EUCLIDEAN SPHERICAL REPRESENTATION OF A 3-D OBJECT

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ABSTRACT

In this paper we present a new packing algorithm. It fills a 3-D space with tangent spheres of different radii. The spheres and the tangency relation form a graph representing the volume; from this graph we can extract volume properties as: elongated parts, flat parts, part-whole, etc.... The computation performed in three steps: computerization of the euclidean map of the objects using the adaptation of Danielson's algorithm to 3-D space, extraction of the tangent spheres, computation of the volumes properties.

RÉSUMÉ

Cet article présente un algorithme de représentation sphérique. Il remplit un objet tridimensionnel avec des sphères tangentes de différents rayons. Les sphères et la relation de tangence forment un graphe correspondant au volume initial; on peut en extraire des propriétés du volume telles que: parties allongées, parties plates, connectivité, ... L'algorithme se déroule en trois étapes: calcul de la carte euclidienne de l'objet par une adaptation au cas tridimensionnel de l'algorithme de Danielson, extraction des sphères tangentes, calcul des propriétés du volume.

1. INTRODUCTION

Spherical representations have been used successfully for the display of moving 3-D objects [1]. Spheres have nice properties: they can be represented by only four numbers, they are convex, rotationnaly invariant and always viewed with the same shape. These properties were of utmost importance for such applications. In order to have the smoothest display the sphere were overlapping. This paper is involved with machine analysis of three dimensional objects with the goal of generating a useful symbolic description and recognizing the objects. For this new goal a representation with non overlapping spheres is more adequate : each point belongs to only one sphere and this make the interpretation easier.

The classical pattern recognition techniques are usually inadequate for 3-D scene analysis . The features to be extracted need to be invariant under the six degrees of freedom of a solid object. The moments of inertia are such mesure but provide poor indications of the objects; on the other hand they fail completely if we have articulated objects. The generalized cones introduced by Binford [2] were the first attempt to deal with such a goal. They have a large descriptive power and they have been succesfully used for recognition of real objects [8].

The work described here differs in three significant ways from previous work :

- first the input data is supplied in a voxel representation. Such a complete 3-D data can be provided by a tactile sensor or by several views with depth information.
- instead of extracting directly high level primitives - i.e. the generalized cones - we construct first a spherical representation :filling the objects with spheres of different radii we obtain a tangency graph representing a compact approximation of the volume.
- using this graph the algorithm then extracts the description of the objects: elongated parts,flat parts, types of cross section.

The second section of the paper explain how we compute the Euclidean map of a 3-D object. Then we explain the construction of the tangency graph. Section 4 shows how the tangency graph can be used for extracting information from the initial object and point out some of the difficulties. Finally, section 5 discusses the advantages and drawbacks of this approach.

2. COMPUTATION OF THE EUCLIDEAN DISTANCE MAP

Given a 2-D object, that is a picture and its background, the Euclidean map indicates for each point its distance to the closest background point. Rosenfeld and Pfaltz [10] have proposed a sequential algorithm which is able to compute such a distance by scanning the image twice. But the chosen metric was the city block distance defined as

$$d((i,j),(k,l)) = |i-k| + |j-l|$$

Recently Danielsson [5] has proposed a sequential algorithm which is able to compute an approximation of the Euclidean distance with the same basic technique. This method also scan the image twice and produces results very close to the exact distance. The error present is less than the error introduced by the discreet representation used (0.9 for the algorithm 8SED).

We extend the 8SED algorithm to the 3-D case and following the Danielsson denomination we call it 26SED. For each point of the space we compute the relative position of its closest neigboor backgroud point by using the computation done for its 26 neighbors. Therefore we need for each point three integers for saving these coordinates.

In what follows, d is defined by
$d(i,i')$: <u>if</u> $i=i'$ <u>then</u> 0 <u>else</u> 1

and $\min((a,b,c),(e,f,g))$ by

<u>if</u> $\sqrt{a^2+b^2+c^2} > \sqrt{e^2+f^2+g^2}$ <u>then</u> (e,f,g)
<u>else</u> (a,b,c)

(of course the square root can be omitted for saving computation).

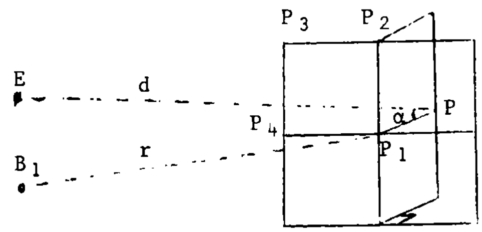

Fig.1 - The worst case error.

26SEWD : initialisation :

$L(i,j,k)=(0,0,0)$ if (i,j,k) belongs to the background

$L(i,j,k)=(M,M,M)$ if (i,j,k) belongs to the volume

(M is a maximum value)

apply 8SED to L(.,.,0);

first scan :

for k=1,2,...,n

for all i and j :

$$L(i,j,k)=\min\begin{cases} L(i,j,k) \\ L(x,y,k+1)+(d(i,x),d(j,y),1) \\ x=i-1,i,i+1 \quad y=j-1,j,j+1 \end{cases}$$

apply 8SED to L(.,.,k)

second scan :

for k=n-1,...1,0

for all i and j :

$$L(i,j,k)=\min\begin{cases} L(i,j,k) \\ L(x,y,k+1)+(d(i,x),d(j,y),1) \\ x=i-1,i,i+1 \quad y=j-1,j,j+1 \end{cases}$$

apply 8SED to L(.,.,k)

If $L(i,j,k)=(a,b,c)$ then the result for the point (i,j) is

$$\sqrt{a^2+b^2+c^2}.$$

As for the 2-D case some errors can occur; but the same reasoning of [5] shows that there is no propagation of these errors. Following Danielsson's technique, we can then prove that errors can occur in the case shown by the fig.1. The distance for P is computed using P1 (the discussion is similar for P2,P3,P4) which is at the distance r from the closest backgroundpoint B ; the real distance between P and the background is $|PE| = d$; then the error is

$r+1 - \cos(\alpha) - r-1+\cos(\alpha)$

If r is large enough the worst case is obtained for $\alpha = 31.172...$ and then the error will be bounded by 0.15.

As shown for the 2-D case, the number of errors in the real computation is very small. Our experiment (not a extensive search) allows us to conclude that less than 2% of the distances are in error ; all the errors found are of the type

$$d_{assigned} = \sqrt{d_{exact}^2 + 1}$$

We did not find an error larger than the error Danielsson found in his extensive search in the 2-D case : 0.027. An extensive search would probably give us a larger one.

3. CHOOSING THE SPHERES

The goal is to fill "in the best way" the volume with tangent spheres of radii greater than a minimum radius Rmin. A measure of performance could be the number of remaining points after this packing. But to write a non combinatorial algorithm computing an optimum packing seems very hard if not impossible. Therefore the heuristic approach for finding the spheres was choosen. After some attempts the selection was made using the following three criterions :

- maximum value of the radius,
- maximum number of already tangent spheres,
- minimum distance to the center of gravity.

The second criterion insure the construction of a highly connected graph. The last criterion is only helpful for choosing the starting sphere in order to insure the same starting point. Fig.2 and fig.3 show such filling.

The result of this packing is a tangency graph where each node is a sphere, each link a tangency relation. In order to take in account the errors introduced by the discretisation exact tangency is not required. Fig.4 displays one resulting graph. The solid lines connects the center of the biggest spheres, dotted lines display the other tangent relation. The smallest spheres which fill the spaces between the big ones are very sensitive to perturbation. Small changes can affect completly their positions but they still continue to be placed in the same shape of repartition, for instance in an anulus between two spheres.

The voxel representation is used as an approximation of the real continuous object. Such an approximation will be troublesome if is too large with respect to the thickness. Experiments lead to the conclusion that at least a thickness of 6 is necessary for having results not affected by discretisation.

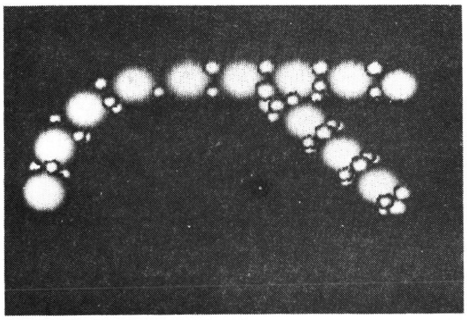

Fig.2 - Sphere packing for an elongated c

Fig.4 - Tangency graph for an other packing for the same shape as fig.2

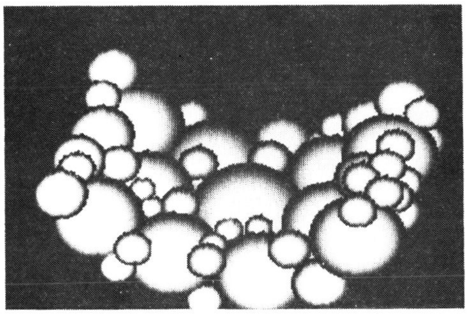

Fig.3 - Packing for a flat object

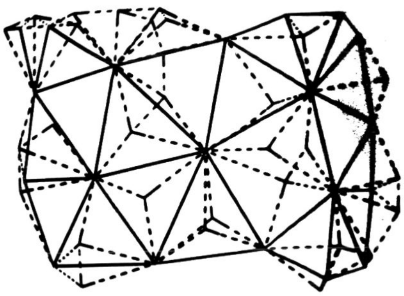

Fig.5 - Tangency graph for fig.3

4. EXTRACTING PROPERTIES AND MATCHING TO A REPRESENTATION

The biggest spheres -radius greater than the half of the radius of its neighboors- give compact and concise information about the rough shape. Fig.4 and fig.5 display respectively a typical elongated shape and a typical flat shape, both are bented. Specialized programs are written for finding each of the properties we want to extract. For extracting flat parts of a volume for example, the algorithm constructs first all the triangles as shown in fig.6; the connections between these triangles permit the extraction of the flat parts: if an edge is common to only two triangle the two triangle belongs to the same flat part, if an edge is common to more than two triangles then it is at an intersection of flat parts. is the end of a flat part.

Looking for more details is possible too. The disposition of the small spheres provides us with information of the section of the elongated parts; looking at the end of an elongated part allows us to conclude what kind of end it has : sharp, flat, ... For the 3-D lambda shown at fig.6 the result is : 3 elongated parts, each cylindrical with radius 5, one flat part that connects the 3 elongated ones. Here the intersection has been assimlated to a flat part.

In some simple cases, the matching to a higher level representation would only be a matching to a graph representing the object.

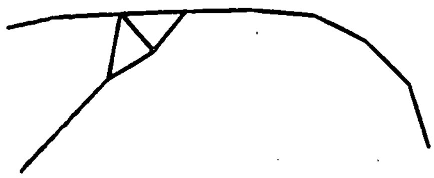

Fig.6 - The tangent big spheres of fig.4

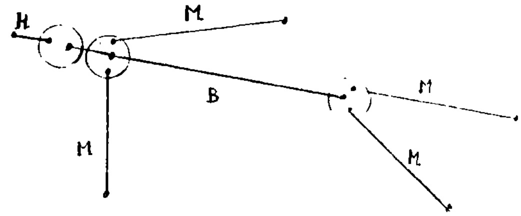

Fig.7 - High level representation of a human body

The human body can be represented by fig. 7 where H will be a spherelike object, M elongated objects, B an elongated slightly flat object.

For other objects such information is not sufficient : we need information about the size, the relative position of the parts and so on, but they are present in the graph too. However if we need information about the surface it is harder (but not impossible) to extract it. But it would be better to extract such information , if needed , from a surface oriented model (see for instance [4] and [6]).

5. DISCUSSION

The medial axis transform (or Blum transform) of a two dimensionnal shape allows one to extract shape information like the elongated parts. In the 3-D case, this axis may become a surface and the amount of information it contains makes it harder to find the same informations. This work is concerned with reducing the quantity of information. Notice that almost all the biggest spheres are centered on the medial axis.

Computing the Blum transform with the Euclidean metric can be done in a time proportionnal to the size of space by using the 26SED algorithm. So it becomes comparable to the Pfalz and Rosenfeld algorithm [10] for the city block distance even if it makes 9 times more comparisons. O'Rourke and Badler's algorithm compute the medial axis from the object boundary [9]; using the refinement suggested in [7] it works in a time $O(m^2)$ where m is the number of points of the boundary. In the 3-D case we can assume that $m=O(n^2)$ where n is the size of the space and therefore this algorithm works in a time equal to $O(n^4)$ instead $O(n^3)$. But in the 2-D case we can assume that $m=O(n)$ and then all these algorithms work in $O(n^2)$.

The generalized cone representation differs in conception from ours. Generalized cones are high level primitives which may be directly matched to a high level description. However the computation is very hard if the object is an overlapping union and difference of several cones. On the other hand the spherical representation can be computed directly but we need more sophisticated tools for extracting higher level information from it. It works fairly well for big objects but is inadequate for objects whoses properties lie more in the surface, like thin volumes.

Nevatia and Binford [8] are computing the generalized cones from a single view with depth information. For the hidden surface, they assume implicitly a shape symetrical to the viewed one. Our approach needs to know all the 3-D information about the object and therefore the voxel representation is used. Making the same regularity assumption, O'Rourke's spherical representation can be computed from a single view surface. From it the voxel representation can be extracted easily and then this approach can be used again.

As it was outlined in ACRONYM [3], feature extraction is more efficient if it is guided by a higher level representation asking for specific information. For instance having found the fuselage of an airplane the system can ask for the wings specifying their shape and orientations. These paper has demonstrated the feasability of the property extraction mechanism. Now it should be reprogrammed as an interaction between different levels of representation making use of this perspective.

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