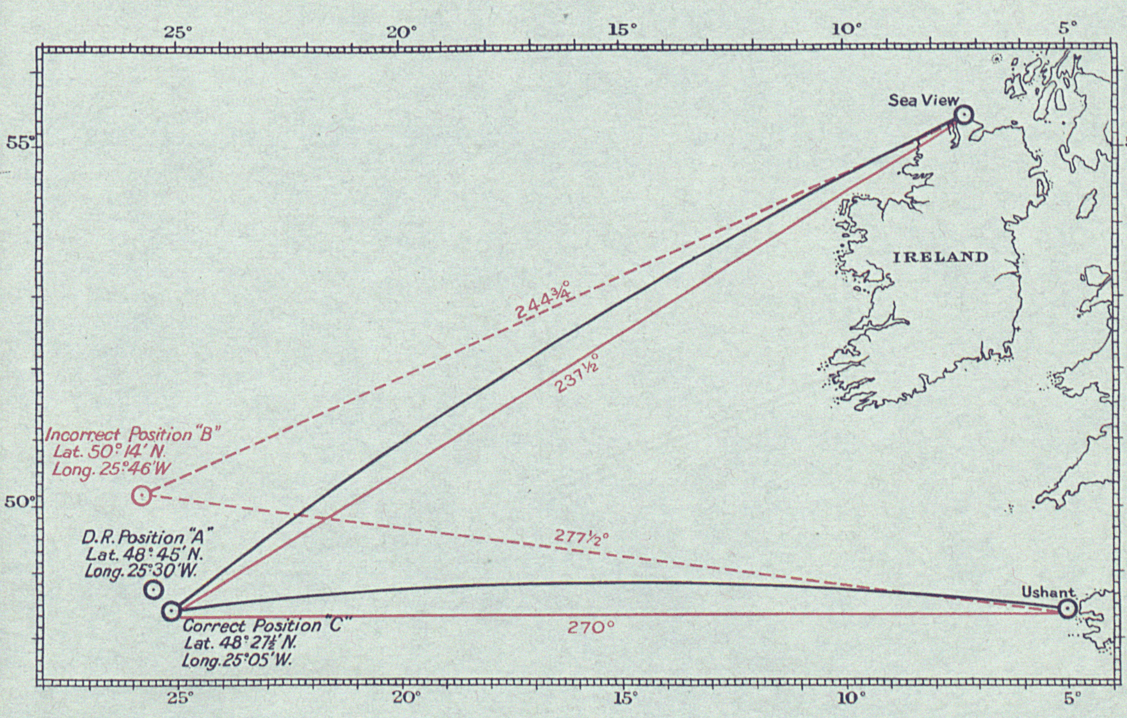
The true bearing signaled by Ushant was 277½°; as ship is west of the station (north lat., see Par. V) the ½ convergency will be "minus" to the true bearing signaled. Therefore the mercatorial bearing will be 270° nearly.

Laying off 237½° and 270° on the chart from Sea View and Ushant, respectively, the intersection will be in:

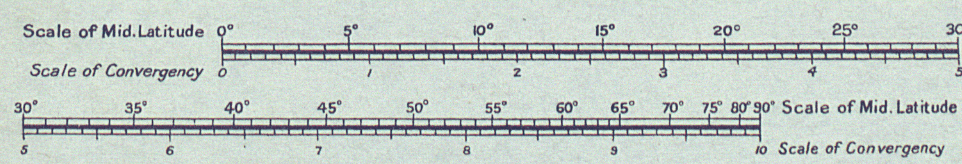
Lat. 48° 27½′ N., long. 25° 05′ W., which is the ship's position.

NOTE.—In plotting the positions the largest scale chart available that embraces the area should be used. A station pointer will be found convenient for laying off the bearings where the distances are great.

[35] Dead reckoning.



Scales for obtaining the Convergency for 10' Diff. Longitude in different Latitudes.



Example:- *Mid. Lat. 50°30', diff. long. 282'; To find the Convergency.*
Under 50°30', on Mid. Lat. scale read 7.7 on scale of Convergency
which multiplied by 28.2 gives 217' the Convergency.

Fig 65

C. & G. S. Print – 99 (20) ++

The accompanying chartlet (see Fig. 65), drawn on the Mercator projection, shows the above positions and the error involved by laying off the true bearings as signaled from Sea View W/T station and Ushant W/T station.

The black curved lines are the great circles passing through Sea View and ship's position, and Ushant and ship's position.

The red broken lines are the true bearings laid off as signaled, their intersection "B" being in latitude 50° 14′ N., longitude 25° 46′ W., or approximately 110′ from the correct position.

The red firm lines are the mean mercatorial bearings laid off from Sea View and Ushant and their intersection "C" gives the ship's position very nearly; that is, latitude 48° 27½′ N., longitude 25° 05′W.

Position "A" is the ship's D. R. position, latitude 48° 45′ N., longitude 25° 30′ W., which was used for calculating the ½ convergency.

NOTE.—As the true position of the ship should have been used to obtain the ½ convergency, the quantity found is not correct, but it could be recalculated using lat. and long. "C" and a more correct value found. This, however, is only necessary if the error in the ship's assumed position is very great.

VII.—ACCURACY OF THIS METHOD OF PLOTTING

Although this method is not rigidly accurate, it can be used for all practical purposes up to 1,000 miles range, and a very close approximation found to the lines of position on which the ship is, at the moment the stations receive her signals.

VIII.—USE OF W/T BEARINGS WITH OBSERVATIONS OF HEAVENLY BODIES

It follows that W/T bearings may be used in conjunction with position lines obtained from observations of heavenly bodies, the position lines from the latter being laid off as straight lines (although in this case also they are not strictly so), due consideration being given to the possible error of the W/T bearings. Moreover, W/T bearings can be made use of at short distances as "position lines," in a similar manner to the so-called "Sumner line" when approaching port, making the land, avoiding dangers, etc.

IX.—CONVERSE METHOD

When ships are fitted with apparatus by which they record the wireless bearings of shore stations whose positions are known, the same procedure for laying off bearings from the shore stations can be adopted, but it is to be remembered that in applying the ½ convergency to these bearings it must be applied in the converse way, in both hemispheres to that laid down in paragraph V.

A new presentation of the problem of conversion of radio bearings to mercatorial bearings with simplified tables is given in Association of Field Engineers, U. S. Coast and Geodetic Survey Bulletin, June, 1932, pages 101–108, by A. L. Shalowitz, cartographic engineer.

THE GNOMONIC PROJECTION

DESCRIPTION

[See Plate IV and table on p. 214.]

Of the many projections in common use today, it is plausible that the origin of the gnomonic projection dates farthest back into antiquity. It has been ascribed to the period of Thales, *ca.* B. C. 550, and has been much employed to outline the trace of celestial phenomena on the surface of the earth.

The gnomonic projection of the sphere is a perspective projection upon a tangent plane, with the point from which the projecting lines are drawn situated at the center of the sphere. This may also be stated as follows:

The eye of the spectator is supposed to be situated at the center of the terrestrial sphere, *from whence, being at once in the plane of every great circle,* it will see these circles projected as straight lines where the visual rays passing through them intersect the plane of projection. A straight line drawn between any two points or places on this chart represents an arc of the great circle passing through them, and is, therefore, the shortest possible *track line* between them and shows at once all the geographical localities through which the most direct route passes.

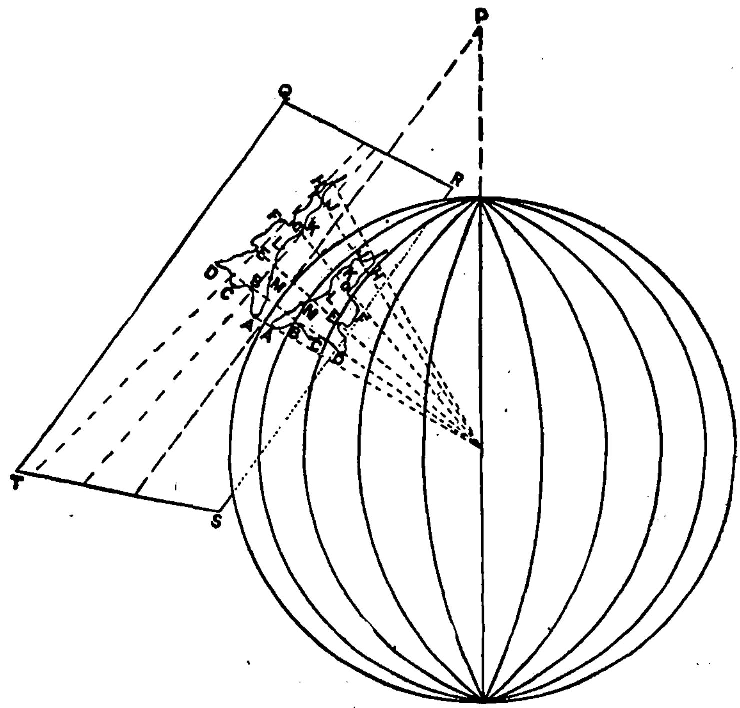


FIG. 66.—*Diagram illustrating the theory of the gnomonic projection.*

The four-sided figure QRST is the imaginary paper forming a "tangent plane," which touches the surface of the globe on the central meridian of the chart. The N.-S. axis of the globe is conceived as produced to a point P on which all meridians converge. Where imaginary lines drawn from the center of the earth through points on its surface fall on the tangent plane, these points can be plotted. The tangent paper being viewed in the figure from underneath, the outline of the island is reversed as in a looking glass; if the paper were transparent, the outline, when seen from the further side (the chart side) would be in its natural relation.—From Charts: Their Use and Meaning, by G. Herbert Fowler, Ph. D., University College, London.

Obviously a complete hemisphere can not be constructed on this plan, since, for points 90° distant from the center of the map, the projecting lines are parallel to the plane of projection. As the distance of the projected point from the center of the map approaches 90° the projecting line approaches a position of parallelism to the plane of projection and the intersection of line and plane recedes indefinitely from the center of the map.

The chief fault of the projection and the one which is incident to its nature is that while those positions of the sphere opposite to the eye are projected in approximately their true relations, those near the boundaries of the map are very much distorted and the projection is useless for distances, areas, and shapes.

The one special property, however, that any great circle on the sphere is represented by a straight line upon the map, has brought the gnomonic projection into considerable prominence. For the purpose of facilitating great-circle sailing the Hydrographic

Office, U. S. Navy, and the British Admiralty have issued gnomonic charts covering in single sheets the North Atlantic, South Atlantic, Pacific, North Pacific, South Pacific, and Indian Oceans.

This system of mapping is now frequently employed by the Admiralty on plans of harbors, polar charts, etc. Generally, however, the area is so small that the difference in projections is hardly apparent and the charts might as well be treated as if they were on the Mercator projection.

The use and application of gnomonic charts as supplementary in laying out ocean sailing routes on the Mercator charts have already been noted in the chapter on the Mercator projection. In the absence of charts on the gnomonic projection, great-circle courses may be placed upon Mercator charts either by computation or by the use of tables, such as Lecky's General Utility Tables. It is far easier and quicker, however, to derive these from the gnomonic chart, because the route marked out on it will show at a glance if any obstruction, as an island or danger, necessitates a modified or composite course.

WIRELESS DIRECTIONAL BEARINGS

The gnomonic projection is by its special properties especially adapted to the plotting of positions from wireless directional bearings.

Observed directions may be plotted by means of a protractor, or compass rose, constructed at each radiocompass station. The center of the rose is at the radio station, and the true azimuths indicated by it are the traces on the plane of the projection of the planes of corresponding true directions at the radio station.

MATHEMATICAL THEORY OF THE GNOMONIC PROJECTION

A simple development of the mathematical theory of the projection will be given with sufficient completeness to enable one to compute the necessary elements.

In figure 67, let $PQP'Q'$ represent the meridian on which the point of tangency lies; let ACB be the trace of the tangent plane with the point of tangency at C; and let the radius of the sphere be represented by R; let the angle COD be denoted by p; then, $CD = OC \tan COD = R \tan p$

All points of the sphere at arc distance p from C will be represented on the projection by a circle with radius equal to CD, or

$$\rho = R \tan p.$$

To reduce this expression to rectangular coordinates, let us suppose the circle drawn on the plane of the projection. In figure 68, let YY' represent the projection of the central meridian and XX' that of the great circle through C (see fig. 67) perpendicular to the central meridian.

If the angle XOF is denoted by ω, we have

$$x = \rho \cos \omega = R \tan p \cos \omega$$

$$y = \rho \sin \omega = R \tan p \sin \omega:$$

or,

$$x = \frac{R \sin p \cos \omega}{\cos p}$$

$$y = \frac{R \sin p \sin \omega}{\cos p}.$$

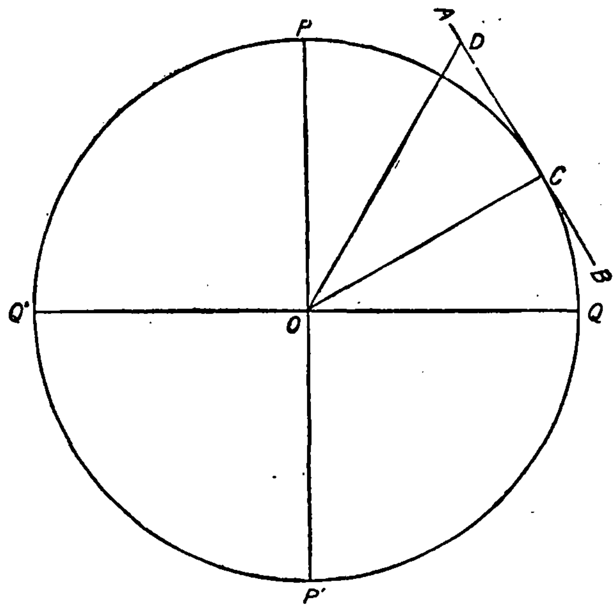


FIG. 67.—Gnomonic projection—determination of the radial distance.

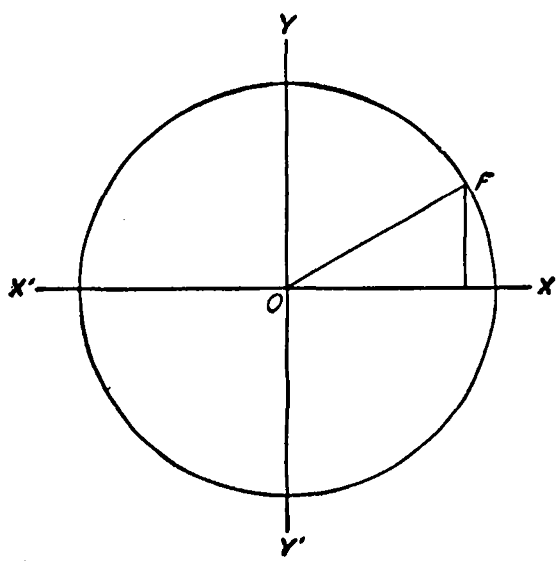


FIG. 68.—Gnomonic projection—determination of the coordinates on the mapping plane.

Now, suppose the plane is tangent to the sphere at latitude α. The expression just given for x and y must be expressed in terms of latitude and longitude, or ϕ and λ, λ representing, as usual, the longitude reckoned from the central meridian.

In figure 69, let T be the pole, Q the center of the projection, and let P be the point whose coordinates are to be determined.

The angles between great circles at the point of tangency are preserved in the projection so that ω is the angle between QP and the great circle perpendicular to TQ at Q;

or,

$$\angle TQP=\frac{\pi}{2}-\omega.$$

Also,

$$TQ=\frac{\pi}{2}-\alpha.$$

$$TP=\frac{\pi}{2}-\phi,$$

$$QP=p,$$

and,

$$\angle QTP=\lambda$$

From the trigonometry of the spherical triangle we have

$$\cos p=\sin\alpha\sin\phi+\cos\alpha\cos\lambda\cos\phi,$$

$$\frac{\sin p}{\cos\phi}=\frac{\sin\lambda}{\cos\omega},\text{ or }\sin p\cos\omega=\sin\lambda\cos\phi,$$

and

$$\sin p\sin\omega=\cos\alpha\sin\phi-\sin\alpha\cos\lambda\cos\phi.$$

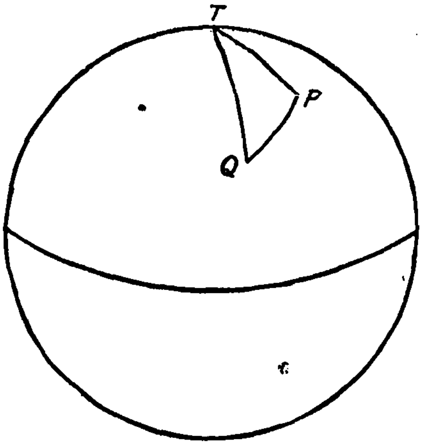


FIG. 69.—Gnomonic projection—transformation triangle on the sphere.

On the substitution of these values in the expressions for x and y, we obtain as definitions of the coordinates of the projection—

$$x=\frac{R\sin\lambda\cos\phi}{\sin\alpha\sin\phi+\cos\alpha\cos\lambda\cos\phi},$$

$$y=\frac{R(\cos\alpha\sin\phi-\sin\alpha\cos\lambda\cos\phi)}{\sin\alpha\sin\phi+\cos\alpha\cos\lambda\cos\phi}.$$

The Y axis is the projection of the central meridian and the X axis is the projection of the great circle through the point of tangency and perpendicular to the central meridian.

These expressions are very unsatisfactory for logarithmic computation purposes. To put them in more convenient form, we may transform them in the following manner:

$$x=\frac{R \sin \lambda \cos \phi}{\sin \alpha\ (\sin \phi+\cos \phi \cot \alpha \cos \lambda)}$$

$$y=\frac{R \cos \alpha\ (\sin \phi-\cos \phi \tan \alpha \cos \lambda)}{\sin \alpha\ (\sin \phi+\cos \phi \cot \alpha \cos \lambda)}.$$

Let

$$\cot \beta=\cot \alpha \cos \lambda,$$

$$\tan \gamma=\tan \alpha \cos \lambda,$$

then

$$x=\frac{R \sin \lambda \cos \phi}{\frac{\sin \alpha}{\sin \beta}\ (\sin \phi \sin \beta+\cos \phi \cos \beta)}$$

$$y=\frac{\frac{R \cos \alpha}{\cos \gamma}\ (\sin \phi \cos \gamma-\cos \phi \sin \gamma)}{\frac{\sin \alpha}{\sin \beta}\ (\sin \phi \sin \beta+\cos \phi \cos \beta)}.$$

But

$$\cos (\phi-\beta)=\sin \phi \sin \beta+\cos \phi \cos \beta,$$

and

$$\sin (\phi-\gamma)=\sin \phi \cos \gamma-\cos \phi \sin \gamma.$$

Hence

$$x=\frac{R \sin \beta \sin \lambda \cos \phi}{\sin \alpha \cos (\phi-\beta)}$$

$$y=\frac{R \cot \alpha \sin \beta \sin (\phi-\gamma).}{\cos \gamma \cos (\phi-\beta)}$$

These expressions are in very convenient form for logarithmic computation. For any given meridian β and γ are constants; hence the coordinates of intersection along a meridian are very easily computed. It is known, *a priori*, that the meridians are represented by straight lines; hence to draw a meridian we need to know the coordinates of only two points. These should be computed as far apart as possible, one near the top and the other near the bottom of the map. After the meridian is drawn on the projection it is sufficient to compute only the y coordinate of the other intersections. If the map extends far enough to include the pole, the determination of this point will give one point on all of the meridians.

Since for this point $\lambda=0$ and $\phi=\frac{\pi}{2}$, we get

$$\beta=\alpha,$$
$$\gamma=\alpha,$$
$$x=0,$$
$$y=R \cot \alpha.$$

If this point is plotted upon the projection and another point on each meridian is determined near the bottom of the map, the meridians can be drawn on the projection.

If the map is extensive enough to include the Equator, the intersections of the straight line which represents it, with the meridians can be easily computed. When $\phi=0$, the expressions for the coordinates become

$$x=R \tan \lambda \sec \alpha$$

$$y=-R \tan \alpha$$

A line parallel to the X axis at the distance $y=-R \tan \alpha$ represents the Equator. The intersection of the meridian λ with this line is given by

$$x=R \tan \lambda \sec \alpha$$

When the Equator and the pole are both on the map, the meridians may thus be determined in a very simple manner. The parallels may then be determined by computing the y coordinate of the various intersections with these straight-line meridians.

If the point of tangency is at the pole, $\alpha=\frac{\pi}{2}$ and the expressions for the coordinates become

$$x=R \cot \phi \sin \lambda$$

$$y=-R \cot \phi \cos \lambda$$

In these expressions λ is reckoned from the central meridian from south to east. As usually given, λ is reckoned from the east point to northward. Letting $\lambda=\frac{\pi}{2}+\lambda'$ and dropping the prime, we obtain the usual forms:

$$x=R \cot \phi \cos \lambda$$

$$y=R \cot \phi \sin \lambda$$

The parallels are represented by concentric circles each with the radius

$$\rho=R \cot \phi$$

The meridians are represented by the equally spaced radii of this system of circles.

If the point of tangency is on the Equator, $\alpha=0$, and the expressions become

$$x=R \tan \lambda$$

$$y=R \tan \phi \sec \lambda$$

The meridians in this case are represented by straight lines perpendicular to the X axis and parallel to the Y axis. The distance of the meridian λ from the origin is given by $x=R \tan \lambda$.

Any gnomonic projection is symmetrical with respect to the central meridian or to the Y axis, so that the computation of the projection on one side of this axis is sufficient for the complete construction. When the point of tangency is at the pole, or on the Equator, the projection is symmetrical both with respect to the Y axis and to the X axis. It is sufficient in either of these cases to compute the intersections for a single quadrant.

Another method for the construction of a gnomonic chart is given in the Admiralty Manual of Navigation, 1915, pages 31 to 38.

WORLD MAPS

HISTORICAL NOTE

When we contemplate the splendors and glories of ancient Greece, we should not forget its contributions to cartographic science and utilitarian needs. We should at least mention Hipparchus (*ca.* B. C. 160–125) as one of the first great contributors to the world, in its mathematical and geographical control. He applied astronomic methods to mark the positions of places on the earth's surface, and is recorded as having invented trigonometry and as having devised the stereographic and orthographic projections for maps.

In mathematical cartography he has, therefore, a direct appeal as one of the landmarks of the science, and the one person who gave us the first solution for the development of the earth's surface upon a plane in a true picture which we all can read.

His stereographic projection like the Parthenon stands out as a monument to all ages and is used at the present day in atlases and geographics as the only *conformal* representation of the hemispheres.

With no intent to present a general review of the history of cartography and the decline of this science during the middle ages, we may state that no developments of monumental importance were achieved until the reform of cartography by Gerhard Mercator, dating from his equal-area map of the world in 1538. Then followed his map of Europe in 1554, and his nautical chart of 1569 in which he added a new meaning to the map projection.

Even if the underlying principle of the new system of navigation was not entirely his own, he is the one person who effectively pronounced it to the world "like some watcher of the skies when a new planet swims into his ken."

In the representation of moderate areas no great difficulties are encountered, but any attempt to map the world in one continuous sheet presents difficulties that are almost insurmountable.

For conformal mapping of the world the Mercator projection, for many purposes, is as good as any, in that it gives a definite measure of its faults in the border scale; for equal-area mapping, Professor Goode's interrupted homalographic projection accomplishes a great deal toward the solution of a most difficult problem.

As stated concisely by Prof. Hinks, "the problem of showing the sphere on a single sheet is intractable," and it is not the purpose of the authors to enter this field to any greater extent than to present a few of the systems of projection that have at least some measure of merit. The ones presented are either conformal or equal-area projections.

MERCATOR PROJECTION

This projection, already described in detail on pages 103–124 was primarily designed for the construction of nautical charts, and in this field has attained an importance beyond all others.

While other projections may contribute their portion in special properties from an educational standpoint, they cannot entirely displace the Mercator projection which has stood the test for over three and a half centuries. It is the opinion of the authors that the Mercator projection not only is a fixture for nautical charts, but that it plays a definite part in giving us a continuous conformal mapping of the world. Its use, however, for world maps has brought forth continual criticism in that the projection is responsible for many false impressions of the relative size of countries differing in latitude.

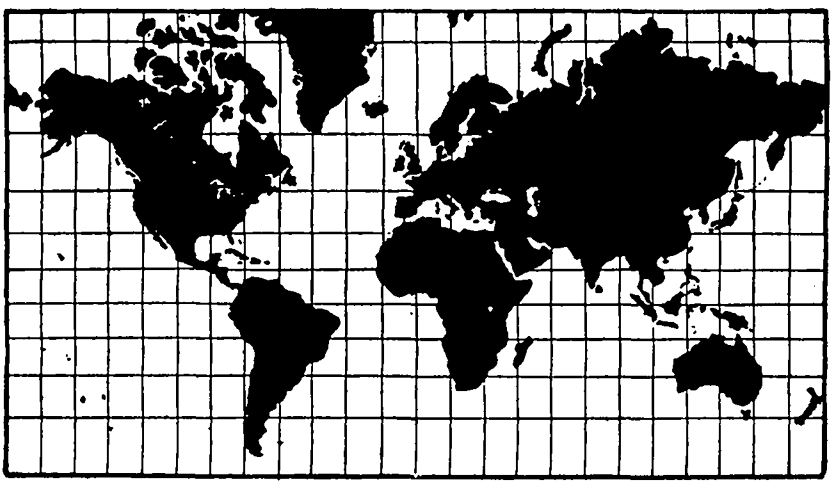


FIG. 70.—Mercator projection, from latitude 60° south, to latitude 78° north.

However, let us not overlook the inadequacies and frequent impossibilities in other systems of projection when extended to world proportions. The restrictions of relative size in the Mercator projection may be more or less disturbing, but likewise so are the violent interruptions of continental alignments with curves running unnaturally as they do in many other projections. In the many pros and cons for simplicity and continuity in mapping the world in one sheet, the Mercator projection still prevails as one that can seriously be worked upon and not as one merely to be looked at.

As proof of the impossibilities of a Mercator projection in world maps, the critics invariably cite the exaggeration of Greenland and the polar regions. In the consideration of the various evils of world maps, the polar regions are, after all, the best places to put the maximum distortion. Generally, our interests are centered between 65° north and 55° south latitude, and it is in this belt that other projections present difficulties in spherical relations which in many instances are not readily expressed in analytic terms.

Beyond these limits a circumpolar chart like the one issued by the Hydrographic Office, U. S. Navy, No. 2560, may be employed. Polar charts can be drawn on the gnomonic projection, the point of contact between plane and sphere being at the pole. In practice, however, they are generally drawn, not as true gnomonic projections, but as polar equidistant projections, the meridians radiating as straight lines from the pole, the parallels struck as concentric circles from the pole, with all degrees of latitude of equal length at all parts of the chart.

However, for the general purposes of a circumpolar chart from latitude 60° to the pole, the polar stereographic projection or the Lambert conformal with two standard parallels would be preferable. In the latter projection the 360 degrees of longitude would not be mapped within a circle, but on a sector greater than a semicircle.

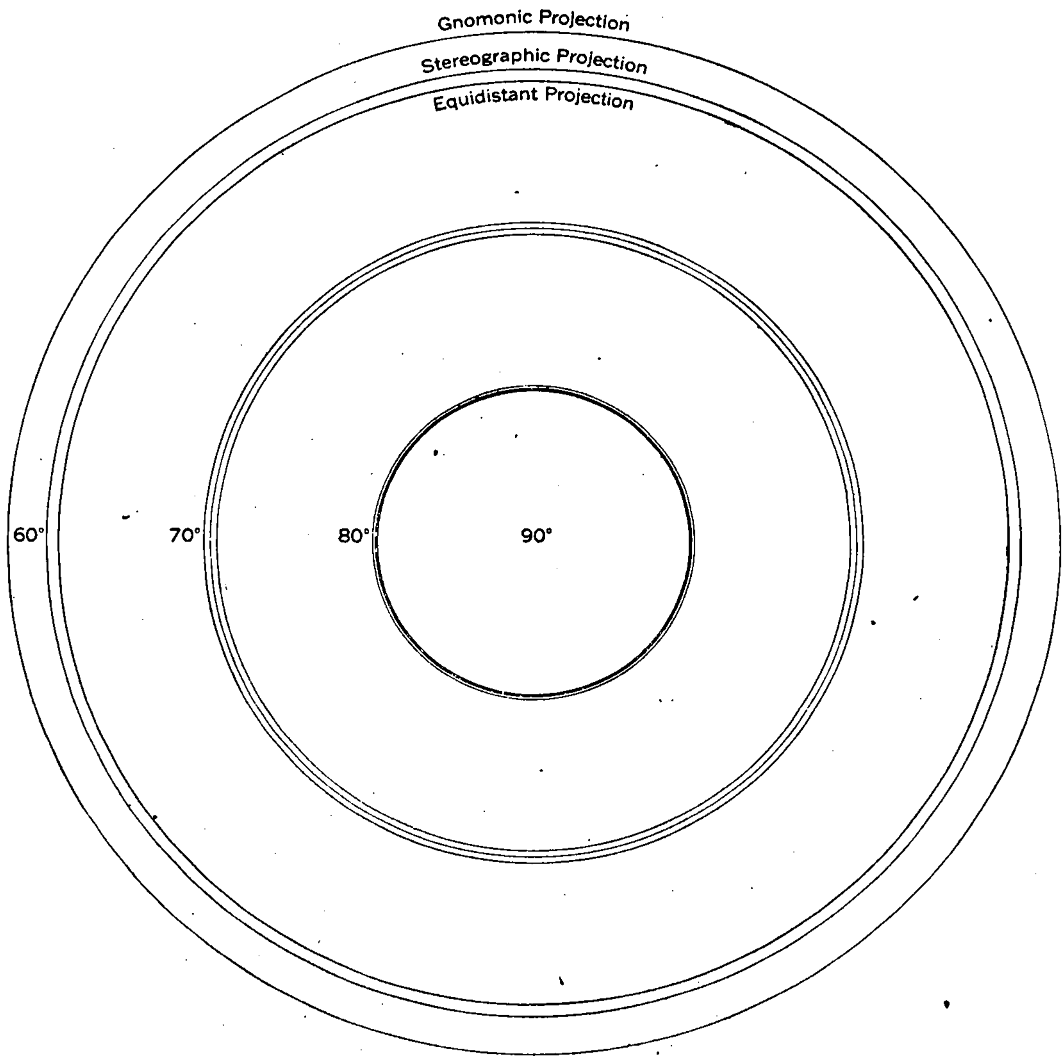


Fig. 71.—Comparison of the equidistant, stereographic, and gnomonic polar projections on parallels 80°, 70°, and 60°.

Inner circles represent the equidistant projection; intermediate circles, the stereographic projection; outer circles, the gnomonic projection.

In the figure giving a comparison of three polar projections, the Lambert zenithal (or azimuthal) projection has not been included, but it may be stated that the radii of its parallels are shorter than those of the others which are represented.

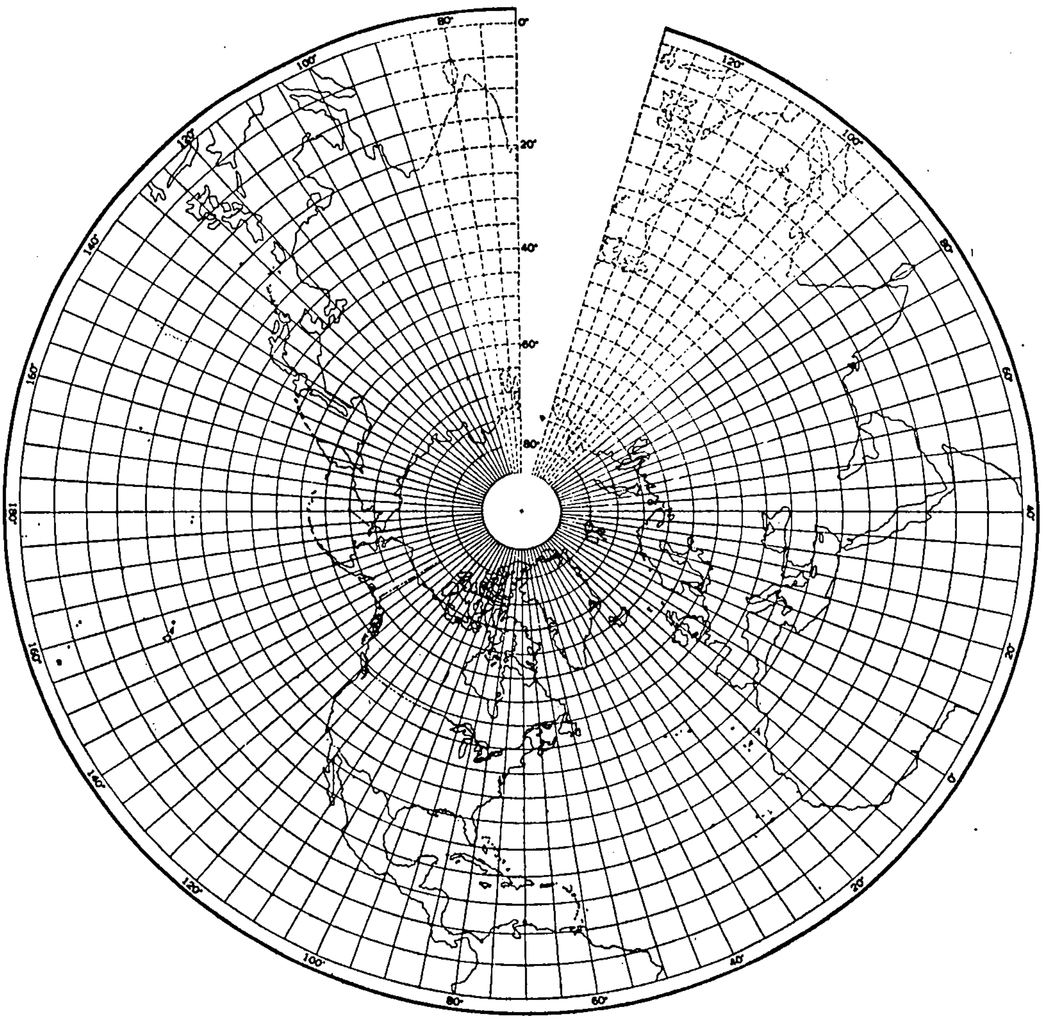


FIG. 72.—Lambert conformal conic projection.
Northern Hemisphere
With standard parallels at 30° and 75°.

NOTE.—The Mercator projection has been employed in the construction of a hydrographic map of the world in 24 sheets, published under the direction of the Prince of Monaco under the title "Carte Bathymétrique des Océans." Under the provisions of the Seventh International Geographic Congress held at Berlin in 1899, and by recommendation of the committee in charge of the charting of suboceanic relief, assembled at Wiesbaden in 1903, the project of Prof. Thoulet was adopted. Thanks to the generous initiative of Prince Albert, the charts have obtained considerable success, and some of the sheets of a second edition have been issued with the addition of continental relief. The sheets measure 1 meter in length and 60 centimeters in height. The series is constructed on 1:10,000,000 equatorial scale, embracing 16 sheets up to latitude 72°. Beyond this latitude the gnomonic projection is employed for mapping the polar regions in four quadrants each.

THE STEREOGRAPHIC PROJECTION

The most widely known of all map projections are the Mercator projection already described, and the stereographic projection which dates back to ancient Greece.

The stereographic projection is one in which the eye is supposed to be placed at the surface of the sphere and in the hemisphere opposite to that which it is desired to project. The exact position of the eye is at the extremity of the diameter passing through the point assumed as the center of the map.

It is the only azimuthal projection which has no angular distortion and in which every circle is projected as a circle. It is a conformal projection and the most familiar form in which we see it is in the *stereographic meridional* as employed to represent the Eastern and Western Hemispheres. In the stereographic meridional projection the

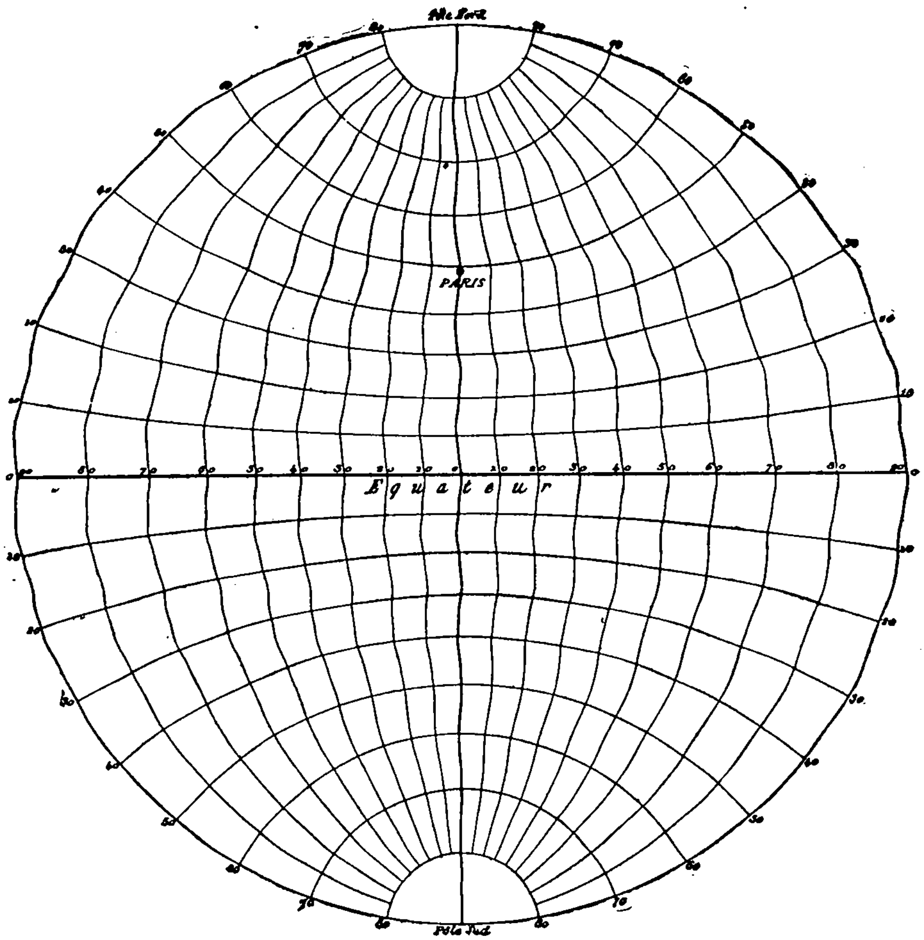


FIG. 73.—Stereographic meridional projection.

center is located on the Equator; in the stereographic horizon projection the center is located on any selected parallel.

Another method of projection more frequently employed by geographers for representing hemispheres is the globular projection, in which the Equator and central meridian are straight lines divided into equal parts, and the other meridians are circular arcs uniting the equal divisions of the Equator with the poles; the parallels, except the Equator, are likewise circular arcs, dividing the extreme and central meridians into equal parts.

In the globular representation, nothing is correct except the graduation of the outer circle, and the direction and graduation of the two diameters; distances and directions can neither be measured nor plotted. It is not a projection defined for the preservation of special properties, for it does not correspond with the surface of the sphere according to any law of cartographic interest, but is simply an arbitrary distribution of curves conveniently constructed.

The two projections, stereographic and globular, are noticeably different when

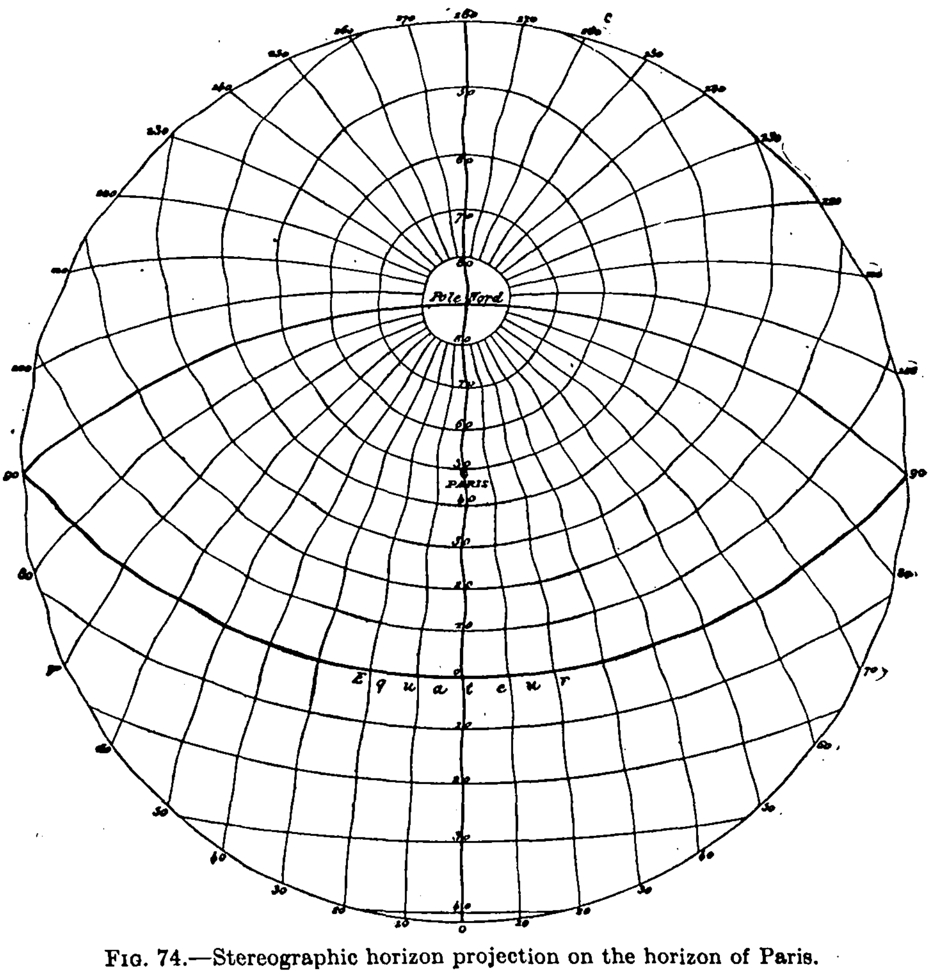


FIG. 74.—Stereographic horizon projection on the horizon of Paris.

seen side by side. In the stereographic projection the meridians intersect the parallels at right angles, as on the globe, and the projection is better adapted to the plotting and measurement of all kinds of relations [36] pertaining to the sphere than any other projection. Its use in the conformal representation of a hemisphere is not fully appreciated.

[36] An interesting paper on this projection appeared in the American Journal of Science, Vol. XI, February, 1901, The Stereographic Projection and its Possibilities from a Graphical Standpoint, by S. L. Penfield.

The application of this projection to the solution of spherical problems is given in Notes on Stereographic Projection, by Prof. W. W. Hendrickson, U. S. N., Annapolis, U. S. Naval Institute, 1905.

A practical use of the stereographic projection is illustrated in the Star Finder devised by G. T. Rude, hydrographic and geodetic engineer, U. S. Coast and Geodetic Survey, and issued by the U. S. Hydrographic Office as H. O. No. 2102a.

In the stereographic projection of a hemisphere we have the principle of Tchebicheff, namely, that a map constructed on a conformal projection is the best possible when the scale is constant along the whole boundary. This, or an approximation thereto, seems to be the most satisfactory solution that has been suggested in the problem of conformal mapping of a hemisphere.

The solution of various problems, including the measurement of angles, directions, and distances on this projection, is given in U. S. Coast and Geodetic Survey Special Publication No. 57. The mathematical theory of the projection, the construction of the stereographic meridional and stereographic horizon projection, and tables for the construction of a meridional projection are also given in the same publication.

THE AITOFF EQUAL-AREA PROJECTION OF THE SPHERE

(See Plate V and fig. 75.)

The projection consists of a Lambert azimuthal hemisphere converted into a full sphere by a manipulation suggested by Aitoff.[37]

It is similar to Mollweide's equal-area projection in that the sphere is represented within an ellipse with the major axis twice the minor axis; but, since the parallels are curved lines the distortion in the polar regions is less in evidence. The representation of the shapes of countries far east and west of the central meridian is not so distorted, because meridians and parallels are not so oblique to one another. The network of meridians and parallels is obtained by the orthogonal or perpendicular projection of a Lambert meridional equal-area hemisphere upon a plane making an angle of 60° to the plane of the original.

The fact that it is an equivalent, or equal-area, projection, combined with the fact that it shows the world in one connected whole, makes it useful in atlases on physical geography or for statistical and distribution purposes. It is also employed for the plotting of the stars in astronomical work where the celestial sphere may be represented in one continuous map which will show at a glance the relative distribution of the stars in the different regions of the expanse of the heavens.

Observations on ellipsoidal projections.—Some criticism is made of ellipsoidal projections, as indeed, of all maps showing the entire world in one connected whole. It is said that erroneous impressions are created in the popular mind either in obtaining accuracy of area at the loss of form, or the loss of form for the purpose of preserving some other property; that while these are not errors in intent, they are errors in effect.

It is true that shapes become badly distorted in the far-off quadrants of an Aitoff projection, but the continental masses of special interest can frequently be mapped in the center where the projection is at its best. It is true that the artistic and mathematically trained eye will not tolerate "the world pictured from a comic mirror," as stated in an interesting criticism; but, under certain conditions where certain properties are desired, these projections, after all, play an important part.

The mathematical and theoretically elegant property of conformality is not of sufficient advantage to outweigh the useful property of equal area if the latter property is sought, and, if we remove the restriction for *shape of elementary figures* as applying to conformal projections, the general shape is often better preserved in projections that are not conformal.

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[37] Also written, D. Aitow. A detailed account of this projection is given in Petermanns Mitteilungen, 1892, vol. 38, pp. 85–87. The projection may justly be termed the Hammer-Aitoff projection, since the result is actually due to Dr. Hammer of Stuttgart.

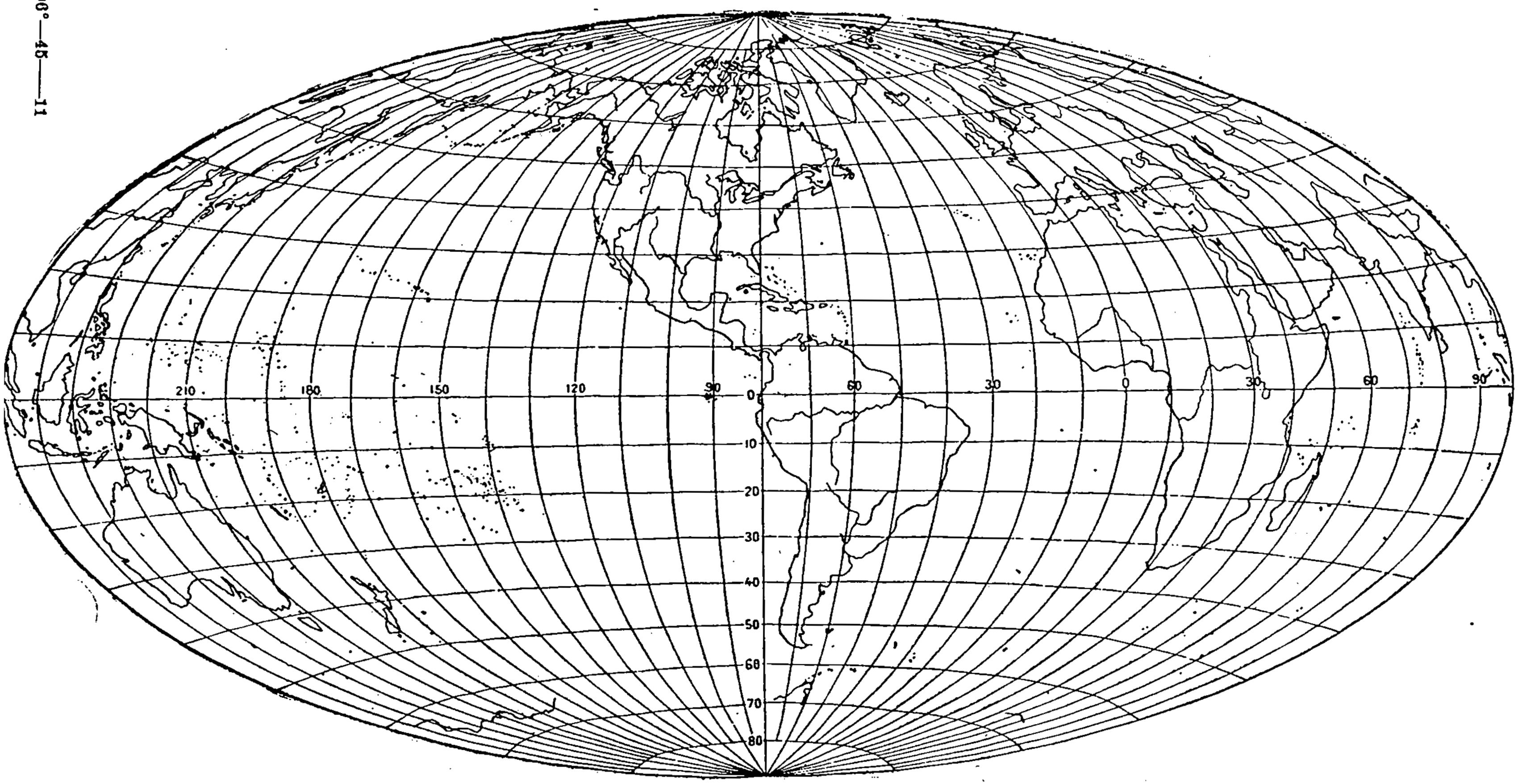

Fig. 75.—The Aitoff equal-area projection of the sphere with the Americas in center.

609106°—45——11

Table for the construction of an Aitoff equal-area projection of the sphere

[Radius of projected sphere equals 1 decimeter. Rectangular coordinates in decimillimeters.]

Longitude		0°	10°	20°	30°	40°	50°	60°	70°	80°	90°	100°	110°	120°	130°	140°	150°	160°	170°	180°
Equator	x	0.0	174.5	348.6	522.1	694.6	865.7	1035.3	1202.8	1368.1	1530.7	1690.5	1847.0	2000.0	2149.2	2294.3	2435.0	2571.2	2702.4	2828.4
	y	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Latitude 10°	x	0.0	172.5	344.6	516.1	686.6	855.7	1023.2	1188.6	1351.8	1512.2	1669.8	1824.0	1974.6	2121.3	2263.8	2401.8	2534.9	2663.2	2785.5
	y	174.3	174.5	175.0	175.8	177.0	178.5	180.4	182.7	185.4	188.6	192.2	196.3	201.0	206.4	212.4	219.2	226.9	235.7	245.6
Latitude 20°	x	0.0	166.5	332.6	498.1	662.5	825.5	986.7	1146.0	1302.7	1456.7	1607.6	1755.0	1898.6	2037.9	2172.7	2302.5	2426.9	2545.6	2657.9
	y	347.3	347.6	348.6	350.2	352.5	355.5	359.1	363.6	368.8	374.9	381.9	389.9	399.0	409.2	420.8	433.8	448.5	465.0	483.7
Latitude 30°	x	0.0	156.4	312.5	467.8	622.1	774.9	925.8	1074.6	1220.8	1364.0	1504.0	1640.1	1772.1	1899.4	2021.7	2138.5	2249.1	2353.0	2449.5
	y	517.6	518.1	519.5	521.8	525.0	529.3	534.5	540.8	548.3	556.9	566.7	578.0	590.7	605.0	621.1	639.1	659.3	681.8	707.1
Latitude 40°	x	0.0	142.2	284.1	425.1	565.1	703.5	840.0	974.2	1105.6	1233.9	1358.7	1479.4	1595.6	1706.8	1812.4	1911.9	2004.6	2089.8	2166.7
	y	684.0	684.6	686.3	689.2	693.2	698.4	704.8	712.6	721.6	732.1	744.1	757.7	773.0	790.1	809.2	830.4	854.0	880.1	909.0
Latitude 50°	x	0.0	123.7	247.0	369.6	491.0	610.8	728.6	844.0	956.6	1066.0	1171.6	1273.0	1369.7	1461.2	1546.8	1626.1	1698.2	1762.5	1818.1
	y	845.2	845.9	847.8	850.9	855.4	861.2	868.3	876.8	886.8	898.3	911.3	926.0	942.4	960.7	980.9	1003.1	1027.5	1054.2	1083.4
Latitude 60°	x	0.0	100.7	201.0	299.9	399.0	495.8	590.7	682.7	773.0	859.5	942.4	1021.2	1095.4	1164.6	1228.1	1285.4	1335.9	1379.1	1414.2
	y	1000.0	1000.6	1002.5	1005.7	1010.2	1016.0	1023.1	1030.8	1041.4	1052.7	1065.4	1079.7	1095.4	1112.8	1131.8	1152.4	1174.8	1198.9	1224.7
Latitude 70°	x	0.0	72.8	145.3	217.1	287.8	357.2	424.8	490.4	553.5	613.8	669.4	724.5	774.2	819.5	860.1	895.6	925.6	949.6	967.4
	y	1147.2	1147.7	1149.4	1152.2	1156.1	1161.1	1167.3	1174.5	1183.0	1192.5	1203.2	1215.1	1228.1	1242.2	1257.4	1273.7	1291.1	1309.6	1328.9
Latitude 80°	x	0.0	39.5	78.8	117.6	155.8	192.9	229.0	263.6	296.6	327.8	356.8	383.7	408.0	429.6	448.4	464.1	476.6	485.6	491.2
	y	1285.6	1285.9	1287.0	1288.8	1291.4	1294.6	1298.5	1303.1	1308.4	1314.4	1321.0	1328.2	1335.9	1344.3	1353.1	1362.4	1372.2	1382.3	1392.7
Latitude 90°	x	0.0	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------
	y	1414.2	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------

The need of critical consideration of the system of projection to be employed in any given mapping problem applies particularly to the equal-area mapping of the entire sphere, which subject is again considered in the following chapters.

A base map without shoreline, size 11 by 22½ inches, on the Aitoff equal-area projection of the sphere, is published by the U. S. Coast and Geodetic Survey, the radius of the projected sphere being 1 decimeter. Tables for the construction of this projection directly from x and y coordinates follow. These coordinates were obtained from the Lambert meridional projection by doubling the x's of half the longitudes, the y's of half the longitudes remaining unchanged.

Thus, in the Lambert meridional projection, the coordinates at latitude 20°, longitude 20°, are

$x=0.33123$ decimeter, or 331.23 decimillimeters.

$y=0.35248$ decimeter, or 352.48 decimillimeters.

For the Aitoff projection, the coordinates at latitude 20°, longitude 40°, will be

$x=2\times331.23=662.5$ decimillimeters.

$y=$ 352.5 decimillimeters.

The coordinates for a Lambert equal-area meridional projection are given on page 77.

THE MOLLWEIDE HOMALOGRAPHIC PROJECTION

This projection is also known as Babinet's equal-surface projection and its distinctive character is, as its name implies, a proportionality of areas on the sphere with the corresponding areas of the projection. The Equator is developed into a straight line and graduated equally from 0° to 180° either way from the central merid-

FIG. 76.—The Mollweide homalographic projection of the sphere.

ian, which is perpendicular to it and of half the length of the representative line of the Equator. The parallels of latitude are all straight lines, on each of which the degrees of longitude are equally spaced, but do not bear their true porportion in length to those on the sphere. Their distances from the Equator are determined by the law of equal surfaces, and their values in the table have been tabulated between the limits 0 at the Equator and 1 for the pole.

The meridian of 90° on either side of the central meridian appears in the projection as a circle, and by intersection determines the length of 90° from the central meridian on all the parallels; the other meridians are parts of elliptical arcs.

Extending the projection to embrace the whole surface of the sphere, the bounding line of the projection becomes an ellipse; the area of the circle included by the meridians of 90° equals that of the hemisphere, and the crescent-shaped areas lying outside of this circle between longitudes ± 90° and ± 180° are together equal to that of the circle; also the area of the projection between parallels ± 30° is equal to the same.

In the ellipse outside of the circle, the meridional lengths become exaggerated and infinitely small surfaces on the sphere and the projection are dissimilar in form.

The distortion in shape or lack of conformality in the equatorial belt and polar regions is the chief defect of this projection. The length which represents 10 degrees of latitude from the Equator exceeds by about 25 per cent the length along the Equator. In the polar regions it does not matter so much if distortions become excessive in the bounding circle beyond 80 degrees of latitude.

The chief use of the Mollweide homalographic projection is for geographical illustrations relating to area, such as the distribution and density of population or the extent of forests, and the like. It thus serves somewhat the same purpose as the Aitoff projection already described.

The mathematical description and theory of the projection are given in Lehrbuch der Landkartenprojectionen by Dr. Norbert Herz, 1885, pages 161 to 165; and Craig (Thomas), Treatise on Projections, U. S. Coast and Geodetic Survey, 1882, pages 227 to 228.

Construction of the Mollweide Homalographic Projection of a Hemisphere

Having drawn two construction lines perpendicular to each other, lay off north and south from the central point on the central meridian the lengths, sin θ, which are given in the third column of the tables (p. 165) and which may be considered as y coordinates, these lengths being in terms of the radius as unity. The points so obtained will be the points of intersection of each parallel of latitude with the central meridian.

With a compass set to the length of the radius and passing through the upper and lower divisions on the central meridian, construct a circle, and this will represent the outer meridian of a hemisphere. Through the points of intersection on the central meridian previously obtained, draw lines parallel to the Equator and they will represent the other parallels of latitude.

For the construction of the meridians, it is only necessary to divide the Equator and parallels into the necessary number of equal parts which correspond to the unit of subdivision adopted for the chart.

Homalographic Projection of the Sphere

In the construction of a projection including the entire sphere (fig. 76), we proceed as before, excepting that the parallels are extended to the limiting ellipse, and their lengths may be obtained by doubling the lengths of the parallels of the hemisphere, or by the use of the second column of the tables under the values for cos θ, in which cos θ represents the total distance out along a given parallel from the central to the

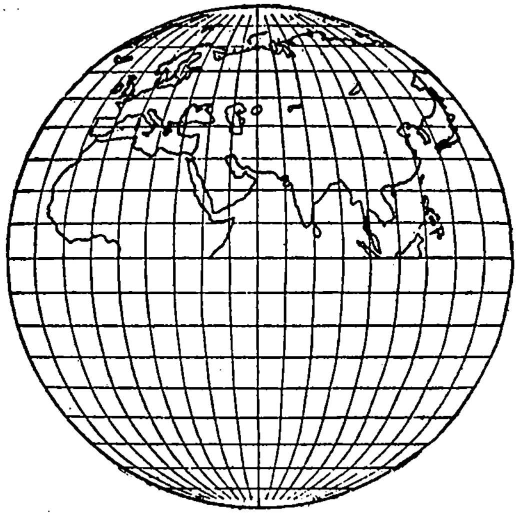

FIG. 77.—The Mollweide homalographic projection of a hemisphere.

outer meridian of the hemisphere, or 90 degrees of longitude. In the projection of a sphere these distances will be doubled on each side of the central meridian, and the Equator becomes the major axis of an ellipse.

Equal divisions of the parallels corresponding to the unit of subdivision adopted for the chart will determine points of intersection of the ellipses representing the meridians.

Table for the construction of the Mollweide homalographic projection [38]

[$\pi \sin \phi = 2\theta + \sin 2\theta$.]

Latitude ϕ	cos θ	sin θ	Difference sin θ	Latitude ϕ	cos θ	sin θ	Difference sin θ
° ′				° ′			
0 00	1.0000000	0.00000000	685431	15 00	0.9788217	0.20471500	676090
0 30	0.9999767	0.00685431	685382	15 30	0.9773830	0.21147590	675460
1 00	0.9999060	0.01370813	685331	16 00	0.9758970	0.21823050	674795
1 30	0.9997884	0.02056114	685279	16 30	0.9743637	0.22497845	674115
2 00	0.9996240	0.02741423	685199	17 00	0.9727827	0.23171960	673430
2 30	0.9994127	0.03426622	685088	17 30	0.9711537	0.23845390	672730
3 00	0.9991542	0.04111710	684950	18 00	0.9694770	0.24518120	672000
3 30	0.9988489	0.04796660	684805	18 30	0.9677529	0.25190120	671250
4 00	0.9984067	0.05481465	684650	19 00	0.9659809	0.25861370	670470
4 30	0.9980970	0.06166115	684485	19 30	0.9641609	0.26531840	669680
5 00	0.9976507	0.06850600	684280	20 00	0.9622929	0.27201520	668880
5 30	0.9971572	0.07534880	684070	20 30	0.9603770	0.27870400	668030
6 00	0.9966169	0.08218950	683830	21 00	0.9584130	0.28538430	667180
6 30	0.9960289	0.08902780	683560	21 30	0.9564009	0.29205610	666340
7 00	0.9953942	0.09586340	683270	22 00	0.9543409	0.29871950	665440
7 30	0.9947127	0.10269610	682970	22 30	0.9522324	0.30537390	664550
8 00	0.9939839	0.10952580	682655	23 00	0.9500756	0.31201940	663620
8 30	0.9932080	0.11635235	682330	23 30	0.9478704	0.31865560	662650
9 00	0.9923847	0.12317565	681980	24 00	0.9456170	0.32528210	661650
9 30	0.9915144	0.12999545	681610	24 30	0.9433152	0.33189860	660660
10 00	0.9905970	0.13681155	681195	25 00	0.9409646	0.33850520	659630
10 30	0.9896322	0.14362350	680745	25 30	0.9385654	0.34510150	658580
11 00	0.9886204	0.15043095	680285	26 00	0.9361174	0.35168730	657520
11 30	0.9875614	0.15723380	679810	26 30	0.9336210	0.35826250	656430
12 00	0.9864550	0.16403190	679330	27 00	0.9310754	0.36482680	655320
12 30	0.9853012	0.17082520	678845	27 30	0.9284809	0.37138000	654200
13 00	0.9841004	0.17761365	678345	28 00	0.9258374	0.37792200	653040
13 30	0.9828517	0.18439710	677825	28 30	0.9231446	0.38445240	651880
14 00	0.9815556	0.19117535	677275	29 00	0.9204030	0.39097120	650720
14 30	0.9802124	0.19794810	676690	29 30	0.9176119	0.39747840	649540

[38] See footnote at end of table.

Table for the construction of the Mollweide homalographic projection [18]—Continued

Latitude φ	cos θ	sin θ	Difference sin θ	Latitude φ	cos θ	sin θ	Difference sin θ
° ′				° ′			
30 00	0.9147706	0.40397380	648290	60 00	0.6471191	0.76238870	528080
30 30	0.9118800	0.41045670	647010	60 30	0.6408456	0.76766950	525170
31 00	0.9089400	0.41692680	645720	61 00	0.6345019	0.77292120	522190
31 30	0.9059504	0.42338400	644400	61 30	0.6280869	0.77814310	519140
32 00	0.9029108	0.42982800	643040	62 00	0.6216001	0.78333450	516070
32 30	0.8998216	0.43625840	641670	62 30	0.6150407	0.78849520	512950
33 00	0.8966820	0.44267510	640300	63 00	0.6084076	0.79362470	509820
33 30	0.8934924	0.44907810	638910	63 30	0.6016988	0.79872290	506610
34 00	0.8902524	0.45546720	637520	64 00	0.5949143	0.80378900	503400
34 30	0.8869620	0.46184240	636110	64 30	0.5880519	0.80882300	500120
35 00	0.8836206	0.46820350	634670	65 00	0.5811107	0.81382420	496830
35 30	0.8802282	0.47455020	633220	65 30	0.5740894	0.81879250	493410
36 00	0.8767850	0.48088240	631680	66 00	0.5669870	0.82372660	489940
36 30	0.8732908	0.48719920	630160	66 30	0.5598024	0.82862600	486440
37 00	0.8697454	0.49350080	628590	67 00	0.5525339	0.83349040	482900
37 30	0.8661484	0.49978670	627000	67 30	0.5451794	0.83831940	479300
38 00	0.8625002	0.50605670	625420	68 00	0.5377379	0.84311240	475580
38 30	0.8588002	0.51231090	623760	68 30	0.5302071	0.84786820	471840
39 00	0.8550482	0.51854850	622130	69 00	0.5225861	0.85258660	468080
39 30	0.8512442	0.52476980	620440	69 30	0.5148715	0.85726740	464320
40 00	0.8473879	0.53097420	618740	70 00	0.5070603	0.86191060	460420
40 30	0.8434792	0.53716160	617010	70 30	0.4991511	0.86651480	456440
41 00	0.8395179	0.54333170	615280	71 00	0.4911423	0.87107920	452380
41 30	0.8355020	0.54948450	613510	71 30	0.4830314	0.87560300	448160
42 00	0.8314364	0.55561960	611700	72 00	0.4748167	0.88008460	443940
42 30	0.8273120	0.56173660	609870	72 30	0.4664942	0.88452400	439640
43 00	0.8231420	0.56783530	608020	73 00	0.4580613	0.88892040	435260
43 30	0.8189142	0.57391550	606160	73 30	0.4495146	0.89327300	430720
44 00	0.8146326	0.57997710	604300	74 00	0.4408511	0.89758020	426160
44 30	0.8102966	0.58602010	602360	74 30	0.4320659	0.90184180	421440
45 00	0.8059058	0.59204370	600390	75 00	0.4231614	0.90605620	416800
45 30	0.8014604	0.59804760	598410	75 30	0.4141156	0.91022420	412100
46 00	0.7969604	0.60403170	596360	76 00	0.4049354	0.91434520	407080
46 30	0.7924049	0.60999530	594340	76 30	0.3956158	0.91841600	401860
47 00	0.7877940	0.61593870	592320	77 00	0.3861534	0.92243460	396550
47 30	0.7831270	0.62186190	590220	77 30	0.3765409	0.92640010	391140
48 00	0.7784035	0.62776410	588130	78 00	0.3667705	0.93031150	385710
48 30	0.7736235	0.63364540	586020	78 30	0.3568322	0.93416860	380200
49 00	0.7687865	0.63950560	583800	79 00	0.3467146	0.93797060	374350
49 30	0.7638925	0.64534360	581600	79 30	0.3364137	0.94171410	368190
50 00	0.7589409	0.65115960	579310	80 00	0.3259234	0.94539600	361990
50 30	0.7539317	0.65695270	577080	80 30	0.3152285	0.94901590	355430
51 00	0.7488643	0.66272350	574850	81 00	0.3043189	0.95257020	348820
51 30	0.7437375	0.66847200	572510	81 30	0.2921755	0.95605840	342180
52 00	0.7385513	0.67419710	570200	82 00	0.2817703	0.95948020	334980
52 30	0.7333054	0.67989910	567830	82 30	0.2701079	0.96283000	327470
53 00	0.7279995	0.68557740	565440	83 00	0.2581516	0.96610470	319470
53 30	0.7226332	0.69123180	562950	83 30	0.2458837	0.96929940	311150
54 00	0.7172058	0.69686130	560450	84 00	0.2332737	0.97241090	302800
54 30	0.7117175	0.70246580	557880	84 30	0.2022700	0.97543890	293630
55 00	0.7061676	0.70804460	555370	85 00	0.2068365	0.97837520	284000
55 30	0.7005550	0.71359830	552820	85 30	0.1929149	0.98121520	273550
56 00	0.6948790	0.71912650	550270	86 00	0.1784407	0.98395070	261900
56 30	0.6891390	0.72462920	547650	86 30	0.1633412	0.98656970	249500
57 00	0.6833342	0.73010570	545000	87 00	0.1474833	0.98906470	236180
57 30	0.6774641	0.73555570	542300	87 30	0.1306660	0.99142650	220970
58 00	0.6715285	0.74097870	539480	88 00	0.1126372	0.99363620	203020
58 30	0.6655270	0.74637350	536670	88 30	0.0929062	0.99566640	180630
59 00	0.6594590	0.75174020	533880	89 00	0.0710530	0.99747270	152500
59 30	0.6533232	0.75707900	530970	89 30	0.0447615	0.99899770	100230
				90 00	0.0000000	1.00000000	

[18] These tables were computed by Jules Bourdin.

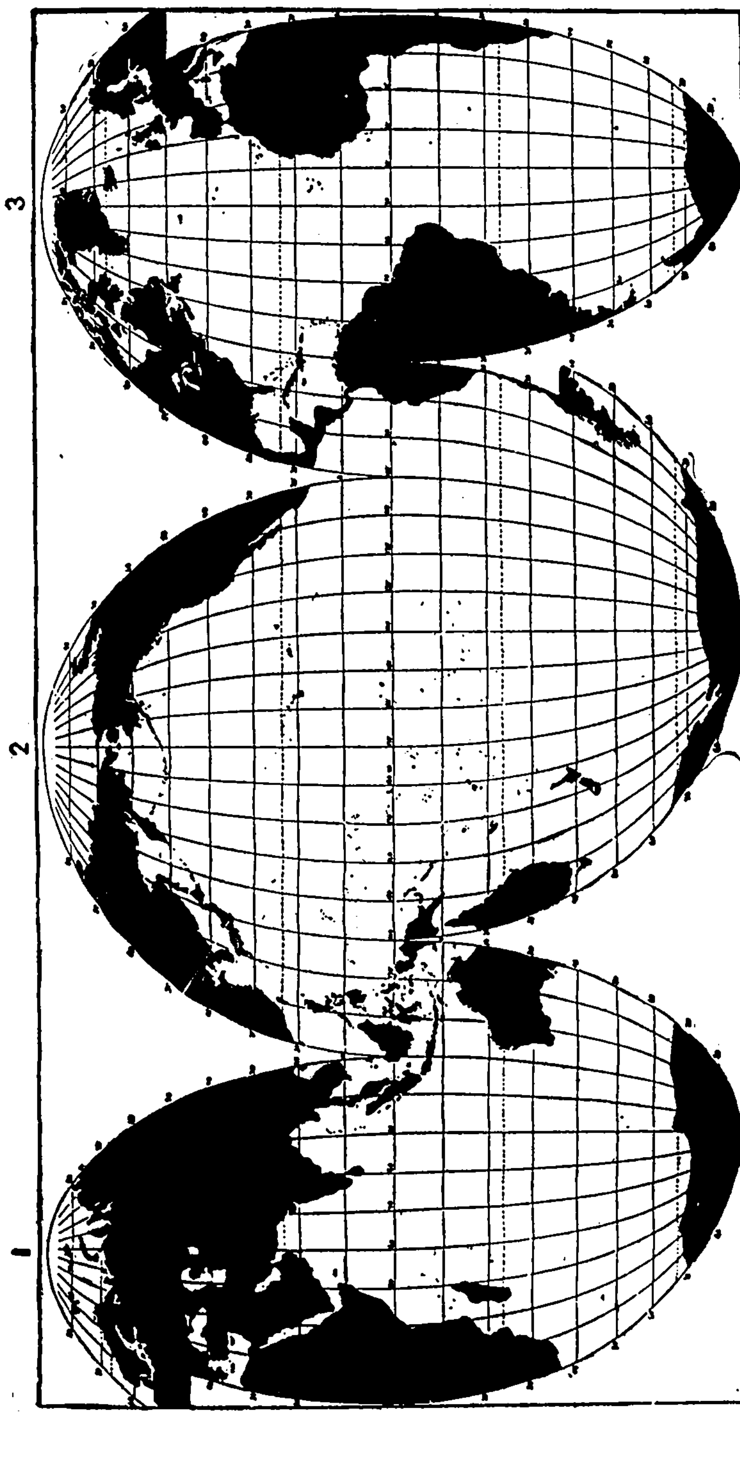

Fig. 78.—Homalographic projection (interrupted) for ocean units.

GOODE'S HOMALOGRAPHIC PROJECTION (INTERRUPTED) FOR THE CONTINENTS AND OCEANS

[See Plate VI and fig. 78.]

Through the kind permission of Prof. J. Paul Goode, Ph. D., we are able to include in this paper a projection of the world devised by him and copyrighted by the University of Chicago. It is an adaptation of the homalographic projection and is illustrated by Plate VI and by figure 78, the former study showing the world on the *homalographic projection (interrupted) for the continents*, the latter being the same projection interrupted for *ocean units*.

The homalographic projection (see fig. 76) which provides the base for the new modification was invented by Prof. Mollweide, of Halle, in 1805, and is an equal-area representation of the entire surface of the earth within an ellipse of which the ratio of major axis to minor axis is 2:1. The first consideration is the construction of an equal-area hemisphere (see fig. 77) within the limits of a circle, and in this projection the radius of the circle is taken as the square root of 2, the radius of the sphere being unity. The Equator and mid-meridian are straight lines at right angles to each other, and are diameters of the map, the parallels being projected in right lines parallel to the Equator, and the meridians in ellipses, all of which pass through two fixed points, the poles.

In view of the above-mentioned properties, the Mollweide projection of the hemisphere offers advantages for studies in comparative latitudes, but shapes become badly distorted when the projection is extended to the whole sphere and becomes ellipsoidal. (See fig. 76.)

In Prof. Goode's adaptation each continent is placed in the middle of a quadrillage centered on a mid-meridian in order to secure for it the best form. Thus North America is best presented in the meridian 100° west, while Eurasia is well taken care of in the choice of 60° east; the other continents are balanced as follows: South America, 60° west; Africa, 20° east; and Australia, 150° east.

Besides the advantage of equal area, each continent and ocean is thus balanced on its own axis of strength, and world relations are, in a way, better shown than one may see them on a globe, since they are all seen at one glance on a flat surface.

In the *ocean units* a middle longitude of each ocean is chosen for the mid-meridian of the lobe. Thus the North Atlantic is balanced on 30° west, and the South Atlantic on 20° west; the North Pacific on 170° west, and the South Pacific on 140° west; the Indian Ocean, northern lobe on 60° east, and southern lobe 90° east.

We have, then, in one setting the continents in true relative size, while in another setting the oceans occupy the center of interest.

The various uses to which this map may be put for statistical data, distribution diagrams, etc., are quite evident.

Section 3 (the eastern section) of figure 78, if extended slightly in longitude and published separately, suggests possibilities for graphical illustration of long-distance sailing routes, such as New York to Buenos Aires with such intermediate points as may be desired. While these could not serve for nautical charts—a province that belongs to the Mercator projection—they would be better in form to be looked at and would be interesting from an educational standpoint.

As a study in world maps on an equal-area representation, this projection is a noteworthy contribution to economic geography and modern cartography.

LAMBERT'S PROJECTION OF THE NORTHERN AND SOUTHERN HEMISPHERES

[See Plate VII.]

This projection was suggested by Commander A. B. Clements of the U. S. Shipping Board and first constructed by the U. S. Coast and Geodetic Survey. It is a conformal conic projection with two standard parallels and provides for a repetition of each hemisphere, of which the bounding circle is the Equator.

The condition that the parallel of latitude 10° be held as one of the standards combined with the condition that the hemispheres be repeated, fixes the other standard parallel at 48° 40′.

The point of tangency of the two hemispheres can be placed at will, and the repetition of the hemispheres provides ample room for continuous sailing routes between any two continents in either hemisphere.

A map of the world has been prepared for the U. S. Shipping Board on this system, scale 1:20,000,000, the diameter of a hemisphere being 54 inches. By a gearing device the hemispheres may be revolved so that a sailing route or line of commercial interest will pass through the point of contact and will appear as a continuous line on the projection.

Tables for the construction of this projection are given on page 89. The scale factor is given in the last column of the tables and may be used if greater accuracy in distances is desired. In order to correct distances measured by the graphic scale of the map, divide them by the scale factor. Corrections to area may be applied in accordance with the footnote on page 83. With two of the parallels true to scale, and with scale variant in other parts of the map, care should be exercised in applying corrections.

In spite of the great extent covered by this system of projection, the property of form, with a comparatively small change of scale, is retained, and a scale factor for the measurement of certain spherical relations is available.

CONFORMAL PROJECTION OF THE SPHERE WITHIN A TWO-CUSPED EPICYCLOID

[See Plate VIII and table on p. 219.]

The shape of the sphere when developed on a polyconic projection (see fig. 48) suggested the development of a conformal projection within the area inclosed by a two-cusped epicycloid. The distortions in this case appear in the distant quadrants, or regions of lesser importance.

Notwithstanding the appearance of similarity in the bounding meridians of the polyconic and the conformal development, the two projections are strikingly different and present an interesting study, the polyconic projection, however, serving no purpose in the mapping of the entire sphere.

For the above system of conformal representation we are indebted to Dr. F. August and Dr. G. Bellermann. The mathematical development appears in Zeitschrift der Gesellschaft für Erdkunde zu Berlin, 1874, volume 9, part 1, No. 49, pages 1 to 22.

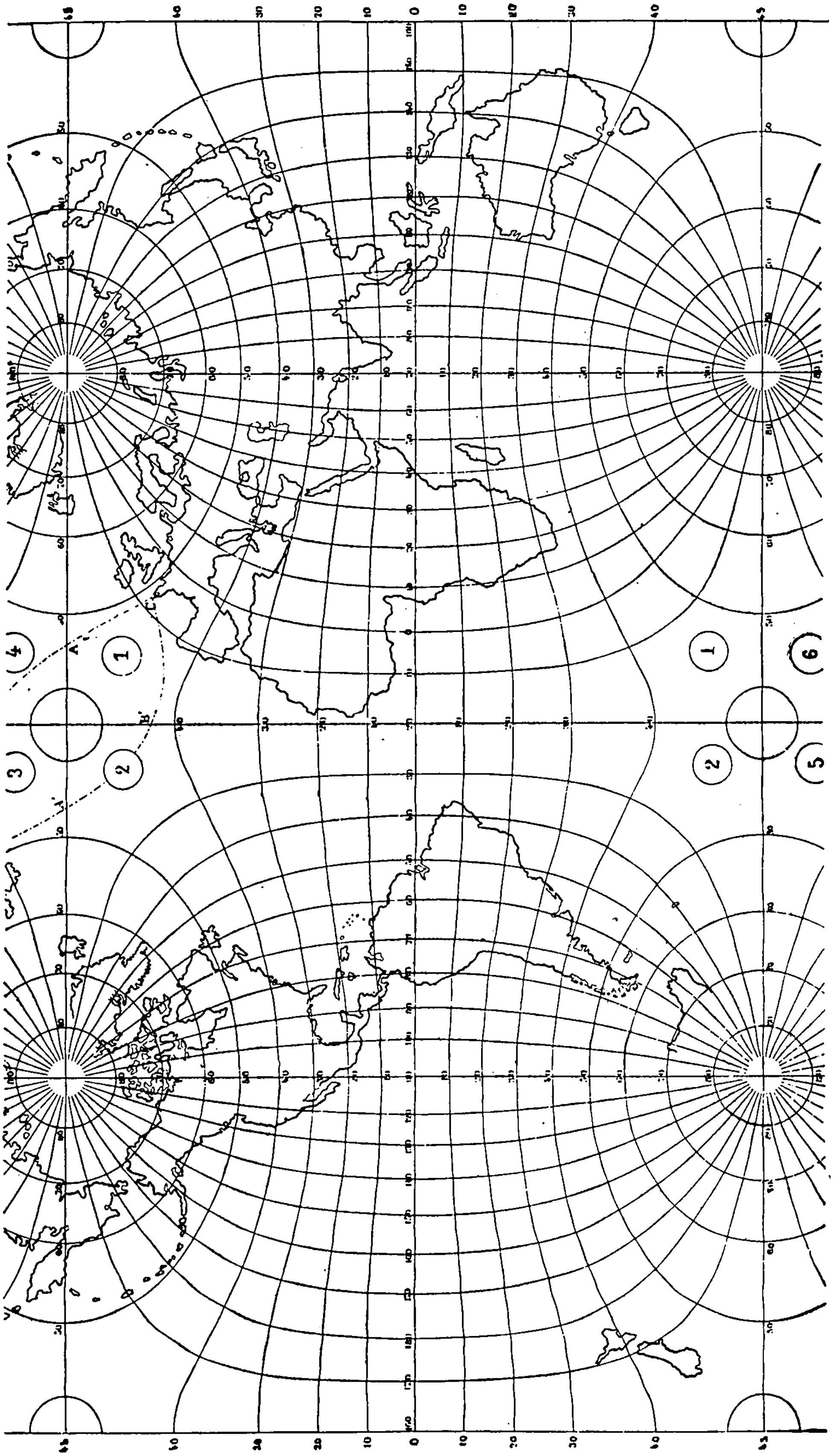

FIG. 79.—Guyou's doubly periodic projection of the sphere.

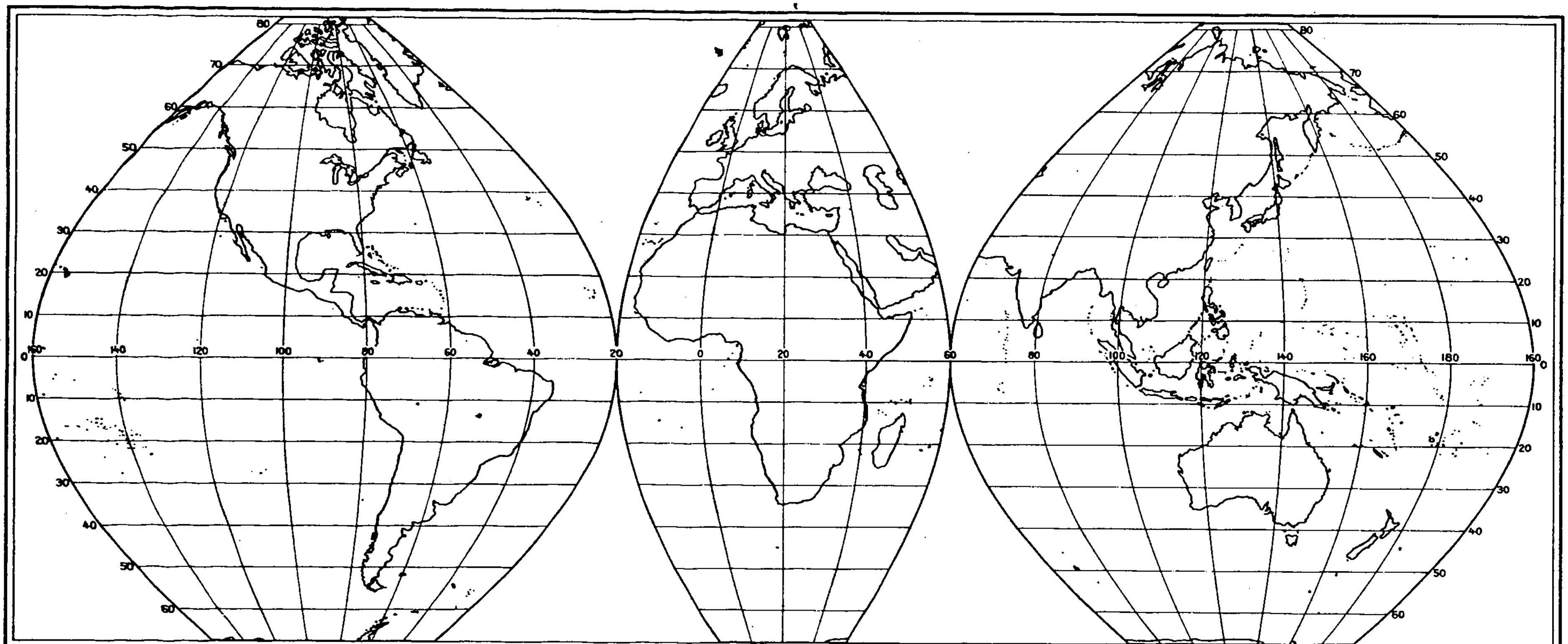

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FIG. 80 -- SINUSOIDAL OR MERCATOR EQUAL-AREA PROJECTION

GUYOU'S DOUBLY PERIODIC PROJECTION OF THE SPHERE

[See fig. 79.]

In Annales Hydrographiques, second series, volume 9, pages 16–35, Paris, 1887, we have a description of an interesting projection of the entire sphere by Lieut. E. Guyou. It is a conformal projection which provides for the repetition of the world in both directions—east or west, north or south, whence the name *doubly periodic*. The necessary deformations are, in this projection, placed in the oceans in a more successful manner than in some other representations.

The accompanying illustration shows the Eastern and Western Hemispheres without the duplicature noted above.

CONFORMAL PROJECTIONS BY LAGRANGE

Two interesting projections for conformal mapping of the world should be mentioned in this review as having already been discussed in United States Coast and Geodetic Survey Special Publication No. 57, pages 111 to 114. Both of these are by Lagrange, one being a double circular projection in which Paris is selected as center of least alteration with variation as slow as possible from that point; the other shows the earth's surface within a circle with the center on the Equator, the variations being most conspicuous in the polar regions. [See fig. 89 and table on p. 220].

RECENT CONTRIBUTIONS TO WORLD MAPPING

The growing interrelation of countries of the Orient and Occident and the tremendous importance of scientific research have recently given an intellectual stimulus to the art and science of cartography, especially as applied to the greater problems which arise in the mapping of areas of continental or world proportions.

Brief mention will here be made of a few of the more important studies or contributions to the subject which have appeared since the publication of the first edition of this book. The first projection to be described with its application to world mapping is the so-called sinusoidal.

SINUSOIDAL OR MERCATOR EQUAL-AREA PROJECTION

This projection has already been mentioned on page 71 under the heading "Sanson-Flamsteed projection." It was employed in the Mercator-Hondius atlases as early as 1606 for a map of South Amercia. In the following years its use was extended to the mapping of Africa and to groups of continents of hemispherical proportions.

The projection as employed in the Mercator atlases appeared without a name but was subsequently called the Sanson-Flamsteed, and, later on, the sinusoidal projection, the latter term being appropriate only from the viewpoint that the meridians are sine curves. Sanson, however, did not use this projection until 1650 and Flamsteed used it first in 1729.

Such, then, is its brief history and one of its applications is illustrated by the accompanying diagram (see fig. 80) as taken from the Nordisk Världs Atlas, Stockholm, 1927.

Description

1. The sinusoidal is an equal-area projection; that is, the projection is true for area scale throughout. A square inch on any part of the map covers the same ground area as a square inch on any other part.

2. It gives true linear scale for all measurements paralleling the Equator and on the vertical (or, as in this case, the central) meridian of each figure. Other linear measurements, however, are variable.

3. It provides an unusually convenient base for plotting data, since the intervals between parallels are true and the meridional divisions along each parallel are true and equal.

4. It allows the map to be divided into three tangential groupings—North and South America, Europe and Africa, Asia and Australia—thus covering as much of the earth's surface as may be desired.

This same system of projection is employed by the Bureau of Foreign and Domestic Commerce, Washington, D. C., for purposes of statistical comparisons, such as population densities, natural resources, etc. The arrangement employed by the American bureau is slightly different and perhaps better than that of the atlas mentioned, due to the fact that this system adapts itself to an interrupted alignment as between the northern and southern continents. Whereas in the atlas we have three vertical groupings, in the bureau's we have the benefit of five different orientations to the north and south, North America being centered on meridian 100° west of Greenwich, South America on 70° west, Europe and Africa on 20° east, Asia on 120° east, and Australia on 140° east.

In the various groupings of continental masses to be charted, to which this system of projection is applicable, it may be suggested that inasmuch as our interest in world mapping is generally confined within the limits of 70° north latitude and 70° south latitude, ordinary maps need not be extended beyond those latitudes. Polar caps covering the remaining 20° can be placed in the vacant spaces of the map on a circular equal-area projection on the same scale as the general map, or the main projection itself may be continued to include the poles.

Construction of the Projection

The construction of this projection is exceedingly simple and is carried out as follows: Draw a straight line to represent the Equator and erect a perpendicular to it at the center of the sheet. On this central meridian lay off to the given scale the true distances of latitude north and south for every parallel to be represented on the map. These distances are given in Tables for a Polyconic Projection of Maps, U. S. Coast and Geodetic Survey Special Publication No. 5 (sixth edition). For small-scale maps, intervals of 1° are given on page 7; for large-scale maps the general tables which follow may be used. Through the points thus determined draw lines parallel to the Equator and these will represent the parallels of latitude of the map.

The meridians can be located by laying off to the given scale east and west from the central meridian their true intervals on each parallel. These intervals are given on page 6 of the publication mentioned above. By connecting the corresponding points thus established on the successive parallels, all of the meridians can be drawn and the projection completed.

This useful projection which has appeared under so many names, none of which are definitely satisfactory either from the viewpoint of authorship or description, might well have been termed "Mercator equal-area projection" in the first place, from the fact that the early atlases bearing his name gave us the first substantial maps in which it was employed. Mercator's name has, however, been so clearly linked with his nautical conformal projection that it becomes necessary to include with his name the words *equal-area* if we wish to disregard the later claimants of its invention, and call it the *Mercator equal-area* projection. At the present day the name sinusoidal, though meager in definition, is rather customary and can not be disregarded.

On any projection that has variation of scale in different directions, it is possible to compute an ellipse at any point that indicates the scale and its direction at that point. This is called Tissot's indicatrix, the principles of which are explained in Special Publication No. 57—General Theory of Polyconic Projections. A new publication in preparation by the Coast and Geodetic Survey shows the application of this indicatrix to several of the equal-area projections. The Coast and Geodetic Survey has constructed for the U. S. Army Air Forces a sinusoidal projection with many of these ellipses shown directly on the projection.

See Fig. 81, the original of which appears as frontispiece in *The Theory of Map Projections*, 1910, by J. I. Craig, Cairo.

MODIFIED SINUSOIDAL AND MOLLWEIDE PROJECTIONS

An interesting projection for world maps somewhat similar to the one just described has been devised by S. W. Boggs, of the Department of State, Washington, D. C. This projection is in a sense a mathematical mean between the sinusoidal and Mollweide projections. Its appearance at the poles is more pleasing than the sinusoidal in that the polar points are not so conspicuous as in the former. While this feature is an improvement to the general appearance of the map, the scale and shape of land masses in the equatorial regions are not as good as in the sinusoidal projection. The properties of the projection are in a general way a mean of the properties of the two projections from which it is derived.

The mathematical theory that forms the basis of the projection was developed by Dr. Adams, one of the authors of this publication, and the first computations were made by Mr. Boggs under Adams' supervision.

LAMBERT EQUAL-AREA MERIDIONAL PROJECTION

This projection has already been described on page 75 of this publication. It has been used successfully for hemispheres, and we find an interesting example in the Eastern and Western Hemispheres placed tangent to each other in Comptes Rendus, L'Association Géodésique Internationale, vol. III, 1911. This arrangement of the land masses of the world probably presents a more graceful appearance than any other.

The Nordisk Världs Atlas shows a map of the Pacific Ocean where the projection is extended to cover 210° of longitude and serves as an ideal arrangement for this particular extensive configuration.

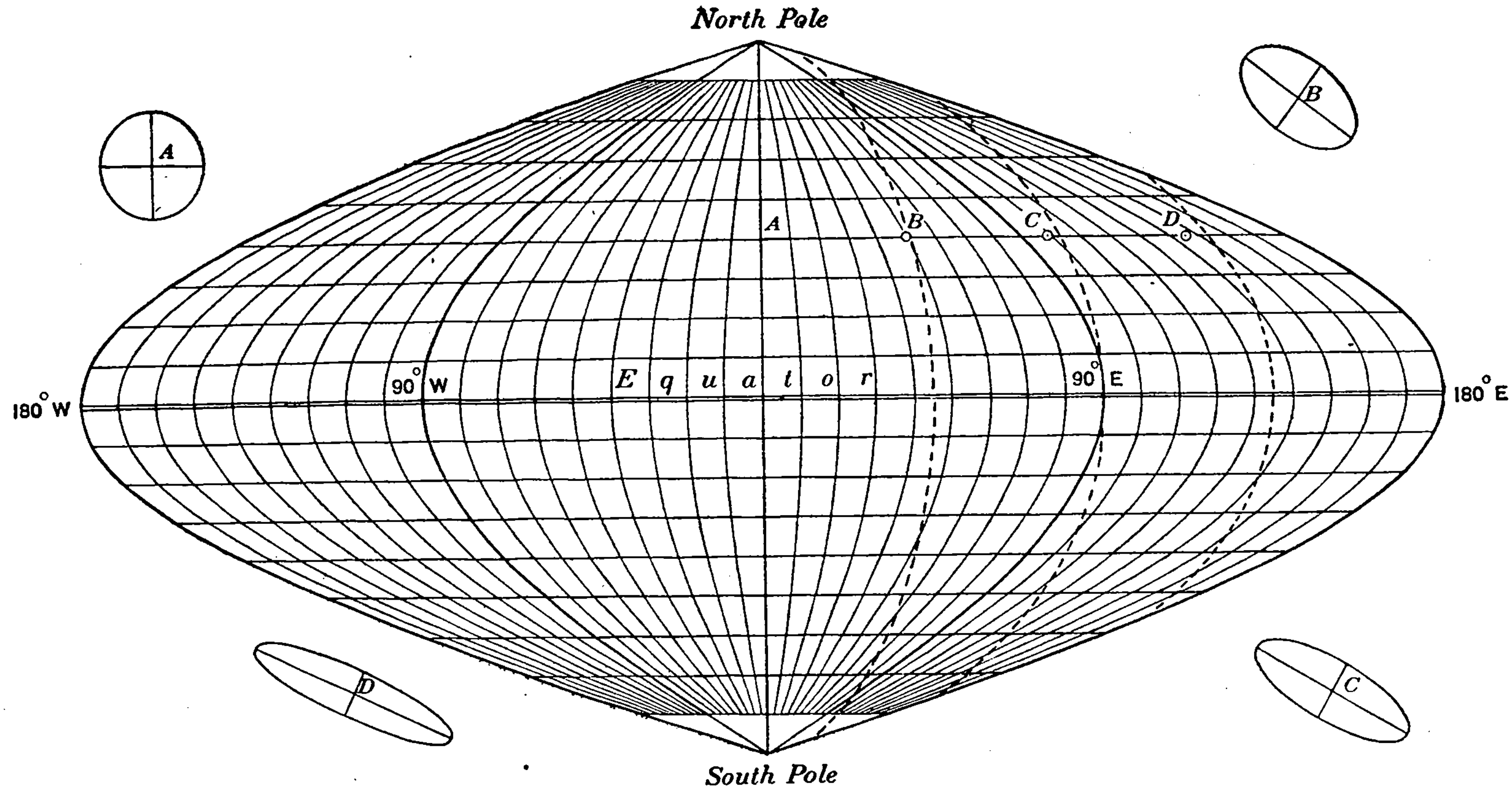

Fig. 81.—Sinusoidal or Mercator equal-area projection.

Diagram showing Tissot's Indicatrix at A, B, C, and D, and isoperimetric (true scale) curves in dash lines.

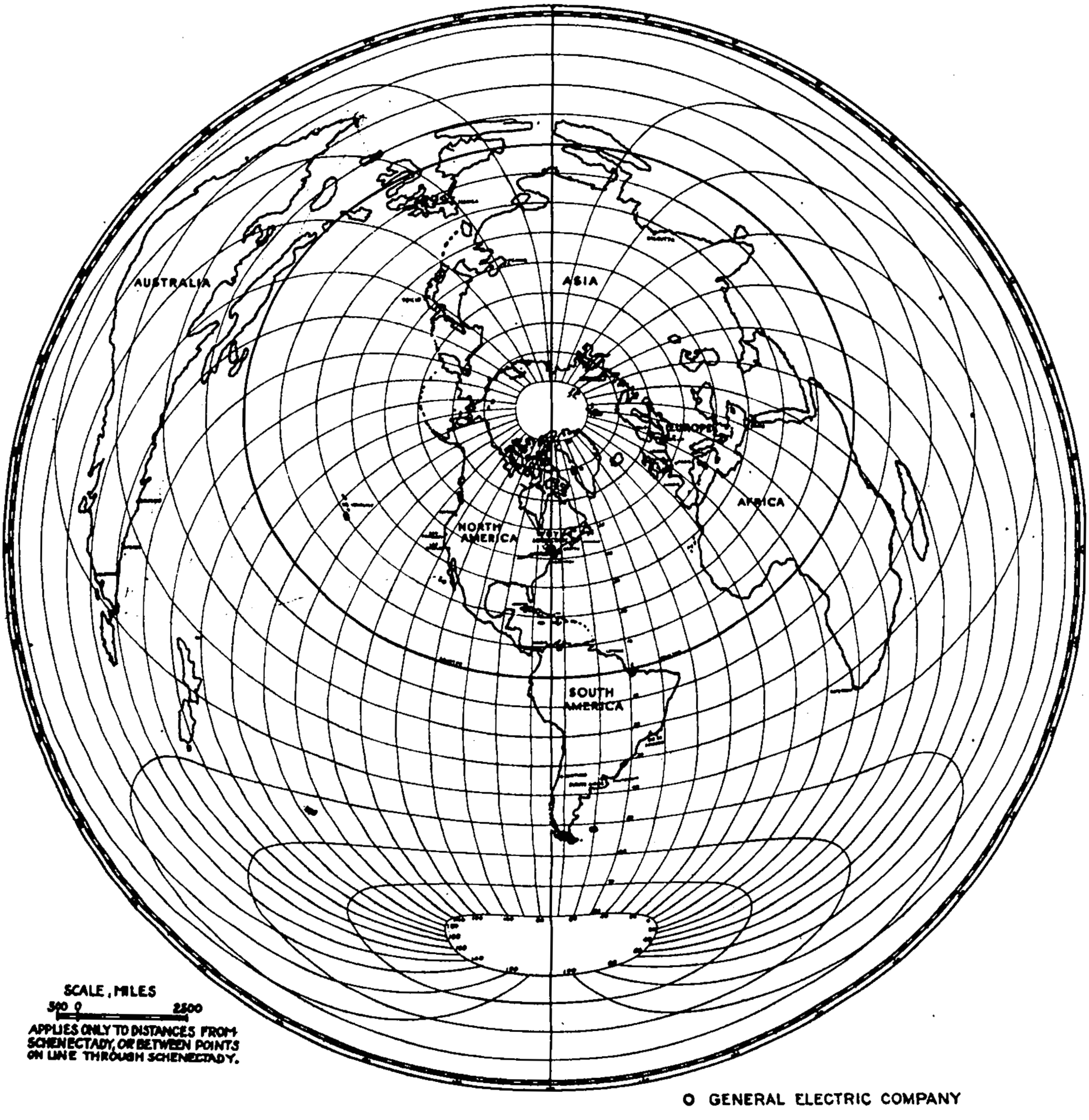

FIG. 82 -- AZIMUTHAL EQUIDISTANT PROJECTION OF THE WORLD

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AZIMUTHAL EQUIDISTANT PROJECTION

In the solution of various problems for determining true distances and azimuths from a given point, the azimuthal equidistant projection has in recent years come into prominent use. This type of projection was first employed by Postel in 1581; afterwards by Lambert in 1772, who regarded it as a zenithal projection; then again in 1799 by Antonio Cagnoli, who believed that he had invented it and gave it his name. We shall here adopt the name *Azimuthal equidistant projection* because the straight lines radiating from the center represent great circles in their true azimuths from it, and the distances along these lines are true to scale. The projection is neither equal-area nor conformal.

No projection can be devised which will give all distances to the correct scale, but this projection gives the distances correct from one point, the center of the map; and for a limited area, even one as large as the United States which has arc distances of 21°, the maximum error in distance is much less on this than on many other projections, not excepting the polyconic.

The question often arises as to the degree of success of a given projection and the percentage of error of the bearings and distances from a given point. Such a question is ofttimes difficult to answer and we may find it useful to test each projection by finding out how nearly it approximates the azimuthal equidistant projection.

The greatest errors in scale on this projection are found along the bounding almucantar or circle of greatest radial distance. Along this circle distance measurements are in excess of true distances, and on all almucantars the scale is increasingly too large as the distance from the center increases.

A map on this projection is easy to construct when the center is one of the poles, since all that is required is to take to the given scale a radius equal to the distance along a meridian from the pole to a given parallel and construct a circle with the pole as center. This circle then represents the parallel.

When the center of the map is any other point, the arc distances and the azimuths from the central point must be computed for the various intersections of the meridians and parallels. Coordinates for these intersections can then be computed to facilitate the construction of the projection. This is generally done on the supposition of the earth as a sphere. The computation for the spheroid is laborious and would only be warranted in cases where extreme accuracy is sought and when the compilation of the map in other respects is done with the utmost care. By considering the earth as a sphere with a mean radius, sufficient accuracy can generally be attained.

The projection is of use for maps from which it is desired to scale accurately the distance and azimuth from the central point to any other point, and its use in aeronautics and for various other purposes is apparent. The chief difficulty lies in the fact that a separate computation and map are required for each central city. The computation is simple, however, and when once made for any given city would serve for all time for the construction of maps on any desired scale.

The projection has been employed in celestial maps, in problems of crystalloptics by the Carnegie Geophysical Laboratory, by the National Geographic Society for maps of the Arctic Regions, North America, Asia, and Africa, and by the General Electric Co. for a map of the world.

The accompanying illustration (see fig. 82) shows, in a diagrammatic form, a map of the world on this projection with center at Schenectady, N. Y., and is used by permission of the General Electric Co. This map was prepared for the convenience of radio engineers of the General Electric Co. in interpreting transmission tests. Besides affording an easy means for measuring the distances from the point which is represented by the center of the map to any other point on the earth's surface, the map is supplied with a periphery scale so that any bearing from the center may be read directly in degrees. The map is valuable also in that it shows the nature of the intervening territory in a straight line between Schenectady and any other points. This feature is important for the radio engineer inasmuch as the distance over which radio signals can be transmitted depends, among other things, on the nature of the intervening territory, especially as to whether it is land or water.

True Great Circle Distances From Two Points

A projection can be constructed on which great circle distances from two points can be mapped as straight lines true to scale but the azimuths will not be true. For example, a doubly equidistant projection based on New York and San Francisco can be computed so that the distances of all other cities from these two points will be true to scale but the azimuths will not be correct from either of these two points to any other point.

In a similar way true azimuths from two points can be maintained but the distances will not then be true to scale.

Possibilities of the extensive use of the gyroscopic compass and the employment of radio signals in the fixing of positions by wireless directional bearings present new problems in the navigation of ships and aircraft. The various uses and tests to which nautical and aerial charts are subjected may still further try the patience of the mathematical cartographer in devising more suitable systems of projection, or in supplying short-cut mathematical or automatic expedients as convenient adjuncts to the Mercator chart. Additional special charts on the gnomonic projection or the azimuthal equidistant projection may serve a useful purpose in the solution of the various problems of great-circle sailing or in determining true distances and azimuths.

There is a modern tendency in many places to exploit projections of various types, generally known as "balance of error," or "minimum error" projections, projections sacrificing conformality, equal-area, and other useful properties, and for which there is no easy geometric accounting. These degenerating types can serve no useful purpose other than picture maps, which become untrustworthy in the derivation or application of spherical relations.

The day may come when the preparation of our charts and their use may be further simplified by the employment of the centesimal system for the graduation of the circumference of the circle, and for the expression of latitudes and longitudes, in place of the sexagesimal system of usual practice. In this the French have taken the lead as they did in the introduction of the metric system, the commensurable advantages of which are universally recognized.

THE PARABOLIC EQUAL-AREA PROJECTION FOR A MAP OF THE WORLD

[See Plate IX.]

In the Geographical Journal for November, 1929, Lieut. Col. J. E. E. Craster suggested the possibility of the construction of several equal-area maps that have much resemblance to the sinusoidal projection. In all of these the parallels are represented by a system of straignt lines parallel to the straight-line Equator. Of these the one that seems of most interest has the meridians represented by parabolas and, hence, it may be called the parabolic equal-area projection.

Mathematical Theory

Let us take the origin of coordinates at the vertex of the parabola and let us represent the outer meridian by equation $y^2=\frac{1}{2}mx$. Then if we represent the central meridian by the straight line $x=2m$, the meridian from pole to pole will equal $2m$, which is half of the whole Equator which will have the length $4m$. For simplicity of computation we may assign the length 4 to the Equator and 2 to the meridian and then the outer meridian will have the equation $y^2=\frac{1}{2}x$. With this simplification one-quarter of the area of the sphere to be mapped will be represented by the area between the axis of x and the curve from $x=0$ to $x=2$.

One-half of the area of the zone between the Equator and any given parallel will be represented on the map by the integral

$$\int_0^y (2-x)dy,$$

or

$$\int_0^y (2-2y^2)dy.$$

The value of this integral in terms of y is, of course,

$$A=2y-\frac{2}{3}y^3.$$

Since, when y is equal to 1, this map area must equal one-fourth of the area of the given sphere or πR^2, we must have the equality

$$\pi R^2=\frac{4}{3}$$

Hence

$$R=0.651470.$$

Now the area of a zone on the sphere from the Equator to latitude ϕ is known to be equal to $2\pi R^2 \sin \phi$. Then the half of this zone must be represented by the expression for A in terms of y as derived above. Hence the equation for the determination of the values of y for the various parallels is given by the solution of the equation

$$2y-\frac{2}{3}y^3=\pi R^2 \sin \phi.$$

But we have already derived the value of πR^2 and found it equal to $\frac{4}{3}$. With this value the equation in y becomes

$$y^3-3y+2\sin\phi=0.$$

Now it happens that this equation is satisfied by the value $y=2\sin\frac{\phi}{3}$. With this simple expression for y a table of y values can readily be computed to any desired degree of accuracy. This table has been computed for every 5° of latitude to six places of decimals and is published at the end of this section.

The length of each parallel between the central meridian and the outer meridian can now be computed from the formula $L=2-2y^2$. Since, however, $y=2\sin\frac{\phi}{3}$, we can use the trigonometric table again by introducing this value of y and transforming the result.

$$L=2\left(1-4\sin^2\frac{\phi}{3}\right),$$

$$=2\left(2\cos\frac{2\phi}{3}-1\right),$$

$$=4\cos\frac{2\phi}{3}-2.$$

By use of this formula a table for the lengths of the parallels has been computed to six places of decimals and is published at the end of this section.

These parallels are divided proportionately for the intersections of the various meridians. If the 10-degree meridians are to be drawn, the space between adjacent meridians will be equal to one-eighteenth of the tabular value for the length of the given parallel and all of the spacings are equal along any given parallel. This makes the projection one that can be constructed very easily, in fact just as easily as the sinusoidal projection which is noted for its facility of construction.

As far as we know, no previous use of this projection has been made either by Colonel Craster or by anyone else for the actual construction of a map. In his article Colonel Craster merely outlined the nature of the projection, together with a number of others of similar kind; some abbreviated tables of the y values were given that could be used as a basis for the construction of maps, but no table of the lengths of the parallels was given for any one of the number. In the actual construction from the elements given in his article it would be necessary either to compute a table for the lengths of the parallels or to construct the outer meridian from coordinates computed from its equation in x and y. It would seem, therefore, that this map is the first example of the use of this method of projection for mapping purposes.

Some time ago a representative of the Bureau of Foreign and Domestic Commerce of this department, consulted with Mr. Deetz concerning the need for a suitable equal-area projection for a world map to be used in the work of that bureau. Mr. Deetz suggested that such a map, constructed on this projection, might meet their needs in a very satisfactory manner. After due consideration it was decided to construct a study for such a map with this projection as basis, and Mr. Deetz compiled and constructed the map as shown in Plate IX.

It was decided to construct the map symmetrically in three sections by making the central section include an extent of longitude of 160° and the two outer equal sections include 120° of longitude each. This gave an excess of longitude of 40° with 20° on the east side of the map and 20° on the west side. This is, in fact, an advantage as can be seen by consulting the map, for it shows India both in relation to Europe and also in relation to eastern Asia, China, Japan, etc. As the continents are placed on the projection, they are all shown with a small amount of distortion. North and South America, Africa, and Australia are in the most favorable positions but Europe is also fairly well represented. Asia suffers most because of the necessary division, but the additional 20° of longitude on the east and the west of the map contributes much toward lessening this defect as can easily be seen from the map itself.

This symmetrical tripartite construction makes it possible to place the Americas in the central section in such a way as to take advantage of the most favorable part of the projection. This is a feature that was very much desired by the Bureau of Foreign and Domestic Commerce in connection with their statistical diagrams. Other arrangements or other divisions could be made to suit the convenience of the maker or, if desired, the whole world could be represented on a continuous undivided projection and we believe it would show superiority to most other such extended projections of the sphere.

On the sinusoidal projection the outer meridians intersect at a very acute angle at the poles, but on the parabolic projection a considerable improvement in this feature is noticeable. All of the meridians, except the central ones, are represented as parabolas of which the equations in coordinates could be very easily derived. The parabola is a very graceful curve and so the resulting graticule is, on the whole, very pleasing to the eye. The perfect symmetry of the whole map can not fail to be appreciated by anyone who may find use for such a delineation of the earth's surface.

The map is strictly equal area in all of its parts and could be shown to be so by mathematical analysis, but we have not attempted to give the analysis in this article. Our object has been to give only enough mathematics to lead to the construction of the map through the computation of the necessary tables. These tables could be used for the construction of a map on any required scale. Colonel Craster gave his table of "y" values to only four places of decimals and a comparison of his table with our 6-place table will show that some of his results were uncertain by several units in the fourth place of decimals. This could be detected by a careful differencing of his table in the above-cited number of the Geographical Journal.

These differences are not great enough to be detected in a small-scale map but they would show in one of larger size. Colonel Craster's object was merely to suggest the possibility of the construction of such a map and he probably did not go into the mathematics of the projection with any great care. Our discovery that the value of "y" is given by $2 \sin \frac{\phi}{3}$ makes it possible to compute the table with great accuracy even to many decimal places. Much credit should be given to Colonel Craster for his suggestion of the possibility of computing an equal-area projection of this kind that may be seen to have so many admirable qualities.

Equal-area maps are especially valuable for illustrating statistical data, either in regard to physical features of the countries or in regard to commercial development. The Bureau of Foreign and Domestic Commerce is, of course, interested in the commercial use of such world maps and, hence, its desire to get one with valuable features as a map as well as one having the equal-area property. We believe that this study indicates that the desired end has been attained in the use of a map on this parabolic equal-area projection.

A second set of tables is appended that are so modified as to give a resulting map that has an area equal to 1 part in 50,000,000 squared of the area of the earth. In figures this means that any section of the map is 1 part in 2.5×10^{15} of the area of the earth's surface that it represents. This exact surface relation may be of special interest to some cartographers. A square inch on any part of such a map will represent 622,744 square miles of the surface of the earth. The linear scale, however, will obviously be variant but along the Equator it may be expressed as 1:51,225,000. If the values in this table are divided by 2 throughout, they will give a map having an area scale ratio of 1 part in 10^{15}, or 10 with 14 additional ciphers after it.

Tables

Distance of parallels from Equator

Latitude (degrees)	Direct computation	For area scale 1 in 2.5×10^{15}	Latitude (degrees)	Direct computation	For area scale 1 in 2.5×10^{15}
		cm.			cm.
0	0	0	50	0.573606	11.21907
5	0.058169	1.13772	55	0.629090	12.30427
10	0.116290	2.27450	60	0.684040	13.37903
15	0.174311	3.40932	65	0.738412	14.44248
20	0.232186	4.54129	70	0.792160	15.49373
25	0.289864	5.66940	75	0.845237	16.53185
30	0.347296	6.79271	80	0.897598	17.55597
35	0.404435	7.91028	85	0.949201	18.56527
40	0.461232	9.02116	90	1.000000	19.55884
45	0.517638	10.12440			

Length of parallels from central meridian to outer meridian

Latitude (degrees)	Direct computation	For area scale 1 in 2.5×10^{15}	Latitude (degrees)	Direct computation	For area scale 1 in 2.5×10^{15}
		cm.			cm.
0	2.000000	39.11767	50	1.341951	26.24700
5	1.993233	38.98532	55	1.208493	23.63672
10	1.972954	88.58868	60	1.064178	20.81408
15	1.939231	37.92910	65	0.909494	17.78864
20	1.892180	37.00884	70	0.744966	14.57067
25	1.831958	35.83096	75	0.571150	11.17103
30	1.758770	34.39949	80	0.388634	7.60123
35	1.672864	32.71927	85	0.198036	3.87335
40	1.574530	30.79597	90	0	0
45	1.464102	28.63613			

A new and more complete table for the construction of a parabolic equal-area projection is given on page 218.

CONFORMAL MAP OF THE WORLD IN A SQUARE POLES IN THE MIDDLE OF OPPOSITE SIDES

Theory

There is a set of isometric curvilinear coordinates for the sphere that may be derived from two spherical ellipses. The existence of these coordinates was pointed out by Lieutenant Guyou in *Annales hydrographiques*, second series, volume 9, 1887. The theory of these coordinates is discussed fully in that publication and it is also given in detail in United States Coast and Geodetic Survey Special Publication No. 112: *Elliptic Functions Applied to Conformal World Maps*. In discussing the theory of this map, we shall confine ourselves to the special problem required for the same.

A spherical ellipse is the locus of the points on the sphere such that the sum of the two great circle arcs which join them to two fixed points is constant. The two fixed

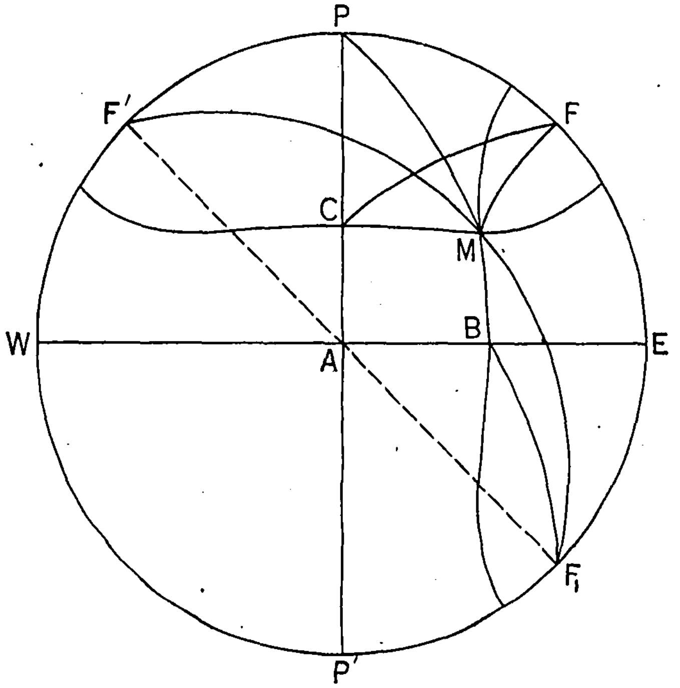

Fig. 83.—Elliptic Coordinates for a conformal map of the world in a square.

points are called the foci of the ellipse. In this discussion we shall confine ourselves to the case in which the foci are located on a great circle at an arc distance of 45° from the pole and situated on opposite sides of the pole. We shall consider two spherical ellipses that have one focus in common and the other foci at opposite ends of a diameter. The foci of the second ellipse will thus be symmetrical with respect to the equator at the arc distance of 45°.

In the diagram F is the common focus and F′ and F_1 are the other foci. Let $F'M=a$ and $FM=b$, then $F_1M=\pi-a$; $PF=PF'=\frac{\pi}{4}$. Let PCAP′ be a meridian the plane of which is perpendicular to the plane of PEP′W. This meridian will thus be the central meridian of the figure. Let λ represent the longitude reckoned out from this central meridian and let ϕ represent the latitude. In the triangle F′PM we have

$$PM=\frac{\pi}{2}-\phi,\ F'M=a,\ \text{and}\ PF'=\frac{\pi}{4}\ \text{and}\ \widehat{F'PM}=\frac{\pi}{2}+\lambda.$$

From the formula for the third side when two sides and the included angle are given, we obtain

$$\cos a=\frac{1}{\sqrt{2}}\sin\phi-\frac{1}{\sqrt{2}}\cos\phi\sin\lambda.$$

Similarly from the triangle FPM, we get

$$\cos b=\frac{1}{\sqrt{2}}\sin\phi+\frac{1}{\sqrt{2}}\cos\phi\sin\lambda.$$

The parametric equations of the unit sphere in terms of ϕ and λ are

$$x=\cos\phi\sin\lambda,$$
$$y=\cos\phi\cos\lambda,$$
$$z=\sin\phi.$$

From the equations for cos a and cos b, we get

$$z=\frac{\cos a+\cos b}{\sqrt{2}}=\sqrt{2}\cos\frac{a+b}{2}\cos\frac{a-b}{2},$$

$$x=\frac{\cos b-\cos a}{\sqrt{2}}=\sqrt{2}\sin\frac{a+b}{2}\sin\frac{a-b}{2}.$$

Now let

$$\frac{a+b}{2}=u\ \text{and}\ \frac{a-b}{2}=v.$$

Then

$$x=\sqrt{2}\sin u\sin v,$$
$$z=\sqrt{2}\cos u\cos v,$$

and from the relation $y=\sqrt{1-x^2-z^2}$,

we get

$$y=\sqrt{1-2\sin^2 u\sin^2 v-2\cos^2 u\cos^2 v}$$
$$y=\sqrt{(1-2\cos^2 u)(1-2\sin^2 v)},$$

in which the radical can assume either the plus or minus sign. These equations give the unit sphere expressed parametrically with u and v as curvilinear coordinates.

We can now express the element of arc upon the sphere in terms of these parameters:

$$\frac{\partial x}{\partial u}=\sqrt{2}\cos u\sin v,$$

$$\frac{\partial x}{\partial v}=\sqrt{2}\sin u\cos v,$$

$$\frac{\partial y}{\partial u}=\frac{+2\sin u\cos u\sqrt{1-2\sin^2 v}}{\sqrt{1-2\cos^2 u}},$$

$$\frac{\partial y}{\partial v}=\frac{-2\sin v\cos v\sqrt{1-2\cos^2 u}}{\sqrt{1-2\sin^2 v}},$$

$$\frac{\partial z}{\partial u}=-\sqrt{2}\sin u\cos v,$$

$$\frac{\partial z}{\partial v}=-\sqrt{2}\cos u\sin v,$$

From these values we can compute the E, F and G for the element of arc, ds.

$$\mathrm{E}=\left(\frac{\partial x}{\partial u}\right)^2+\left(\frac{\partial y}{\partial u}\right)^2+\left(\frac{\partial z}{\partial u}\right)^2=\frac{2(\sin^2 u\cos^2 v-\cos^2 u\sin^2 v)}{2\sin^2 u-1}$$

$$=\frac{2\sin(u+v)\sin(u-v)}{2\sin^2 u-1}$$

$$=\frac{2\sin a\sin b}{2\sin^2 u-1},$$

$$\mathrm{F}=\left(\frac{\partial x}{\partial u}\right)\left(\frac{\partial x}{\partial v}\right)+\left(\frac{\partial y}{\partial u}\right)\left(\frac{\partial y}{\partial v}\right)+\left(\frac{\partial z}{\partial u}\right)\left(\frac{\partial z}{\partial v}\right)=0,$$

$$\mathrm{G}=\left(\frac{\partial x}{\partial v}\right)^2+\left(\frac{\partial y}{\partial v}\right)^2+\left(\frac{\partial z}{\partial v}\right)^2=\frac{2(\sin^2 u\cos^2 v-\cos^2 u\sin^2 v)}{1-2\sin^2 v}$$

$$=\frac{2\sin(u+v)\sin(u-v)}{1-2\sin^2 v}$$

$$=\frac{2\sin a\sin b}{1-2\sin^2 v}.$$

Therefore the element of arc becomes

$$ds^2=\sin a\sin b\left(\frac{du^2}{\sin^2 u-\frac{1}{2}}+\frac{dv^2}{\frac{1}{2}-\sin^2 v}\right).$$

Since the F coefficient is zero the u and v curves are everywhere mutually perpendicular, that is they form a system of orthogonal curvilinear coordinates for the sphere.

Now in the figure let AC be denoted by n and AB by m with WABE representing the equator. Then

$$FC=\frac{a+b}{2}=u,$$

$$F_1B=\tfrac{1}{2}(F_1M+FM),$$

but

$$F_1M=\pi-F'M,$$

so

$$F_1B=\frac{\pi}{2}-\frac{a-b}{2}=\frac{\pi}{2}-v.$$

From the right spherical triangle FCP, we have

$$\sin n=\sqrt{2}\cos u,$$

and from the right spherical triangle F_1BE, we get

$$\sin m=\sqrt{2}\sin v.$$

By differentiating this last equation, we obtain

$$\cos m\,dm=\sqrt{2}\cos v\,dv,$$

but

$$\cos m=\sqrt{1-2\sin^2 v},$$

hence

$$dm=\frac{\cos v\,dv}{\sqrt{\tfrac{1}{2}-\sin^2 v}},$$

but

$$\cos v=\sqrt{1-\tfrac{1}{2}\sin^2 m},$$

therefore

$$\frac{dv}{\sqrt{\tfrac{1}{2}-\sin^2 v}}=\frac{dm}{\sqrt{1-\tfrac{1}{2}\sin^2 m}}.$$

Now in turn by differentiating the equation for sin n, we get by a similar process

$$\frac{du}{\sqrt{\sin^2 u-\tfrac{1}{2}}}=-\frac{dn}{\sqrt{1-\tfrac{1}{2}\sin^2 n}}.$$

The expression for the differential element of arc now becomes

$$ds^2=\sin a\sin b\left(\frac{dm^2}{1-\tfrac{1}{2}\sin^2 m}+\frac{dn^2}{1-\tfrac{1}{2}\sin^2 n}\right).$$

We can now obtain a set of isometric coordinates by the following relations:

$$p=\int_0^v\frac{dv}{\sqrt{\tfrac{1}{2}-\sin^2 v}}=\int_0^m\frac{dm}{\sqrt{1-\tfrac{1}{2}\sin^2 m}},$$

and

$$q=\int_u^{\pi/2}\frac{du}{\sqrt{\sin^2 u-\tfrac{1}{2}}}=-\int_m^0\frac{dn}{\sqrt{1-\tfrac{1}{2}\sin^2 n}}=\int_0^m\frac{dn}{\sqrt{1-\tfrac{1}{2}\sin^2 n}}.$$

With these coordinates the differential element of arc now becomes

$$ds^2 = \sin a \sin b \ (dp^2 + dq^2),$$

with p and q expressed as elliptic integral of the first kind; that is

$$p = F\left(m, \frac{1}{\sqrt{2}}\right),$$

$$q = F\left(n, \frac{1}{\sqrt{2}}\right).$$

After m and n are computed, the values of p and q can be found from Legendre's table of the elliptic integral of the first kind with $k = \frac{1}{\sqrt{2}}$.

This relation gives Lieutenant Guyou's map of a hemisphere in a square with sides equal to $2F\left(\frac{\pi}{2}, \frac{1}{\sqrt{2}}\right)$ or $2K$ for $k = \frac{1}{\sqrt{2}}$.

In order to map the whole sphere in the above square it is necessary first to map the whole sphere conformally on the hemisphere. By starting with the polar stereographic and by denoting the co-latitude by w, we get the complex variable

$$z = \tan \frac{w}{2} e^{i\lambda}.$$

By assuming a new variable

$$z' = \sqrt{z} = \tan^{\frac{1}{2}} \frac{w}{2} e^{\frac{i\lambda}{2}},$$

we get the polar stereographic mapped on the upper half plane or a Lambert conformal conic projection with $l = 1/2$.

Now on another sphere, let us assume the relation

$$\tan \frac{w'}{2} e^{i\lambda'} = \tan^{\frac{1}{2}} \frac{w}{2} e^{\frac{i\lambda}{2}}.$$

Then if we let

$$\tan \frac{w'}{2} = \tan^{\frac{1}{2}} \frac{w}{2},$$

and

$$e^{i\lambda'} = e^{\frac{i\lambda}{2}},$$

we can get by letting λ' and λ vanish together

$$\lambda' = \frac{\lambda}{2},$$

and

$$1 + \tan^2 \frac{w'}{2} = \sec^2 \frac{w'}{2} = 1 + \tan \frac{w}{2},$$

$$\cos^2 \frac{w'}{2} = \frac{1}{1+\tan \frac{w'}{2}},$$

$$1+\cos w' = \frac{2}{1+\tan \frac{w'}{2}},$$

but

$$\cos w' = \sin \phi',$$

$$\sin \phi' = \frac{1-\tan \frac{w}{2}}{1+\tan \frac{w}{2}}$$

$$= \frac{\cos \frac{w}{2} - \sin \frac{w}{2}}{\cos \frac{w}{2} + \sin \frac{w}{2}}$$

$$= \frac{\cos^2 \frac{w}{2} - \sin^2 \frac{w}{2}}{1+2 \sin \frac{w}{2} \cos \frac{w}{2}}$$

$$= \frac{\cos w}{1+\sin w}$$

$$= \frac{\sin \phi}{1+\cos \phi}$$

$$= \tan \frac{\phi}{2}.$$

From this relation we get

$$\cos \phi' = \sqrt{1-\tan^2 \frac{\phi}{2}}$$

$$= \frac{\sqrt{\cos \phi}}{\cos \frac{\phi}{2}}.$$

By this process we have mapped the whole sphere conformally on a hemisphere. At this point we could compute ϕ' and λ' and then use the formulas already derived. However it is more convenient to substitute these values directly in the formulas. In this way we get

$$\cos a = \frac{1}{\sqrt{2}}\left(\tan\frac{\phi}{2} - \frac{\sqrt{\cos\phi}}{\cos\frac{\phi}{2}}\sin\frac{\lambda}{2}\right),$$

$$\cos b = \frac{1}{\sqrt{2}}\left(\tan\frac{\phi}{2} + \frac{\sqrt{\cos\phi}}{\cos\frac{\phi}{2}}\sin\frac{\lambda}{2}\right).$$

By means of these formulas we compute a and b.

Then

$$v = \frac{1}{2}(a - b)$$

and

$$u = \frac{1}{2}(a + b).$$

Furthermore

$$\sin m = \sqrt{2}\sin v$$

and

$$\sin n = \sqrt{2}\cos u.$$

With the values of m and n thus obtained, we can compute from Legendre's table the values of p and q.

Construction of the Table

We can then assume as our coordinates in the plane

$$x = p,$$

$$y = q,$$

or we can multiply the p and q values by any constant we may wish. In the map as constructed the p and q values were multiplied by 20 and the resulting coordinates were constructed as centimeters. The table of coordinates is published in this form because of the convenience in construction. The computation was made for ten-degree intersections of meridians and parallels; in addition, the coordinates for the parallels of 85° and 89° were computed to aid in the correct construction of the meridians near the poles.

As given, the coordinates of the northeast quadrant alone are tabulated. From consideration of symmetry, the coordinates for all of the quadrants can be obtained by a proper distribution of the signs, plus and minus.

After the map was constructed with the dimensions as shown in the table, a photolithographic print was made of it reduced to approximately two-thirds of the linear dimensions of the actually constructed map. The table may be used of course for the construction of a map of any desired size by multiplying the coordinates by the necessary ratio.

Table of coordinates for a conformal map of the world in a square

Latitude.	Longitude 0°		Longitude 10°		Longitude 20°		Longitude 30°		Longitude 40°	
	x cm.	y cm.	x cm.	y cm.	x cm.	y cm.	x cm.	y cm.	x cm.	y cm.
0°	0	0	1.7464	0	3.4996	0	5.2660	0	7.0524	0
10	0	1.7532	1.7460	1.7564	3.4928	1.7666	5.2558	1.7832	7.0390	1.8048
20	0	3.5544	1.7326	3.5612	3.4718	3.5816	5.2242	3.6154	6.9066	3.6632
30	0	5.4588	1.7144	5.4684	3.4338	5.5004	5.1656	5.5524	6.9200	5.6258
40	0	7.5370	1.6836	7.5514	3.3730	7.5944	5.0754	7.6664	6.7972	7.7679
50	0	9.8936	1.6364	9.9122	3.2788	9.9682	4.9332	10.0626	6.6060	10.1956
60	0	12.7008	1.5626	12.7236	3.1292	12.7956	4.7070	12.9148	6.3010	13.0836
70	0	16.2968	1.4348	16.3258	2.8734	16.4132	4.3194	16.5604	5.7774	16.7690
80	0	21.5556	1.1796	21.5890	2.3596	21.6896	3.5406	21.8584	4.7230	22.0976
85	0	25.7268	0.9172	25.7584	1.8324	25.8534	2.7434	26.0126	3.6480	26.2372
89	0	31.8422	0.4490	31.8610	0.8950	31.9166	1.3356	32.0094	1.7680	32.1386
90	0	37.0814	0	37.0814	0	37.0814	0	37.0814	0	37.0814

Latitude.	Longitude 50°		Longitude 60°		Longitude 70°		Longitude 80°		Longitude 90°	
	x cm.	y cm.	x cm.	y cm.	x cm.	y cm.	x cm.	y cm.	x cm.	y cm.
0°	8.8656	0	10.7124	0	12.5996	0	14.5334	0	16.5204	0
10	8.8488	1.8372	10.6922	1.8742	12.5760	1.9182	14.5066	1.9624	16.4918	2.0248
20	8.7958	3.7248	10.6284	3.8004	12.5018	3.8896	14.4226	3.9924	16.3976	4.1078
30	8.6994	5.7208	10.5126	5.8374	12.3664	5.9756	14.2684	6.1354	16.2252	6.3134
40	8.5450	7.8994	10.3260	8.0612	12.1478	8.2542	14.0182	8.4778	15.9460	8.7320
50	8.3038	10.3682	10.0336	10.5818	11.8028	10.8374	13.6208	11.1368	15.4964	11.4798
60	7.9174	13.3038	9.5624	13.5764	11.2436	13.9076	12.9692	14.2970	14.7496	14.7496
70	7.2510	17.0418	8.7450	17.3826	10.2648	17.7966	11.8162	18.2898	13.4068	18.8700
80	5.9062	22.4068	7.0918	22.7996	8.2776	23.2718	9.4528	23.8334	10.6466	24.4908
85	4.5430	26.5284	5.4264	26.8884	6.2950	27.3442	7.1346	27.8240	7.9590	28.4052
89	2.1888	32.3046	2.5958	32.5054	2.9862	32.7414	3.3566	33.0114	3.7034	33.3132
90	0	37.0814	0	37.0814	0	37.0814	0	37.0814	0	37.0814

Latitude	Longitude 100°		Longitude 110°		Longitude 120°		Longitude 130°		Longitude 140°	
	x *cm.*	*y* *cm.*	*x* *cm.*	*y* *cm.*	*x* *cm.*	*y* *cm.*	*x* *cm.*	*y* *cm.*	*x* *cm.*	*y* *cm.*
0°	18.5658	0	20.6742	0	22.8486	0	25.0894	0	27.3944	0
10	18.5340	2.0864	20.6408	2.1516	22.8148	2.2192	25.0568	2.2858	27.3646	2.3488
20	18.4334	4.2340	20.5352	4.3680	22.7076	4.5086	24.9508	4.6480	27.2694	4.7798
30	18.2470	6.5136	20.3382	6.7268	22.5060	6.9500	24.7552	7.1746	27.0872	7.3902
40	17.9402	9.0148	20.0104	9.3226	22.1662	9.6492	24.4168	9.9844	26.7688	10.3130
50	17.4406	11.8666	19.4674	12.2954	21.5914	12.7610	23.8284	13.2530	26.1984	13.7532
60	16.5970	15.2690	18.5272	15.8588	20.5614	16.5204	22.7270	17.2536	25.0630	18.0490
70	15.0460	19.5476	16.7452	20.3368	18.5210	21.2556	20.3978	22.3300	22.4142	23.5988
80	11.8032	25.2596	12.9864	26.1474	14.1206	27.1724	15.2348	28.3538	16.2766	29.7138
85	8.7448	29.0654	9.4882	29.8072	10.1786	30.6322	10.8030	31.5404	11.3468	32.5288
89	4.0242	33.6452	4.3164	34.0060	4.5756	34.3926	4.8014	34.8028	4.9888	35.2332
90	0	37.0814	0	37.0814	0	37.0814	0	37.0814	0	37.0814

Latitude	Longitude 150°		Longitude 160°		Longitude 170°		Longitude 180°	
	x *cm.*	*y* *cm.*	*x* *cm.*	*y* *cm.*	*x* *cm.*	*y* *cm.*	*x* *cm.*	*y* *cm.*
0°	29.7576	0	32.1696	0	34.6164	0	37.0814	0
10	29.7328	2.4038	32.1516	2.4470	34.6070	2.4746	37.0814	2.4842
20	29.6532	4.8960	32.0938	4.9878	34.5762	5.0468	37.0814	5.0672
30	29.4992	7.5828	31.9814	7.7366	34.5168	7.8364	37.0814	7.8712
40	29.2246	10.6134	31.7776	10.8590	34.4078	11.0220	37.0814	11.0788
50	28.7152	14.2318	31.3858	14.6424	34.1908	14.9260	37.0814	15.0286
60	27.6218	18.8800	30.4680	19.6802	33.6444	20.3108	37.0814	20.5614
70	24.6320	25.1250	27.1890	27.0252	30.4504	29.5690	37.0814	33.1400
80	17.2136	31.2730	17.9834	33.0420	18.5020	35.0020	18.6870	37.0814
85	11.7938	33.5910	12.1282	34.7158	12.8354	35.8868	12.4058	37.0814
89	5.1374	35.6800	5.2450	36.1386	5.3100	36.6088	5.3318	37.0814
90	0	37.0814	0	37.0814	0	37.0814	0	37.0814

RÉSUMÉ

GENERAL STATEMENT

From experience in meeting a frequent demand for a brief outline on the subject of map projections in general use, it is thought that a résumé of those systems which are of special interest in problems of the present day may serve a useful purpose.

While the statements made here have been taken largely from the preceding chapters of this book and are repetitions to a certain extent, it is intended to bring out more clearly the special uses and comparisons of the several systems and to note the more recent observations and studies.

Under the subject, "Thoughts on Map Construction" by Francis Bacon, we find the following pertinent passage: "All depends on keeping the eye steadily fixed upon the facts of nature, and so receiving their images simply as they are; for God forbid that we should give out a dream of our own imagination for a pattern of the world."

As a spheroidal surface can not be spread out upon a plane without distortion, any representation of an extensive part of the earth's surface must necessarily involve a certain amount of approximation or compensation systematically accomplished, or it must be restricted to the desired special property that will meet a problem under consideration. A globe is, of course, the only possible method of truly representing the earth's surface, and on it the problems of the geographer and navigator are easily visualized. Directions, distances, forms, and areas which comprise its elements are correct, and its scale is constant. Nevertheless in many instances it is impracticable. By use of the developable surface of a cone or cylinder as an intermediate step in the construction of a projection, and with the use of mathematical analysis it is possible to design a map which is accurate in respect to certain properties. It is this consideration that becomes the purpose of the following review.

A map projection is an orderly arrangement of the two sets of lines or curves that constitute the framework of a map, one set to represent parallels and the other set to represent meridians. By means of these two sets of basic lines, places on the earth's surface can be located by latitude and longitude, that is, they can be assigned a definite position corresponding to the network of parallels and meridians that are conceived as covering the surface of the earth. This orderly arrangement or sequence may come from direct geometric projection or it may be expressed in mathematical terms. Of course, all projections can be stated in mathematical terms, but sometimes a projection is more in evidence when considered as a true geometric projection or development. It is rather unfortunate that all possible schemes are called projections, for in some cases it is rather difficult to interpret them in the way of geometric projections.

The errors which arise in maps are those of distortion which implies deviation from right shape in the meridians and parallels, involving curvature in these reference lines; deformation of angles; changes of scale and errors of distances, errors of bearings, and errors of area.

PROPERTIES OF PROJECTIONS

The principal properties that govern an orderly arrangement of meridians and parallels, and which we aim to obtain at the expense of other properties are:

1. The correct angles between meridians and parallels, and true shape for restricted areas as found in the conformal projections.

2. Equivalence of area as represented in the equal-area projections. If we take a coin, a paper weight, or, if you like, a small irregular piece of cardboard and place it on an equal-area projection, the same piece of paper in any orientation whatever on any other part of the map will cover the same amount of area of the country represented.

Other potential properties of projections are:

1. The representation of the rhumb line as a straight line, as in the Mercator projection.

2. The representation of the great circle as a straight line as in the gnomonic projection.

3. The representation of true azimuths and distances from a given point as in the azimuthal equidistant projection.

On páge 54 there is shown an ideal head drawn on a globular projection. By plotting the outline of this head into the corresponding graticules of an orthographic, stereographic, and Mercator system, the properties of the globular projection are shown in a distorted picture in these other three systems. This does not imply that the globular projection has any intrinsic value (in fact it has not), being nothing more than a geometrical design easily constructed. The normal head for the Mercator might as well have been used as a base at the expense of the others. The diagram merely illustrates how it is impossible to hold more than one or two desirable properties in a given projection.

The selection of a projection is, therefore, the first important step in the compilation of any map or chart, as the value of the product to the particular purpose it is designed to serve, depends largely on the projection upon which the chart is constructed.

THE MERCATOR PROJECTION

In this Résumé, the Mercator projection has not been included on account of the desirability of having in one place, as much as possible, its history, description, computation, construction; its uses, and comparisons with other projections, etc.

The reader is therefore referred to pages 103–124 for the various details covering the true story from its invention to the present day. Even if some repetitions occur, this thought may be justified by a quotation from the Roman writer Seneca, as follows: "A thing is never too often repeated which is never sufficiently learned."

THE GNOMONIC PROJECTION

In this projection the eye of the spectator is supposed to be situated at the center of the terrestrial sphere, whence, being in the plane of every great circle, it will see these circles projected as straight lines upon a plane tangent at a selected central point. It follows then that a straight line between any two points on a gnomonic chart represents the track line or shortest route on the earth's surface between them.

The projection is used chiefly as an adjunct to the Mercator system to which the finally selected route can be transferred by corresponding graticules of latitude and longitude. The great circle thus transferred becomes a curved line on the Mercator projection, where it may be resolved into convenient sailing rhumbs so that the port bound for may be reached by the shortest practicable route.

Between a great circle and a rhumb line, the actual difference in distance is frequently immaterial, being but one-fourth mile in 500 statute miles along parallel 40°

and diminishing to zero along the meridian. For greater lengths the difference increases very rapidly.

The gnomonic projection requires a considerable amount of computation, is difficult to construct, and the scale of distances is complicated. Its chief fault is that localities near the boundary of the map become greatly distorted in distances, areas, and shapes. This is because near 90° from the center of the map the projecting line approaches parallelism to the plane of projection and its intersection recedes to infinity. The area which can be mapped on this projection is less than a hemisphere.

A gnomonic chart will serve as a general control map for many purposes and for office use especially. It is useful in the study of air routes and in the ready solution of problems involving the definite location of the minimum line between two points on the earth's surface. In other projections the localities through which a great circle passes are a matter of approximation or conjecture. At the present time, excepting for a small number of harbor charts by the British Admiralty, it has hardly been more than an auxiliary to the Mercator chart.

However, the laying down of a true azimuth from a given radio station is another problem. On account of angular distortion in a gnomonic projection, a specially computed compass rose at any given station (except at the point of tangency) is necessary in order to obtain a true bearing from the station. The problem may be simplified either by including on the chart, compass roses computed for important stations or by supplying for the stations on any particular chart the gnomonic azimuth tables such as are given in Special Publication No. 75, U. S. Coast and Geodetic Survey.

A base chart of the United States, No. 3074a, on a gnomonic projection has been constructed by the U. S. Coast and Geodetic Survey. The point of tangency is placed at latitude 40° and the center of longitude at 96°. On this projection, the meridians being great circles, are represented by straight lines. The parallels are conic sections, being ellipses above latitude 50° and hyperbolas below latitude 50°, the conic section changing from ellipse to hyperbola by passing through the parabola at latitude 50°.

THE POLYCONIC PROJECTION

The polyconic projection possesses great popularity on account of mechanical ease of construction and the fact that a general table for its use has been calculated.

In this system the reference lines, excepting the equator and central meridian, are curved, the parallels being nonconcentric circles. The central meridian is truly spaced and the even divisions of the parallels are true to scale. The lengths of the arcs of latitude increase as we recede from the central meridian, but in the United States, for a longitudinal extent within 560 statute miles on either side of a central meridian, the error in scale and areas does not exceed 1 per cent. In latitude, the projection can be carried as far as may be desired. The use of the projection for extensive areas should thus be restricted to maps of wide latitudes and narrow longitudes.

Within the limits described the projection attains an accuracy that meets all general purposes by reason of compromising various conditions impossible to be represented on any one map or chart, and is sufficiently close to other types possessing special properties to determine its choice in many instances.

For isolated or restricted maps not part of a general scheme, the polyconic projection which is easily constructed, will generally be satisfactory. Due to the curvature of its outer meridians, adjoining maps constructed on their own central meridians, will present curvature in opposite directions, east and west, thus interfering with a neat junction when the projection is carried too far.

For a map of the United States with its wide longitudinal extent, the Albers and Lambert conformal projections are better suited, either of them offering certain additional or special properties of their own which are only approximated in the polyconic projection.

The errors of the polyconic projection increase approximately as the square of the distance from the central meridian. We observe that this increase amounts to as much as 500 per cent in the outer meridian of a map of the world.

THE TRANSVERSE POLYCONIC PROJECTION

The polyconic projection can be transversed in a way to meet a configuration of predominating east-and-west dimension. A great circle at right angles to a central meridian at the middle part of the map can be made to play the part of the central meridian in the ordinary polyconic projection, the poles being transferred (in construction only) to the equator.

Plate II features an area adapted to this transverse arrangement. On other maps and charts, this important section of the North Pacific Ocean can not readily be visualized on account of excessive scale variations. The map is not intended for navigational purposes but is of great interest from a geographic viewpoint as exhibiting in their true relations, important localities covering a wide expanse. Figure 84, showing the North Pacific region on both the Mercator and transverse polyconic projections, explains the reason for the remark, "The Mercator projection puts Alaska too far north."

THE LAMBERT CONFORMAL CONIC PROJECTION AND THE ALBERS EQUAL-AREA PROJECTION

These two projections are somewhat similar in appearance but different in their properties. Points in common are: Both are conic projections and both have two standard parallels of true scale; the projections being conic, their meridians are straight lines converging in the direction of the pole, and the parallels are concentric circles intersecting the meridians at right angles. On the selected parallels, arcs of longitude are represented in their true lengths, or to exact scale.

Their differences are: In the former projection the intervals of the parallels depend upon the condition of conformality; in the latter they depend upon the condition of equal-area.

Definition of conformality: If at any point the scale along the meridian and the parallel is the same (not necessarily correct, but the same in both directions) and the meridians and parallels are at right angles to one another, then the shapes of all elementary figures on the map are the same as their corresponding areas upon the earth.

In the Lambert projection the scale is too small between the standard parallels and too large beyond them; in the Albers projection the meridional scale between the standard parallels is too large and the scale along the parallels correspondingly too small, the reverse condition existing beyond the standard parallels.

The Lambert projection was used in France for military maps during the first World War. Our chart No. 3070 (see Plate I, reduced scale), North Atlantic Ocean

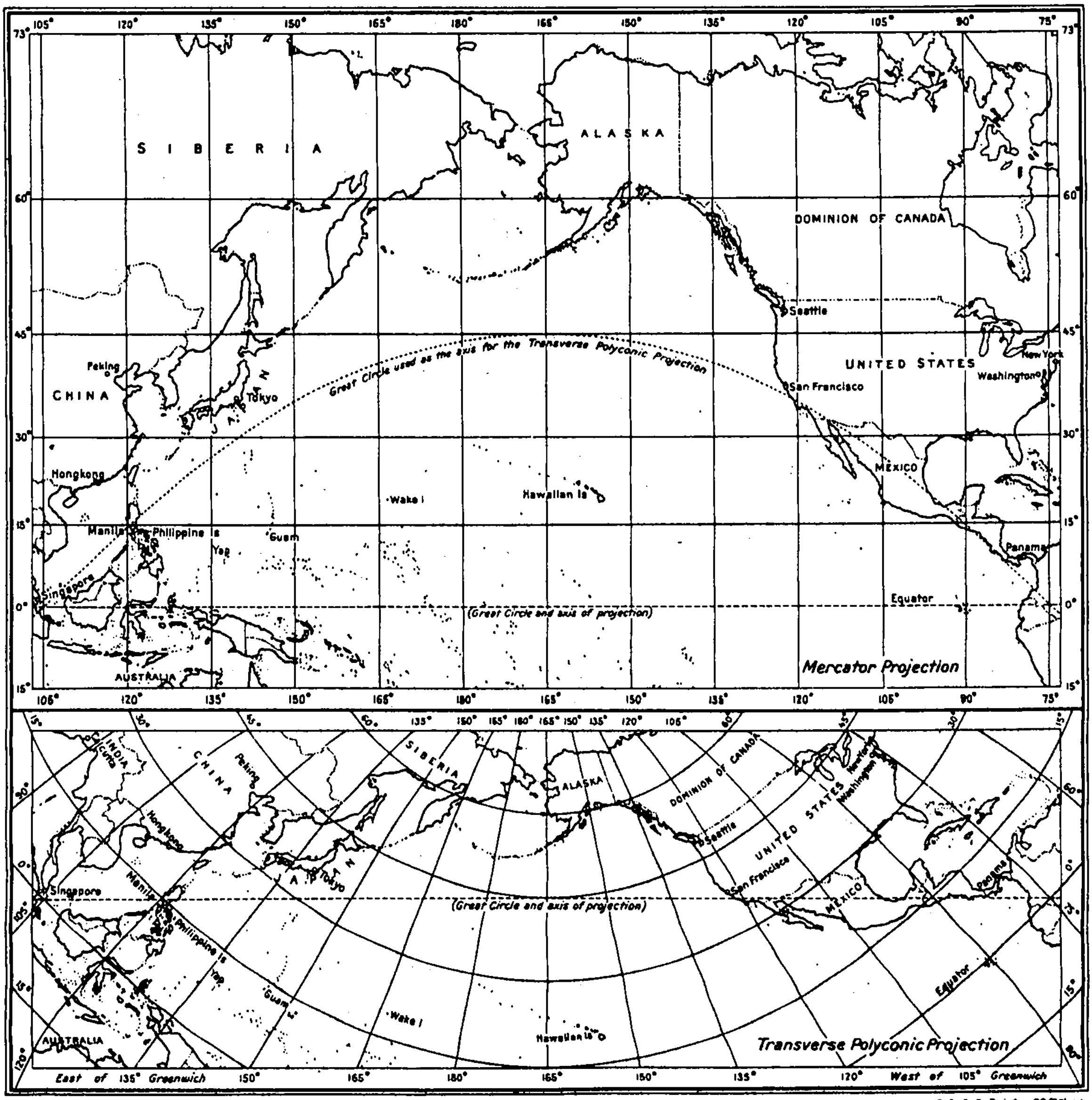

FIG. 84 - THE NORTH PACIFIC REGIONS ON THE MERCATOR AND THE TRANSVERSE-POLYCONIC PROJECTIONS

The rapidly increasing scale of the Mercator projection, amounting to 100 per cent in latitude 60° apparently "puts Alaska to far north." In contrast the transverse polyconic projection shows geographical features more nearly in their true relative positions, sizes and shapes.

with Eastern North America and Europe, is constructed on this projection, having been prepared for the Army War College and also used by the Shipping Board during the first World War.

Former maps and charts of this area showed excessive variations in scale. On a polyconic projection for the same limits, the maximum scale error would be 100 per cent; on a Mercator projection the scale variations would be about 180 per cent. On our Lambert map, the scale shows a maximum increase of only 11 per cent, and that occurs in latitude 70°, an unimportant part of the map. The great central portion of the map, from latitude 30° to latitude 60° shows scale varying from zero to 2 per cent only.

MERITS OF THE LAMBERT PROJECTION

In a map of the United States with standard parallels at 33° and 45° the greatest change of scale is but 2½ per cent. As employed by the U. S. Geological Survey for State maps, 1:500,000, the projection is ideal and should meet engineering, military, and geographic requirements. It is also employed in the construction of aeronautical charts by the Coast and Geodetic Survey for the Civil Aeronautics Administration of the Department of Commerce.

The graphic scale of any State map constructed from the general tables of the United States, should be based upon the local or middle latitude scale as given for different latitudes. This operation will assist in bringing the excess or reduced scale of any State still closer to true scale.

For the measurement of distances in all directions within the United States, the Lambert projection is superior to any other excepting possibly the Albers projection. However, by use of a scale factor (ordinarily not necessary), the Lambert projection provides for exact scale; and if exact scale is not obtained, the reasons for it are found in errors of compilation, method of printing, shrinkage or expansion of paper in opposite directions.

In errors of azimuth as compared with the polyconic, Lambert zenithal, and Albers, it is superior to the others here mentioned. To obtain an azimuth on a Mercator projection requires a special computation or the use of tables.

We find in projections like the polyconic, the Bonne, and the Mercator that, after proceeding about 25 degrees from one of the axes, the scale errors or scale variations become so increasingly greater that these projections totter and can no longer be utilized for geographic purposes. The Mercator projection is at its best from, say, 15° north latitude to 15° south latitude, and for the rhumb line property alone, it may be carried to higher latitudes. The United States is, then, beyond the zone of usefulness of the Mercator projection except for the property of the rhumb line, but the rhumb is retained at the expense of the great circle, directions, and scale.

For a line from New York to San Francisco, we find the great circle passing through its extremities has departed from the rhumb 181 miles. Higher up, before reaching the forty-ninth parallel, the difference in a 2,700-mile line is 240 miles. It is evident, in these higher latitudes especially, that the Mercator chart may require an auxiliary chart or special computations for meridional crossings of the great circle. In many instances it may necessitate the conversion of true bearings to mercatorial bearings.

If radio signals following the great circle can be utilized on the Lambert projection, it might seem that the Lambert projection and the gnomonic projection are identical. They are not identical, but numerous tests show that on the Lambert projection of the United States, by reason of conditions imposed by its standard parallels of true scale, a straight line may follow the great circle closely, it may cross it and recross it, never receding from it excessively.

Distance Comparisons, New York to San Francisco

[New York (lat. 40° 45′, long. 73° 59′); San Francisco (lat. 37° 47′, long. 122° 25′)]

	Statute miles
True distance [39] New York to San Francisco, by long distance formula	2, 572. 3
Straight line distance on C. & G. Survey Chart No. 3060b (Lambert projection)	2, 575
Rhumb line (or straight line on Mercator chart), computed	2, 609. 3
The rhumb line is longer than the true distance	37. 0
Distance of straight line (New York to San Francisco) below the great circle on the Lambert projection at a central meridian is only	9. 5
Distance of rhumb line below the great circle on the Mercator projection (also the actual distance between them on the earth), at a central meridian, is	181. 0

Due to the location of the central parallel on the Lambert projection, the straight line from New York to San Francisco follows the geodesic closely, crossing and recrossing it, with results in distance measurement differing less than 2 miles from actual computation.

A geodesic on the Lambert projection, in the case of two standard parallels, is always concave toward the *central* parallel; consequently in crossing the central parallel it changes its curvature in order to maintain its concavity toward the said parallel. The term geodesic, also termed a geodetic line, is the shortest or minimum line that can be drawn between any two points on the *ellipsoidal surface* of the earth.

The difference between the length of a great circle as obtained by the solution from a spherical triangle, and that obtained from a geodesic, leads to a refinement difficult to compute and ordinarily of no great importance.[39]

Great Circle and Rhumb Line Comparisons

From the many other tests in the measurement of distances, a few more may be given:

Boston to Seattle, great circle distance	2492.5 stat. mi.
Boston to Seattle, Lambert straight line	2503.5 stat. mi.
Excess of Lambert	11 stat. mi.

Washington to Los Angeles

Mercator, rhumb line below great circle	125 stat. mi., approx.
Lambert, straight line departs only	2 stat. mi., approx.

New York to Seattle

Mercator, rhumb line below great circle	180 stat. mi., approx.
Lambert, straight line below great circle	21 stat. mi., approx.

Quebec to Seattle

Mercator, rhumb line below great circle	200 stat. mi., approx.
Lambert, straight line below great circle	37 stat. mi., approx.

[39] The final test of measured distances from any map or chart depends upon the system of projection, the accuracy of compilation method of printing, etc. It may be advisable in many instances where longer distances are involved not to use the map or chart, nor even the solution from a spherical triangle. For example, the distance from New York to San Francisco by solving the spherical triangle, *using sphere equivalent in area to spheroid*, is 2,566 statute miles—or 6 miles less than the actual distance. For the computation of long distances, see Special Publication No. 138, pp. 207–208. Still greater refinement may be obtained by use of formulas in Special Publication No. 100.

The following are approximate distances by use of Weather Plotting charts, Alaska and North America, 1–10,000,000 (Lambert projection, standard parallels 30° and 75°); H. O. Track chart No. 1262, (Mercator projection), and H. O. great circle sailing chart No. 5300:

Seattle to Yokohama at 173° East Longitude

Mercator, rhumb line below great circle .... 780 stat. mi., approx.
Lambert, straight line above great circle .... 60 stat. mi., approx.
The distance between these points on H. O. Track chart No. 1622, is 4276 naut. mi.

Panama to Yokohama at 130° West Longitude

Mercator, rhumb line below great circle .... 1850 stat. mi., approx.
Lambert, straight line above great circle .... 350 stat. mi., approx.
The distance between these points on H. O. Track chart No. 1262, is 7682 naut. mi.

It has been stated that a straight line on the Lambert chart of the United States *approximates* a great circle. While it is only an approximation, as the gnomonic projection alone represents the straight line exactly as a great circle, it is a very close one.

From the preceding summary which represents a few of numerous tests that have been made, it is evident that the approximation is of such an order as to serve most practical purposes for distance measurements and for a great-circle track.

The listed departures of a straight line from the great circle as given in the above examples are not as important as the measured distance between points. The distance along curved lines becomes greater than that along adjacent straight lines, only as the departure is sufficient. Within the limits of the United States, from the standpoint of the added distance involved, the departure is not sufficient to be taken into consideration in the use of a Lambert projection for air navigation.

The last example, Panama to Yokohama, is an unusually long distance, and represents a great-circle track of more than 8800 statute miles. In such an instance it would be advisable to transfer the great circle from the gnomonic to the Lambert chart, just as it is advisable, in much lesser distances, to transfer it to the Mercator chart.

If a straight line be drawn on a Mercator chart and a plane sets a course to follow that line, assuming no leeway, it will pass over the features through whose charted positions the line passes, providing that corrections are continually made for ever-changing magnetic variation. However, the following data, taken from a Lambert projection covering latitude 47° to 49°, show the small amount that a plane would depart from charted features under similar conditions. This particular chart was chosen as showing the most unfavorable conditions in both the Lambert and Mercator projections:

In 100 statute miles (Lambert projection) the departure is only ¼ mile.
In 200 statute miles (Lambert projection) the departure is only 1¼ miles.

As the plane flying a rhumb line would theoretically pass directly over the charted features, the above comparison favors the Mercator projection by the small amounts indicated. In practice, however, when a mean magnetic course is applied, the plane will no longer track a straight line even on the Mercator chart, the departure at the middle of a 280-mile course, in an east-and-west direction in lat. 48°, being as much as 3.7 miles. This departure is unrelated to the projection used, being due entirely to the practice of flying a mean magnetic course. Conditions of this order exist also along the New York-Pennsylvania boundary and westward for a considerable extent.

On the other hand, in a radio range, assuming a line 200 miles long, in the same locality and direction, the Lambert projection straight line departs from the great circle one-third statute mile—an error quite negligible. In contrast, on the Mercator projection, unless the conversion from true to mercatorial bearing is applied, the error is 5.6 miles. At 300 miles this becomes 12.6 miles, and at much greater distances, unless conversions are applied, these errors of the Mercator become excessive.

The use of conversion tables is thus necessary at times to determine the plus or minus corrections to directional bearings on the Mercator chart, and the utility of the latter projection is lessened comparatively by the self-contained safety properties embodied in the Lambert projection.

All indications point to an increase in use of radio ranges to guide planes. While in general, up to this time, the distances used have been short, there have been cases, as in that of the *Norge* in its flight across the North Pole, where radio bearings have been taken to stations hundreds of miles distant.

A report from the North German Lloyd steamship *Bremen* indicates the great distances over which radiobeacons are sometimes heard under unusual conditions. The *Bremen*, when in latitude 42°28′ and longitude 58°11′, on December 10, took several bearings on the Colon (Canal Zone) radiobeacon, at a distance of 2,289 miles. The minimum was sharp and the bearing varied not more than one-half degree from the great-circle course from the ship's position.

In connection with nautical charts, Admiral Tonta, of the International Hydrographic Bureau, Monaco, says: "We are convinced that in very many circumstances, the Lambert projection can, with appreciable advantage, replace Mercator's projection as a basis for nautical charts."

Radio Bearing Comparisons

Plotting radio bearings on a Lambert projection from three assumed positions of radio stations in Wisconsin, Minnesota, and North Dakota, the stations being distant 161, 227, and 315 statute miles, respectively, the fix obtained from plotted bearings, as well as from a 3-arm protractor, is practically perfect either way, being less than one-half a statute mile.

On the Mercator projection when the bearings are plotted without correction, there is a triangle of error from which an adjusted position indicates an error of 7.7 statute miles from true position, and when plotted with a 3-arm protractor, if the bearings are taken by the plane, the error is 9.7 statute miles.

Using only the two more distant radio stations, the intersection of the bearings in a Mercator projection is distant from the true position 18½ statute miles; on a Lambert projection the error is only 1.7 statute miles.

On a single bearing the maximum error on a Mercator projection is 3½ statute miles; on a Lambert, less than ½ statute mile. A single bearing does not itself determine position, but it serves as a check on the progress of a plane along a postulated dead-reckoning line, if the bearing makes an angle with that line such as to give a definite intersection.

In a number of tests that have been taken with three bearings in the locality from Newfoundland to Labrador, Greenland, Iceland, and Ireland, on Lambert charts of scales somewhat larger than that of the Lambert chart of the United States, very satisfactory results have been obtained where distances are within 500 statute miles from stations.

Even to 750 miles bearings have been determined within an error of one degree, an amount equal to about one-half of the error inherent in the observations of the directions themselves. In the more favorable central portions of the chart, bearings may be plotted with satisfactory accuracy for much greater distances.

While other examples for determining and comparing position from radio bearings may be better or even worse than the ones given, it is evidently necessary to determine carefully the plus or minus corrections to directional bearings on a Mercator chart.

It is expected in the near future to have an arrangement of stations and schedules such as to facilitate the use of three bearings at almost any place and time.

Celestial Navigation

In regard to the advantages of the Lambert projection in celestial navigation, statements relating to the subject appear in the special report of 1918 by Captain J. P. Ault who also commanded the nonmagnetic ship CARNEGIE of the Department of Terrestrial Magnetism. The elaborate program of the continuous cruises for a number of years embraced the seven seas and covered several hundred thousand nautical miles. Having given considerable attention to charts and maps for celestial navigation in the air and having himself constructed a Lambert projection, scale 1:1,000,000, to test its adaptability in various ways, he states that "it was found to meet the requirements of the problem in every respect," and that it would have several advantages over the Mercator projection.

Junction of Small-Scale Charts

In extended areas on a Lambert projection when an upper sheet is joined to an adjacent lower sheet, both sheets on the same scale but on their own standard parallels, caution must be exercised in providing sufficient overlap. While each sheet is in itself conformal, the continuity of conformality ceases at the junction of the sheets.

Thus, in the smaller scales, in the problem covering a wide latitude or extensive area on one sheet, the determination of position from radio bearings is not always as precise nor as simple as might be desired. If the bearing is to be plotted from a point far removed from the center of the projection, a correction may have to be applied if errors greater than about 1° are considered objectionable. If a bearing observed at the plane must be plotted from a station on an adjoining chart, the reverse of the bearing must be corrected for convergence, plotted from the station to the edge of the chart, and continued on the second chart at the same angle at which the last meridian of the first chart was intersected. This applies to any system of projection where a general chart is not available, and may introduce the need of a series of general charts, or of an intermediate or gnomonic projection with specially computed compass roses constructed at the various stations that may be available.

THE LAMBERT PROJECTION FOR SPEED AND ACCURACY

For speed and accuracy the great circle or shortest distance between points should be represented by lines that are straight, all angles as nearly true as possible, and the scale the same at all points of the map and in all directions.

For all of these properties the Lambert projection provides the best pattern that can possibly be attained—a projection that will facilitate the accurate and immediate solution of problems of distance and direction—a projection superior to others in its adaptability to all radio navigational methods. It generally avoids the unnecessary introduction or plotting of a great circle, a property contained with sufficient closeness within the limits of the United States. It generally avoids increments in scale, and angular variations involving a correction, the sign of which, plus or minus, may be confusing.

The amount of scale variation wholly unfits the Mercator projection for land maps—see Fig. No. 85—and gives rise to needless inconvenience when applied to many other purposes.

With the distinctive features of land surface relatively represented in detail on an accurate chart and with the various other aids to guide the mariner and aviator, the Lambert projection is the leading instrument of today for safe and rapid methods of navigation.

OTHER ADVANTAGES OF THE LAMBERT PROJECTION

It has the advantages of practical modern tests and thorough investigation into present day requirements. Such needs must not be left to chance, in following like fashion an old custom, but must be dictated with analytic rigor. The Mercator projection has stood the test for nearly three centuries, but is it not a fact that "use doth breed a habit in a man", and create an unwillingness to further improvement without investigation, and to the ultimate decay of scientific advancement? In the words of Herbert Spencer: "There is a principle which is a bar against all information, which is proof against all argument and which can not fail to keep a man in everlasting ignorance; this principle is contempt prior to examination."

The Mercator projection is a mathematical navigational *device,* but for a map of an area the size of the United States with its longitudinal predominance, for example, it serves no general useful purpose. The Lambert projection for air navigation is superior to the Mercator projection, having certain important practical advantages over the latter in this respect. Besides this, the Lambert projection is strictly the best projection attainable for engineering, tactical, and other wider uses. For a geographic comparison see Plate XII based on parallel 39°. The total scale variation of the Lambert projection of the United States is slightly less than 3%, and in the wide central section between latitudes 30½° and 47½° the variation is within one-half of 1 percent.

Directions and distances, the fundamentals of navigation must be accurate beyond question, for together they give position, while accurate shape and uniform scale facilitate identification.

As all nautical charts and nautical literature are based on the Mercator projection, it is possible that organizations finding it necessary to use both projections may experience difficulties, due to the different properties and methods of use of the two systems.

The Mercator projection cannot be used, of course, in high latitudes, and it is impossible to use any one system of projection for the entire world. In each case, therefore, the system which offers the greatest advantage for a particular area and problem should be used, instead of following an old custom.

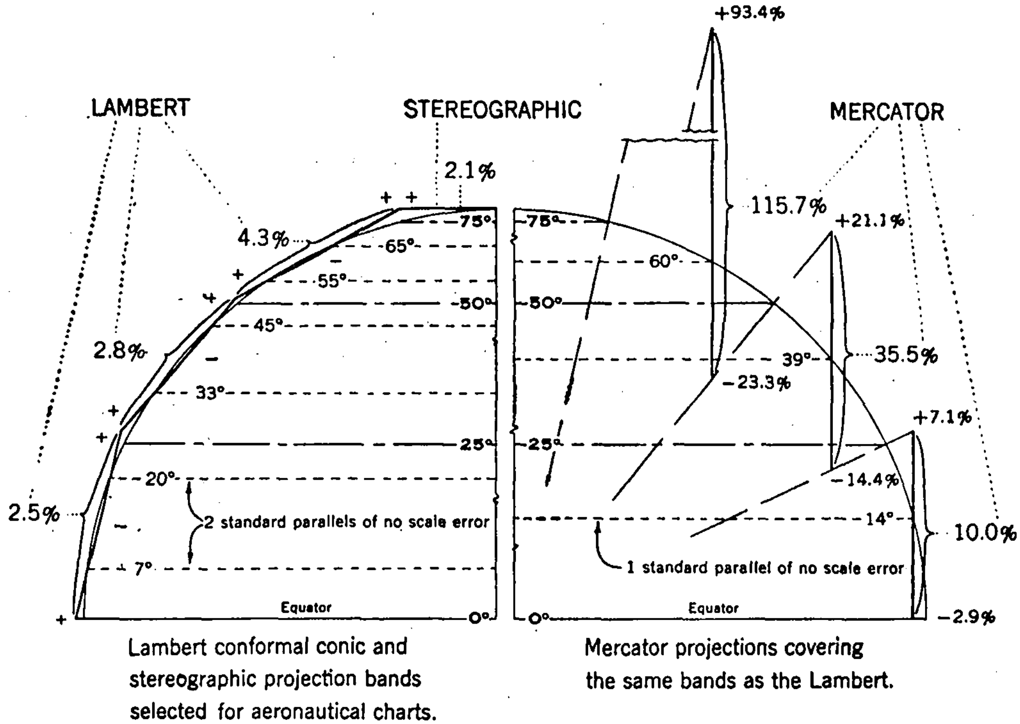

FIG. 85.—Variation in scale of each projection band.

THE LAMBERT OR THE MERCATOR FOR AERONAUTICAL CHARTS

The question arises in aeronautical charts whether we prefer a projection like the *Mercator which has a certain special property* and conveniences and which will permit, if necessary, the incorporation of navigational fundamentals from an auxiliary chart, the gnomonic; and, furthermore, which will follow the style set by nautical charts; or, whether we prefer a projection like the Lambert which, besides its own properties, is not too remote in other requirements, but is sufficiently accurate to be used directly for the receipt of radio signals emanating from long distances; a projection which structurally provides for the best pattern of the United States that can possibly be attained, and therefore well adapted for pilotage; a projection which will facilitate the accurate and immediate solution of problems of distance and direction, and in this respect far superior to the Mercator; a projection with minimum scale variations (the Mercator having an increment of 40 percent in the United States at its northern border, necessitating adjustment to these increments when making compilations); a projection which, in addition, will serve for engineering, tactical or other wider uses.

Considering the predominating east-and-west extent of the United States—a ratio of about 9:5 compared with its meridional extent, the adaptability of the Lambert system to one complete map of the *country and its configuration*, is almost exceptional, the greatest scale variation being but 2½ per cent, while the wide central section between latitude 30½° and 47½° is favored with a scale variation within one-half of 1 per cent. Direction and distances, the fundamentals of navigation, must be accurate beyond question, for together they give position, while accurate shape and uniform scale facilitate identification.

The better facilities at this day for determining location and direction, as furnished by radiobeacons, lights, prominent objects, terrain, etc., bring air navigation methods in the United States more and more into the plane of pilotage than they are over the open oceans where landmarks are seldom available and the problem is largely one of celestial navigation. The Lambert's chief point of superiority lies in its peculiar suitability to all radio navigational methods—a consideration of primary importance since it is evident that radio navigation is destined to assume a dominating place.

Aeronautical charts within the United States present a problem distinctly of their own. For these charts, the sufficiently close approximation to the great circle as supplied by the Lambert projection, together with pilotage, outweigh the rhumb line. It may be well to remember that the Lambert projection is based upon two standard parallels of true scale within our own domain. It is not based upon a distant equatorial line causing increasing scale variations, etc., as previously stated.

In this summation we have endeavored to present the salient features of the Mercator and Lambert projections, and in this final analysis, the object has been to show in a few sentences the changed conditions from the days of the Portolan charts to the day of the Lambert projection, conditions as new as the airplane itself.

MATHEMATICAL RELATION BETWEEN THE LAMBERT, THE STEREOGRAPHIC, AND THE MERCATOR PROJECTIONS

These three projections are conformal projections, and the last two may be considered as special cases of the Lambert. When the tangent cone of the Lambert projection becomes a tangent plane at the pole, we have a projection of the spheroid that merges into the stereographic projection; and when the parallel of tangency approaches the equator, the Lambert projection of the spheroid merges into the Mercator projection. A more complete discussion of the relation between these three projections is given on pages 32–35, Special Publication No. 53, U. S. Coast and Geodetic Survey.

MERITS OF THE ALBERS EQUAL-AREA PROJECTION

In the wall map of the United States constructed by the U. S. Geological Survey on the Albers projection, scale 1:2,500,000 with standard parallels at 29° 30′ and 45° 30′, the greatest scale error is but 1¼ per cent. It may be noted that the former wall map of the United States on the polyconic projection had a maximum scale error of 7 per cent.

It is in keeping with the practice followed by the best European atlases that political divisions, continents, and hemispheres should be based upon equal-area representation, a property which best meets the problems of geography. Where the area scale is constant, direct comparisons for area and distribution become available.

If the Lambert projection as used for States were extended to a general map of the United States, the scale variation would indicate an excess in areas of as much as 5 per cent along the southern border. If the Albers projection were used for the separate States, a better area representation would have been obtained but at the expense of conformality which is a desirable property in engineering projects. We must remember that in conformal maps, conditions at any point on the map are better than in equal-area maps in which the scale, when 1 per cent too large in one direction, is 1 per cent too small in the direction approximately at right angles to it; so that at any point, or from any point, conformality better meets engineering requirements. In units of the size of States these requirements have certain rights and the Lambert projection serves them better. In the larger general map of the United States 1:2,500,000, the requirements are mostly those of geography, in which minimum scale error and equal-area representation govern.

The Albers projection with its two standard parallels of true scale has, in addition, at every point on the map two intersecting lines or curves of true length scale, equally inclined to the meridian and approximately at right angles to each other, see plate III and Fig. 59. These curves of true scale are termed isoperimetric curves running at oblique directions through the map. These curves are sufficiently flat to indicate directions of true scale in such a way that straight lines which lie approximately northeasterly or northwesterly, or their opposites, along the path of these curves are practically true to scale. Besides its special equal-area property, there is in the Albers projection a liberality of true scale not to be found in other systems.

In a comparison of maps of the United States on a Lambert conformal projection, scale 1:5,000,000, dimensions 25 by 39 inches and an Albers projection, if constructed to the same scale and limits, the difference at the corners of the two maps amounts to one-seventh inch only. This and other similarities indicate that the Lambert projection for the United States is nearly an equal-area projection, and that the Albers projection which is an equal-area projection is nearly conformal.

THE AZIMUTHAL EQUIDISTANT PROJECTION

This projection takes its name from the fact that straight lines radiating from the center of the map represent great circles in their true azimuths from it, and the distances, along these lines are true to scale. The projection is neither equal-area nor conformal, and further description is given on pages 175–176. In addition to its use as noted on the foregoing pages, two charts have been issued on this projection by the Hydrographic Office, U. S. N., one of these with the center at Washington and another with its center at San Francisco. Any important city can well afford to have a map constructed on this projection for general use.

It is doubtful, however, whether for geographic purposes this projection should be used for continents, as its center in most instances would be a point of no cartographic interest, and the true scaling properties from such a point would be obtained at the expense of greater scale errors in other parts of the map. The Lambert azimuthal equal-area is, for geographic purposes, more desirable in having besides its special property a lesser maximum scale error than the equidistant projection.

ERRORS OF DISTANCE MEASUREMENTS

The question frequently arises as to the best projection for scaling distances between any two points within the United States. The following study is based solely on the system of projection and shows to some extent the errors due to scale variations in the several projections that are in use at the present day. In this analysis no account is taken of paper distortion, process errors in reproduction, or errors due to an assumed position of a city center.

It has already been stated in this publication that it is impossible to scale true distances from more than one or two points. The azimuthal equidistant projection has been employed for such purposes when one point only has been considered. A two-point projection is also possible but apparently no map of this kind has ever been constructed. As for the thousands of points or places within the United States, only an approximation in distances can be obtained; and each projection has its merits and defects according to the relative geographic position of the points involved. Resultant errors are occasionally so great as to make long distance errors inadmissible unless corrections are applied.

The Polyconic Projection

Due to the increasing divergence in its parallels, measured meridional distances along the eastern and western coasts of the United States may be too great by as much as 7 per cent. Along a line from San Francisco to Seattle the scaling error due to projection alone is about 4½ per cent and the measured distance between these two cities is 29 miles greater than the true distance. Even at New York there is a meridional excess error of 4¼ per cent. In the coastal belts the errors are inadmissible. However, directions that are closely east-and-west in the United States provide distances within reasonable accuracy and are generally better than those obtained from either the Lambert or the Albers projection when uncorrected for scale factors.

Errors in the meridional scale may be very closely expressed in percentage by the formula: $E=+\left(\frac{l^\circ \cos \phi}{8.1}\right)^2$, in which l°=distance of point from central meridian expressed in degrees of longitude, and ϕ=latitude.

By the use of the above formula, long meridional or diagonal distances may be corrected or approximated on the polyconic projection.

The Lambert Conformal Conic Projection With Two Standard Parallels

By reason of the adaptability of this projection to the configuration of the United States, and the use of two standard parallels of true scale, the errors in general scaling properties are minimized. However, due to the placement of the standard parallels at 33° and 45° in the map of the United States referred to, a line from New York to San Francisco, midway between the standards, indicates a shortage in measured distance of 12 miles due to the projection alone. A line from easternmost Maine to Cape Flattery, Wash., is too long by as much as 28 miles. Meridional measurements, except those including southern Florida and southern Texas, generally do not indicate serious errors.

The use of the table of relative scale factors along the parallels as given in the following table, will provide a correction to measured distances where greater precision is desired. It should be observed, however, that the values given in the table should be used as a divisor in order to obtain the corrected measurements.

Relative scale factors along the parallels for the Lambert Conformal Conic Projection for a map of the United States

Latitude	Scale factor	Latitude	Scale factor	Latitude	Scale factor
° ′		° ′ ″		° ′	
24 00	1.02759	34 00	0.99837	43 30	0.99755
24 30	1.02545	34 30	.99766	44 00	.99828
25 00	1.02337	35 00	.99702	44 30	.99910
25 30	1.02137	35 30	.99645	45 00	1.00000
26 00	1.01944	36 00	.99596	45 30	1.00099
26 30	1.01759	36 30	.99554	46 00	1.00206
27 00	1.01581	37 00	.99519	46 30	1.00322
27 30	1.01410	37 30	.99492	47 00	1.00446
28 00	1.01246	38 00	.99472	47 30	1.00580
28 30	1.01089	38 30	.99459	48 00	1.00723
29 00	1.00940	39 00	.99454	48 30	1.00875
29 30	1.00797	39 05 13.27	.99454	49 00	1.01037
30 00	1.00662	39 30	.99457		
30 30	1.00534	40 00	.99467		
31 00	1.00413	40 30	.99484		
31 30	1.00299	41 00	.99510		
32 00	1.00192	41 30	.99543		
32 30	1.00092	42 00	.99584		
33 00	1.00000	42 30	.99633		
33 30	0.99915	43 00	.99690		

The scale of this projection remains constant along each parallel but varies along the meridian. At any given point, however, the scale is the same in all directions. Corrections to measured distances along the meridian depend upon scale factors corresponding to the different latitudes and can be taken from the same table.

On recent editions of this chart, a diagrammatic scale, 2500 statute miles in length, has been constructed for latitudes from 25° to 50° at intervals of 5°. Smooth curves have been drawn through corresponding graduations at each latitude, facilitating interpolation for scaling distances at intermediate latitudes also. The scale factor for each latitude was taken into account in graduating the scales, making it possible to obtain very accurate results without further recourse to the scale factors.

The Albers Equal-Area Projection With Two Standard Parallels

A wider separation of the standard parallels in a map of the United States on an Albers projection as compared with the Lambert conformal conic projection described above, results in scaling properties somewhat different in the two systems of projection. In the Lambert projection the standard parallels are placed at 33° and 45°, and in the Albers projection the standards are placed at 29½° and 45½°. Approximately east-and-west measurements between the standards on an Albers projection may be too short by nearly 1 per cent, and a line from New York to San Francisco indicates a shortage of 19 miles. Only in this narrow belt between the standards do we find the Albers projection noticeably inferior to the Lambert projection. The scaling properties of the Albers projection on the other hand are superior to the Lambert in the southern portion of the United States. Other lines of true scale appear in the Albers projection not to be found in either of the systems already considered. These are the families of curves running northeasterly or northwesterly, or their opposites, through the map, in such a way that straight lines near the path of these curves are practically true to scale. (See "Merits of the Albers equal-area projection," p. 200.)

Regardless of the liberality of true scale in the Albers projection, its chief fault is in the measurement of east-and-west distances between the standard parallels. However, corrections can be made to measured distances by the use of scale factors given in the following table. The factors should be used as a divisor to measured distances.

Relative scale factors for the Albers Projection for a map of the United States

Latitude (degrees)	Scale factor		Latitude (degrees)	Scale factor		Latitude (degrees)	Scale factor		Latitude (degrees)	Scale factor	
	Along parallel	Along meridian		Along parallel	Along meridian		Along parallel	Along meridian		Along parallel	Along meridian
52	1.0286	0.9722	45	0.9987	1.0013	35	0.9916	1.0085	25	1.0122	0.9879
51	1.0226	.9779	44	.9964	1.0036	34	.9925	1.0075	24	1.0155	.9847
			43	.9945	1.0056	33	.9937	1.0063	23	1.0191	.9813
50	1.0172	.9830	42	.9929	1.0071	32	.9952	1.0048	22	1.0228	.9777
49	1.0125	.9876	41	.9918	1.0083	31	.9970	1.0031	21	1.0268	.9739
48	1.0083	.9918	40	.9910	1.0091	30	.9989	1.0011	20	1.0310	.9700
47	1.0046	.9954	39	.9905	1.0096	29½	1.0000	1.0000			
46	1.0014	.9986	38	.9903	1.0098	29	1.0011	0.9989			
45½	1.0000	1.0000	37	.9904	1.0097	28	1.0036	.9964			
			36	.9900	1.0092	27	1.0062	.9938			
						26	1.0091	.9910			

The Mercator Projection

On account of the increment in scale in a map of the United States, reaching approximately 40 per cent at the northern limit, the use of scale-units derived from the vertical border scale is not always satisfactory. The use of a diagram showing a long horizontal scale for the different latitudes will provide better scaling units if taken with sufficient frequency when wide extent of latitude is involved in a given line.

The scale increments in a Mercator projection must not be considered as an error of the system of projection. These increments are entirely due to an orderly change of scale, and if a diagrammatic scale is employed as stated, distance measurements will be available except in the higher latitudes of charts where the greater changes in scale may render distance measurements difficult.

In the measurement of long lines on a Mercator chart it may be necessary to compute, or transfer the apex (or several points) of a great circle from a gnomonic chart, and to measure along the path of the great circle instead of the rhumb line, the measuring unit changing with the latitude.

From this summary it is evident that in the problem of measuring long distances between any two points within the United States, any given projection has definite limitations. However, by the use of scale factors in portions of the map where scale variations are considerable, or by the use of suitable diagrammatic scales, the errors due to the system of projection itself may be very much reduced.

Although the polyconic projection indicates serious errors in long-distance measurements in the United States—possible errors in excess of 40 miles—the following procedure in cases where base maps on different projections are available, will provide fairly good results when suitable diagrammatic scales are absent, without resort to scale factors:

The use of the polyconic projection for all distance measurements in directions east-and-west or close thereto, and the Albers or Lambert projection in directions approximating north-and-south. Intermediate directions present no very decided choice, nor one that is easily formulated. The frontispiece of this publication shows the nature of the three projections as far as their scaling properties are concerned.

WORLD MAPS

In regard to world maps again, the difficulties are not solely due to the attempt to depict the spheroidal surface of the earth upon a plane. It is the great land masses of the Eastern Hemisphere with their political considerations that operate against evenly balanced alignments in tripartite or quadripartite units. In the middle ground between units larger than continents and smaller than hemispheres, such studies as are lacking in orderly arrangement, i. e., essentially different from the picture on a globe, are unsuited for geographic purposes. It has been stated in language terse and effective, that the best way of judging a world projection is to look at it.

It is well to remember that when meridians are discontinuous, it naturally follows that the configuration is distorted in the corresponding localities. In this respect it would be better for map makers to follow more closely the demands of the science of mathematics.

Projection lines should appear at reasonable intervals as they are the control and key which tell us that the structure is sound. They should be lightly drawn so as not to detract from the general picture, and the name of the projection should appear on the map as an index to the purpose it may serve.

For equal-area world mapping in hemispheres, the following plates on the Lambert meridional equal-area projection are of a high order of excellence:

Atlante Internazionale del Touring Club Italiano, Milano, 1929, Plates 1, 2, and 3, diameter of hemispheres, 15¾ in.

Nordisk Världsatlas, Plates 1, 2, 3, and 4, 14 inches by 16 inches, showing more than hemispheres in east-and-west dimensions.

The above-mentioned projection as employed for the configuration of the oceans, in the Nordisk Atlas, is now being used in the preparation of world maps for seismological studies. It is useful for plotting positions of earthquake epicenters, and serves to eliminate the effect of distortion of other projections, especially in the polar regions. The arrangement is well adapted to show the earthquake belts with the fewest possible breaks in their continuity.

Two projections of the sphere that are found in some atlases, as giving a fair representation of the world, are the Van der Grinten projection and Gall's projection. These two projections are neither conformal nor equal-area and may be classed as intermediates having no properties of definite scientific value. They present a fair uniformity in the configuration of the world, avoiding the excessive scale increments of the Mercator in the higher latitudes and lessening the distortions of equal-area projections. Their utility nevertheless is pictorial and their practical importance is limited.

RECENT CONTRIBUTIONS TO AERONAUTICAL CHARTS

The development of great circle navigation as effecting the elimination of circuitous routes by the use of auxiliary charts and tables, has received considerable attention in recent years. Through radio communication and the adaptability of other than Mercator projections, new methods have been offered.

For planning and carrying out continental and intercontinental flights, a type of projection, new in its application to navigation, has been brought into use by Louis Kahn. Strip maps which cover some of the principal routes of the world have been constructed by him and published by Ed. Blondel la Rougery, Paris, 1932, scale 1 : 10,000,000. They are constructed on a transverse Mercator projection (also known as the Gauss conformal) and are intended specially for great circle navigation. These maps also contain a system of squaring and a system of declination circles or star paths, with a border scale subdivided into hour angles, facilitating position finding astronomically by graphic means with no tables necessary other than those contained in the nautical almanac. The development of the circle of contact which represents the theoretical route is used as axis of the chart, and the map is useful for 15 degrees on either side of this axis to the extent that the ordinary Mercator projection is without criticism or need of an auxiliary chart or tables from 15° north latitude to 15° south latitude. In this transverse system of projection both meridians and parallels become curved lines, but due to the fact that the charts are ordinarily so limited in their width, the curvature of the meridians is rather limited and straight lines are approximately the paths of great circles throughout the chart. The rhumb line is no longer a straight line and the method for laying off courses from a meridian central to the course will have to be followed. Better maps, especially in the higher latitudes, are thus obtained at the expense of the rhumb line and the old method of laying off courses from the initial meridians.

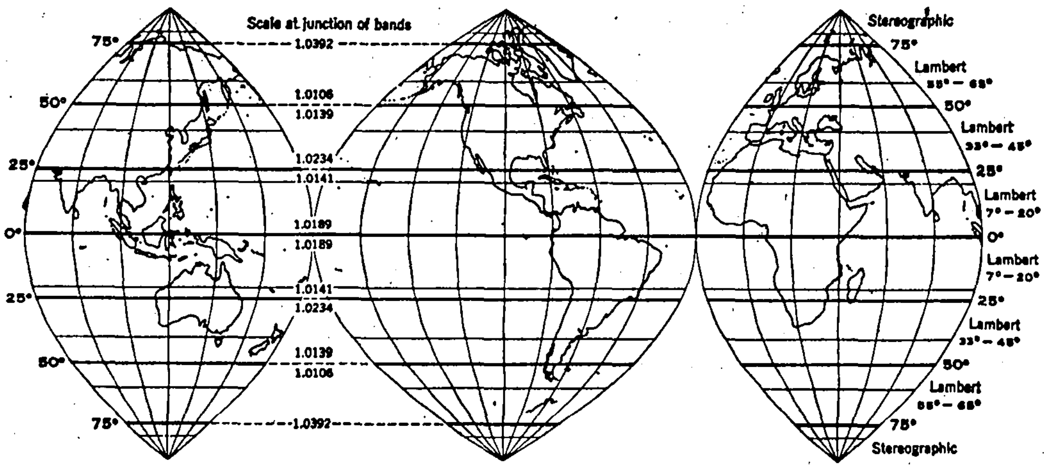

FIG. 86—Projection scheme for aeronautical charts of the World.
Scale of Series 1:5,000,000

Showing types of projection and notation of standard parallels for the various latitude bands

In the useful and timely textbook on projections by Driencourt and Laborde, 4 volumes, Paris, 1932, we find a number of solutions of the problem of world flights, and the following is of special interest:

Employment of Mercator projection from lat. 15° N. to lat. 15° S,
Employment of Lambert projection from lat. 10° to lat. 40°
Employment of Lambert projection from lat. 30° to lat. 60°,
Employment of Lambert projection from lat 50° to lat. 80°,

concluding with charts of the polar regions on the stereographic projection with 15 degree radius.

It is seen from the above arrangement that the third chart of the series corresponds to Coast and Geodetic Survey Chart No. 3070, covering a part of the North Atlantic Ocean with Eastern North America and Europe, scale 1 : 10,000,000.

The arrangement, in toto, offers an instrument for general navigation with overlaps (or recoveries) of 5 degrees to 10 degrees, and is well suited to aerial traffic which has its greatest development in an east-and-west direction. These overlaps serve as convenient liaisons for routes where origin or destination are beyond the limits of any one chart.

It is believed that the several conformal systems of projection above mentioned, are in their latitudinal extent sufficiently limited so that radio bearings may generally be utilized directly without correction. It is not the process of such corrections as much as the possibility of applying them erroneously and in the wrong direction, that calls for methods making the benefits of great-circle navigation more readily available.

A new series of projections, in some respects similar to the preceding arrangement, has been designed by the U. S. Coast and Geodetic Survey and is now employed for the World Aeronautical Charts of the U. S. Army Air Forces. The scheme provides for a total of 43 charts, and a diagram showing the main features of this arrangement is given in Fig. 86.

In connection with the new series, it is evident that the system is readily adapted to subdivisions of larger scale.

Considerable study is given at the present day to the subject of radiogoniometry for the purpose of devising simpler methods and instruments for plotting directional bearings and great-circle routes more readily. In this conception of the problem, the need of simple and rapid methods is greatest in charts of large geographic extent covering long distance routes in all directions.

THE OBLIQUE CONIC CONFORMAL PROJECTION

A new and interesting type of projection covering an extensive area has been devised by O. M. Miller of the American Geographical Society of New York. The projection has been utilized and published by the Society in three sheets, extending from New York to San Diego on the north, and southward to include all of South America, scale 1:50,000,000.

In a bipolar oblique arrangement, Pole 1 of the projection is assumed at 20° S. and 110° W. of Greenwich, and Pole 2 at 45° N. and 19° 59′ 36″ W. In each section there are two small circles in respect to the given pole of the projection where the scale is true.

Being oblique, the projection necessitates several transformations of coordinates in its computation and its relation to a conventional geographic projection. As the two parts of the bipolar projection are fitted together on a line that cuts the Americas into two parts, a systematic adjustment is made in the vicinity of Panama.

The three sheets being parts of one projection may be fitted together. They contain a key of scale departures. The maximum (uncorrected) linear scale error on land is less than 4%. Details and illustrations of the various elements appear in the Geographical Journal, January 1941.

ADAMS' EQUAL-AREA PROJECTION OF THE HEMISPHERES

[See Plate XIV]

A new equal-area projection with straight line parallels and with Lambert equivalent spacings on the central meridian has recently been devised by Dr. Oscar S. Adams, co-author of this publication. The projection is of special interest for showing the world in equal-area hemispheres or other desired arrangement such as interrupted alignments, or the world in a continuous map.

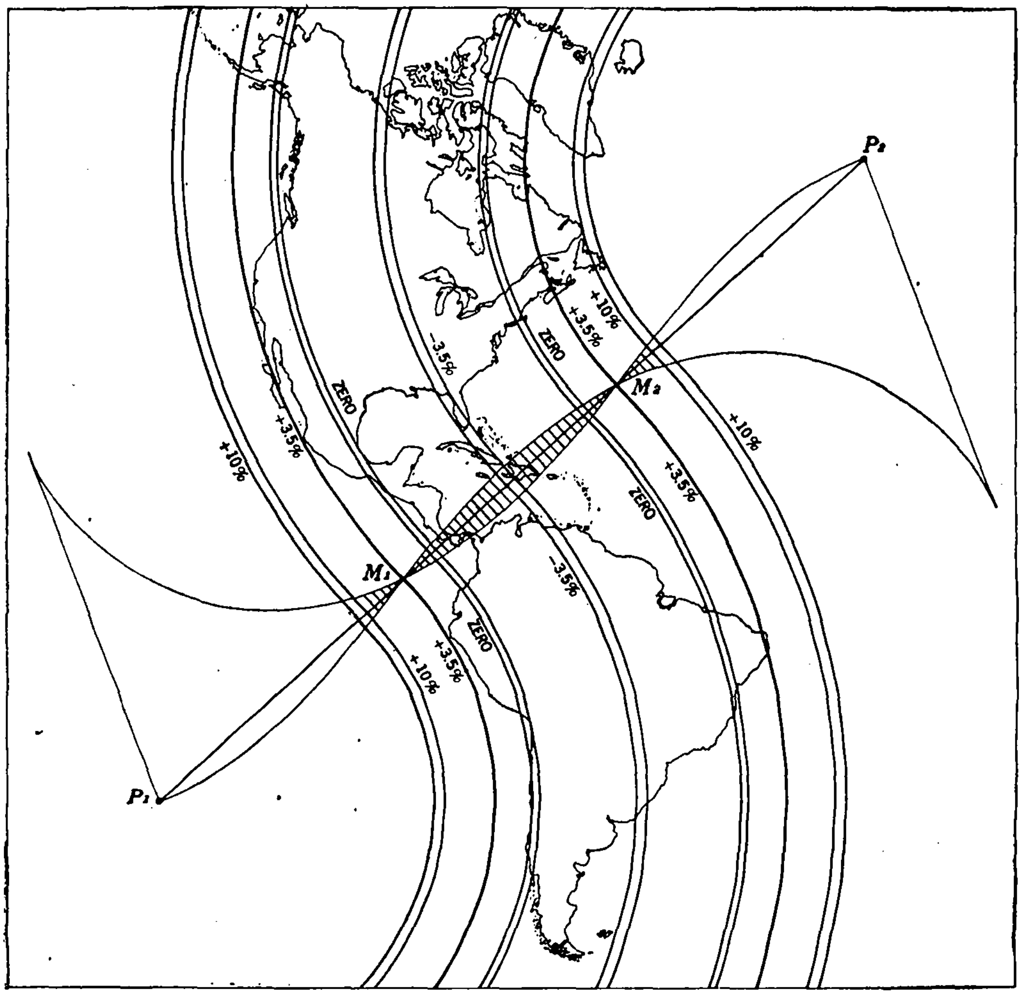

FIG. 87.—Bipolar oblique conic conformal projection.

Showing scale departures and zone of adjustment of the two parts of the projection.

It avoids the sharp points at the poles which are so objectionable in the sinusoidal projection, and the meridional scale excesses of the Mollweide projection in the equatorial region. It avoids the curvature of the parallels of the Lambert equal-area meridional projection, and as a whole presents a pleasing picture, not only new but mathematically and cartographically important.

The manuscript for a new publication entitled *Equal-Area Projections* has recently been prepared by Dr. Adams. Tables for the construction of the new projection as well as illustrations of the tangent hemispheres are included. The projection is illustrated by Plate XIV.

RECTANGULAR COORDINATES

An important aid to practical cartography has been supplied by the establishment of plane coordinate systems for all of the States of the United States. In response to the demand for coordinate systems that may be applied to extensive areas, the control data are now put into such form as to be most readily available to the engineers in cadastral and public works surveys. Each system consists of one or more zones, each of which is referred to one origin and extended over the whole area of the original projection so that every point on the map is coordinated both with respect to its position in a given square as well as to its position in latitude and longitude.

The trigonometric computations and refinements of a geodetic survey which takes into account the spheroidal shape of the earth need no longer concern the local surveyor who, with the simpler rectangular coördinates, automatically fits his local operations on the plane into the general geodetic control of a State-wide unit.

When the coordinate system is based on a definite system of projection, the scale distortion becomes known for all parts of the region and a relationship with the meridians and parallels is established. Definite scale factors for different parts of the map can be furnished and readily applied so as to bring the computations practically within geodetic accuracy.

With local surveys fitted into such a rectangular control, they can serve as a basis for further work in the same region, and will be coordinated with the work in any other region of the State. The triangulation stations are furnished for latitude and longitude by the Coast and Geodetic Survey, and in most cases now have an azimuth mark. The same stations are also furnished with their plane coordinates which, when plotted on the rectangular grid, will provide the control for a traverse from a station by turning off an angle from the azimuth mark to the next station. The work can then be continued on the plane and checked with other control points as the survey proceeds. Discrepancies can be distributed throughout the traverse and the survey coordinated with the general control. It is evident that such a system adapts itself to the quick computation of distances between two points whose rectangular coordinates are given, as well as the determination of an azimuth of a line joining any two points.

As the subject of rectangular coordinates is directly related to the subject of map projections, a careful study is necessary in the choice of projection suited to the configuration of any locality or State. The properties and advantages of conformal projections for this purpose have already been noted under their subject chapters of this publication. An added superiority is obvious in the use of two standard parallels, or, as in the case of transverse projections, the reduction of the scale along the central meridian so as to give a balance of scale within the limits of the projection. States and localities of predominating east-and-west dimension, such as North Carolina, Tennessee, and Long Island, N. Y., naturally call for a Lambert conformal projection with two standard parallels. (See plate X.) New Jersey with its greater dimension in a north-and-south direction is better served with a transverse Mercator projection with coordinates related to a central meridian. (See fig. 88.) To give balance to the scale in the latter system, the scale is reduced along the central meridian so as to provide for exact scale along two circles formed by planes parallel to the plane of the central meridian. The process is similar to what we should have in the ordinary Mercator projection if the scale were held exact at certain parallels north and south of the Equator. For some of the States, owing to their size and shape, systems with two or more zones have been employed in order to avoid the larger scale errors which might arise in the use of a single zone.

The quadrillage or system of kilometric squares based on the Lambert projection, so successfully used by the army in France, has thus gradually come into more general use in the United States.

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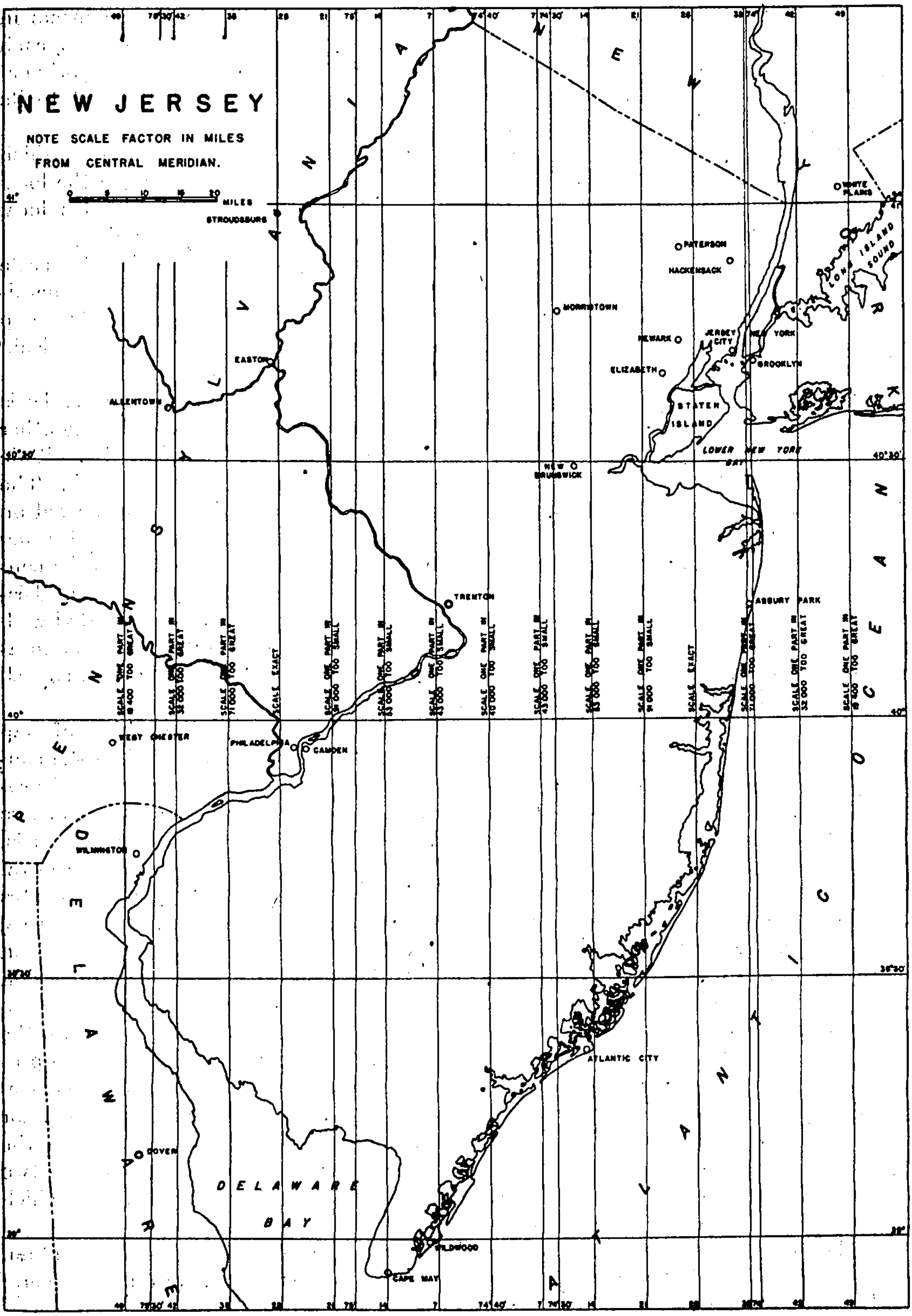

Fig. 88.—Transverse Mercator projection for rectangular coordinates.

Successfully employed in this country in small units by the Corps of Engineers, United States Army, the extension of such a system to units of larger dimensions, will serve a most useful purpose in the coordination of the work of State, city, county, and public works surveys.

Publications on the subject of plane coordinates have been prepared by the United States Coast and Geodetic Survey.

AUXILIARY LATITUDES

In all problems of map projection it should be borne in mind that the adopted figure of the earth is not that of a true sphere, but of an ellipsoid of revolution. In cases where great accuracy in the computations is required it is necessary to take this matter into consideration. If one develops formulas for different projections directly, making use of this fact, the formulas become quite complicated. There is a scheme by which this difficulty can be avoided to a large extent. We can first project the spheroid on the sphere holding some given property that will enable us to consider the earth as a sphere in further computations.

With this in mind such computations have been made, and they result in five different kinds of latitude. In each case it amounts to the mapping of the spheroid on some adopted sphere. Two of these latitudes are especially useful in the computations of projections. These are the isometric or conformal latitude and what we have called the authalic or equal-area latitude. The isometric or conformal latitude results from a mapping of the spheroid conformally upon the sphere. After this is done all computations of conformal projections can be made from this sphere. The authalic or equal-area latitude results from an equal-area projection of the spheroid upon a sphere which has exactly the same area as that of the spheroid.

Besides these two there are three other varieties of latitude that are useful in questions of geodesy and cartography. These are the geocentric latitude, the parametric latitude, and the rectifying latitude. The geocentric latitude is, of course, the angle between the equatorial radius and a radius vector from the center of the ellipsoid to the given point. The parametric latitude is the one used in the ordinary parametric equations of the meridian of the ellipsoid. In the rectifying latitude we project the spheroid upon the sphere which has a length of meridian exactly equal to the length of the meridian of the spheroid.

For these five kinds of latitude, developments have been made expressing the value of each in terms of the geodetic latitude together with additional terms expressed in signs of the multiple arcs. In Special Publication No. 67 these developments have been given in full and the development has been carried far enough in each case to give all the accuracy that is required for the most exact computations. Tables for each of the latitudes have been computed from these developments for every half degree of latitude. These are included in the above-mentioned special publication. If further values are needed for any case the developments are such that any required accuracy may be obtained.

In the computation of any conformal or equal-area projection, if it is desired to take into consideration the ellipticity of the earth by the use of these latitudes, a computation can be made just as if the earth were a sphere. In this way the latitudes are very important in such computations. The rectifying latitude also makes it possible to compute readily and accurately any arc of the meridian that may be desired.

The Coast and Geodetic Survey has computed systems of plane coordinates for all of the States, to be used in cadastral and local surveys. In all of these computations the conformal or isometric latitude is employed in computing the elements for the tables. Also the rectifying latitude is made use of in determining the arcs of the meridian which are required in the computations for the given systems. In this work these latitudes have been found to be of very great value in shortening and simplifying the computations.

REVIEW IN GENERAL

Following the descriptive and theoretical outlines of map projection, it remains for the cartographer to select that one which is best suited for the framework of his particular problem, the ultimate purpose of which can only be accomplished by honest thought and skill—factors significant and essential in map construction as in any other enduring art.

Simplicity in a map will scarcely be denied to be the best recommendation, and may be accomplished in part at least by avoiding as much as possible distortions of forms and positions of regions—forms that are beyond endurance due to the oblique incidence of meridians and parallels, and forms that are essentially different from the picture on a globe.

Where the chief end is to present to the eye a picture of what appears on the surface of the earth, we should limit ourselves to projections which are conformal, projections composed of the most easily drawn curves in which the straight line is included as being a circle whose center is at infinity.

However, for geographic purposes, especially in atlas maps, equal-area projections must be given preference. Such is a map of the United States on an Albers projection, the difference between it and the Lambert conformal projection being scarcely noticeable.

In addition to these, the stereographic, gnomonic, equidistant, Lambert zenithal, and other azimuthal projections have their potential properties and consequently their special uses and applications.

In regard to interrupted projections, such as the sinusoidal, the parabolic equal-area, and the Mollweide, considerable attention has been given of late to their use in continuous equal-area maps of the world with respect to divisions less than hemispheres. These projections usually appear in a tripartite alignment where the equator is the only continuous parallel. The outer meridians in these arrangements are split into inversely similar curves.

Of the various types of interrupted projections, the sinusoidal (Mercator equal-area) is quite popular. In this projection the outer meridians come to a very acute angle, and for this reason the map is not usually extended beyond 80 degrees of latitude. In the parabolic projection, a considerable improvement in this feature is noticeable. The parabola being considered the most graceful curve in art generally, the polar representation is less acute, and the resulting graticules are on the whole more pleasing to the eye. The perfect symmetry of the whole map can not fail to be appreciated by anyone who may find use for such a delineation of the earth's surface. The Mollweide projection has already been sufficiently described under its subject heading. Distortion of area in the equatorial region is its principal defect. In respect to finding a projection adapted to a strictly continuous map of the world, no satisfactory solution has been offered, and a resort to a division of the earth's surface into hemispheres is preferable.

A new equal-area projection of cartographic importance and one that is specially suited to mapping of the hemispheres deserves notice in a general review. Description of this recently devised arrangement is given on page 208.

The subject of scale variations in equal-area projections is receiving considerable attention of late as exemplified in Tissot's Indicatrix and illustrated in Fig. No. 81. Likewise the nature and usefulness of isoperimetric (true scale) curves inherent in equal-area projections, are a feature that has heretofore received rather scant attention. The application of these curves to an Albers projection of the United States appears in Plate III of this publication.

Many other projections for world maps have no special properties and their use is limited. Representation in them appears in a middle-ground between conformality and equal-area, and difficulties arise in expressing any desired properties in analytic terms. The Van der Grinten projection thus presents a rather pleasing picture of the world as a whole and is one of the best projections of this class. Variations of scale or errors of scale are ever present in all maps of considerable extent of the earth's surface, and these errors are generally better accounted for in conformal projections where their properties are clearly defined and spherical relations more readily applied.

Through field surveys and by air photographs we obtain the data for the physical narration of facts which are to be embodied in a map of the earth's surface. A truthful representation of these facts rests upon the mathematics of map projection as the only possible means of furnishing pictorially, in one medium, the true story regarding the earth's surface or any portion of it. The projection for this narration should be a classic in its conception and appearance, playing the same part in a map that the steel framework does in a building, the structural units of the latter being represented by the grids of the map.

The lines of projection, having served their useful purpose in the compilation of the map content, should be *lightly drawn* and at reasonable intervals only, so as not to detract from the general picture. The name of the projection should appear as it is an index to the purpose the map may serve.

It has been the aim of the authors to trace in a general way the evolution of map structure, to elucidate the original formulas, and to present the salient features of important projections in such a manner as to convey to the cartographer and the public a thorough appreciation of the properties involved and their application to chart construction. Some of the best projections have remained in obscurity for a century or more because they were not understood. Noticeable among these are the Lambert conformal conic projection and the Albers equal-area projection, both of which have had scant notice until comparatively recent years. As stated by Admiral Tonta, the excellent qualities of the Lambert projection, although not unknown, had been systematically forgotten for some time and were evidently not fully appreciated until the beginning of the first World War.

The growing interrelation of countries and communities, and the various uses and tests to which nautical and aeronautical charts are subjected at the present day, may bring forth new devices or mathematical expedients in the chart itself or as an adjunct to it. In order to meet any new or special requirements, the matter should not be left to chance, nor like fashion to a matter of custom, but careful planning for the purpose of the chart is always necessary. It is hoped that the above information may be useful in explaining the conditions and restrictions under which this branch of science is operating, its magnitude and character, and its usefulness to national development.

ADDITIONAL TABLES

The following tables are included for possible special uses.

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GNOMONIC PROJECTION TABLE.

[Center of projection is in latitude 40°. Middle longitude for the United States is 96° West.]

Latitude (Degrees)	Longitude 0°		Longitude 1°		Longitude 2°		Longitude 3°	
	X	Y	X	Y	X	Y	X	Y
	Meters	*Meters*	*Meters*	*Meters*	*Meters*	*Meters*	*Meters*	*Meters*
53	0	+1, 470, 859. 1	68, 680. 4	+1, 471, 350. 3	137, 369. 5	+1, 472, 824. 4	206, 076. 2	+1, 475, 282. 1
52	0	+1, 354, 195. 9	69, 989. 3	+1, 354, 687. 9	139, 988. 2	+1, 356, 164. 4	210, 006. 0	+1, 358, 626. 1
51	0	+1, 238, 395. 1	71, 288. 6	+1, 238, 887. 7	142, 587. 5	+1, 240, 365. 5	213, 906. 9	+1, 242, 829. 7
50	0	+1, 123, 377. 5	72, 579. 1	+1, 123, 870. 2	145, 169. 2	+1, 125, 348. 6	217, 781. 4	+1, 127, 813. 5
49	0	+1, 009, 065. 8	73, 861. 6	+1, 009, 558. 3	147, 735. 1	+1, 011, 036. 2	221, 632. 3	+1, 013, 500. 4
48	0	+895, 384. 3	75, 137. 1	+895, 876. 4	150, 286. 9	+897, 353. 0	225, 462. 0	+899, 814. 9
47	0	+782, 259. 3	76, 406. 4	+782, 750. 6	152, 826. 3	+784, 224. 9	229, 283. 2	+786, 683. 1
46	0	+669, 618. 1	77, 670. 2	+670, 108. 4	155, 354. 8	+671, 579. 5	233, 068. 0	+674, 032. 5
45	0	+557, 389. 5	78, 929. 4	+557, 878. 4	157, 874. 1	+559, 345. 5	236, 849. 1	+561, 791. 8
44	0	+445, 503. 1	80, 184. 8	+445, 990. 3	160, 385. 7	+447, 452. 5	240, 618. 7	+449, 890. 7
43	0	+333, 889. 5	81, 437. 1	+334, 374. 8	162, 891. 2	+335, 831. 2	244, 379. 2	+338, 259. 8
42	0	+222, 479. 9	82, 687. 2	+222, 963. 0	165, 392. 2	+224, 412. 8	248, 132. 9	+226, 830. 3
41	0	+111, 206. 1	83, 935. 7	+111, 686. 7	167, 890. 1	+113, 128. 9	251, 882. 2	+115, 533. 9
40	0	0	85, 183. 4	+477. 8	170, 386. 6	+1, 911. 7	255, 629. 2	+4, 302. 7
39	0	−111, 206. 1	86, 431. 2	−110, 731. 3	172, 888. 1	−109, 306. 7	259, 376. 4	−106, 931. 1
38	0	−222, 479. 9	87, 679. 7	−222, 008. 5	175, 381. 1	−220, 594. 0	263, 125. 9	−218, 235. 3
37	0	−333, 889. 5	88, 929. 7	−333, 421. 8	177, 882. 2	−332, 018. 3	266, 880. 1	−329, 677. 9
36	0	−445, 503. 1	90, 182. 1	−445, 039. 3	180, 387. 9	−443, 647. 8	270, 641. 2	−441, 327. 3
35	0	−557, 389. 5	91, 437. 5	−556, 930. 0	182, 899. 7	−555, 551. 3	274, 411. 6	−553, 252. 1
34	0	−669, 618. 1	92, 696. 7	−669, 163. 2	185, 419. 3	−667, 798. 3	278, 193. 7	−665, 522. 1
33	0	−782, 259. 3	93, 960. 6	−781, 809. 3	187, 948. 2	−780, 459. 1	281, 989. 7	−778, 207. 4
32	0	−895, 384. 3	95, 229. 9	−894, 939. 6	190, 487. 9	−893, 605. 1	285, 802. 1	−891, 379. 5
31	0	−1, 009, 065. 8	96, 505. 5	−1, 008, 626. 6	193, 040. 2	−1, 007, 308. 7	289, 633. 4	−1, 005, 110. 9
30	0	−1, 123, 377. 5	97, 788. 1	−1, 122, 944. 3	195, 606. 6	−1, 121, 644. 0	293, 486. 0	−1, 119, 475. 7
29	0	−1, 238, 395. 1	99, 078. 7	−1, 237, 968. 1	198, 188. 9	−1, 236, 686. 6	297, 362. 5	−1, 234, 549. 4
28	0	−1, 354, 195. 9	100, 378. 0	−1, 353, 775. 4	200, 788. 9	−1, 352, 513. 7	301, 265. 5	−1, 350, 409. 5
27	0	−1, 470, 859. 1	101, 687. 0	−1, 470, 445. 6	203, 408. 2	−1, 469, 204. 8	305, 197. 7	−1, 467, 135. 4
26	0	−1, 588, 466. 4	103, 006. 7	−1, 588, 060. 2	206, 048. 7	−1, 586, 841. 5	309, 161. 7	−1, 584, 808. 8
25	0	−1, 707, 101. 8	104, 337. 8	−1, 706, 703. 4	208, 712. 4	−1, 705, 507. 8	313, 160. 6	−1, 703, 513. 9
24	0	−1, 826, 852. 2	105, 681. 5	−1, 826, 461. 9	211, 401. 1	−1, 825, 290. 8	317, 197. 1	−1, 823, 337. 6
23	0	−1, 947, 807. 3	107, 038. 7	−1, 947, 425. 6	214, 116. 9	−1, 946, 280. 2	321, 274. 3	−1, 944, 369. 9
22	0	−2, 070, 060. 3	108, 410. 5	−2, 069, 687. 7	216, 861. 9	−2, 068, 569. 8	325, 895. 4	−2, 066, 704. 1

Latitude (degrees)	Longitude 4°		Longitude 5°		Longitude 6°		Longitude 7°	
	X	Y	X	Y	X	Y	X	Y
	Meters	*Meters*	*Meters*	*Meters*	*Meters*	*Meters*	*Meters*	*Meters*
53	274, 809. 3	+1, 478, 724. 8	343, 577. 5	+1, 483, 154. 5	412, 389. 6	+1, 488, 573. 5	481, 254. 6	+1, 494, 985. 1
52	280, 052. 5	+1, 362, 074. 5	350, 137. 1	+1, 366, 511. 6	420, 269. 4	+1, 371, 939. 8	490, 459. 0	+1, 378, 362. 5
51	285, 257. 2	+1, 246, 281. 5	356, 648. 6	+1, 250, 723. 0	428, 091. 6	+1, 256, 157. 0	499, 596. 5	+1, 262, 586. 5
50	290, 426. 9	+1, 131, 266. 6	363, 116. 6	+1, 135, 709. 8	435, 861. 7	+1, 141, 145. 9	508, 673. 5	+1, 147, 578. 2
49	295, 565. 1	+1, 016, 952. 5	369, 545. 3	+1, 021, 394. 6	443, 584. 9	+1, 026, 829. 4	517, 695. 9	+1, 033, 260. 5
48	300, 675. 2	+903, 263. 9	375, 939. 0	+907, 702. 1	451, 266. 3	+913, 132. 3	526, 669. 8	+919, 558. 2
47	305, 760. 5	+790, 126. 9	382, 301. 8	+794, 558. 5	458, 910. 8	+799, 980. 8	535, 600. 9	+806, 397. 4
46	310, 824. 2	+677, 469. 0	388, 637. 8	+681, 891. 3	466, 523. 3	+687, 302. 4	544, 495. 0	+693, 706. 0
45	315, 869. 6	+565, 218. 9	394, 951. 1	+569, 629. 3	474, 108. 6	+575, 026. 0	553, 357. 7	+581, 412. 8
44	320, 899. 9	+453, 306. 5	401, 245. 5	+457, 702. 3	481, 671. 6	+463, 081. 4	562, 194. 5	+469, 447. 4
43	325, 918. 1	+341, 662. 2	407, 524. 9	+346, 040. 9	489, 216. 8	+351, 399. 1	571, 010. 9	+357, 740. 6
42	330, 927. 3	+230, 217. 2	413, 793. 8	+234, 576. 2	496, 749. 0	+239, 910. 3	579, 812. 3	+246, 223. 6
41	335, 930. 7	+118, 903. 4	420, 054. 5	+123, 240. 0	504, 272. 6	+128, 546. 9	588, 604. 1	+134, 828. 2
40	340, 931. 2	+7, 652. 8	426, 312. 3	+11, 964. 3	511, 792. 4	+17, 240. 8	597, 391. 7	+23, 486. 2
39	345, 931. 9	−103, 602. 6	432, 570. 4	−99, 318. 6	519, 313. 0	−94, 075. 8	606, 180. 5	−87, 870. 1
38	350, 935. 9	−214, 930. 4	438, 832. 8	−210, 676. 7	526, 838. 8	−205, 470. 8	614, 975. 7	−199, 308. 6
37	355, 946. 1	−326, 398. 7	445, 103. 3	−322, 178. 0	534, 374. 5	−317, 012. 3	623, 782. 8	−310, 897. 4
36	360, 965. 8	−438, 075. 8	451, 385. 6	−433, 890. 7	541, 924. 7	−428, 768. 6	632, 607. 1	−422, 705. 0
35	365, 998. 0	−550, 030. 6	457, 683. 8	−545, 883. 9	549, 494. 1	−540, 808. 6	641, 454. 2	−534, 800. 4
34	371, 045. 7	−662, 332. 6	464, 001. 6	−658, 227. 1	557, 087. 4	−653, 202. 1	650, 329. 4	−647, 253. 2
33	376, 112. 3	−775, 052. 2	470, 343. 0	−770, 990. 8	564, 709. 3	−766, 019. 6	659, 238. 5	−760, 134. 1
32	381, 200. 8	−888, 260. 8	476, 712. 1	−884, 246. 4	572, 364. 7	−879, 332. 5	668, 187. 0	−873, 514. 7
31	386, 314. 5	−1, 002, 031. 1	483, 113. 0	−998, 066. 6	580, 058. 4	−993, 213. 7	677, 180. 7	−987, 468. 0
30	391, 456. 9	−1, 116, 437. 1	489, 549. 8	−1, 112, 525. 6	587, 795. 6	−1, 107, 737. 4	686, 225. 4	−1, 102, 068. 1
29	396, 631. 2	−1, 231, 554. 5	496, 026. 8	−1, 227, 699. 1	595, 581. 3	−1, 222, 979. 5	695, 327. 2	−1, 217, 391. 2
28	401, 840. 9	−1, 347, 460. 8	502, 548. 3	−1, 343, 664. 7	603, 420. 9	−1, 339, 017. 6	704, 492. 2	−1, 333, 515. 1
27	407, 089. 7	−1, 464, 235. 4	509, 118. 8	−1, 460, 502. 0	611, 319. 5	−1, 455, 931. 6	713, 726. 8	−1, 450, 519. 5
26	412, 381. 2	−1, 581, 960. 3	515, 743. 0	−1, 578, 293. 0	619, 282. 9	−1, 573, 803. 4	723, 037. 3	−1, 568, 486. 9
25	417, 719. 2	−1, 700, 719. 6	522, 425. 5	−1, 697, 122. 0	627, 316. 7	−1, 692, 717. 5	732, 430. 4	−1, 687, 501. 7
24	423, 107. 6	−1, 820, 600. 3	529, 171. 3	−1, 817, 076. 2	635, 426. 8	−1, 812, 761. 4	741, 913. 2	−1, 807, 651. 7
23	428, 550. 5	−1, 941, 692. 7	535, 985. 5	−1, 938, 245. 8	643, 619. 4	−1, 934, 025. 5	751, 492. 7	−1, 929, 027. 4
22	434, 052. 0	−2, 064, 090. 1	542, 873. 3	−2, 060, 724. 3	651, 900. 7	−2, 056, 603. 4	761, 176. 4	−2, 051, 722. 8

GNOMONIC PROJECTION TABLE—Continued.

[Center of projection is in latitude 40°. Middle longitude for the United States is 96° West.]

Latitude (degrees)	Longitude 8°		Longitude 9°		Longitude 10°		Longitude 11°	
	X	Y	X	Y	X	Y	X	Y
	Meters	*Meters*	*Meters*	*Meters*	*Meters*	*Meters*	*Meters*	*Meters*
53	550, 181. 2	+1, 502, 392. 7	619, 178. 3	+1, 510, 800. 6	688, 254. 9	+1, 520, 218. 5	757, 419. 9	+1, 530, 636. 7
52	560, 715. 4	+1, 385, 783. 3	631, 048. 5	+1, 394, 206. 4	701, 467. 8	+1, 403, 636. 9	771, 983. 1	+1, 414, 080. 1
51	571, 173. 6	+1, 270, 015. 6	642, 833. 5	+1, 278, 448. 5	714, 586. 6	+1, 287, 890. 3	786, 443. 4	+1, 298, 346. 7
50	581, 562. 9	+1, 155, 010. 7	654, 541. 4	+1, 163, 447. 9	727, 620. 3	+1, 172, 895. 1	800, 810. 7	+1, 183, 357. 9
49	591, 890. 3	+1, 040, 691. 8	666, 180. 1	+1, 049, 128. 1	740, 577. 4	+1, 058, 574. 6	815, 094. 6	+1, 069, 037. 3
48	602, 162. 4	+926, 983. 7	677, 757. 0	+935, 413. 7	753, 466. 6	+944, 853. 7	829, 304. 3	+955, 309. 8
47	612, 386. 1	+813, 812. 7	689, 279. 9	+822, 231. 3	766, 296. 2	+831, 659. 1	843, 449. 0	+842, 102. 2
46	622, 567. 7	+701, 106. 5	700, 755. 9	+709, 508. 7	779, 074. 3	+718, 918. 5	857, 537. 9	+729, 342. 2
45	632, 713. 8	+588, 793. 9	712, 192. 3	+597, 174. 7	791, 809. 0	+606, 560. 9	871, 579. 6	+616, 959. 1
44	642, 830. 6	+476, 805. 0	723, 596. 4	+485, 159. 3	804, 508. 3	+494, 516. 2	885, 588. 0	+504, 882. 6
43	652, 924. 5	+365, 070. 2	734, 975. 0	+373, 393. 0	817, 179. 9	+382, 715. 2	899, 556. 8	+393, 043. 5
42	663, 001. 7	+253, 520. 8	746, 335. 8	+261, 807. 2	829, 831. 7	+271, 089. 0	913, 509. 5	+281, 373. 1
41	673, 068. 2	+142, 088. 5	757, 684. 1	+150, 333. 5	842, 471. 4	+159, 569. 4	927, 449. 6	+169, 803. 2
40	683, 130. 3	+30, 705. 4	769, 028. 5	+38, 904. 0	855, 106. 8	+48, 088. 3	941, 385. 7	+58, 265. 6
39	693, 194. 1	−80, 696. 5	780, 375. 3	−72, 549. 3	867, 745. 4	−63, 422. 1	955, 326. 2	−53, 307. 7
38	703, 265. 8	−192, 185. 0	791, 731. 4	−184, 094. 8	880, 395. 1	−175, 029. 9	969, 279. 7	−164, 984. 6
37	713, 351. 4	−303, 828. 8	803, 103. 8	−295, 799. 2	893, 063. 6	−286, 803. 4	983, 254. 7	−276, 833. 4
36	723, 457. 2	−415, 694. 9	814, 499. 5	−407, 732. 6	905, 758. 7	−398, 811. 1	997, 259. 8	−388, 923. 0
35	733, 589. 4	−527, 854. 0	825, 925. 5	−519, 968. 6	918, 488. 1	−511, 122. 3	1, 011, 303. 6	−501, 322. 7
34	743, 754. 4	−640, 375. 2	837, 388. 8	−632, 562. 1	931, 259. 9	−623, 807. 1	1, 025, 394. 9	−614, 102. 5
33	753, 958. 4	−753, 329. 2	848, 896. 8	−745, 598. 9	944, 082. 1	−736, 936. 2	1, 039, 542. 6	−727, 333. 4
32	764, 208. 0	−866, 787. 9	860, 456. 8	−859, 145. 8	956, 962. 7	−850, 581. 7	1, 053, 755. 6	−841, 087. 6
31	774, 509. 8	−980, 824. 1	872, 076. 1	−973, 276. 1	969, 910. 2	−964, 816. 9	1, 068, 043. 2	−955, 438. 6
30	784, 870. 5	−1, 095, 512. 5	883, 762. 4	−1, 088, 064. 2	982, 933. 0	−1, 079, 716. 5	1, 082, 414. 8	−1, 070, 461. 3
29	795, 297. 0	−1, 210, 929. 0	895, 523. 4	−1, 203, 586. 6	996, 039. 8	−1, 195, 357. 0	1, 096, 879. 8	−1, 186, 232. 4
28	805, 796. 3	−1, 327, 151. 7	907, 367. 2	−1, 319, 921. 8	1, 009, 239. 5	−1, 311, 816. 8	1, 111, 448. 2	−1, 302, 830. 4
27	816, 375. 7	−1, 444, 260. 6	919, 301. 8	−1, 437, 148. 6	1, 022, 541. 1	−1, 429, 176. 4	1, 126, 130. 0	−1, 420, 336. 2
26	827, 042. 6	−1, 562, 338. 2	931, 335. 7	−1, 555, 351. 1	1, 035, 954. 1	−1, 547, 518. 6	1, 140, 935. 6	−1, 538, 832. 8
25	837, 804. 6	−1, 681, 469. 8	943, 477. 6	−1, 674, 614. 0	1, 049, 488. 2	−1, 666, 928. 8	1, 155, 875. 6	−1, 658, 405. 9
24	848, 669. 8	−1, 801, 741. 7	955, 736. 4	−1, 795, 025. 3	1, 063, 153. 3	−1, 787, 495. 4	1, 170, 961. 3	−1, 779, 144. 2
23	859, 646. 2	−1, 923, 246. 3	968, 121. 3	−1, 916, 676. 1	1, 076, 959. 8	−1, 909, 309. 8	1, 186, 204. 1	−1, 901, 139. 4
22	870, 742. 5	−2, 046, 077. 4	980, 642. 1	−2, 039, 661. 0	1, 090, 918. 5	−2, 032, 466. 8	1, 201, 615. 7	−2, 024, 486. 9

Latitude (degrees)	Longitude 12°		Longitude 13°		Longitude 14°		Longitude 15°	
	X	Y	X	Y	X	Y	X	Y
	Meters	*Meters*	*Meters*	*Meters*	*Meters*	*Meters*	*Meters*	*Meters*
53	826, 682. 3	+1, 542, 076. 1	896, 051. 2	+1, 554, 538. 2	965, 585. 6	+1, 568, 030. 1	1, 035, 144. 6	+1, 582, 559. 5
52	842, 604. 2	+1, 425, 542. 2	913, 340. 9	+1, 438, 029. 8	984, 203. 2	+1, 451, 550. 4	1, 055, 200. 9	+1, 466, 111. 8
51	858, 414. 6	+1, 309, 823. 9	930, 510. 8	+1, 322, 828. 9	1, 002, 742. 8	+1, 335, 869. 2	1, 075, 121. 3	+1, 350, 458. 1
50	874, 124. 3	+1, 194, 843. 0	947, 572. 6	+1, 207, 357. 8	1, 021, 167. 0	+1, 220, 908. 6	1, 094, 919. 3	+1, 235, 505. 4
49	889, 743. 8	+1, 080, 522. 8	964, 587. 4	+1, 093, 038. 5	1, 039, 487. 9	+1, 106, 592. 4	1, 114, 607. 8	+1, 121, 193. 0
48	905, 283. 1	+966, 788. 8	981, 416. 3	+979, 298. 1	1, 057, 717. 4	+992, 846. 1	1, 134, 199. 7	+1, 007, 441. 4
47	920, 752. 4	+853, 567. 5	998, 220. 4	+866, 062. 9	1, 075, 867. 8	+879, 596. 6	1, 153, 707. 7	+894, 177. 7
46	936, 161. 4	+740, 787. 1	1, 014, 960. 1	+753, 260. 8	1, 093, 949. 1	+766, 772. 1	1, 173, 143. 9	+781, 330. 2
45	951, 519. 9	+628, 376. 5	1, 031, 646. 1	+640, 821. 1	1, 111, 974. 2	+654, 301. 8	1, 192, 520. 6	+668, 828. 1
44	966, 837. 5	+516, 265. 7	1, 048, 288. 6	+528, 673. 9	1, 129, 953. 7	+542, 115. 9	1, 211, 849. 9	+556, 601. 6
43	982, 123. 5	+404, 385. 6	1, 064, 898. 1	+416, 749. 8	1, 147, 898. 6	+430, 145. 1	1, 231, 148. 6	+444, 581. 6
42	997, 387. 4	+292, 667. 4	1, 081, 484. 6	+304, 980. 2	1, 165, 820. 2	+318, 320. 8	1, 250, 418. 6	+332, 699. 4
41	1, 012, 638. 6	+181, 042. 8	1, 098, 058. 3	+193, 296. 8	1, 183, 729. 2	+206, 574. 6	1, 269, 671. 6	+220, 886. 6
40	1, 027, 886. 2	+69, 443. 7	1, 114, 629. 4	+81, 631. 4	1, 201, 636. 6	+94, 838. 4	1, 288, 929. 3	+109, 075. 1
39	1, 043, 139. 7	−42, 197. 9	1, 131, 207. 8	−30, 083. 8	1, 219, 553. 3	−16, 955. 8	1, 308, 198. 6	−2, 803. 2
38	1, 058, 408. 2	−153, 950. 0	1, 147, 803. 8	−141, 917. 2	1, 237, 490. 2	−128, 876. 4	1, 327, 491. 2	−114, 816. 7
37	1, 073, 701. 2	−265, 881. 1	1, 164, 427. 5	−253, 937. 2	1, 255, 458. 4	−240, 991. 8	1, 346, 818. 9	−227, 034. 0
36	1, 089, 028. 1	−378, 060. 0	1, 181, 089. 1	−366, 212. 8	1, 273, 468. 8	−353, 371. 2	1, 366, 193. 6	−339, 524. 2
35	1, 104, 398. 3	−490, 556. 1	1, 197, 798. 9	−478, 813. 4	1, 291, 532. 8	−466, 084. 1	1, 385, 627. 3	−452, 357. 2
34	1, 119, 821. 4	−603, 439. 7	1, 214, 567. 5	−591, 809. 3	1, 309, 661. 5	−579, 201. 1	1, 405, 132. 3	−565, 608. 7
33	1, 135, 307. 2	−716, 781. 8	1, 231, 405. 2	−705, 272. 1	1, 327, 866. 4	−692, 793. 7	1, 424, 720. 7	−679, 335. 2
32	1, 150, 865. 5	−830, 654. 9	1, 248, 323. 0	−819, 274. 0	1, 346, 159. 1	−806, 934. 5	1, 444, 405. 8	−793, 624. 8
31	1, 166, 506. 5	−945, 132. 5	1, 265, 331. 8	−933, 889. 1	1, 364, 551. 7	−921, 697. 6	1, 464, 198. 8	−908, 546. 7
30	1, 182, 240. 8	−1, 060, 289. 9	1, 282, 442. 8	−1, 049, 192. 6	1, 383, 056. 0	−1, 037, 158. 7	1, 484, 114. 3	−1, 024, 176. 8
29	1, 198, 077. 4	−1, 176, 203. 8	1, 299, 667. 3	−1, 165, 261. 6	1, 401, 684. 6	−1, 153, 895. 2	1, 504, 165. 2	−1, 140, 592. 8
28	1, 214, 028. 8	−1, 292, 953. 2	1, 317, 017. 2	−1, 282, 175. 4	1, 420, 450. 3	−1, 270, 486. 4	1, 524, 365. 1	−1, 257, 874. 4
27	1, 230, 105. 8	−1, 410, 619. 0	1, 334, 504. 6	−1, 400, 015. 2	1, 439, 366. 0	−1, 388, 514. 0	1, 544, 728. 3	−1, 376, 103. 7
26	1, 246, 318. 4	−1, 529, 284. 7	1, 352, 141. 7	−1, 518, 864. 7	1, 458, 445. 2	−1, 507, 562. 0	1, 565, 269. 2	−1, 495, 365. 0
25	1, 262, 679. 9	−1, 649, 036. 3	1, 369, 941. 6	−1, 638, 810. 4	1, 477, 701. 9	−1, 627, 717. 4	1, 586, 002. 9	−1, 615, 745. 7
24	1, 279, 202. 0	−1, 769, 962. 8	1, 387, 917. 4	−1, 759, 941. 6	1, 497, 150. 4	−1, 749, 069. 8	1, 606, 944. 9	−1, 737, 335. 9
23	1, 295, 897. 1	−1, 892, 156. 4	1, 406, 082. 0	−1, 882, 350. 9	1, 516, 805. 7	−1, 871, 712. 4	1, 628, 111. 4	−1, 860, 229. 8
22	1, 312, 778. 5	−2, 015, 712. 7	1, 424, 452. 2	−2, 006, 184. 5	1, 536, 683. 3	−1, 995, 741. 8	1, 649, 519. 0	−1, 984, 523. 0

GNOMONIC PROJECTION TABLE—Continued.

[Center of projection is in latitude 40°. Middle longitude for the United States is 96° West.]

Latitude (degrees)	Longitude 16°		Longitude 17°		Longitude 18°		Longitude 19°	
	X	Y	X	Y	X	Y	X	Y
	Meters	*Meters*	*Meters*	*Meters*	*Meters*	*Meters*	*Meters*	*Meters*
52	1, 126, 344. 2	+1, 481, 722. 6	1, 197, 643. 1	+1, 498, 392. 1	1, 269, 107. 8	+1, 516, 130. 1	1, 340, 748. 7	+1, 534, 947. 3
51	1, 147, 657. 3	+1, 366, 089. 2	1, 220, 361. 8	+1, 382, 787. 2	1, 293, 245. 7	+1, 400, 557. 3	1, 366, 320. 4	+1, 419, 410. 3
50	1, 168, 841. 2	+1, 251, 156. 9	1, 242, 944. 8	+1, 267, 872. 7	1, 317, 241. 8	+1, 285, 663. 3	1, 391, 744. 6	+1, 304, 540. 2
49	1, 189, 909. 9	+1, 136, 849. 8	1, 265, 406. 9	+1, 153, 572. 8	1, 341, 111. 9	+1, 171, 372. 9	1, 417, 037. 8	+1, 190, 261. 5
48	1, 210, 877. 0	+1, 023, 093. 8	1, 287, 762. 9	+1, 039, 813. 7	1, 364, 871. 3	+1, 057, 612. 0	1, 442, 216. 4	+1, 076, 500. 7
47	1, 231, 755. 9	+909, 816. 2	1, 310, 026. 8	+926, 522. 6	1, 388, 535. 3	+944, 308. 2	1, 467, 296. 5	+963, 185. 4
46	1, 252, 559. 9	+796, 945. 2	1, 332, 212. 9	+813, 627. 9	1, 412, 118. 8	+831, 390. 1	1, 492, 293. 7	+850, 244. 2
45	1, 273, 301. 9	+684, 410. 2	1, 354, 334. 9	+701, 059. 3	1, 435, 636. 4	+718, 787. 3	1, 517, 223. 6	+737, 606. 9
44	1, 293, 994. 9	+572, 141. 5	1, 376, 406. 5	+588, 746. 9	1, 459, 102. 7	+606, 430. 0	1, 542, 101. 7	+625, 203. 9
43	1, 314, 651. 5	+460, 069. 9	1, 398, 441. 3	+476, 621. 7	1, 482, 532. 1	+494, 249. 3	1, 566, 943. 3	+512, 966. 0
42	1, 335, 284. 5	+348, 126. 8	1, 420, 452. 9	+364, 614. 9	1, 505, 939. 0	+382, 176. 4	1, 591, 763. 5	+400, 824. 8
41	1, 355, 906. 3	+236, 243. 8	1, 442, 454. 6	+252, 658. 4	1, 529, 337. 7	+270, 143. 0	1, 616, 577. 4	+288, 711. 7
40	1, 376, 529. 7	+124, 352. 8	1, 464, 459. 8	+140, 683. 7	1, 552, 742. 3	+158, 080. 9	1, 641, 400. 2	+176, 558. 5
39	1, 397, 167. 1	+12, 385. 5	1, 486, 482. 0	+28, 622. 7	1, 576, 167. 3	+45, 921. 7	1, 666, 247. 1	+64, 296. 7
38	1, 417, 831. 2	−99, 726. 5	1, 508, 534. 7	−83, 593. 2	1, 599, 626. 8	−66, 403. 3	1, 691, 133. 1	−48, 142. 3
37	1, 438, 534. 6	−212, 051. 8	1, 530, 631. 3	−196, 032. 7	1, 623, 135. 4	−178, 962. 9	1, 716, 073. 7	−160, 827. 7
36	1, 459, 290. 1	−324, 659. 9	1, 552, 785. 5	−308, 765. 4	1, 646, 707. 5	−291, 826. 8	1, 741, 084. 3	−273, 829. 2
35	1, 480, 110. 6	−437, 620. 6	1, 575, 011. 1	−421, 861. 3	1, 670, 357. 8	−405, 065. 2	1, 766, 180. 4	−387, 217. 2
34	1, 501, 009. 2	−551, 004. 9	1, 597, 322. 0	−535, 391. 5	1, 694, 101. 3	−518, 749. 4	1, 791, 377. 9	−501, 063. 3
33	1, 521, 999. 0	−664, 884. 5	1, 619, 732. 5	−649, 428. 1	1, 717, 952. 9	−632, 951. 7	1, 816, 692. 9	−615, 440. 1
32	1, 543, 093. 7	−779, 332. 6	1, 642, 256. 9	−764, 044. 4	1, 741, 928. 2	−747, 745. 8	1, 842, 141. 6	−730, 421. 4
31	1, 564, 306. 9	−894, 423. 7	1, 664, 909. 9	−879, 315. 3	1, 766, 042. 7	−863, 206. 8	1, 867, 740. 9	−846, 082. 7
30	1, 585, 652. 6	−1, 010, 234. 2	1, 687, 706. 6	−995, 317. 3	1, 790, 312. 6	−979, 411. 5	1, 893, 507. 7	−962, 501. 2
29	1, 607, 145. 4	−1, 126, 841. 8	1, 710, 662. 4	−1, 112, 128. 7	1, 814, 754. 2	−1, 096, 438. 7	1, 919, 459. 7	−1, 079, 756. 0
28	1, 628, 799. 8	−1, 244, 326. 9	1, 733, 793. 0	−1, 229, 830. 1	1, 839, 384. 5	−1, 214, 369. 2	1, 945, 614. 8	−1, 197, 928. 5
27	1, 650, 631. 1	−1, 362, 771. 6	1, 757, 114. 8	−1, 348, 504. 1	1, 864, 220. 8	−1, 333, 286. 2	1, 971, 991. 4	−1, 317, 102. 3
26	1, 672, 654. 9	−1, 482, 260. 9	1, 780, 644. 6	−1, 468, 236. 1	1, 889, 281. 2	−1, 453, 275. 6	1, 998, 608. 8	−1, 437, 363. 7
25	1, 694, 887. 5	−1, 602, 882. 6	1, 804, 399. 5	−1, 589, 114. 3	1, 914, 584. 0	−1, 574, 426. 1	2, 025, 486. 7	−1, 558, 802. 0
24	1, 717, 345. 4	−1, 724, 727. 2	1, 828, 397. 7	−1, 711, 230. 0	1, 940, 148. 4	−1, 696, 829. 5	2, 052, 645. 6	−1, 681, 509. 8
23	1, 740, 046. 2	−1, 847, 889. 0	1, 852, 657. 7	−1, 834, 677. 8	1, 965, 994. 5	−1, 820, 581. 1	2, 080, 106. 6	−1, 805, 583. 0
22	1, 763, 007. 7	−1, 972, 465. 7	1, 877, 198. 7	−1, 959, 556. 3	1, 992, 142. 9	−1, 945, 780. 1	2, 107, 892. 0	−1, 931, 121. 4
21	----------	----------	----------	----------	----------	----------	2, 136, 024. 7	−2, 058, 229. 0

Latitude (degrees)	Longitude 20°		Longitude 21°		Longitude 22°		Longitude 23°	
	X	Y	X	Y	X	Y	X	Y
	Meters	*Meters*	*Meters*	*Meters*	*Meters*	*Meters*	*Meters*	*Meters*
52	1, 412, 576. 0	+1, 554, 854. 7	1, 484, 600. 2	+1, 575, 864. 5	1, 556, 832. 0	+1, 597, 989. 2	1, 629, 281. 9	+1, 621, 242. 2
51	1, 439, 597. 1	+1, 439, 358. 0	1, 513, 087. 2	+1, 460, 412. 5	1, 586, 802. 3	+1, 482, 586. 9	1, 660, 754. 0	+1, 505, 895. 2
50	1, 466, 465. 2	+1, 324, 515. 0	1, 541, 416. 2	+1, 345, 600. 7	1, 616, 610. 0	+1, 367, 810. 7	1, 692, 059. 4	+1, 391, 159. 1
49	1, 493, 197. 9	+1, 210, 251. 0	1, 569, 605. 7	+1, 231, 354. 5	1, 646, 274. 6	+1, 253, 585. 8	1, 723, 218. 4	+1, 276, 959. 7
48	1, 519, 812. 4	+1, 096, 492. 4	1, 597, 673. 7	+1, 117, 600. 5	1, 675, 814. 9	+1, 139, 839. 4	1, 754, 250. 8	+1, 163, 224. 0
47	1, 546, 325. 5	+983, 166. 9	1, 625, 638. 0	+1, 004, 266. 7	1, 705, 249. 6	+1, 026, 499. 4	1, 785, 176. 3	+1, 049, 880. 5
46	1, 572, 753. 9	+870, 203. 5	1, 653, 516. 2	+891, 282. 1	1, 734, 597. 2	+913, 495. 0	1, 816, 014. 1	+936, 858. 2
45	1, 599, 114. 1	+757, 531. 8	1, 681, 325. 5	+778, 576. 4	1, 763, 875. 8	+800, 756. 1	1, 846, 783. 3	+824, 087. 2
44	1, 625, 422. 3	+645, 082. 3	1, 709, 083. 1	+666, 080. 2	1, 793, 103. 5	+688, 213. 3	1, 877, 502. 9	+711, 498. 2
43	1, 651, 694. 6	+532, 786. 1	1, 736, 806. 1	+553, 724. 6	1, 822, 298. 2	+575, 797. 7	1, 908, 191. 7	+599, 022. 3
42	1, 677, 947. 1	+420, 574. 5	1, 764, 511. 3	+441, 441. 0	1, 851, 477. 7	+463, 440. 6	1, 938, 668. 3	+486, 590. 9
41	1, 704, 195. 9	+308, 379. 0	1, 792, 215. 7	+329, 160. 8	1, 880, 659. 7	+351, 073. 6	1, 969, 551. 4	+374, 135. 4
40	1, 730, 456. 9	+196, 131. 4	1, 819, 936. 1	+216, 815. 6	1, 909, 862. 1	+238, 628. 2	2, 000, 259. 9	+261, 587. 2
39	1, 756, 746. 1	+83, 763. 0	1, 847, 689. 4	+104, 336. 9	1, 930, 102. 8	+126, 035. 7	2, 031, 012. 3	+148, 877. 6
38	1, 783, 079. 6	−28, 794. 8	1, 875, 492. 7	−8, 344. 3	1, 968, 399. 5	+13, 226. 9	2, 061, 827. 8	+35, 937. 4
37	1, 809, 473. 7	−141, 611. 4	1, 903, 363. 1	−121, 297, 3	1, 997, 770. 5	−99, 867. 6	2, 092, 725. 1	−77, 303. 2
36	1, 835, 944. 6	−254, 756. 7	1, 931, 317. 9	−234, 592. 4	2, 027, 234. 0	−213, 318. 1	2, 123. 723. 7	−190, 914. 7
35	1, 862, 509. 0	−368, 301. 3	1, 959, 374. 5	−348, 300. 3	2, 056, 808. 4	−327, 195. 9	2, 154, 843. 0	−304, 968. 6
34	1, 889, 183. 6	−482, 317. 0	1, 987, 550. 8	−462, 493. 1	2, 086, 512. 5	−441, 573. 1	2, 186, 102. 7	−419, 537. 2
33	1, 915, 985. 5	−596, 876. 7	2, 015, 864. 7	−577, 243. 9	2, 116, 365. 4	−556, 523. 2	2, 217, 523. 1	−534, 694. 5
32	1, 942, 932. 0	−712, 054. 4	2, 044, 334. 8	−692, 627. 3	2, 146, 386. 5	−672, 121. 1	2, 249, 124. 6	−650, 515. 8
31	1, 970, 040. 8	−827, 926. 2	2, 072, 979. 7	−808, 719. 5	2, 176, 595. 8	−788, 443. 5	2, 280, 928. 2	−767, 077. 9
30	1, 997, 330. 1	−944, 569. 4	2, 101, 818. 8	−925, 598. 3	2, 207, 013. 5	−905, 568. 5	2, 312, 955. 5	−884, 459. 8
29	2, 024, 818. 6	−1, 062, 063. 8	2, 130, 871. 6	−1, 043, 343. 8	2, 237, 660. 7	−1, 023, 576. 9	2, 345, 228. 6	−1, 002, 742. 4
28	2, 052, 525. 3	−1, 180, 490. 9	2, 160, 158. 7	−1, 162, 038. 3	2, 268, 558. 7	−1, 142, 551. 3	2, 377, 770. 2	−1, 122, 009. 1
27	2, 080, 469. 9	−1, 299, 935. 1	2, 189, 700. 8	−1, 281, 766. 5	2, 299, 729. 7	−1, 262, 577. 0	2, 410, 603. 7	−1, 242, 345. 9
26	2, 108, 672. 8	−1, 420, 483. 2	2, 219, 519. 5	−1, 402, 615. 9	2, 331, 196. 7	−1, 383, 742. 2	2, 443, 753. 5	−1, 363, 841. 5
25	2, 137, 155. 0	−1, 542, 225. 1	2, 249, 637. 3	−1, 524, 677. 0	2, 362, 983. 8	−1, 506, 138. 2	2, 477, 244. 5	−1, 486, 587. 9
24	2, 165, 938. 3	−1, 665, 254. 0	2, 280, 077. 1	−1, 648, 043. 7	2, 395, 114. 1	−1, 629, 859. 5	2, 511, 102. 8	−1, 610, 680. 5
23	2, 195, 045. 3	−1, 789, 666. 5	2, 310, 863. 2	−1, 772, 813. 4	2, 427, 614. 6	−1, 755, 004. 3	2, 545, 355. 5	−1, 736, 218. 5
22	2, 224, 499. 6	−1, 915, 563. 3	2, 342, 020. 5	−1, 899, 087. 7	2, 460, 511. 4	−1, 881, 675. 1	2, 580, 030. 7	−1, 863, 305. 1
21	2, 254, 325. 6	−2, 043, 049. 2	2, 373, 575. 1	−2, 026, 972. 1	2, 493, 832. 2	−2, 009, 978. 6	2, 615, 157. 8	−1, 992, 048. 1

GNOMONIC PROJECTION TABLE—Continued

[Center of projection is in latitude 40°. Middle longitude for the United States is 96° West.]

Latitude (degrees)	Longitude 24°		Longitude 25°		Longitude 26°		Longitude 27°	
	X	Y	X	Y	X	Y	X	Y
	Meters	*Meters*	*Meters*	*Meters*	*Meters*	*Meters*	*Meters*	*Meters*
52	1,701,960.8	+1,645,637.6						
51	1,734,954.1	+1,530,351.9	1,809,414.6	+1,555,972.4	1,884,147.7	+1,582,772.8	1,959,165.4	+1,610,770.4
50	1,767,777.1	+1,415,661.2	1,843,776.3	+1,441,332.8	1,920,070.2	+1,468,190.7	1,996,672.0	+1,496,252.3
49	1,800,451.0	+1,301,491.6	1,877,986.7	+1,327,198.0	1,955,839.7	+1,354,096.1	2,034,024.7	+1,382,204.1
48	1,832,996.6	+1,187,770.5	1,912,067.5	+1,213,495.7	1,991,479.0	+1,240,417.2	2,071,247.0	+1,268,554.0
47	1,865,434.3	+1,074,426.3	1,946,040.1	+1,100,154.3	2,027,010.4	+1,127,082.8	2,108,362.2	+1,155,230.8
46	1,897,784.3	+961,388.4	1,979,925.5	+987,103.6	2,062,455.8	+1,014,022.3	2,145,393.3	+1,042,164.5
45	1,930,066.7	+848,586.9	2,013,744.8	+874,273.6	2,097,837.1	+901,166.3	2,182,363.1	+929,285.5
44	1,962,301.2	+735,952.6	2,047,518.7	+761,595.1	2,133,176.0	+788,445.5	2,219,294.2	+816,524.6
43	1,994,507.7	+623,416.5	2,081,267.8	+648,999.4	2,168,494.1	+675,791.2	2,256,209.1	+703,813.0
42	2,026,705.6	+510,910.1	2,115,012.6	+536,417.8	2,203,812.9	+563,134.6	2,293,130.3	+591,082.2
41	2,058,914.7	+398,364.7	2,148,773.8	+423,781.6	2,239,153.9	+450,407.0	2,330,080.4	+478,263.2
40	2,091,154.5	+285,711.8	2,182,572.0	+311,022.1	2,274,538.8	+337,539.8	2,367,081.9	+365,287.3
39	2,123,444.9	+172,882.3	2,216,427.8	+198,070.3	2,309,980.1	+224,463.5	2,404,157.6	+252,084.9
38	2,155,805.6	+59,807.0	2,250,362.1	+84,856.7	2,345,526.8	+111,108.6	2,441,330.2	+138,586.3
37	2,188,256.8	−53,584.1	2,284,395.9	−28,689.0	2,381,174.0	−2,595.3	2,478,623.1	+24,720.9
36	2,220,818.5	−167,361.8	2,318,550.5	−142,637.6	2,416,952.9	−116,719.4	2,516,059.5	−89,582.9
35	2,253,511.4	−281,597.7	2,352,847.5	−257,061.3	2,452,886.2	−231,336.2	2,553,663.2	−204,397.8
34	2,286,356.2	−396,364.6	2,387,308.8	−372,033.0	2,488,997.0	−346,519.0	2,591,458.6	−319,797.5
33	2,319,374.3	−511,736.8	2,421,956.7	−487,627.5	2,525,308.7	−462,342.8	2,629,470.2	−435,857.6
32	2,352,587.2	−627,789.8	2,456,814.0	−603,920.7	2,561,845.4	−578,884.2	2,667,723.2	−552,654.9
31	2,386,017.2	−744,601.2	2,491,904.2	−720,990.5	2,598,631.7	−696,221.5	2,706,243.6	−670,268.4
30	2,419,686.9	−862,250.3	2,527,251.0	−838,916.9	2,635,692.7	−814,435.2	2,745,057.9	−788,779.2
29	2,453,619.6	−980,818.5	2,562,879.3	−957,782.0	2,673,054.4	−933,608.0	2,784,193.3	−908,270.6
28	2,487,839.5	−1,100,380.9	2,598,814.2	−1,077,070.2	2,710,743.5	−1,053,825.3	2,823,678.1	−1,028,828.6
27	2,522,371.2	−1,221,051.1	2,635,082.0	−1,198,669.1	2,748,787.6	−1,175,174.9	2,863,541.2	−1,150,542.1
26	2,557,240.4	−1,342,891.5	2,671,709.7	−1,320,868.7	2,787,215.3	−1,297,748.0	2,903,813.0	−1,273,502.9
25	2,592,473.6	−1,466,003.9	2,708,725.4	−1,444,362.7	2,826,056.2	−1,421,639.1	2,944,524.4	−1,397,806.4
24	2,628,098.4	−1,590,484.6	2,746,158.2	−1,569,248.2	2,865,341.1	−1,546,946.2		
23	2,664,143.5	−1,716,433.7	2,784,038.4	−1,695,626.4	2,905,101.9	−1,673,771.6		
22	2,700,638.6	−1,843,955.4	2,822,397.5	−1,823,602.6	2,945,372.1	−1,802,221.6		

Latitude (degrees)	Longitude 28°		Longitude 29°		Longitude 30°		Longitude 31°	
	X	Y	X	Y	X	Y	X	Y
	Meters	*Meters*	*Meters*	*Meters*	*Meters*	*Meters*	*Meters*	*Meters*
51	2,034,480.4	+1,639,983.0	2,110,105.2	+1,670,429.4				
50	2,073,595.6	+1,525,536.4	2,150,854.6	+1,556,062.3	2,228,463.1	+1,587,850.5	2,306,435.5	+1,620,922.3
49	2,112,556.4	+1,411,541.2	2,191,450.0	+1,442,127.3	2,270,720.7	+1,473,983.5	2,350,384.2	+1,507,132.0
48	2,151,387.5	+1,297,925.6	2,231,916.9	+1,328,552.7	2,312,851.9	+1,360,457.1	2,394,209.4	+1,393,661.6
47	2,190,113.1	+1,184,618.9	2,272,280.6	+1,215,267.0	2,354,882.8	+1,247,201.0	2,437,938.1	+1,280,440.9
46	2,228,757.0	+1,071,551.0	2,312,565.8	+1,102,203.5	2,396,839.3	+1,134,145.3	2,481,597.3	+1,167,400.8
45	2,267,343.0	+958,652.4	2,352,797.4	+989,289.6	2,438,747.1	+1,021,220.7	2,525,213.7	+1,054,470.5
44	2,305,894.8	+845,854.2	2,392,999.9	+876,457.4	2,480,631.9	+908,358.4	2,568,813.9	+941,582.7
43	2,344,435.8	+733,087.4	2,433,197.7	+763,638.0	2,522,519.0	+795,489.5	2,612,424.4	+828,668.1
42	2,382,989.5	+620,283.5	2,473,415.5	+650,762.7	2,564,434.2	+682,545.3	2,656,071.9	+715,657.9
41	2,421,579.4	+507,373.6	2,513,677.8	+537,762.7	2,606,402.9	+569,456.7	2,699,783.0	+602,482.8
40	2,460,229.2	+394,288.5	2,554,009.1	+424,568.7	2,648,450.9	+456,154.4	2,743,584.6	+489,073.5
39	2,498,962.6	+280,958.9	2,594,434.4	+311,111.1	2,690,604.2	+342,568.6	2,787,503.7	+375,359.9
38	2,537,803.5	+167,314.6	2,634,978.7	+197,319.5	2,732,888.8	+228,628.6	2,831,567.8	+261,271.1
37	2,576,776.2	+53,284.7	2,675,667.3	+83,122.8	2,775,331.4	+114,263.1	2,875,804.4	+146,735.4
36	2,615,905.2	−61,202.6	2,716,526.0	−31,551.2	2,817,958.8	−600.4		
35	2,655,215.5	−176,220.2	2,757,580.9	−146,775.8	2,860,798.5	−116,035.7		
34	2,694,732.4	−291,842.4	2,798,858.6	−262,625.7	2,903,878.2	−232,118.0		
33	2,734,481.9	−408,145.1	2,840,386.2	−379,177.2				
32	2,774,490.5	−525,205.8	2,882,191.7	−496,508.4				
31	2,814,785.3	−643,104.0	2,924,303.5	−614,699.4				
30	2,855,394.3	−761,921.3						
29	2,896,346.1	−881,741.9						
28	2,937,670.4	−1,002,652.4						

Latitude (degrees)	Longitude 32°		Longitude 33°		Longitude 34°		Longitude 35°	
	X	Y	X	Y	X	Y	X	Y
	Meters	*Meters*	*Meters*	*Meters*	*Meters*	*Meters*	*Meters*	*Meters*
50	2,384,786.1	+1,655,300.1	2,463,529.9	+1,691,007.8				
49	2,430,456.3	+1,541,595.9	2,510,953.4	+1,577,399.5	2,591,891.8	+1,614,568.1	2,673,288.4	+1,653,128.8
48	2,476,006.7	+1,428,190.1	2,558,261.6	+1,464,067.6	2,640,992.1	+1,501,320.4	2,724,216.6	+1,539,975.9
47	2,521,465.4	+1,315,012.6	2,605,483.8	+1,350,941.9	2,690,013.0	+1,388,255.8	2,775,073.1	+1,426,982.8
46	2,566,860.2	+1,201,994.0	2,652,648.9	+1,237,953.0	2,738,984.5	+1,275,305.2	2,825,889.0	+1,314,079.8
45	2,612,219.1	+1,089,065.1	2,699,785.7	+1,125,032.0	2,787,936.5	+1,162,899.6		
44	2,657,569.5	+976,157.2	2,746,922.9	+1,012,109.9	2,836,898.8	+1,049,470.3		
43	2,702,939.1	+863,201.3	2,794,089.2	+899,117.9				
42	2,748,355.7	+750,128.6	2,841,313.5	+785,987.2				
41	2,793,847.0	+636,869.9						
40	2,839,441.0	+523,355.5						
39	2,885,166.0	+409,515.1						

PARABOLIC EQUAL-AREA PROJECTION TABLE FOR WORLD OR SECTIONAL MAPPING

The following table for the construction of a parabolic equal-area projection for world or sectional mapping is computed by use of the authalic (or equal-area) latitude, the X and Y values being represented by centimeters when the ratio of area is 1 to the square of 10,000,000. The use of authalic latitude takes into consideration the ellipsoidal shape of the earth, and hence gives results of greater precision. The table is computed for intervals of one degree.

A map of Pan-America, scale 1:10,000,000, has been constructed by the Carnegie Institution, Department of Genetics, by the use of this table.

The table given on page 180 was computed for a mean sphere and for 5-degree intervals only.

Latitude (degrees)	x coordinate — Longitude from central meridian					y coordinate
	90°	60°	30°	5°	1°	
	cm.	*cm.*	*cm.*	*cm.*	*cm.*	*cm.*
0	100.0754	66.7169	33.3585	5.5597	1.1119	0
1	100.0620	66.7080	33.3540	5.5590	1.1118	1.1591
2	100.0215	66.6810	33.3405	5.5568	1.1113	2.3183
3	99.9545	66.6363	33.3182	5.5530	1.1106	3.4772
4	99.8606	66.5737	33.2869	5.5478	1.1096	4.6363
5	99.7397	66.4931	33.2466	5.5411	1.1082	5.7952
6	99.5922	66.3948	33.1974	5.5329	1.1066	6.9538
7	99.4175	66.2783	33.1392	5.5232	1.1046	8.1123
8	99.2168	66.1442	33.0721	5.5120	1.1024	9.2706
9	98.9888	65.9925	32.9963	5.4994	1.0999	10.4288
10	98.7338	65.8225	32.9113	5.4852	1.0970	11.5863
11	98.4524	65.6349	32.8175	5.4696	1.0939	12.7436
12	98.1445	65.4297	32.7148	5.4525	1.0905	13.9006
13	97.8097	65.2065	32.6032	5.4339	1.0868	15.0571
14	97.4484	64.9656	32.4828	5.4138	1.0828	16.2134
15	97.0605	64.7070	32.3535	5.3923	1.0784	17.3691
16	96.6462	64.4308	32.2154	5.3692	1.0738	18.5243
17	96.2055	64.1370	32.0685	5.3448	1.0689	19.6792
18	95.7383	63.8255	31.9128	5.3188	1.0638	20.8333
19	95.2446	63.4964	31.7482	5.2914	1.0583	21.9869
20	94.7248	63.1499	31.5749	5.2625	1.0525	23.1396
21	94.1788	62.7859	31.3929	5.2322	1.0464	24.2921
22	93.6063	62.4042	31.2021	5.2004	1.0401	25.4437
23	93.0081	62.0054	31.0027	5.1671	1.0334	26.5944
24	92.3834	61.5889	30.7945	5.1324	1.0265	27.7444
25	91.7329	61.1553	30.5776	5.0963	1.0193	28.8937
26	91.0566	60.7044	30.3522	5.0587	1.0117	30.0424
27	90.3545	60.2363	30.1182	5.0197	1.0039	31.1898
28	89.6265	59.7510	29.8755	4.9793	0.9958	32.3365
29	88.8730	59.2487	29.6243	4.9374	0.9875	33.4824
30	88.0940	58.7293	29.3647	4.8941	0.9788	34.6270
31	87.2894	58.1929	29.0965	4.8494	0.9699	35.7709
32	86.4594	57.6396	28.8198	4.8033	0.9607	36.9135
33	85.6043	57.0695	28.5348	4.7558	0.9512	38.0552
34	84.7237	56.4825	28.2412	4.7069	0.9414	39.1957
35	83.8184	55.8789	27.9395	4.6566	0.9313	40.3349
36	82.8879	55.2586	27.6293	4.6049	0.9210	41.4732
37	81.9326	54.6217	27.3109	4.5518	0.9104	42.6100
38	80.9526	53.9684	26.9842	4.4974	0.8995	43.7457
39	79.9481	53.2987	26.6494	4.4416	0.8883	44.8801
40	78.9189	52.6126	26.3063	4.3844	0.8769	46.0130
41	77.8655	51.9103	25.9552	4.3259	0.8652	47.1448
42	76.7879	51.1919	25.5960	4.2660	0.8532	48.2749
43	75.6861	50.4574	25.2287	4.2048	0.8410	49.4037
44	74.5606	49.7071	24.8535	4.1423	0.8285	50.5312
45	73.4110	48.9407	24.4703	4.0784	0.8157	51.6568
46	72.2379	48.1586	24.0793	4.0132	0.8026	52.7811
47	71.0412	47.3608	23.6804	3.9467	0.7893	53.9035
48	69.8211	46.5474	23.2737	3.8790	0.7758	55.0245
49	68.5777	45.7185	22.8592	3.8099	0.7620	56.1438
50	67.3114	44.8743	22.4335	3.7395	0.7479	57.2612
51	66.0222	44.0148	22.0074	3.6679	0.7336	58.3769
52	64.7102	43.1401	21.5701	3.5950	0.7190	59.4909
53	63.3756	42.2504	21.1252	3.5209	0.7042	60.6029
54	62.0188	41.3459	20.6729	3.4455	0.6891	61.7131
55	60.6396	40.4264	20.2132	3.3689	0.6738	62.8214
56	59.2385	39.4923	19.7462	3.2910	0.6582	63.9278
57	57.8154	38.5436	19.2718	3.2120	0.6424	65.0319
58	56.3706	37.5804	18.7902	3.1317	0.6263	66.1343
59	54.9045	36.6030	18.3015	3.0503	0.6100	67.2343
60	53.4171	35.6114	17.8057	2.9676	0.5935	68.3323
61	51.9086	34.6057	17.3029	2.8838	0.5768	69.4282
62	50.3793	33.5862	16.7931	2.7989	0.5598	70.5220
63	48.8291	32.5527	16.2764	2.7127	0.5425	71.6132
64	47.2587	31.5058	15.7529	2.6255	0.5251	72.7022
65	45.6679	30.4453	15.2226	2.5371	0.5074	73.7890
66	44.0571	29.3714	14.6857	2.4476	0.4895	74.8735
67	42.4263	28.2842	14.1421	2.3570	0.4714	75.9553
68	40.7760	27.1840	13.5920	2.2653	0.4531	77.0349
69	39.1064	26.0709	13.0355	2.1726	0.4345	78.1119
70	37.4177	24.9451	12.4726	2.0788	0.4158	79.1863
71	35.7100	23.8067	11.9033	1.9839	0.3968	80.2581
72	33.9837	22.6558	11.3279	1.8880	0.3774	81.3273
73	32.2388	21.4925	10.7463	1.7910	0.3582	82.3937
74	30.4757	20.3171	10.1586	1.6931	0.3386	83.4576
75	28.6950	19.1300	9.5650	1.5942	0.3188	84.5187
76	26.8962	17.9308	8.9654	1.4942	0.2988	85.5769
77	25.0800	16.7200	8.3600	1.3938	0.2787	86.6328
78	23.2467	15.4998	7.7489	1.2915	0.2583	87.6849
79	21.3965	14.2643	7.1322	1.1887	0.2377	88.7345
80	19.5298	13.0195	6.5098	1.0850	0.2170	89.7811
81	17.6459	11.7639	5.8820	0.9803	0.1961	90.8248
82	15.7462	10.4975	5.2487	0.8748	0.1750	91.8652
83	13.8308	9.2205	4.6103	0.7684	0.1537	92.9028
84	11.8995	7.9330	3.9665	0.6611	0.1322	93.9372
85	9.9531	6.6354	3.3177	0.5530	0.1106	94.9684
86	7.9916	5.3277	2.6639	0.4440	0.0888	95.9963
87	6.0158	4.0102	2.0051	0.3342	0.0668	97.0211
88	4.0242	2.6828	1.3414	0.2236	0.0447	98.0423
89	2.0191	1.3461	0.6730	0.1122	0.0224	99.0604
90	0.0000	0.0000	0.0000	0.0000	0.0000	100.0752

PROJECTION TABLE OF THE SPHERE WITHIN A TWO-CUSPED EPICYCLOID

Longitude (degrees)	Latitude 0°		Latitude 10°		Latitude 20°	
	x	y	x	y	x	y
0	0	0	0.0657005	0	0.1329336	0
10	0	0.0655830	0.0659502	0.0652806	0.1334302	0.0644988
20	0	0.1320679	0.0667068	0.1310564	0.1349544	0.1294722
30	0	0.1986196	0.0679901	0.1977852	0.1375243	0.1954062
40	0	0.2672316	0.0698354	0.2661576	0.1412182	0.2628114
50	0	0.3379900	0.0722946	0.3365854	0.1461885	0.3322379
60	0	0.4115928	0.0754386	0.4097712	0.1524254	0.4042906
70	0	0.4886206	0.0793623	0.4864277	0.1602653	0.4976464
80	0	0.5700642	0.0843123	0.5672926	0.1699033	0.5590835
90	0	0.6568543	0.0900809	0.6535871	0.1816515	0.6434912
100	0	0.7501592	0.0972437	0.7461844	0.1959160	0.7339214
110	0	0.8513849	0.1059496	0.8465500	0.2132368	0.8316716
120	0	0.9612505	0.1165520	0.9563507	0.2342777	0.9381914
130	0	1.0848856	0.1295162	1.0776500	0.2599632	1.0554138
140	0	1.2219628	0.1454616	1.2130316	0.2915024	1.1856574
150	0	1.3768912	0.1652213	1.3657684	0.3304315	1.3317128
160	0	1.5540494	0.1899343	1.5400864	0.3789856	1.4973462
170	0	1.7592016	0.2211277	1.8376844	0.4404290	1.6880806
180	0	2.	0.2611265	1.9770816	0.5180136	1.9074542

Longitude (degrees)	Latitude 30°		Latitude 40°		Latitude 50°	
	x	y	x	y	x	y
0	0.2034338	0	0.2793195	0	0.3635676	0
10	0.2041862	0.0631074	0.2802784	0.0609474	0.3648154	0.0577322
20	0.2064626	0.1316525	0.2833638	0.1222879	0.3685880	0.1157704
30	0.2103218	0.1910839	0.2885056	0.1843829	0.3749738	0.1744240
40	0.2158656	0.2568702	0.2958856	0.2476688	0.3838829	0.2339767
50	0.2232437	0.3245138	0.3050484	0.3130724	0.3962648	0.2948634
60	0.2326608	0.3945624	0.3181886	0.3795353	0.4108000	0.3566911
70	0.2449072	0.4676172	0.3337174	0.4490871	0.4307951	0.4217824
80	0.2588288	0.5443830	0.3527249	0.5217812	0.4540864	0.4885921
90	0.2767904	0.6256337	0.3755278	0.5982518	0.4821972	0.5581998
100	0.2975072	0.7122866	0.4036212	0.6792015	0.5159930	0.6310620
110	0.3231872	0.8054182	0.4369882	0.7654118	0.5564494	0.7076256
120	0.3543094	0.9063060	0.4776436	0.8579324	0.6048868	0.7883974
130	0.3921517	1.0168946	0.5265641	0.9577176	0.6634542	0.8738781
140	0.4383940	1.1378104	0.5859527	1.0620988	0.7327556	0.9645540
150	0.4952640	1.2725474	0.6584734	1.1843322	0.8169787	1.0558424
160	0.5657522	1.4234770	0.7475379	1.3141952	0.9190820	1.1630216
170	0.6039234	1.5940444	0.8577977	1.4574472	1.0435333	1.2710883
180	0.7652722	1.7886287	0.9954774	1.6160449	1.1961326	1.3845351

Longitude (degrees)	Latitude 60°		Latitude 70°		Latitude 80°		Latitude 90°	
	x	y	x	y	x	y	x	y
0	0.4607018	0	0.5787124	0	0.7349414	0	1	0
10	0.4621716	0.0529277	0.5803343	0.0454350	0.7364340	0.0344776	1	0
20	0.4672152	0.1065867	0.5852284	0.0909263	0.7413749	0.0648148	1	0
30	0.4741149	0.1595796	0.5934766	0.1321514	0.7489142	0.0969471	1	0
40	0.4846000	0.2136174	0.6052234	0.1822728	0.7599221	0.1286701	1	0
50	0.4990250	0.2686200	0.6206861	0.2281989	0.7742548	0.1598415	1	0
60	0.5169762	0.3245270	0.6399599	0.2742612	0.7920463	0.1902634	1	0
70	0.5391044	0.3816124	0.6641303	0.3206262	0.8134522	0.2197014	1	0
80	0.5659222	0.4400499	0.6923585	0.3668740	0.8386672	0.2478621	1	0
90	0.5980757	0.5000000	0.7261381	0.4131360	0.8678768	0.2743574	1	0
100	0.6363678	0.5615868	0.7658208	0.4591221	0.9012980	0.2988543	1	0
110	0.6817881	0.6248820	0.8121519	0.5044956	0.9391886	0.3206583	1	0
120	0.7355584	0.6898793	0.8660224	0.5487450	0.9814328	0.3392176	1	0
130	0.7979642	0.7587495	0.9284004	0.5912766	1.0289884	0.3541808	1	0
140	0.8745804	0.8242182	1.0005022	0.6310376	1.0812768	0.3629320	1	0
150	0.9638786	0.8927645	1.0836293	0.6667320	1.1383840	0.3659622	1	0
160	1.0700168	0.9609010	1.1792318	0.6965438	1.2002228	0.3612830	1	0
170	1.1963838	1.0271250	1.2882392	0.7129000	1.2662668	0.3472414	1	0
180	1.3471508	1.0886622	1.4140122	0.7288040	1.3306988	0.3219392	1	0

TABLE OF LAGRANGE'S PROJECTION OF THE SPHERE WITHIN A CIRCLE

Longitude (degrees)	Latitude 0°		Latitude 10°		Latitude 20°	
	x	y	x	y	x	y
0	0	0	0.0438284	0	0.0888596	0
10	0	0.0436609	0.0439117	0.0435769	0.0890217	0.0433150
20	0	0.0874887	0.0441632	0.0873193	0.0895344	0.0867926
30	0	0.1316525	0.0445865	0.1313952	0.0903874	0.1305951
40	0	0.1763270	0.0451883	0.1759778	0.0915999	0.1748918
50	0	0.2216947	0.0459781	0.2212479	0.0931908	0.2198588
60	0	0.2679492	0.0469686	0.2673976	0.0951855	0.2656829
70	0	0.3152988	0.0481763	0.3146331	0.0976168	0.3125639
80	0	0.3639702	0.0495724	0.3628167	0.1005269	0.3607219
90	0	0.4142136	0.0513315	0.4132838	0.1039656	0.4103906
100	0	0.4663077	0.0533363	0.4652177	0.1079961	0.4618327
110	0	0.5205671	0.0556764	0.5192969	0.1127013	0.5153662
120	0	0.5773503	0.0584004	0.5758725	0.1181685	0.5712879
130	0	0.6370703	0.0615685	0.6353504	0.1245251	0.6300212
140	0	0.7002075	0.0652555	0.6982049	0.1319160	0.6919998
150	0	0.7673270	0.0695554	0.7649879	0.1405257	0.7577453
160	0	0.8390996	0.0745872	0.8363638	0.1505874	0.8278711
170	0	0.9163312	0.0804866	0.9130985	0.1624541	0.9034360
180	0	1.0000000	0.0874887	0.9961657	0.1763250	0.9843316

Longitude (degrees)	Latitude 30°		Latitude 40°		Latitude 50°	
	x	y	x	y	x	y
0	0.1364697	0	0.1884436	0	0.2474276	0
10	0.1367251	0.0428463	0.1887939	0.0421076	0.2478703	0.0409832
20	0.1374947	0.0858470	0.1898387	0.0843588	0.2492046	0.0820941
30	0.1387904	0.1291590	0.1915962	0.1268991	0.2514492	0.1234614
40	0.1406313	0.1729429	0.1940931	0.1698780	0.2546356	0.1652176
50	0.1430461	0.2173669	0.1973654	0.2134492	0.2588118	0.2075000
60	0.1460726	0.2626078	0.2014643	0.2577765	0.2635038	0.2499439
70	0.1501045	0.3088547	0.2064532	0.3030319	0.2703818	0.2942080
80	0.1541678	0.3563118	0.2124130	0.3404008	0.2779512	0.3389347
90	0.1593751	0.4052047	0.2194434	0.3970844	0.2868561	0.3847996
100	0.1654740	0.4557775	0.2276665	0.4463015	0.2972716	0.4320093
110	0.1725808	0.5083069	0.2372325	0.4972948	0.3093459	0.4807224
120	0.1808370	0.5631018	0.2483243	0.5503325	0.3233060	0.5311656
130	0.1904178	0.6205149	0.2611607	0.6057158	0.3394148	0.5835687
140	0.2015393	0.6809490	0.2760359	0.6637837	0.3579035	0.6381849
150	0.2144701	0.7448686	0.2932722	0.7249198	0.3794335	0.6952880
160	0.2295464	0.8128137	0.3132981	0.7895589	0.4042149	0.7551781
170	0.2471927	0.8854192	0.3366425	0.8581908	0.4329290	0.8181755
180	0.2679493	0.9634329	0.3639703	0.9314104	0.4663077	0.8846224

Longitude (degrees)	Latitude 60°		Latitude 70°		Latitude 80°		Latitude 90°	
	x	y	x	y	x	y	x	y
0	0.3178372	0	0.4085369	0	0.5434661	0	1	0
10	0.3183819	0.0392428	0.4091855	0.0363622	0.5441959	0.0307481	1	0
20	0.3200224	0.0785897	0.4111388	0.0727936	0.5466232	0.0615355	1	0
30	0.3227810	0.1181460	0.4144191	0.1093630	0.5500690	0.0922957	1	0
40	0.3265266	0.1579375	0.4190637	0.1461390	0.5552642	0.1231173	1	0
50	0.3318110	0.1983143	0.4251363	0.1831939	0.5620183	0.1539807	1	0
60	0.3382041	0.2391464	0.4325842	0.2205338	0.5703908	0.1848886	1	0
70	0.3459602	0.2806290	0.4420034	0.2584946	0.5804409	0.2158359	1	0
80	0.3551891	0.3228807	0.4526497	0.2966636	0.5922910	0.2468141	1	0
90	0.3660254	0.3660254	0.4653061	0.3354736	0.6060012	0.2777961	1	0
100	0.3786317	0.4101908	0.4799520	0.3748748	0.6217106	0.3087521	1	0
110	0.3932042	0.4555098	0.4967776	0.4149168	0.6395515	0.3396315	1	0
120	0.4099776	0.5021180	0.5160082	0.4556399	0.6594753	0.3702514	1	0
130	0.4292856	0.5501568	0.5379080	0.4970707	0.6822531	0.4008564	1	0
140	0.4513158	0.5997654	0.5627856	0.5392167	0.7074731	0.4309861	1	0
150	0.4766271	0.6510848	0.5910015	0.5820588	0.7355386	0.4605926	1	0
160	0.5056567	0.7042429	0.6229746	0.6255417	0.7666786	0.4894771	1	0
170	0.5380943	0.7593943	0.6589776	0.6693438	0.8011176	0.5173782	1	0
180	0.5773503	0.8164966	0.7002073	0.7139397	0.8390998	0.5439778	1	0

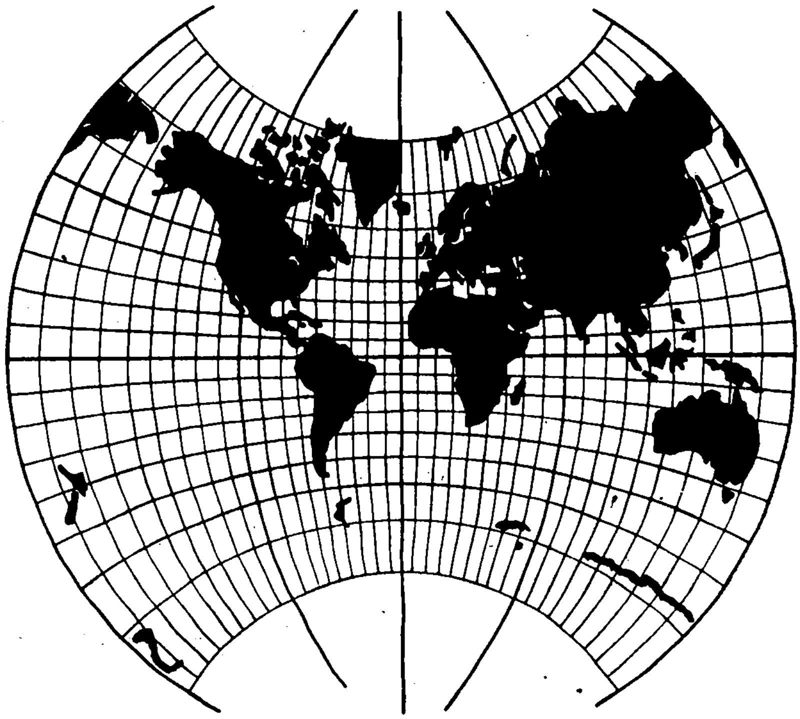

FIG. 89 -- LAGRANGE'S PROJECTION OF THE SPHERE WITHIN A CIRCLE

C. & G. S. Print — 99(20) ++

CARTOGRAPHIC PUBLICATIONS OF THE U. S. COAST AND GEODETIC SURVEY

Special Publication No. 5. Tables for a polyconic projection of maps and lengths of terrestrial arcs of meridian and parallels, based upon Clarke's reference spheroid of 1866, fifth edition, 1930.

Special Publication No. 38. Elements of chart making, 1916. [40]

Special Publication No. 47. The Lambert conformal conic projection with two standard parallels, including comparison of Lambert projection with Bonne and polyconic projections, 1918. [40]

Special Publication No. 49. Lambert projection tables for the war zone in France, with conversion tables, 1918. [40]

Special Publication No. 52. Lambert projection tables for the United States, 1918. [40]

Special Publication No. 53. General theory of the Lambert conformal conic projection, 1918.

Special Publication No. 57. General theory of polyconic projections, 1919.

Special Publication No. 59. Grid system for progressive maps in the United States, 1919. [40]

Special Publication No. 60. A study of map projections in general, 1919.

Special Publication No. 66. Rules and practice relating to construction of nautical charts, 1920. [40]

Special Publication No. 67. Latitude developments connected with geodesy and cartography, 1921. [40]

Special Publication No. 75. Radio-compass bearings, 1921.

Special Publication No. 112. Elliptic functions applied to conformal world maps, 1925. [40]

Special Publication No. 130. Tables for Albers projection for the United States and its possessions, 1927.

Special Publication No. 153. Conformal projection of the sphere within a square, 1929.

Special Publication No. 205. Cartography, A Review and Guide for the Construction and Use of Maps and Charts, 1936.

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[40] This publication is now (1944), at least temporarily, out of print. It is available for reference in most large libraries.

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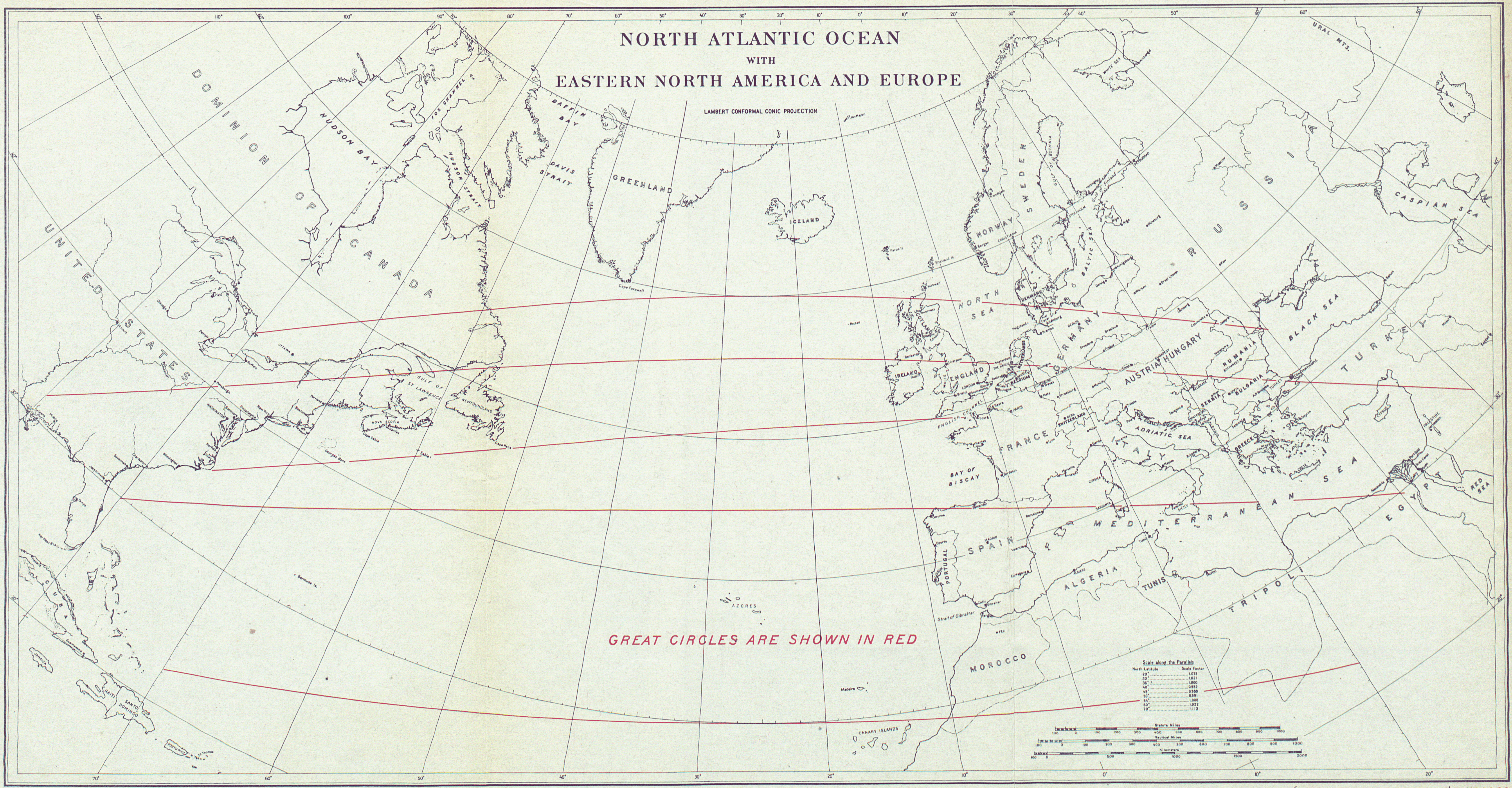

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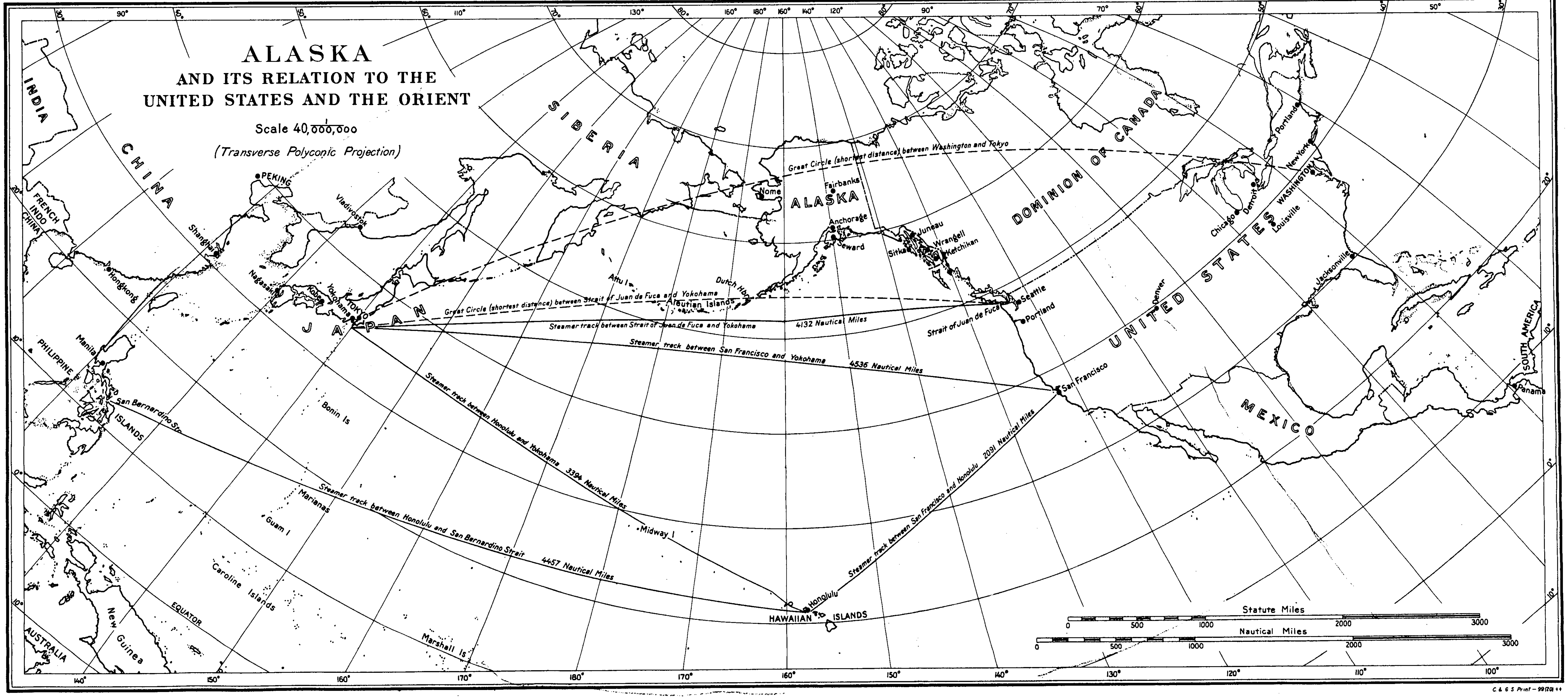
ALASKA
AND ITS RELATION TO THE
UNITED STATES AND THE ORIENT
Scale 1/40,000,000
(Transverse Polyconic Projection)
INDIA
CHINA
FRENCH INDO CHINA
SIBERIA
PEKING
Vladivostok
Shanghai
Hongkong
Nagasaki
Kobe
Yokohama
TOKYO
JAPAN
Manila
PHILIPPINE ISLANDS
San Bernardino Str
Bonin Is
Marianas
Guam I
Caroline Islands
EQUATOR
New Guinea
AUSTRALIA
Marshall Is
Midway I
Attu I
Aleutian Islands
Dutch Hbr
Nome
Fairbanks
ALASKA
Anchorage
Seward
Juneau
Sitka
Wrangell
Ketchikan
DOMINION OF CANADA
Great Circle (shortest distance) between Washington and Tokyo
Great Circle (shortest distance) between Strait of Juan de Fuca and Yokohama
Steamer track between Strait of Juan de Fuca and Yokohama 4132 Nautical Miles
Steamer track between San Francisco and Yokohama 4536 Nautical Miles
Steamer track between Honolulu and Yokohama 3394 Nautical Miles
Steamer track between Honolulu and San Bernardino Strait 4457 Nautical Miles
Steamer track between San Francisco and Honolulu 2091 Nautical Miles
Strait of Juan de Fuca
Seattle
Portland
San Francisco
Denver
UNITED STATES
Chicago
Detroit
Louisville
WASHINGTON
New York
Portland
Jacksonville
MEXICO
Panama
SOUTH AMERICA
Honolulu
HAWAIIAN ISLANDS
Statute Miles
0 500 1000 2000 3000
Nautical Miles
0 500 1000 2000 3000

ALBERS EQUAL AREA PROJECTION

STANDARD PARALLELS 29° 30′ AND 45° 30′

Plate III

100 50 0 500 1000 Statute Miles

Isoperimetric Curves are shown in Red

Due to the symmetry of the map there are an infinite number of isoperimetric (true scale) curves, exact replicas of those shown.

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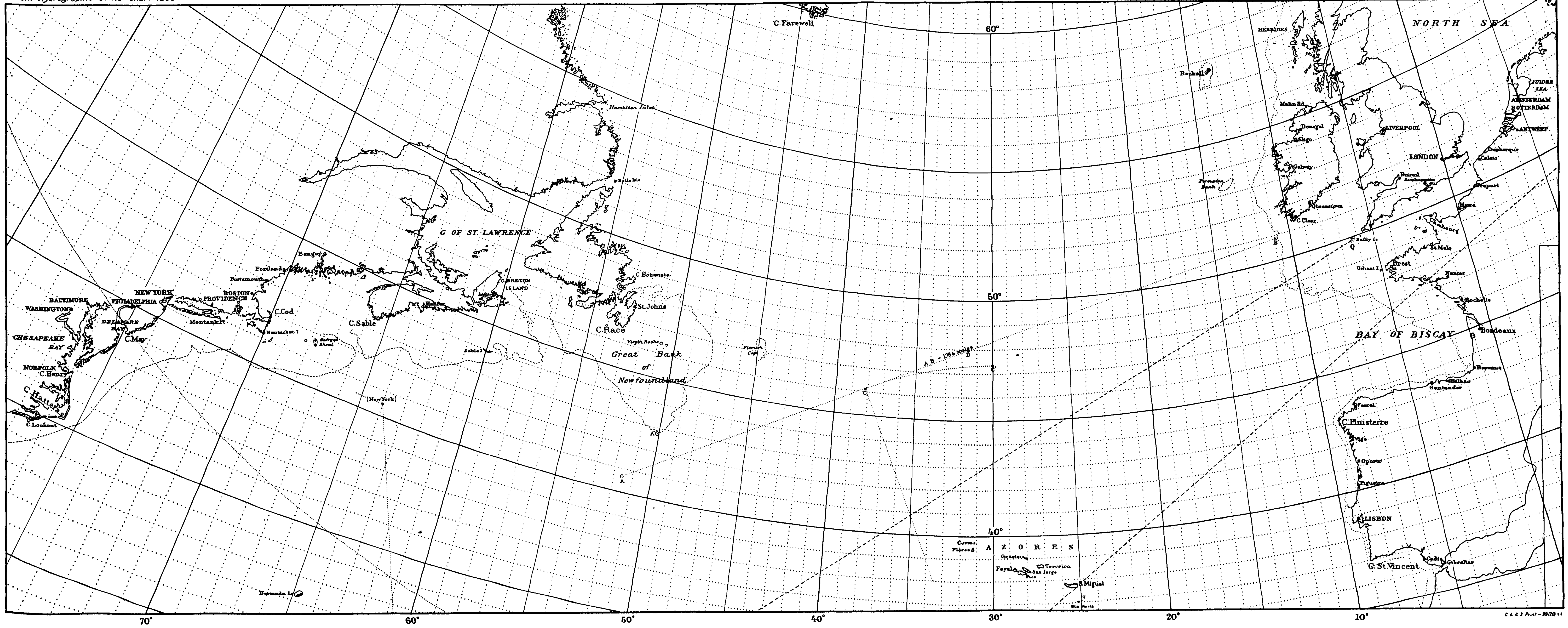
From Hydrographic Office Chart 1280

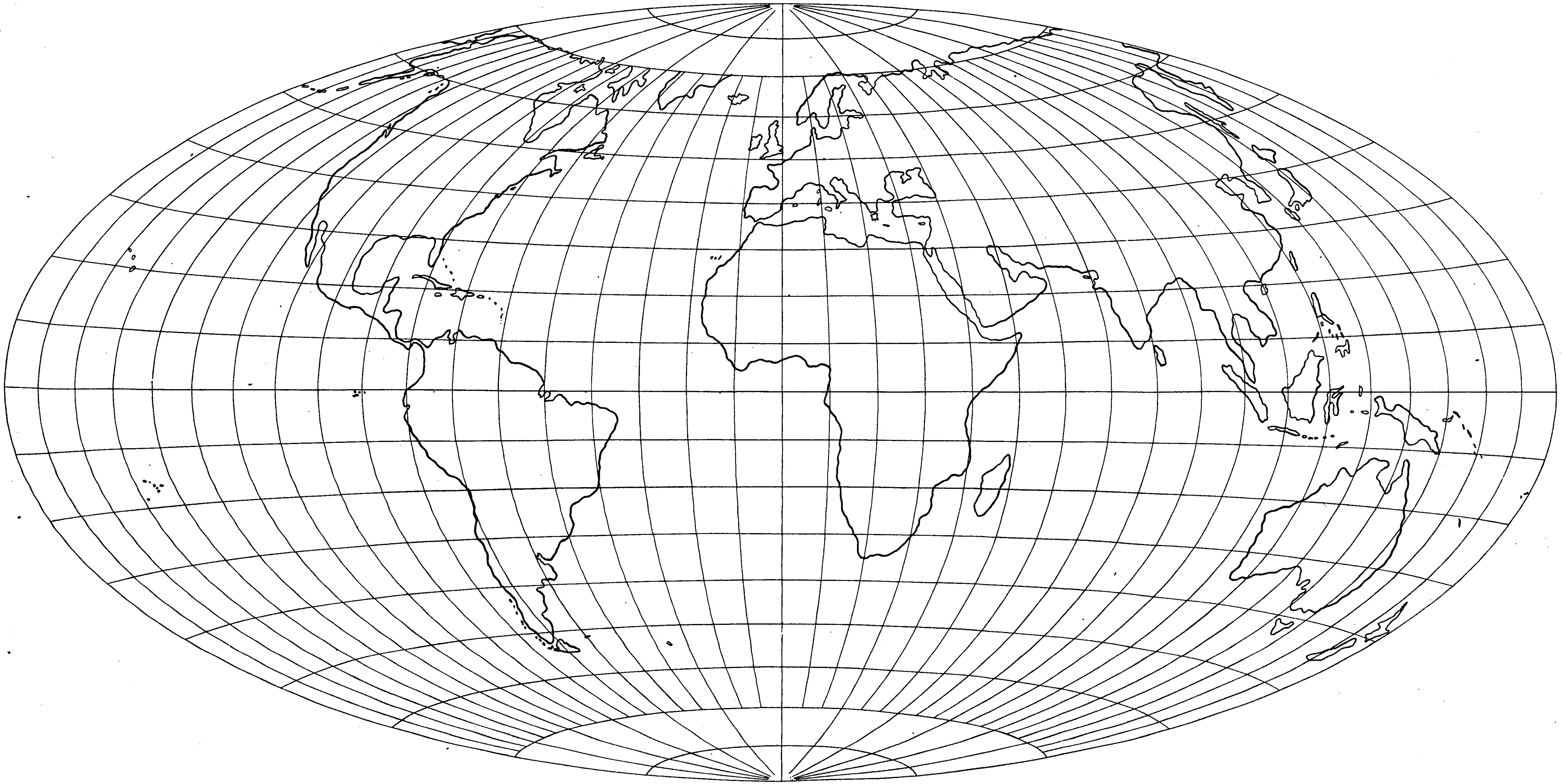
GNOMONIC PROJECTION OF THE NORTH ATLANTIC OCEAN; POINT OF TANGENCY 30°N. AND 30°W.

Plate IV



AITOFF'S EQUAL AREA PROJECTION OF THE SPHERE

Plate V



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THE WORLD ON THE HOMALOGRAPHIC PROJECTION (INTERRUPTED) FOR THE CONTINENTS (GOODE)

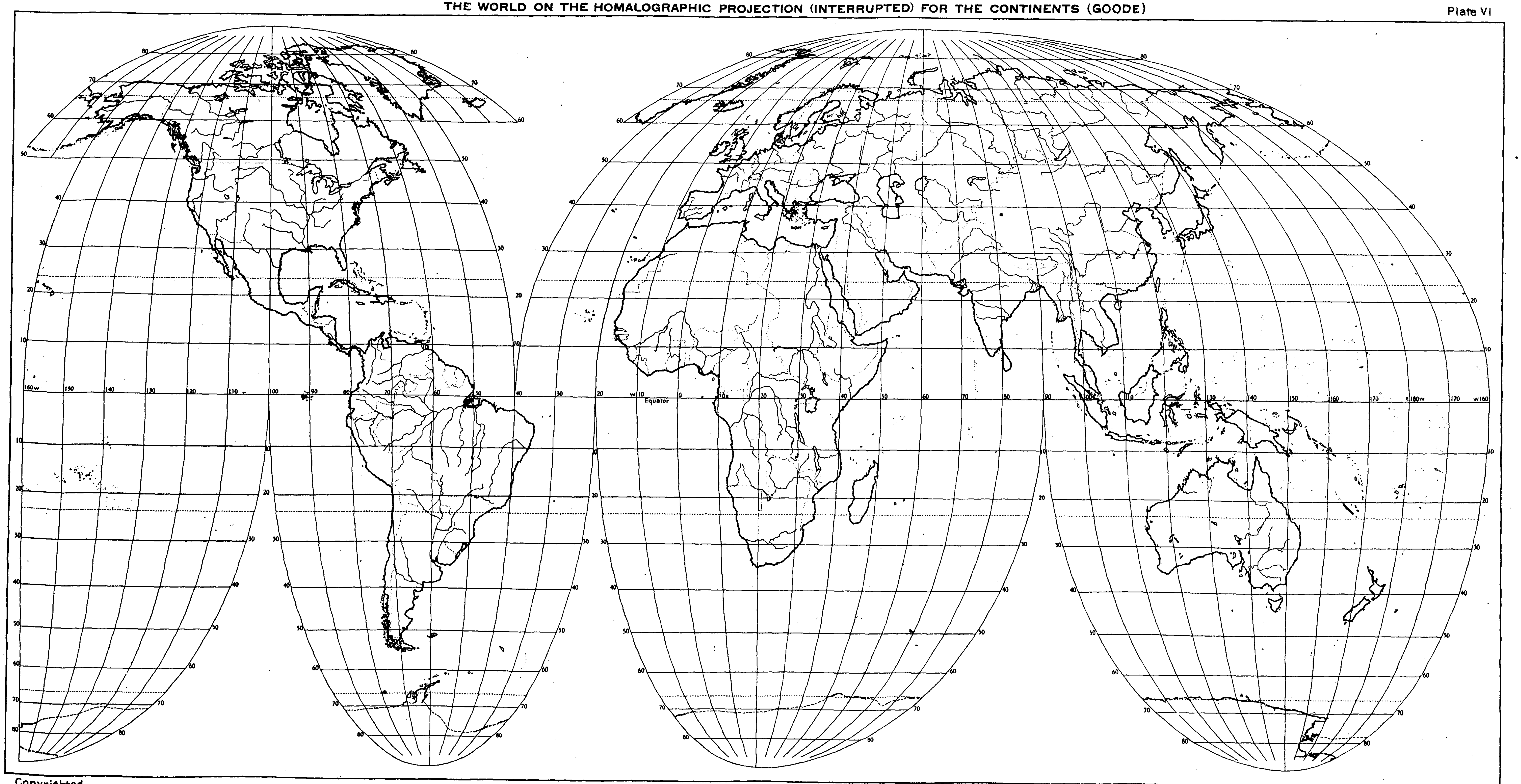

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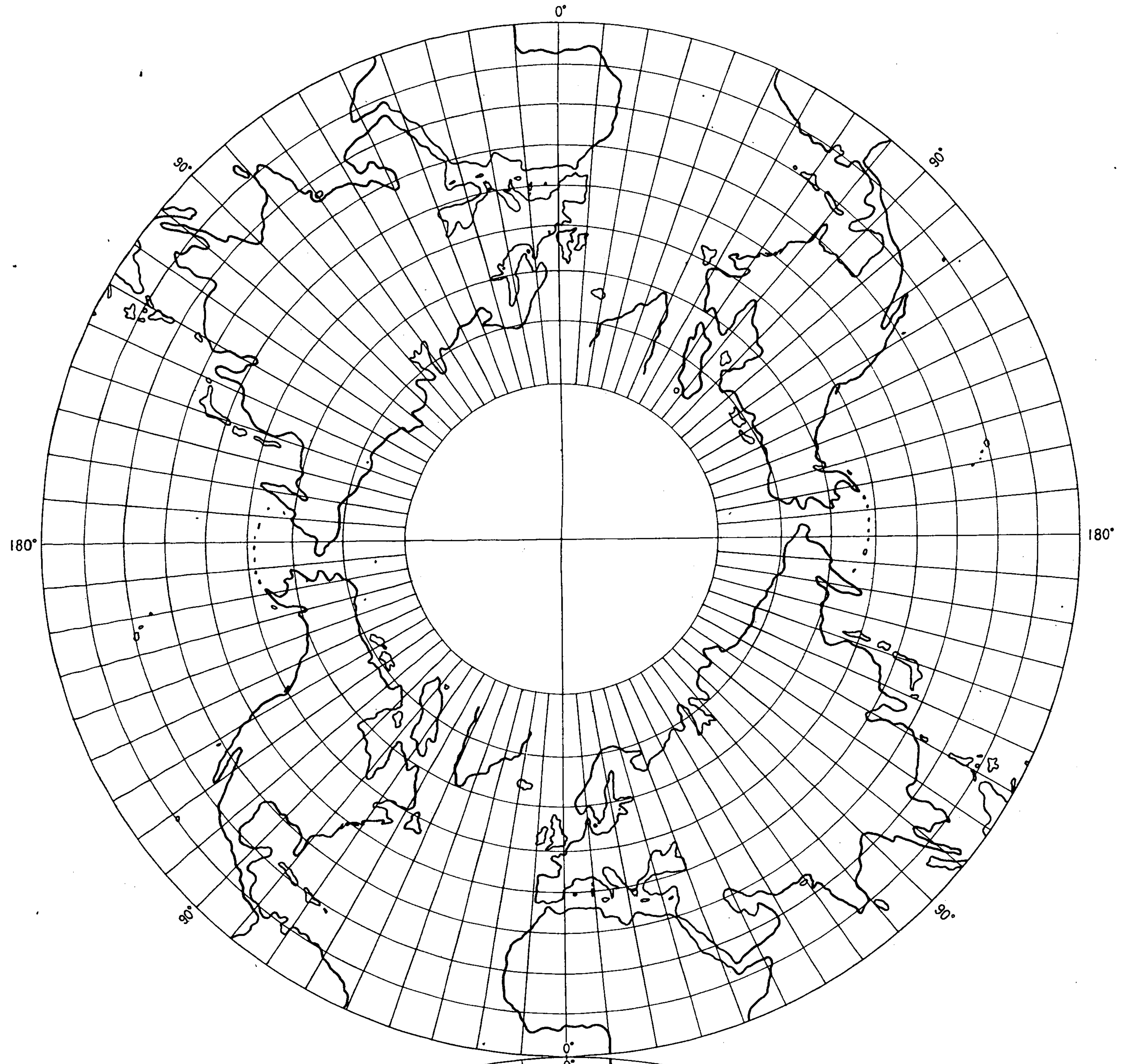
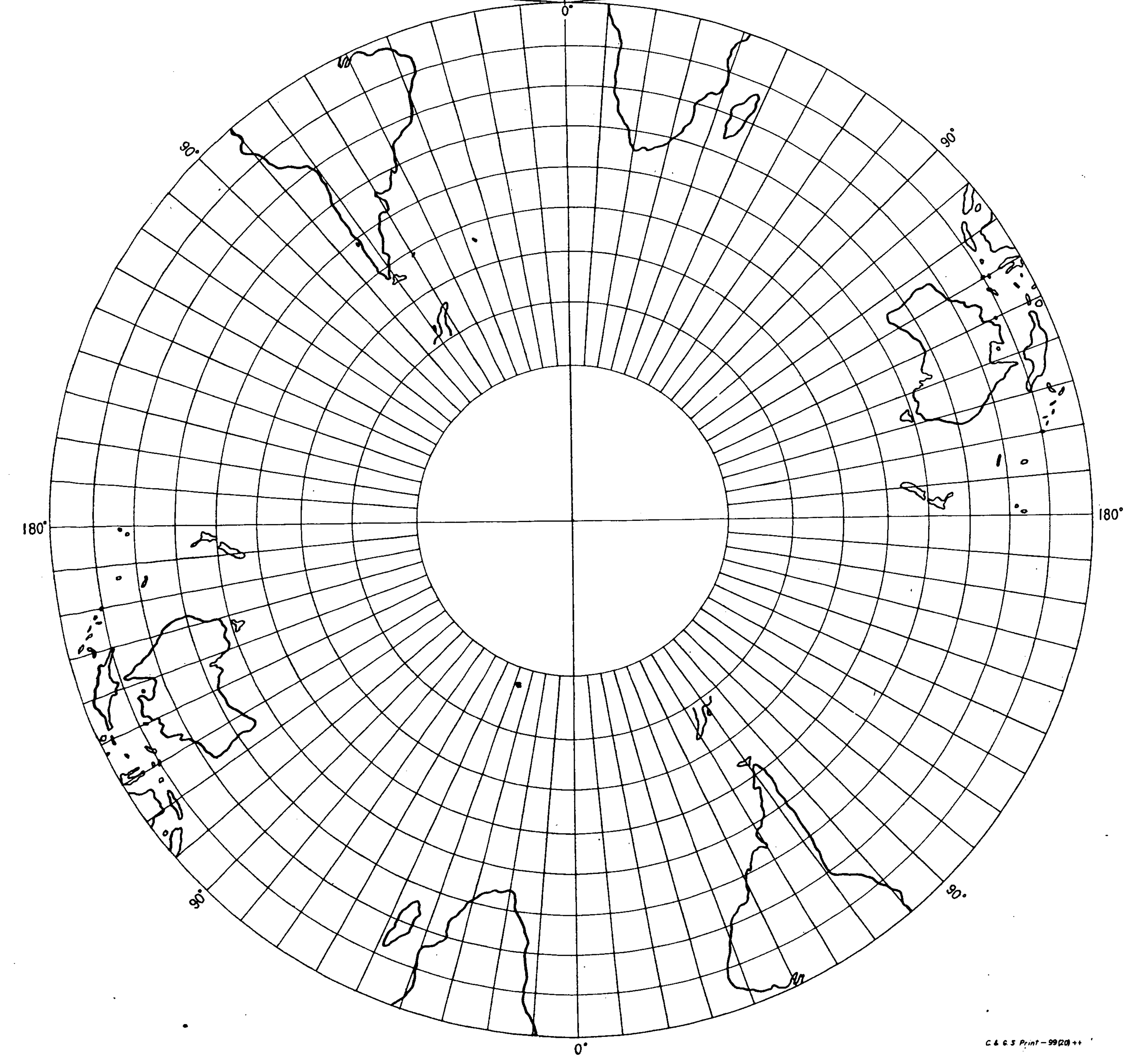
Plate VII

LAMBERT PROJECTION OF THE

NORTHERN AND SOUTHERN HEMISPHERES





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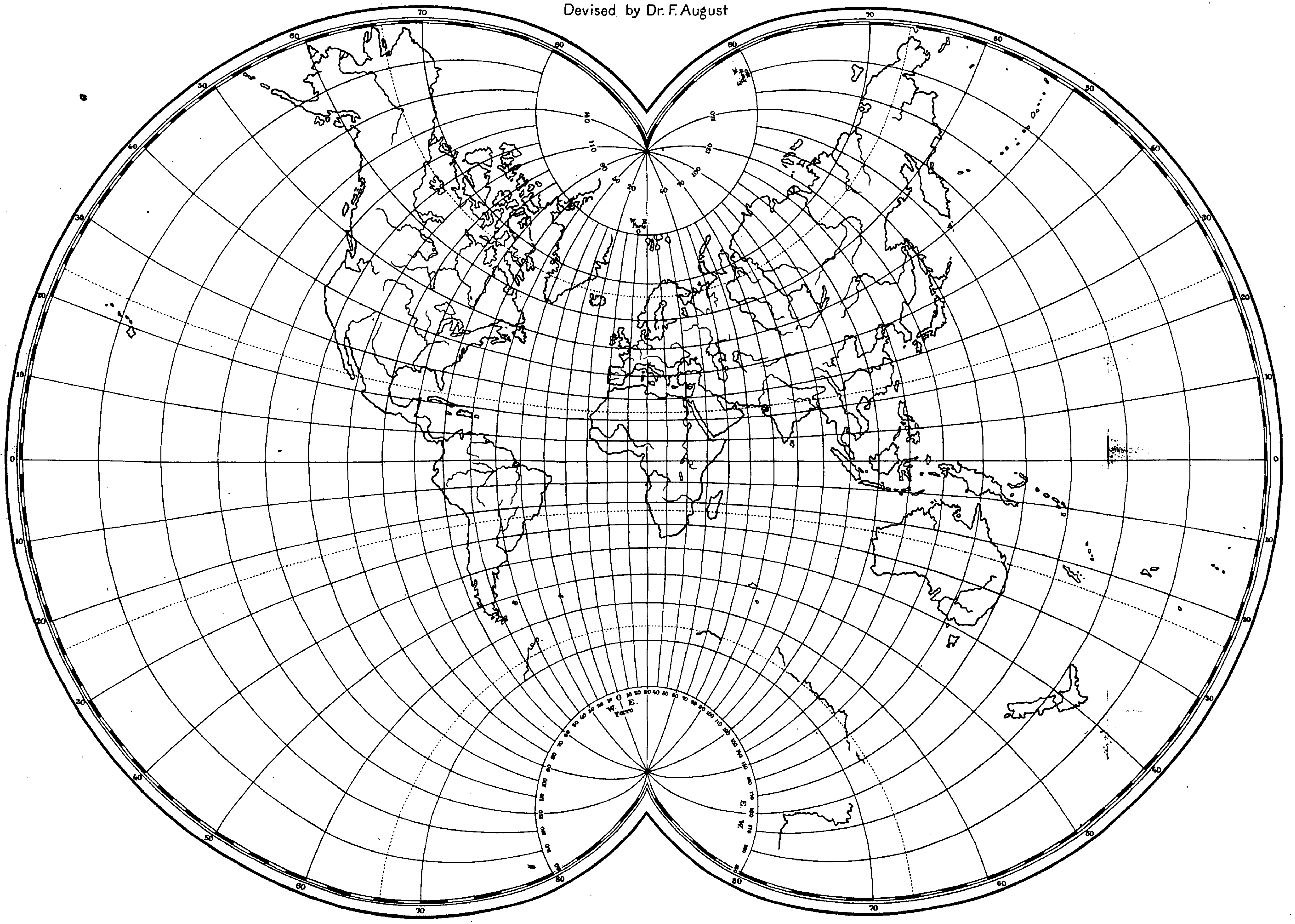
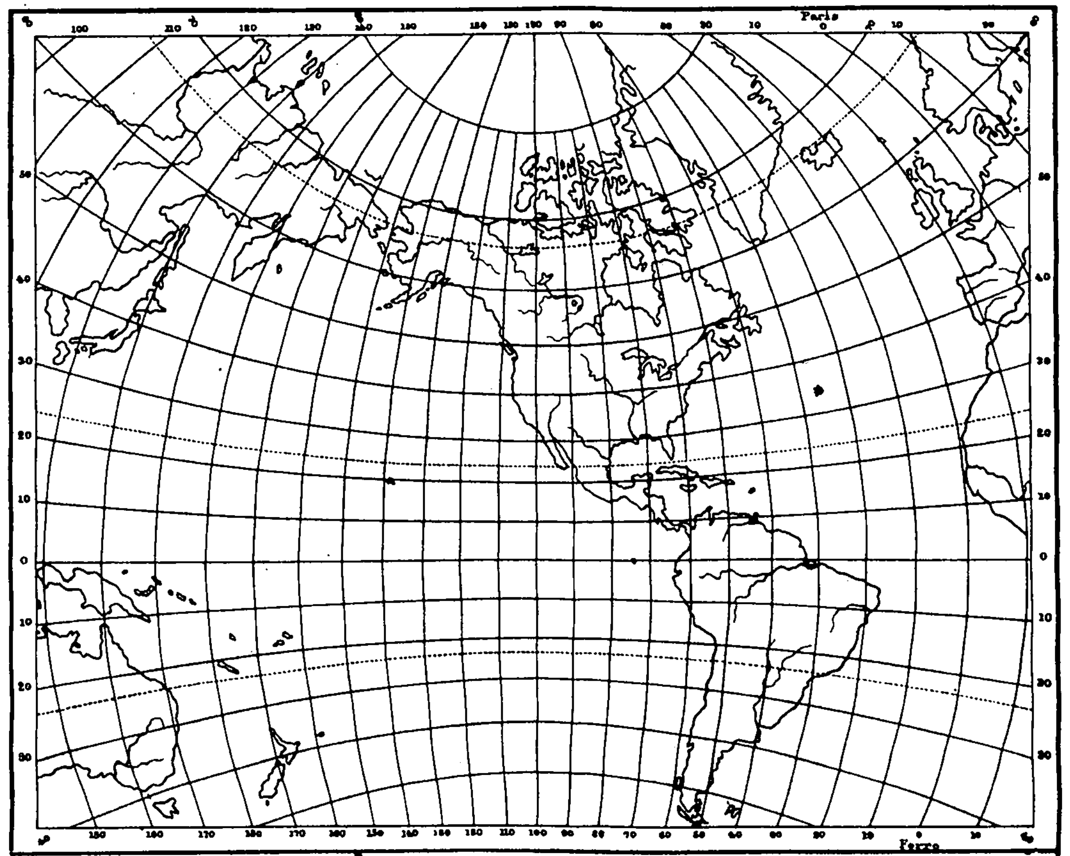
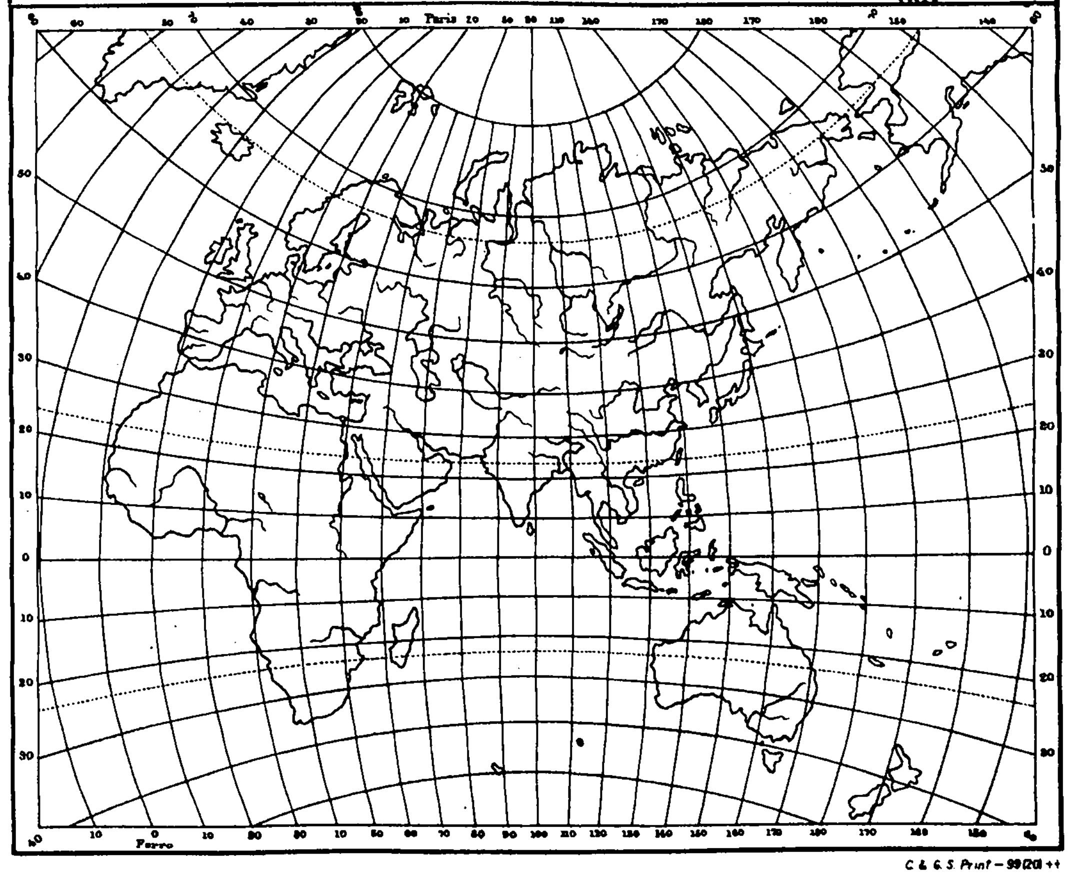
Conformal Projection of the Sphere

Within a two cusped Epicycloid

Devised by Dr. F. August







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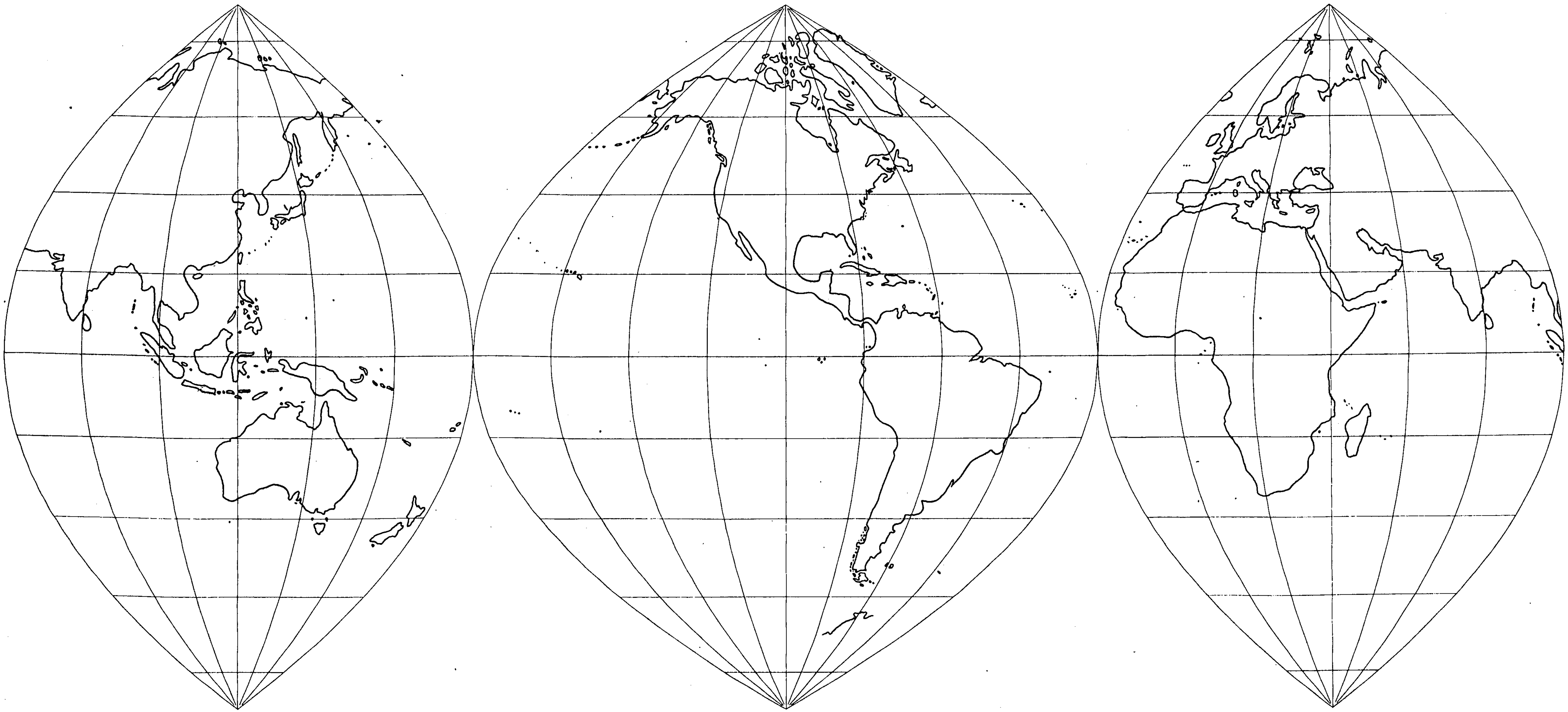


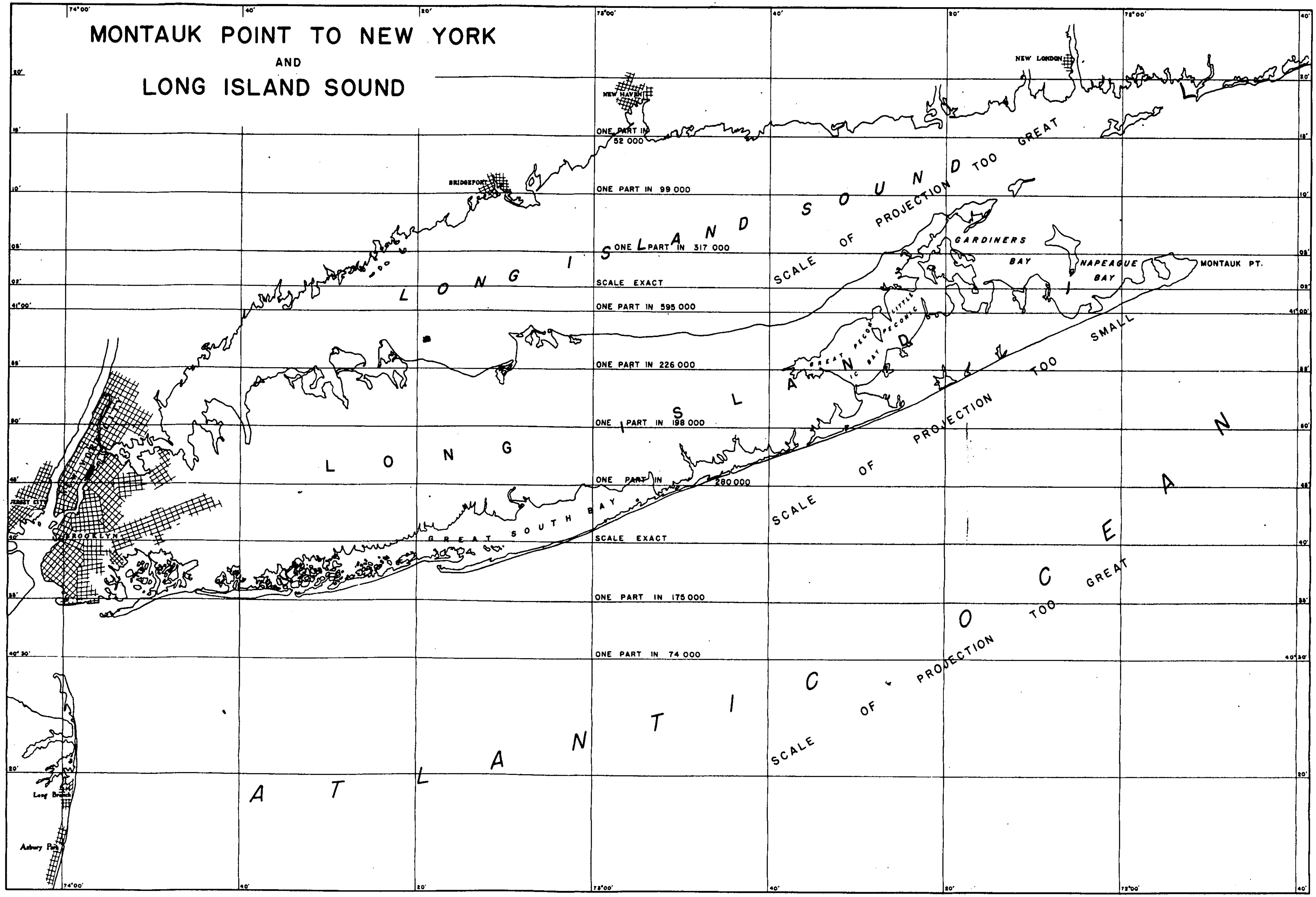
Parabolic Equal Area Projection of the World

Compiled and Arranged by

C. H. Deetz, U. S. Coast and Geodetic Survey

C & G S Print – 99(20)++

Plate X



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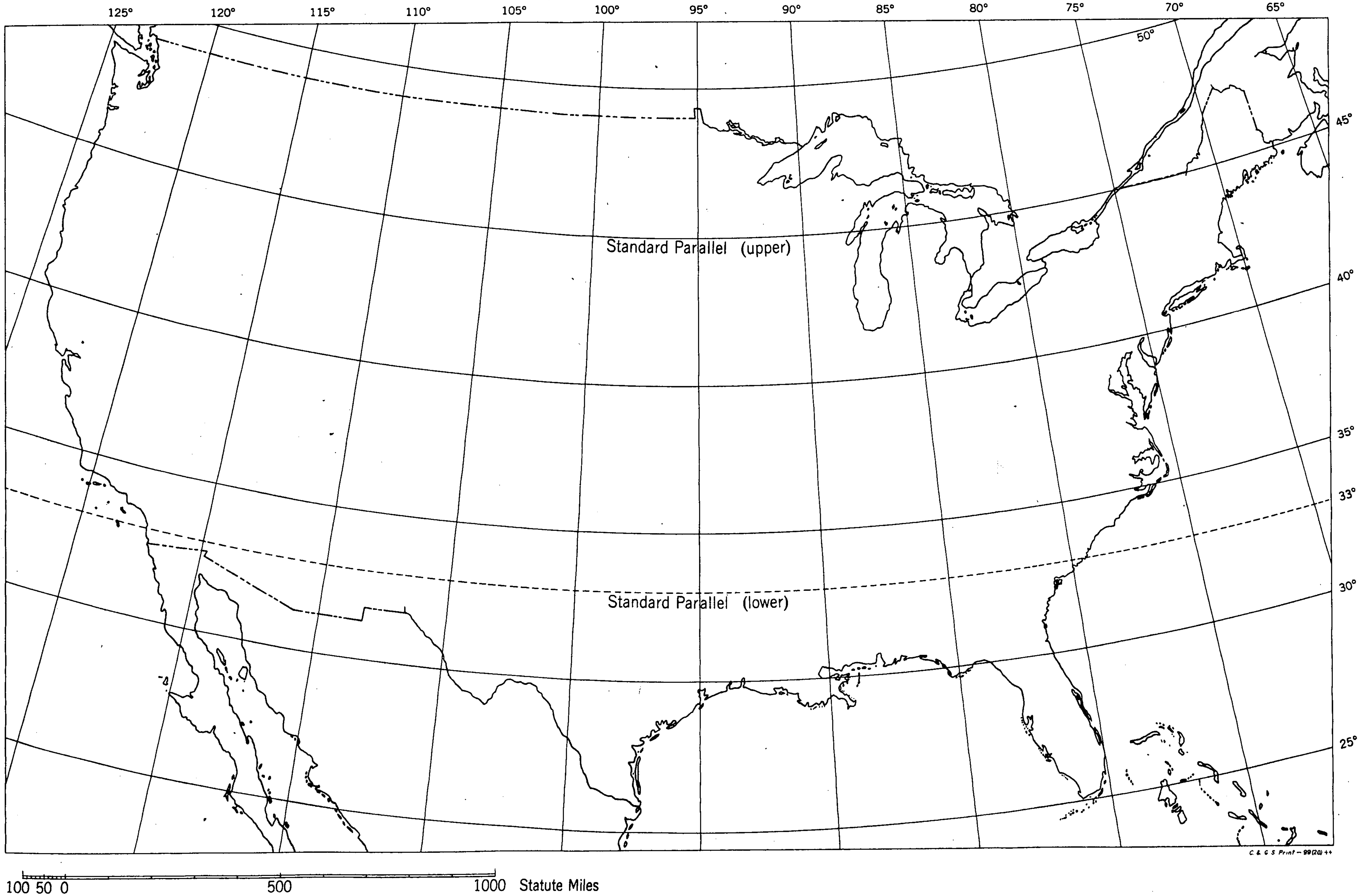
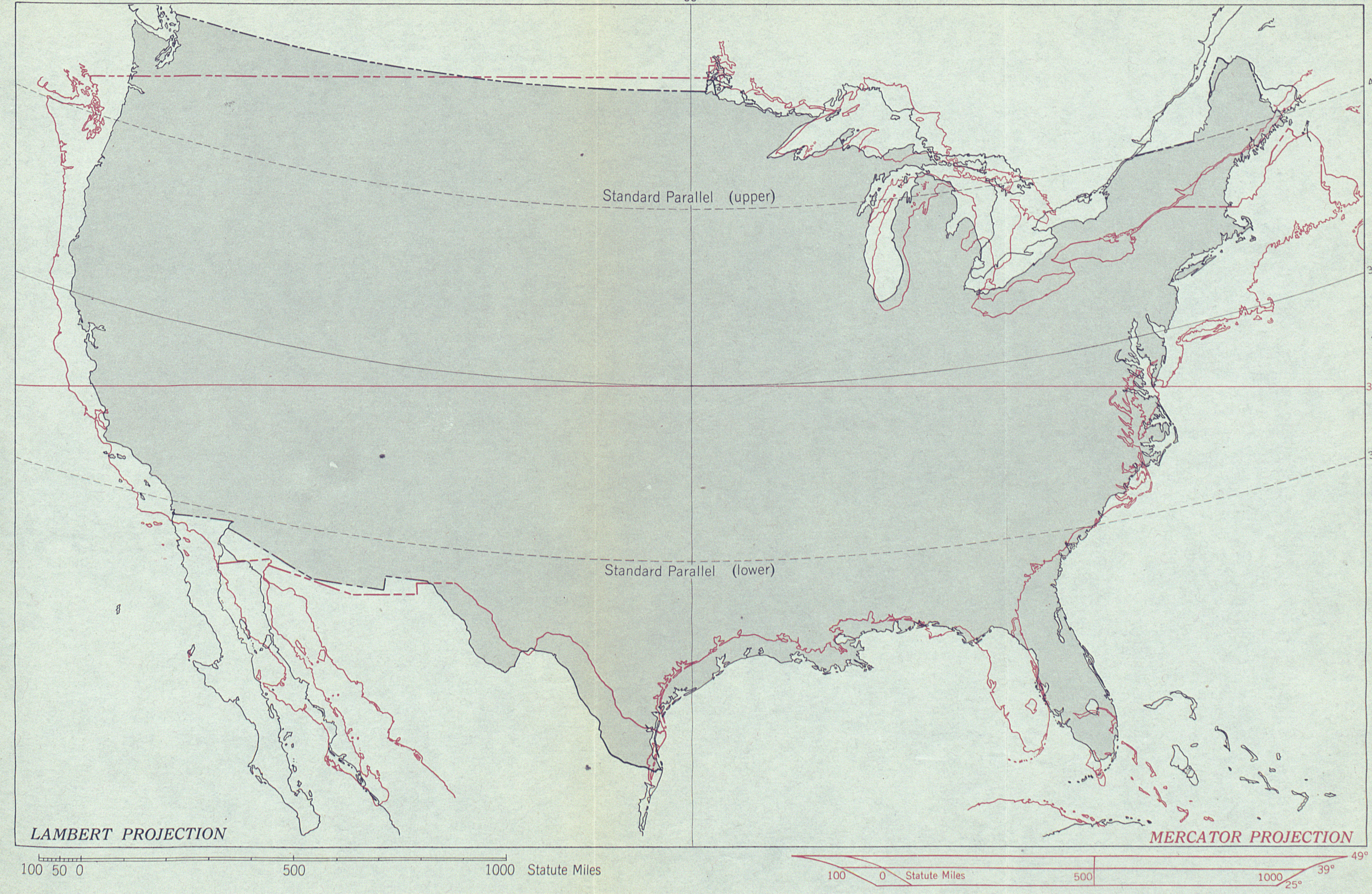
LAMBERT CONFORMAL CONIC PROJECTION FOR RECTANGULAR COORDINATES

LAMBERT CONFORMAL CONIC PROJECTION

STANDARD PARALLELS 33° AND 45°

Plate XI





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A COMPARISON OF THE MERCATOR PROJECTION AND THE LAMBERT CONFORMAL CONIC PROJECTION WITH TWO STANDARD PARALLELS

SUPERIMPOSED AT LAT. 39° NORTH, AND LONG. 96° WEST.

LAMBERT'S ZENITHAL EQUAL AREA PROJECTION

FOR MAP OF THE UNITED STATES

Scale 1:25,000,000

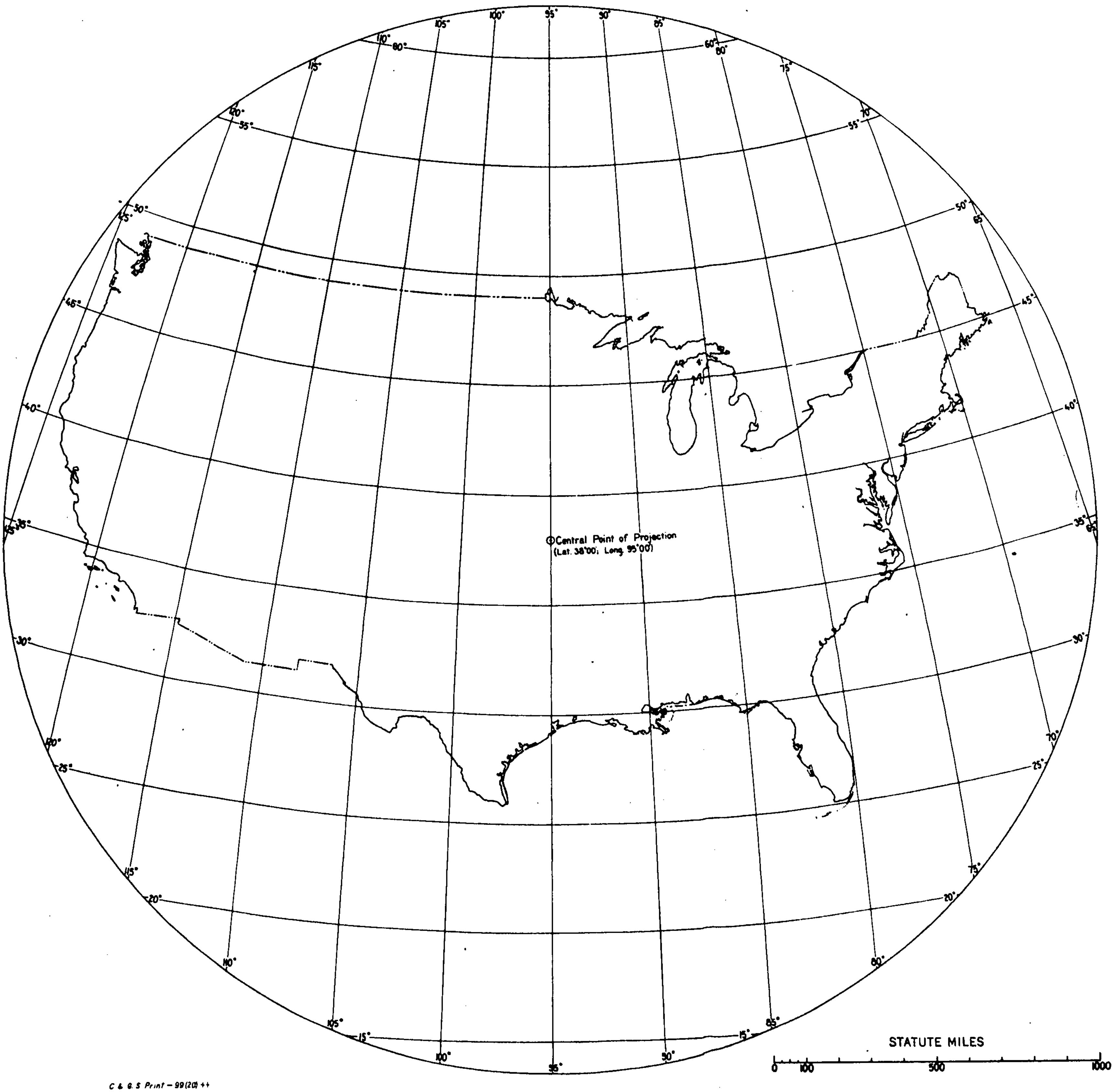

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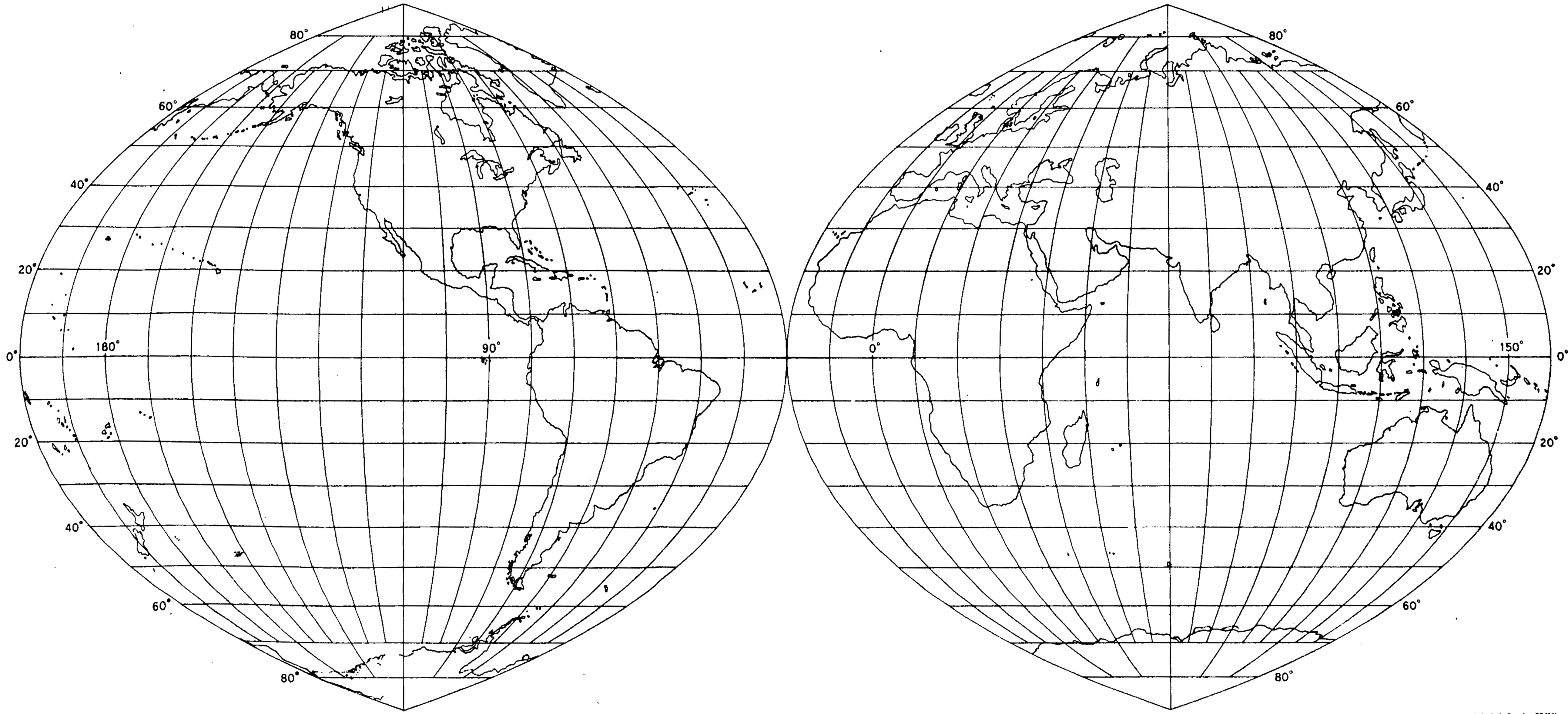
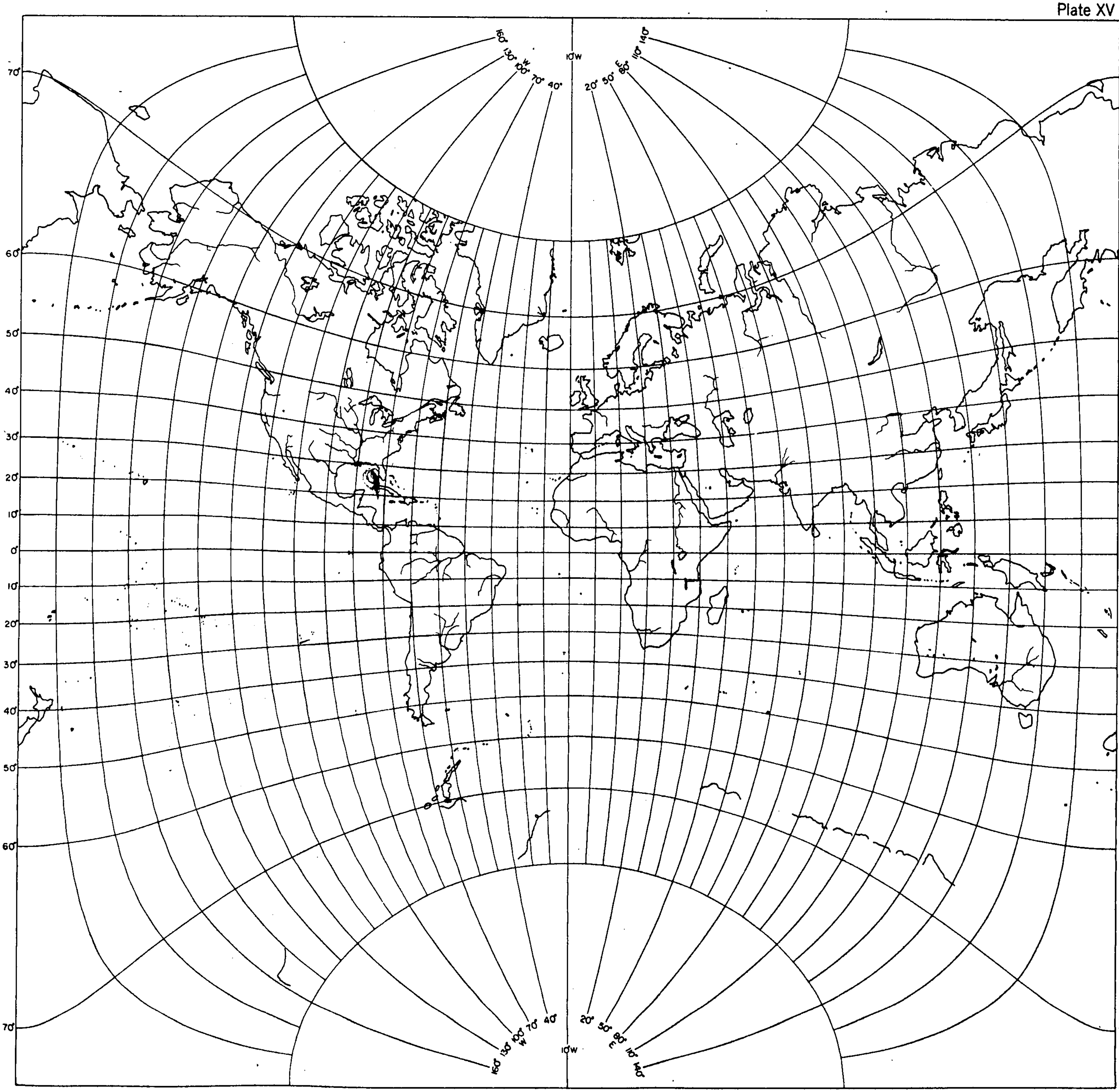
Plate XIV



C & G S Print – 99(20) 44

Equivalent Projection with Straight Line Parallels and with Lambert Equivalent Spacings on the Central Meridian. (Projection limited to hemispheres)



CONFORMAL MAP OF THE WORLD IN A SQUARE