A Piecewise Polynomial Approach to Shading Using Spectral Distributions

Maria G. Raso*
Alain Fournier†

Department of Computer Science
University of Toronto
Toronto, Ontario, Canada
M5S 1A4

Abstract

Shading techniques should be wavelength dependent, a fact which has traditionally been ignored in computer graphics: colour is usually represented in terms of an RGB coordinate system, which is inappropriate and has demonstrable problems. The use of spectral reflectances for materials in place of the RGB values in the shading computations would solve these problems.

Our technique proposes using piecewise cubic polynomials to approximate the reflectance spectra. With this method, we take the entire visible spectrum into account and avoid any wavelength sampling problems. We also propose a solution to the problem of having a large degree polynomial, as a result of successive multiplications in the shading computations.

Résumé

Il est un fait qui souvent été omis en infographie: les techniques d'ombrage devraient être dépendantes des longueurs d'ondes, alors que généralement la couleur est représentées en fonction d'un systême de coordonées RVB (Rouge Vert, Bleu). Ceci est insuffisant, et crée des problèmes réels. L'utilisation des réflectances spectrales pour les matériaux, à la place des valeurs RVB, peut résoudre ces problèmes pour les calculs d'ombrages.

Notre technique propose d'utliser des polynômes cubiques, pour approximer le spectre de réflectance. Cette méthode prend en compte la totalité du spectre visible, et évite ainsi tout problème d'échantillonage des longueurs d'ondes. Nous présentons également une solution au problème des polynômes de degré élevé, suite aux multiplications successives pour le calcul d'ombrage.

Keywords: Spectral Reflectance, Piecewise Polynomial, Chebyshev Polynomial, CIE XYZ.

*Maria can be reached at Alias Research Inc., 110 Richmond Street East, Toronto, Ontario, Canada M5C 1P1

†Alain can be reached at Department of Computer Science, University of British Columbia, Vancouver, British Columbia, Canada V6T 1W5

1 Introduction

Colour is fundamental in computer graphics for displays. Shading techniques have wavelength dependent terms. However, by using RGB values in the shading computations, problems such as undersampling of the spectrum will occur. Colorimetry is used to solve problems related to the use of colour in computer graphics. The CIE XYZ colour system [Wyse82] was developed in 1931 by the CIE (Commission Internationale de l'Eclairage) as a standard universal colour system that takes into account the human visual system. The CIE XYZ colour matching functions are positive across the entire visible spectrum, unlike the CIE RGB colour matching functions which have negative portions and are inconvenient for some computations.

Several techniques which have been proposed for solving the "RGB problem" use measured spectral reflectances of materials in the shading computations. However, these techniques also have problems associated with them. We present a solution to the problems of using RGB values for shading in computer graphics. Our solution uses the spectral reflectances of materials represented as piecewise polynomials, and takes into account the entire visible spectrum.[1]

2 RGB for Illumination and Shading

Illumination models require information about all light sources in an environment. This includes direct light sources, reflected, refracted, and ambient light. In general, the diffuse component of reflected light provides most of the surface colour and the specular component provides the highlights on an object as a result of direct light source reflections. User supplied RGB values are generally (incorrectly) used in place of the wavelength dependent intensities in an illumination model.

[1]The visible spectrum is defined in the range 300–830nm though it is commonly limited to a smaller range such as 380–780nm.

Traditionally in computer graphics, colours are computed in an *RGB space*. The basis for using these three colour values is due to the monitor phosphors which are triads of red, green, and blue dots. However, the values given to each of the RGB terms are usually based on the user's perception of what they should be. These should in fact be *measured* RGB values based on monitor calibrations if the user is actually thinking of the RGB in terms of monitor phosphors.

When specifying colour in terms of RGB values, it should be made apparent which RGB space the shading computations are being performed in. There are many RGB spaces, among them the CIE RGB, the NTSC RGB, and the monitor RGB.

The Commission Internationale d'Eclairage (CIE) developed the CIE RGB system based on three monochromatic colours chosen so that red (R) is at $700nm$, green (G) at $546.1nm$, and blue (B) at $435.8nm$. Chromaticity values are obtained from the CIE RGB tristimulus values in the following way:

$$r = \frac{R}{R+G+B} \qquad g = \frac{G}{R+G+B} \qquad b = \frac{B}{R+G+B}. \quad (1)$$

The NTSC[2] RGB values correspond to the chromaticity values of phosphors for home television sets. Since not all television sets have the same chromaticities, the NTSC signal assumes standard chromaticity values for virtual phosphors. This is the reason that the colour on different televisions do not appear the same.

The RGB values of monitors used for displaying computer graphics images are specific to that monitor, and therefore the same image can be perceived differently on separate monitors. This is due to the non-standard chromaticities of the phosphors among the various screens.

Although other colour systems exist, any system used for computing colour must be converted to a monitor RGB system before displaying the image. An RGB colour space is the conventional method employed in computer graphics for representing colour in images. However, even assuming we know which RGB space is being referred to in the shading computations, it does not solve the problems related to using RGB values. If all colours in the graphics system are to be specified in terms of RGB values, then the colour computation using the monitor RGB space is appropriate. On the other hand, if the user wants the colours in the image to be a true representation of the interaction of light with a material, then spectral distributions are necessary.

3 Problems Related to RGB Spaces

Using RGB values in shading computations can be seen as a sampling of the spectrum at only three wavelengths, resulting in severe undersampling (unless the spectra is a simple curve) or as a filtering of the spectrum through the tristimulus curves.

[2]National Television Standards Committee.

In ray tracing, the multiplications of the wavelength dependent terms in the shading model are accumulated to compute the intensity of a point in the image. If these computations involve RGB values, then colour distortion can occur. It is incorrect to compute reflection and refraction using RGB values. The product of the spectral distribution of the light source and the wavelength dependent reflections of the surface determine the spectral distribution of the reflected light.

In addition, colour information is lost in complex illumination models and reflection and refraction models when only three values of the spectrum are used. For example, a white light reflected off a smooth coloured surface will have a specular highlight the colour of the light source (as is the case with the Phong illumination model)[3]. The diffuse and ambient terms are scaled by each of the RGB values. The highlight should be the desaturated colour of the combination of the light and surface colours.

The space of the spectral light source and material reflectance has infinite dimension. If they are projected to *any* three dimensions, information is lost that can never be recovered. Shading effects cannot be computed accurately after that since these are physical effects in the original infinite dimension spectral space. For example, consider the two reflecting objects in Figure 1. A white light source is reflected off Object 1 onto Object 2, and then off of Object 2 reaching the observer. If Object 1 is a material having a reddish colour of [1,0.5,0] and Object 2 has a bluish colour of [0,0.5,1], the RGB value observed by the viewer would be a dark green colour (*i.e.* [0,x,0] where x is some value between 0 and 1). However, the spectral distribution of the reflectance reaching the observer should actually be black for some specific reflectance curves. In the example of Figure 2 one can see what the spectral distribution of the reflectance reaching the observer would be.

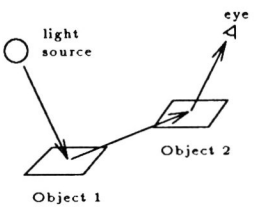

Figure 1: Reflection of light in a scene.

Researchers are now beginning to take into account the wavelength dependencies of illumination models in order to improve shading techniques in rendering systems.

4 Solutions Involving Spectral Distributions

Several techniques have been proposed that use spectral distributions in the models. One solution is a *sampling*

[3]RGB models often render the specular highlight the same colour as the light source, even though this is incorrect.

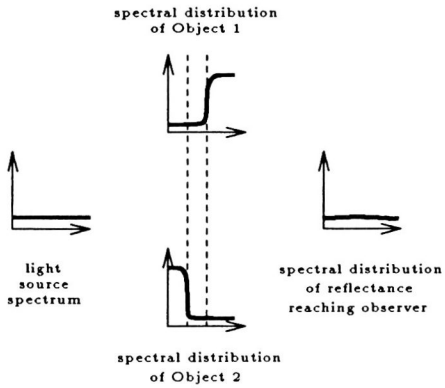

Figure 2: Spectral distributions for Figure 1.

solution that samples the spectrum at various values for colour computations. Prior to image storage or display, some method is used to convert the spectral values to an RGB colour space appropriate for monitor display.[4] Another solution is to use a linear model to represent the reflectance spectra.

4.1 Arbitrary Sampling of the Spectrum

Hall and Greenberg [Hall83] developed a testbed image synthesis system at the Computer Graphics Lab of Cornell University which uses ray tracing techniques to generate realistic images. The testbed system point samples the spectrum at arbitrary intervals. After the shading computations are completed, the resulting samples are transformed into the CIE XYZ space and then by matrix multiplications into the RGB space of the monitor used for display. Although they produced more realistic images, they point sampled the spectrum arbitrarily, which could result in loss of colour information.

4.2 Sampling Using a Gaussian Quadrature Technique

Meyer [Meye88] developed a technique that uses Gaussian quadrature to select the wavelengths to sample. A minimum number of wavelengths[5] are selected by this technique, which are located across the spectrum at the important positions necessary for accurately rendering the material's colour. Meyer used his own colour space AC_1C_2 in order to determine the wavelengths to sample and compared his results to the CIE XYZ space. A problem with this technique is that integrations for some wavelengths may not be possible since portions of his $c_1(\lambda)$ and $c_2(\lambda)$ colour matching curves are negative. The gaussian technique only complicates the computation of chromaticity values.

[4] The computations on the sampled points are all completed *before* the transformation into the RGB space of the monitor.

[5] The illumination model has to be repeated for every wavelength sampled.

4.3 Representing Spectra By a Linear Model

Maloney [Malo87] used linear models for light sources and surfaces over a limited range in order to achieve colour constancy. Surface reflectances are represented by a linear model with a small number of parameters. The linear model is composed of band-limited functions having 5–7 parameters. A problem with this method is that it relies on global optimization of all spectral data which could result in an incorrect linear model for some curves.

5 The Piecewise Polynomial Solution

The method we propose uses piecewise cubic polynomials to represent spectral distributions [Raso90]. The visible spectrum is divided into two spans with a cubic polynomial approximating the spectral data in each span. After all computations are complete, the resulting polynomial representation of the spectra is transformed to the CIE XYZ coordinate system. Matrix multiplications will then convert the XYZ values to the RGB space of the monitor used for display.

Polynomials are a simple way to represent spectral distributions and dividing the spectrum into spans provides more flexibility. It is convenient for our operations to use polynomials. It is easy to fit a polynomial to the original spectral data points. Also, multiplying reflectance and absorptance polynomials and integrating the final polynomials to compute the CIE XYZ chromaticity coordinates are all simple to do.

5.1 Polynomial Degree and Number of Spans

In order to use piecewise polynomials, the appropriate polynomial degree and number of spans to use must be determined. Preliminary tests were performed in order to achieve this. Refer to Figure 3 for the various methods that can be used.

degree	spans	fitting criteria	fitting methods
1	uniform ↗ fixed ↘ non-uniform	- least square error - min max error - least area - least luminance difference - least RGB difference - least perceptual difference	- least square - Chebyshev - Hermite - a weighted method
2			
3	# ↗ variable ↘ bound		
.			
.			
.			

Figure 3: Methods for fitting a curve to data points.

The degree of the polynomials can be varied. The division can be fixed at wavelength positions such that there are equal or different ranges in each span. The number of spans can be varied as well as the bound on each span. In addition, it must be determined beforehand what is

to be minimized. In Figure 3, the various measures are listed under the column on fitting criteria.

5.2 Polynomial Representation

All light sources are described by a spectral energy distribution and surfaces are defined by spectral reflectances for some type of material. In order to simplify the illumination computations, all spectral data are approximated by two spans of a cubic polynomial with the spectrum divided at $550nm$.

The division value $550nm$ was determined by the fact that the wavelength $555nm$ is the point at which the luminous efficiency function of the cones in the eye is maximum.[6] By dividing the spectrum at this maximum, the wavelength value is accounted for twice in the computations (since it is an end point for each span) and it is equivalent to weighting the curve at this point. However, errors in accuracy for approximating the CIE XYZ values may arise if there is a large discontinuity at this point when the polynomials are computed for each span.

We use a least squares algorithm to determine the coefficients for the spectral reflectance polynomials. The cubic polynomials resulting from the least squares algorithm will have the following form (one for each span):

$$P(\lambda) = a_0 + a_1\lambda + a_2\lambda^2 + a_3\lambda^3. \quad (2)$$

We wanted a curve fitting algorithm that would fit a cubic polynomial to the spectral data reasonably well and that would be easy to implement. A least squares algorithm fit our criteria. Refer to Figure 4 for the fit of a cubic polynomial to reflectance data for a blue paint.

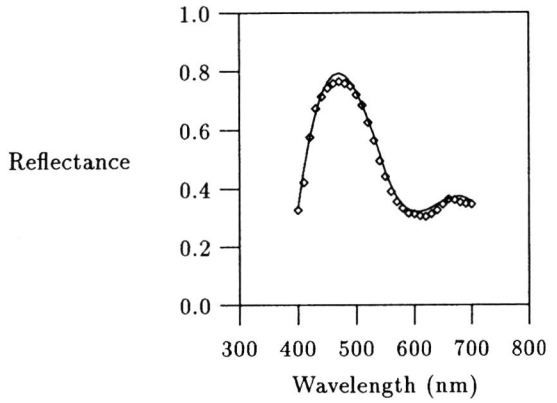

Figure 4: Cubic polynomial fit to the reflectance data for a blue paint.

5.3 The XYZ System

The CIE XYZ colour coordinate system was chosen to compute the colour coordinates from the polynomial representation of the spectra. In the CIE XYZ system, the three primaries X, Y, and Z represent the tristimulus values as computed by the integrals:

$$X = \int_{\lambda=380}^{780} Q(\lambda)\bar{x}(\lambda)d\lambda \quad (3)$$

$$Y = \int_{\lambda=380}^{780} Q(\lambda)\bar{y}(\lambda)d\lambda \quad (4)$$

$$Z = \int_{\lambda=380}^{780} Q(\lambda)\bar{z}(\lambda)d\lambda \quad (5)$$

where $Q(\lambda)$ represents the spectral distribution of some stimulus. In our case, $Q(\lambda)$ is the polynomial representation of the reflectance distribution of materials. The values $\bar{x}(\lambda)$, $\bar{y}(\lambda)$, and $\bar{z}(\lambda)$ are the colour matching functions, each of which is also represented by a cubic piecewise polynomial (the 1964 colour matching functions [Wyse82] were used in this case).

5.4 Advantages and Disadvantages

We present the piecewise polynomial approach as a solution to the problems with using RGB values in shading computations. Our technique avoids the problems that arise with the solutions discussed in Section 4, since we take the entire visible spectrum into account (*i.e.* no sampling errors occur with our method). However, by using piecewise polynomials, discontinuities may arise between the spans. Note that since we do not do "shape fitting", a discontinuity is not a problem. The problem is when the discontinuity introduces significant differences in the final colour values. The approximation errors may be reduced by using some method that is more restrictive on the spans than the simple least squares algorithm we use (refer to Figure 3 for *fitting criteria*). However, since most of the material reflectance curves are smooth, the least squares approximation does not yield significant errors.

6 The Closure Problem

The use of polynomials in computations results in a closure problem. There are many multiplications in the illumination model with each multiplication doubling the degree in the resulting polynomial. The degree of the final polynomial for the integration computations for each X, Y, and Z could end up being a very large value, especially in the context of ray tracing. This problem is solved by keeping the degree of the polynomials constant by using a degree "reduction" technique. The "reduction" is accomplished by use of Chebyshev polynomials to approximate large degree polynomials.

6.1 The Chebyshev Polynomial Solution

Chebyshev polynomials are defined in the region $-1 \leq x \leq 1$. Therefore, it was necessary that the degree reduction technique convert the polynomials which represent

[6] The cones are the cells of the eye responsible for colour vision. Since the method we propose has spectral data in increments of $10nm$, we went down to $550nm$ from $555nm$.

the spectral data in the region 380–780nm to the range $[-1, 1]$.

The new reduced polynomial is then computed by the formula:

$$P_3(\lambda \in [380, 780]) = [1 \quad \lambda \quad \lambda^2 \quad \lambda^3]R \quad T \; M \; T^{-1} \; R^{-1} \; P_6 \tag{6}$$

where P_6 is the vector matrix of the coefficients for the polynomial to be reduced, R is the matrix to convert the polynomials in the spectral range 380–780nm to the Chebyshev range $[-1, 1]$, T is the Chebyshev polynomial matrix, and M is a variation on the identity matrix.[7] The matrix M is used to select the first four coefficients for the new polynomial.

In effect, what Equation 6 does is first change the range of the polynomial from [380,780] to $[-1, 1]$ (by multiplying P_6 by R^{-1}). This result is then multiplied by the inverse Chebyshev polynomial matrix T^{-1}. The M matrix selects the first four coefficients. Then in order to get the new coefficients back to the original range, these values are multiplied by the Chebyshev polynomial matrix T and then the R matrix to get back to the original range again. The resulting vector matrix which holds the new coefficients has four values in the first 4 rows and 0's in the remaining rows (the 0's are due to the multiplication by M). The multiplication by M is similar to truncating the degree 6 polynomial. Note that Equation 6 will have to be applied for each span.

6.2 Results From Degree "Reduction"

Figure 5 illustrates how close two spans having degree 3 polynomials can approximate two spans having degree 6 polynomials (with the subdivision at 550nm). The solid line in each span is the degree 6 polynomial and the dashed line in each span is the approximated degree 3 polynomial.

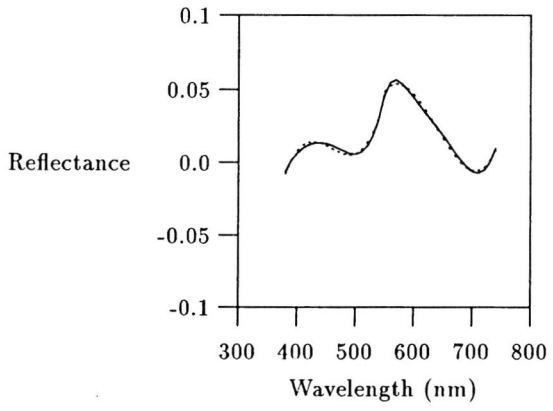

Figure 5: Approximating a degree 6 polynomial by a degree 3 polynomial.

[7] M is a 7 x 7 matrix having 1's in the first four diagonal positions.

By using this Chebyshev method to approximate large degree polynomials, the computations in the illumination model are "reduced" and made easier. Also, the error in reducing the polynomial is generally less than the initial error of approximating the spectral data points. Refer to Table 1 for the error values. The table gives the errors for fitting a cubic to the reflectance data for a green shingle under the first error column. The second error column lists the errors in fitting a curve to the x colour matching functions. The last column gives the errors from reducing the polynomial as a result of multiplying the reflectance and x colour matching cubic polynomials.

span #	errors		
	reflectance	x	reduced
one	3.532443e-5	6.953191e-3	4.819727e-6
two	2.381345e-6	2.124504e-2	3.651482e-6

TABLE 1 : Comparison of errors in "reducing" a polynomial.

7 Implementation and Results

Our method was implemented in Optik, a ray tracing program developed at the University of Toronto's **D**ynamic **G**raphics **P**roject (DGP). The illumination model in Optik is represented by:

I = ambient + Lambertian diffuse + reflected + Phong specular + refracted

The usual way in which surfaces and light colours are specified in Optik (and many other rendering systems) is by RGB components: **red green blue**. In our implementation, this triple is replaced internally by two cubic polynomials representing the surface reflectance or the spectral energy distribution of the light source. The spectral reflectance distributions of materials were obtained from the collection of natural phenomena data by Krinov [Krin47] and from spectral data in [Wyse82].

The reflectance values for a surface material are used to determine the polynomial representation of the spectral data. In addition, the surface colour transmittance is represented by a spectral distribution, where the spectral information is defined by a colour filter distribution. The polynomial representations of the filter distributions can also be multiplied by the light source distributions to simulate a coloured light source.

7.1 Representing Light Sources

It should be noted that light source spectral data does not have to be approximated by polynomials. In fact, it is difficult to approximate light source data with sufficient precision using *any* method since light spectra are so irregular in shape.[8] Some light source spectral data is

[8] The light source spectral data can be approximated by polynomials *if* the spectrum is subdivided into enough spans to reasonably fit a curve to the data in each span. This may also require a different degree polynomial for each span, which would further complicate the computations.

more complex than surface reflectances since it has high frequencies at certain wavelengths and is less smooth. For example, the spectral distribution for a fluorescent light is difficult to approximate because of the spiky frequencies, whereas sunlight is smoother and can be represented more easily by a polynomial.

It is not necessary to represent the light sources in ray tracers with the same method used to define the reflectances and colour matching functions. A different representation can be used for lights while still keeping the polynomial representation for surface reflectances. This is possible because in ray tracing after all reflection and refraction computations are complete, we are left with a cubic polynomial which is then convolved with the light source. It does not matter how this light source is represented (for example, as samples, basis functions, or even polynomials) as long as we know how to convolve it.[9]

7.2 Polynomial Representation to CIE XYZ Space

After all computations and degree reductions are completed, we are left with one polynomial for each span as a representation of the spectral distribution:

$$\text{span } \#1\text{—} \quad a_0 + a_1\lambda + a_2\lambda^2 + a_3\lambda^3$$
$$\text{span } \#2\text{—} \quad b_0 + b_1\lambda + b_2\lambda^2 + b_3\lambda^3$$

These polynomials are multiplied by the $\bar{x}(\lambda)$, $\bar{y}(\lambda)$, and $\bar{z}(\lambda)$ colour matching functions. The result is integrated using Equations 3 - 5 to determine the X, Y, and Z tristimulus values (similarly for Y and Z):

$$\begin{aligned} X &= \int_{380}^{550} (a_0 + a_1\lambda + a_2\lambda^2 + a_3\lambda^3)\bar{x}_1(\lambda)d\lambda \\ &+ \int_{550}^{780} (b_0 + b_1\lambda + b_2\lambda^2 + b_3\lambda^3)\bar{x}_2(\lambda)d\lambda \end{aligned} \tag{7}$$

where $\bar{x}_i(\lambda)$, (similarly for $\bar{y}_i(\lambda)$, and $\bar{z}_i(\lambda)$) is the polynomial representation of the colour matching functions for span #1 and span #2. The multiplication of the polynomials under the integration will result in degree 6 polynomials which are not required to be "reduced" because this is one of the final steps in determining the chromaticity values. There will no longer be any polynomial multiplication with these polynomials after this point.

Equation 7 is easily solved using any integral solving technique of calculus. Once each of the XYZ values have been computed, Equations 8 are used to determine the xyz chromaticity values.

$$x = \frac{X}{X+Y+Z} \qquad y = \frac{Y}{X+Y+Z} \qquad z = \frac{Z}{X+Y+Z}. \tag{8}$$

[9]Unfortunately, the current version of Optik computes intensities from the light source to the eye, forcing us to represent the light source spectral data in some form from the beginning.

7.3 CIE XYZ Space to Monitor RGB Space

After the CIE XYZ chromaticity values are determined they are converted to rgb chromaticity values, taking into account the chromaticities of the monitor phosphors.[10] Once all previous steps are completed, the RGB values that are to be written to the monitor can be determined. This requires that the luminance for each red, green, and blue phosphor be known.

It is possible that some of the RGB values will be outside the range [0,1]. This will be a result of the values being outside the gamut of realizable colours or there being too much luminance. In this case, the out of range R, G or B values are clamped to the [0,1] range.[11]

Finally, the shading computation is completed by multiplying each of these RGB values by 256, resulting in the intensity values that will be written to the display file or the display monitor.

7.4 Results

In terms of computation time, it will obviously take longer to run a ray tracer using the spectral reflectance distributions rather than the RGB values. On average, the ratio in total computation time to render an image is about two. This is due to the eight values that are required to be computed (each coefficient for the cubic polynomial of each span) whereas the RGB method computes three values. There is no initial overhead, however, in determining the polynomial representation of the spectral reflectance data of a material.

The differences in using the RGB method versus the spectral distributions can be seen in the following examples. Example 1 is composed of two highly specular red spheres. These spheres were shaded using the spectral reflectance for a red paint from the spectra in Figure 6. The specular component of the Phong model was set to a value of 60 in both cases. The highlight is larger for the sphere rendered using reflectance spectra. This is due to the problem that shading information is lost when using an RGB model.

Example 2 uses the spheres from Example 1 inside a transparent blue cube. The cube was shaded using the spectral reflectance for a blue filter for the transmissivity. Refer to Table 2 for the approximate times in rendering each of these examples.

example	shading method	time (hh:mm:ss)
Example 1	RGB	00:52:55
	Reflectance Spectra	01:59:38
Example 2	RGB	02:23:44
	Reflectance Spectra	04:28:39

TABLE 2: Approximate total rendering times.

[10]These values are necessary to determine the rgb colours produced by the monitor on which the image will be displayed.

[11]In computer graphics, the RGB values are incorrectly clamped to the [0,1] range. However, this problem is a separate issue to this paper and will not be addressed here.

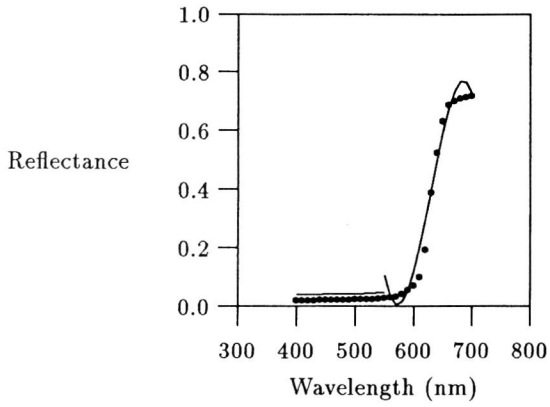

Figure 6: Reflectance curve for a red paint.

8 Conclusions

We have presented a simple solution to the problems with using RGB values in shading computations. Our method involves representing spectral data by piecewise cubic polynomials. Having two spans of cubic polynomials is not guaranteed to be an optimal solution, but it does solve the problems of RGB values in shading computations.

The method we have proposed models the interaction (*i.e.* reflection and absorption) between light and a material which cannot be treated accurately with RGB triples. We avoid the problems of sampling that other researchers have, by taking the entire visible spectrum into account. In addition, we have suggested a solution to the closure problem by using Chebyshev polynomials for degree "reduction". The final images are correctly converted from the CIE XYZ space to the RGB space of the monitor used for display. Although our implementation increases the computation time, the traditional method using RGB values (which is faster) does not give accurate results in some cases.

References

[CIE71] Commission Internationale de l'Eclairage, "Colorimetry - Official Recommendations of the International Commission on Illumination", Bureau Central de la CIE, Paris, France, 1971.

[Cook82] Cook, R. L., and Torrance, K. E., "A Reflectance Model for Computer Graphics", *ACM Transactions on Computer Graphics*, Vol. 1, No. 1, January 1982.

[Cowa83] Cowan, W. B., "An Inexpensive Scheme for Calibration of a Colour Monitor in Terms of Terms of CIE Standard Coordinates", *Computer Graphics*, Vol. 17, No. 3, July 1983.

[Fox68] Fox, L., and Parker, I. B., "Chebyshev Polynomials in Numerical Analysis", Oxford University Press, London, 1968.

[Hall83] Hall, R. A. and Greenberg, D. P., "A Testbed for Realistic Image Synthesis", *IEEE Computer Graphics and Applications*, Vol. 3, No. 8, November 1983.

[Hall89] Hall, R. A., "Illumination and Color in Computer Generated Imagery", Springer-Verlag, New York, 1989.

[Judd75] Judd, D. B. and Wyszecki, G., "Color in Business, Science, and Industry", John Wiley & Sons, Inc., New York, 1975.

[Krin47] Krinov, E. L., translated by Belkov, G., "Spectral Reflectance Properties of Natural Formations", *translated by Belkov, G.*, Laboratoria Aerometodov, Akad. Nauk SSSR, Moscow, 1947.

[Malo87] Maloney, L. T., "Evaluation of Linear Models of Surface Spectral Reflectance With Small Numbers of Parameters", *Computer Graphics - Modelling Natural Phenomena*, pp. 2-39, Winter Quarter, 1987.

[Meye88] Meyer, G. W., "Wavelength Selection for Synthetic Image Generation", *Computer Vision, Graphics, and Image Processing*, Vol. 41, 1988.

[OSA53] Committee on Colorimetry, Optical Society of America, "The Science of Color", Thomas Y. Crowell Company, New York, 1953.

[Phon75] Bui-Tuong, Phong, "Illumination for Computer Generated Pictures", *Communications of the ACM*, Vol. 18, No. 6, June 1975.

[Raso90] Raso, M., "A Piecewise Polynomial Approach To Shading Using Spectral Distributions", M.Sc. Thesis, Department of Computer Science, University of Toronto, 1990.

[Wyse82] Wysecki, G. and Stiles, W. S., "Color Science : Concepts and Methods, Quantitative Data and Formulae", 2nd edition, John Wiley & Sons, Inc., New York, 1982.