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Mechanics of the Fouetté turn

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Abstract

The Fouetté turn in classical ballet is performed repeatedly on one leg with swinging of the free limbs, producing a continued sequence of turns with one turn leading into the next. The purpose of this study was to determine the possible time history profiles of the twisting torque between the supporting leg and the remainder of the body that will allow continued performances of the Fouetté turn. Simulations were performed using a model which comprised the supporting leg and the remainder of the body to find torque profiles that maintain the initial angular velocity so that the state after one revolution is the same as the initial state. The solution space of torque profiles was determined for various rotation times and coefficients of friction between foot and floor. As the time for one revolution became shorter the solution space became smaller and for a given turn time there was a lower limit on the coefficient of friction. As the frictional coefficient became smaller the solution space became smaller and for a given coefficient there was a lower limit on the turn time. Turns of a given tempo can be performed on floors with different friction by modifying the twisting torque profile. When a turn is completed with a net change in angular velocity this can be compensated for in the next turn by adjusting the twisting torque profile.

KEY WORDS: Turn; Simulation; Angular Momentum; Ballet.

PsycINFO classification code: 2330, 3720, 3740

1. Introduction

Skilled ballet dancers can continuously perform repeated Fouetté turns (Fig. 1) and while there are various performance styles, such as Italian and Russian Fouetté turns according to the ballet style of the dancer (Warren, 1990), the basic technique is the same. The turn is started from one or two revolutions of the pirouette which is initiated with both feet in contact with the floor to produce the initial angular momentum. The dancer then keeps turning to music, swinging the arms and the free leg while the supporting foot is stationary in full contact with the floor (Figs.1 A-D). After the swinging, the dancer adopts the pirouette position during which the foot slips (Fig.1 E-I) before starting to swing the free leg again (Fig. 1 J-K). The dancer regains the angular momentum lost due to friction during the slipping phase by swinging the free limbs when the foot is stationary, which enables the floor to exert a large frictional torque T_F on the foot in the same direction as the swinging (Laws, 1984, 1998; Imura, Iino, & Kojima, 2008). The dancer can keep turning for more than 30 revolutions by repeating these movements.

The frictional torque T_F is the only external torque during the Fouetté turn and consequently determines the changes in the angular momentum of the whole body. This frictional torque is dependent on the limiting frictional torque (limiting T_F) and the twisting torque T used to swing the free limbs. The supporting foot during the Fouetté turn is essentially on tiptoe during the slipping or in full contact with the floor when the foot stops turning.

*** insert Fig. 1 here ***

Dancers have to perform the Fouetté turn in time to the music in the choreography, facing the front at the same position for the aesthetics of ballet (Laws, 1984). However, they sometimes turn to music tempo that is too fast or too slow and struggle with performing successful turns. The friction coefficient between the shoes and the floor of the performance stage may be different to that of the practice studio and this will require technique to be modified.

The purpose of this study was to determine the possible time history profiles of the twisting torque between the supporting leg and the remainder of the body that will allow continued performances of the Fouetté turn. Techniques for coping with changes in tempo and friction were also investigated.

2. Methods

A computer simulation model of the Fouetté turn was used to investigate the solution space of twisting torque profiles that permitted performances of successful turns for various coefficients of friction between foot and floor and various time periods of turn. The body (mass 49.5 kg) of a typical dancer who participated in a previous study (Imura et al., 2008) was modeled as two cylinders (Fig. 2): the supporting leg L and the remainder of the body B whose moment of inertia I_B about a vertical axis changed according to the positions of the free leg and arms. The time profiles of the foot radius r, the moment of inertia I_B, and the normal ground reaction force N were based on experimental data from the study of Imura et al. (2008) and were represented by joining adjacent maximum and minimum values using monotonic quintic functions with zero first and second time derivatives at the endpoints (Fig. 3).

*** insert Fig. 2 here ***

The radius r was taken to be 0.12 m at the maximum when the supporting foot is fully in contact with the floor and to be 0.05 m at the minimum when the dancer stands on tiptoe (Fig. 3a). These bounding values were determined using the distance between the toe and the center of the pressure (CoP) from the experimental data of Imura et al. (2008). The foot radius time profile was matched to the experimental data with time normalized to the turn time, recognizing that CoP locations were unreliable when the normal reaction force was small.

The time profile of I_B (Fig. 3b) was calculated using the theorem of parallel axes by scaling inertia data (Ae, Tang, & Yokoi, 1992) of a subject with similar body mass as the model and using arm and leg positions based upon those exhibited in the experimental study of Imura et al. (2008). The maximum and minimum I_B were calculated to be 2.67 and 1.06 kg.m^2, respectively. The moment of inertia of the leg I_L was assumed to be constant during the turn and was calculated to be 0.085 kg.m^2. The normal ground reaction force N was determined by the following four values based on experimental data from the study of Imura et al. (2008): 2.68 body weights at the time of full foot contact, 0.0 body weights once slipping had started, 1.24 and 0.82 body weights during slipping on tiptoe. It was assumed that the fitted profile was symmetrical about the mid-turn time (Fig. 3c). The average N was one body weight during one turn.

The friction coefficient μ between the shoes and the floor was estimated to be 0.2, calculated from the slipping phase of the experimental data of Imura et al. (2008) using the equation $\mu = T_F / Nr$. The leg L was rotated further than B (Fig. 2) by 0.22 radians at the start of the turn according to the experimental data of Imura et al. (2008).

Simulations were performed for 1 s starting with the supporting foot in full contact with the floor (Fig. 1A), with the requirement that the dancer rotates 2π in 1 s using an appropriate twisting torque T. Three variables were used to define the time profile of T: T_{max} the initial (positive) torque to swing the free limbs and trunk, T_{min} the (negative) torque which is maintained during the sliding to reverse the swinging, and the time t_1 at which T becomes T_{min}. T_{max}, and T_{min} were joined by monotonic quintic functions, assuming the profile of T to be symmetric since the dancer swings the limbs at the end of the turn as at the start (Fig. 3d).

*** insert Fig. 3 here ***

The frictional torque T_F while the dancer is slipping can be assumed to be the limiting T_F, which is the product of the friction coefficient μ, the normal ground reaction force N, and the radius r of the foot contact area with the floor. When the foot is stationary, the frictional torque T_F acting on the supporting foot is equal to T. The frictional torque T_F was defined as $T_F = T$ when $T \leq \mu Nr$ and $T_F = \mu Nr$ once L slipped. Values for μ, N, r, I_B, I_L, and T were input into the equations of motion.

The angular momentum of the body B is given by $h = I_B\dot{\phi}_B$ where ϕ_B is the angle turned by the body B. The torque T applied to the body B by the supporting leg L is equal to the rate of change of h. Thus:

$T = I_B\ddot{\phi}_B + \dot{I}_B\dot{\phi}_B$ from which:

$$\ddot{\phi}_B = (T - \dot{I}_B\dot{\phi}_B)/I_B \quad (1)$$

The net torque acting on the leg L in the direction of the turn is T_F - T and so:

$T_F - T = I_L \ddot{\phi}_L$ from which:

$$\ddot{\phi}_L = (T_F - T) / I_L \qquad (2)$$

The angles and angular velocities of B and L ($\phi_B, \dot{\phi}_B$ and $\phi_L, \dot{\phi}_L$) were calculated from the accelerations derived in (1), (2) using stepwise integration.

A grid search was made for T_{max} between 10 and 30 Nm, T_{min} between –1 and –10 Nm and t_1 between 0.1 and 0.4 s to find the possible time profiles of T for turns which satisfied all of the following conditions. These were: (a) the leg L rotates 2π radians and stops by the end of the simulation, (b) the body B rotates 2π radians and the angular velocity of B at the end of the simulation is the same as at the start, (c) the foot does not slip more than 0.13 radians in the direction opposite to that of the turn (as for the experimental data of Imura et al., 2008). The coefficient of friction μ = 0.2 and time for one turn was 1.0 s. After determining the bounding time profiles of minimum and maximum T_{max} from the solution space of T, additional cases were considered in which the friction coefficient ranged from 0.1 to 0.3 (t_{end} = 1.0 s) and the time for one turn ranged from 0.7 to 1.0 s (μ = 0.2), spanning the experimentally determined values of μ = 0.2 and t_{end} = 0.85 s.

3. Results

For each combination of frictional coefficient μ and turn time t_{end} there were maximum and minimum values for the initial (maximum) value T_{max} of the twisting torque T (Tables 1-4). Each pair of max-min solutions gave rotation angle time histories similar to experimental data and had similar angular velocity time histories (Figs. 4 and 5). For each value of T_{max} lying between the maximum and minimum values there existed unique values of T_{min} and t_1 for which the body and supporting leg each rotated one revolution and any counter-slipping was less than 0.13 radians as shown in the example (Table 2, Fig. 6). With μ fixed at 0.2, smaller values of t_{end} lead to larger values of minimum T_{max} and consequently to a smaller range of solutions (Table 1). For t_{end} = 0.76 there was a narrow range of solutions with T_{max} lying between 28.1 Nm and 28.3 Nm and for t_{end} = 0.75, or smaller, there were no solutions. For decreasing t_{end} the maximum angle difference ϕ_d between the body and the supporting leg increased (Table 1). The initial angular velocity $\dot{\phi}_B$ of the body remained essentially constant for a given t_{end} and was inversely proportional to t_{end} (Table 1).

*** insert Table 1 here ***

*** insert Table 2 here ***

With t_{end} fixed at 1.0 s smaller values of μ lead to smaller values of T_{max} and

T_{min} (Table 3). As μ decreased the maximum and minimum values of T_{max}, T_{min}, and t_1 became closer (Table 3), giving a narrower range of solutions and for $\mu < 0.12$ there were no solutions. The initial angular velocity $\dot{\phi}_B$ remained essentially constant for the various values of μ.

For each solution with given values of μ and t_{end}, there existed solutions with equivalent torque profiles for different combinations of μ and t_{end}. For example if μ increased from 0.20 to 0.25, t_{end} decreased from 1.0 to 0.9 in the equivalent solution (Table 4). The torque profiles of these equivalent solutions have the same t_1/t_{end} values and merely have different scaling factors for torque and time.

*** insert Table 3 here ***

*** insert Table 4 here ***

*** insert Fig. 4 here ***

*** insert Fig. 5 here ***

4. Discussion

The purpose of this study was to determine the possible time history profiles of T for various tempos and friction. For each combination of time of turn and frictional coefficient there is a range of solutions for T (Tables 1-4). The solutions for T satisfy the requirement that the net change in the angular velocity of the body B is zero and that the leg L and body B each rotate 2π at t_{end}. Because the body B rotates under the action of only T, the time integral of T should be zero after one revolution so that there is no net change in angular momentum. The rotation of the leg L is dependent on the net torque T_F - T, so the foot should rotate 2π during slipping as a consequence of sufficient acceleration produced by the net torque. Again the time integral of T_F - T must be zero since the start and end velocities are zero. Hence, T_{max}, T_{min}, and t_1 are such that the integral of T is zero and T_F - T rotates L just one revolution during slipping. For a given value of T_{max} these two constraints give a unique solution for the remaining two degrees of freedom (T_{min} and t_1). Thus for a given set of conditions (μ, t_{end}) there is a set of torque profiles defined by T_{max}, T_{min}, and t_1 each of which lie within the bounds shown in Tables 1-4. While there is a range of solutions for a given set of conditions (μ, t_{end}), the tight specification of any individual solution may make the Fouetté turn difficult to perform in a steady state since precise timing would be required by the dancer. On the other hand since there are various time histories within the general profiles which produce one revolution of the Fouetté turn (Fig. 6), such time profiles may represent the different styles from different schools.

*** insert Fig. 6 here ***

Initially the twisting torque is positive and less than the limiting frictional torque, accelerating the upper body in the direction of twist while the supporting leg remains stationary with the twisting torque and frictional torque cancelling each other (Figs. 4a and 5a). If the twisting torque exceeds the limiting frictional torque in this phase ($T > 0$) the foot will slip in the direction opposite to that of the twist. Soon after the upper body starts to decelerate relative to the supporting leg and T becomes negative, the magnitude of T exceeds that of limiting T_F and the foot slips in the direction of twist (Figs. 4a and 5a). Once the magnitude of T falls below that of limiting T_F, the angular velocity of the foot decreases until the foot stops rotating at which time $T = T_F$.

For a given coefficient of friction there is a lower bound on the time of the turn since faster turns require larger torques (Table 1) and these are bounded by the limiting frictional torque. This explains why the range of solutions narrows for faster turns (Table 1). There will also be an upper bound on the turn time arising from the assumed time profile of the vertical reaction force N (Fig. 3) since the supporting leg must flex in order to reduce N below one body weight and there is a limit to the amount of flexion possible. The average angular velocity of the body will be inversely proportional to the turn time, and since the angular velocity profiles of the body are similar in different solutions (Figs. 4c and 5c) the initial angular velocity $\dot{\phi}_B$ will be approximately inversely proportional to t_{end} (Table 1). Thus if a dancer completes a turn with a net decrease in $\dot{\phi}_B$ this could be compensated for in the next turn by choosing to continue to turn at the new angular velocity and selecting a twisting torque profile with the corresponding t_{end}. Alternatively, a larger T_{max} for the same T_{min} could be used to produce a net gain in angular momentum in the next turn while keeping the same turn time. Real time adjustments for ϕ_B and ϕ_L could also be made by modifying the time profiles of I_B and r.

As the friction coefficient becomes smaller, the magnitude of T also decreases but so does the range of possible profiles for T so that for a given t_{end} there is a lower limit for μ below which there are no solutions. This is a consequence of not having sufficient T_F - T to produce the required rotation of the foot. For values of μ below this limit there are solutions with longer turn times since for a given solution there are equivalent solutions with smaller μ and larger t_{end} (Table 4). For larger values of μ, solutions require larger values of T (Table 3), and so there will be an upper limit on μ imposed by the ability of the dancer to exert large torques. For floors with different μ it will be possible to turn at a given tempo providing μ lies within a certain range. For μ below the lower bound slower turns will be possible and for μ above the upper bound faster turns will be possible up to a limit.

For a given pair of values of μ and t_{end} and a particular solution there are other corresponding pairs of values with an equivalent solution for which t_l/t_{end} is the same (Table 4). In comparing these solutions a change of t_{end} by a scaling factor k will correspond to a change in each of $\dot{I}_B$, $\dot{\phi}_B$, and $\dot{\phi}_L$ by a factor of 1/k and a change in each of $\ddot{\phi}_B$ and $\ddot{\phi}_L$ by a factor $1/k^2$. As a consequence of equations (1) and (2) T and T_F will change by a factor $1/k^2$.

There are a number of simplifications associated with the model. The moment of inertia about the longitudinal axis of the supporting leg is assumed to be constant. In an actual performance the knee bends and extends in order to stand on tiptoe and so the moment of inertia will vary. However knee flexion occurs primarily during full foot contact and changing the moment of inertia in this phase would have minimal effect on a simulation since the foot slips very little during this phase. During the majority of the slipping phase the leg is straight and the assumption of constant moment of inertia is reasonable. Although the time profiles of the variables r, I_B, and N are simplifications they were based on the experimental data of a dancer and the calculated leg and body rotation angles were similar to performance data (Figs. 4 and 5). The coefficient of static friction has been assumed to be the same as that of dynamic friction rather than a little larger. The effect of this will have been to reduce the range of possible solutions slightly. The limit of 0.13 radians on foot slippage in the direction opposite to the turn constrains the maximum angle difference between body and support leg to anatomically feasible values of around 1 radian or less in the simulations (Tables 1-4). If this constraint is removed the solution space is much larger but includes simulations with large relative rotations between body and support leg which are beyond anatomical limits.

This simple model has been used to describe the solution space of the possible time profiles of the twisting torque T that produce the required rotation about the longitudinal axis in a Fouetté turn. A change of floor to one with increased friction will require a larger twisting torque to turn at the same tempo. A net reduction in the angular velocity of the body B after one turn can be compensated for by increasing T_{max} for the same T_{min}. The model could be applied not only to the Fouetté turn but also to other ballet turns such as the pirouette and Grand Fouetté Italien.

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Table 1. Ranges of the twisting torque parameters T_{max}, T_{min}, t_1 for which the dancer can turn in steady state (μ = 0.2)

T	t_{end}	T_{max}	$-T_{min}$	t_1/t_{end}	ϕ_d	$\dot{\phi}_B$
max	1.00	28.3	4.9	0.15	0.66	4.49
min	1.00	18.4	5.5	0.23	0.76	4.49
max	0.90	28.3	5.4	0.16	0.68	4.97
min	0.90	20.2	6.0	0.23	0.80	4.97
max	0.80	28.3	6.0	0.18	0.75	5.57
min	0.80	23.9	6.7	0.22	0.91	5.58

Notes: ϕ_d is the maximum difference between leg and body angles,
$\dot{\phi}_B$ is the initial angular velocity of the body B

Table 2. An example of an intermediate twisting torque profile lying between maximum and minimum solutions

T	μ	t_{end}	T_{max}	$-T_{min}$	t_1	ϕ_d	$\dot{\phi}_B$
max	0.30	1.0	42.4	6.5	0.13	0.65	4.52
int	0.30	1.0	32.3	6.6	0.17	0.65	4.53
min	0.30	1.0	22.2	8.1	0.27	1.05	4.55

Notes: ϕ_d is the maximum difference between leg and body angles,
$\dot{\phi}_B$ is the initial angular velocity of the body B

Table 3. Ranges of the twisting torque parameters T_{max}, T_{min}, and t_1 for which the dancer can turn in steady state (t_{end} = 1.0 s)

T	μ	T_{max}	$-T_{min}$	t_1	ϕ_d	$\dot{\phi}_B$
max	0.15	21.2	4.2	0.16	0.69	4.46
min	0.15	15.9	4.7	0.23	0.86	4.47
max	0.20	28.3	4.9	0.15	0.66	4.49
min	0.20	18.4	5.5	0.23	0.76	4.49
max	0.25	35.3	5.7	0.14	0.65	4.51
min	0.25	20.1	6.8	0.25	0.89	4.52
max	0.30	42.4	6.5	0.13	0.65	4.52
min	0.30	22.2	8.1	0.27	1.05	4.55

Notes: ϕ_d is the maximum difference between leg and body angles,
$\dot{\phi}_B$ is the initial angular velocity of the body B

Table 4. Equivalent solutions for twisting torque profiles with different combinations of friction and turn time

T	μ	t_{end}	T_{max}	$-T_{min}$	t_l/t_{end}	ϕ_d	$\dot{\phi}_B$
max	0.20	1.0	28.3	4.9	0.15	0.66	4.49
min	0.20	1.0	18.4	5.5	0.23	0.76	4.49
max	0.25	0.9	35.4	6.2	0.15	0.66	5.02
min	0.25	0.9	22.9	6.8	0.23	0.76	5.02

Notes: ϕ_d is the maximum difference between leg and body angles,
$\dot{\phi}_B$ is the initial angular velocity of the body B

List of Figure Captions

Fig. 1. Sequential view of one revolution of Fouetté turn. Each picture is shown every 10% time of one revolution (adapted from Imura et al., 2008).

Fig. 2. The model comprises the supporting leg L and the remainder of the body B. Initial torque directions are shown.

Fig. 3. Time profiles of (a) radius of the foot contact area, (b) moment inertia of body B, (c) normal ground reaction force, and (d) a representative example of the twisting torque T. The abscissae show one time unit for one revolution of the turn and experimental data (normalized to the time of the turn) are shown using dashed lines.

Fig. 4. Time profiles of a successful Fouetté turn ($\mu = 0.2$) for which the twisting torque parameter T_{max} is minimum: (a) twisting torque (dashed line), frictional torque (thick line), limiting frictional torques (both directions, thin lines), (b) rotation angles of body B (thin line) and leg L (thick line) with experimental data (dashed lines), and (c) angular velocity of body B (thin line) and leg L (thick line).

Fig. 5. Time profiles of a successful Fouetté turn ($\mu = 0.2$) for which the twisting torque parameter T_{max} is maximum: (a) twisting torque (dashed line), frictional torque (thick line), limiting frictional torques (both directions, thin lines), (b) rotation angles of body B (thin line) and leg L (thick line) with experimental data (dashed lines`), and (c) angular velocity of body B (thin line) and leg L (thick line).

Fig. 6. An example of an intermediate twisting torque profile (dashed line) between maximum T_{max} (thick line) and minimum T_{max} (thin line) corresponding to Table

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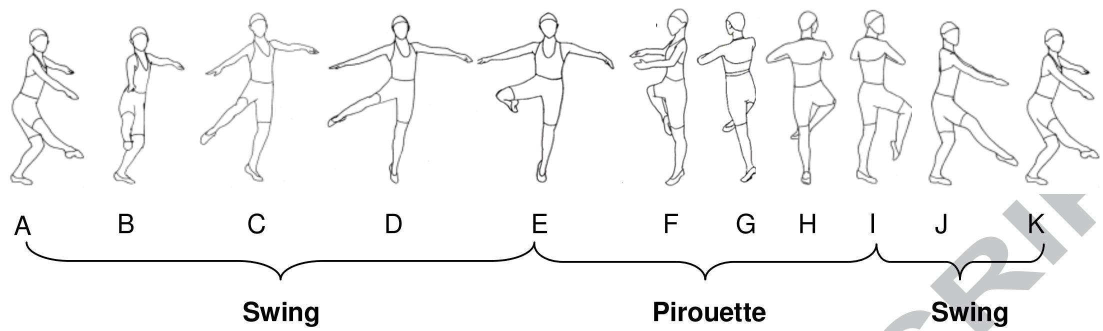


Fig. 1.

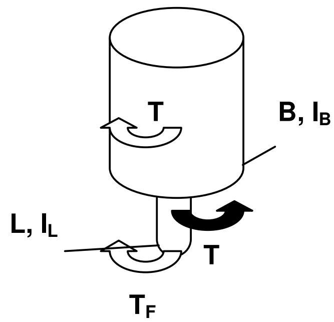
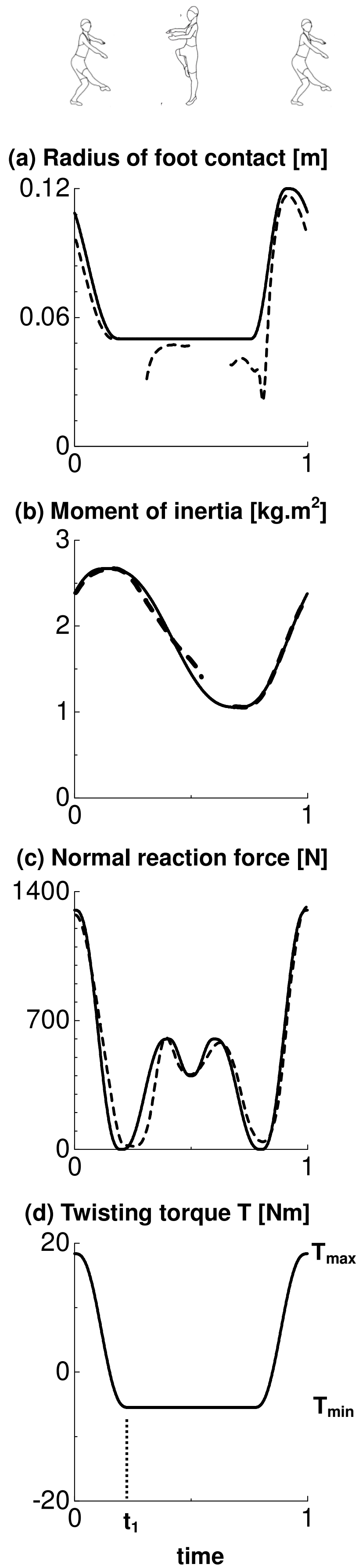


Fig. 2.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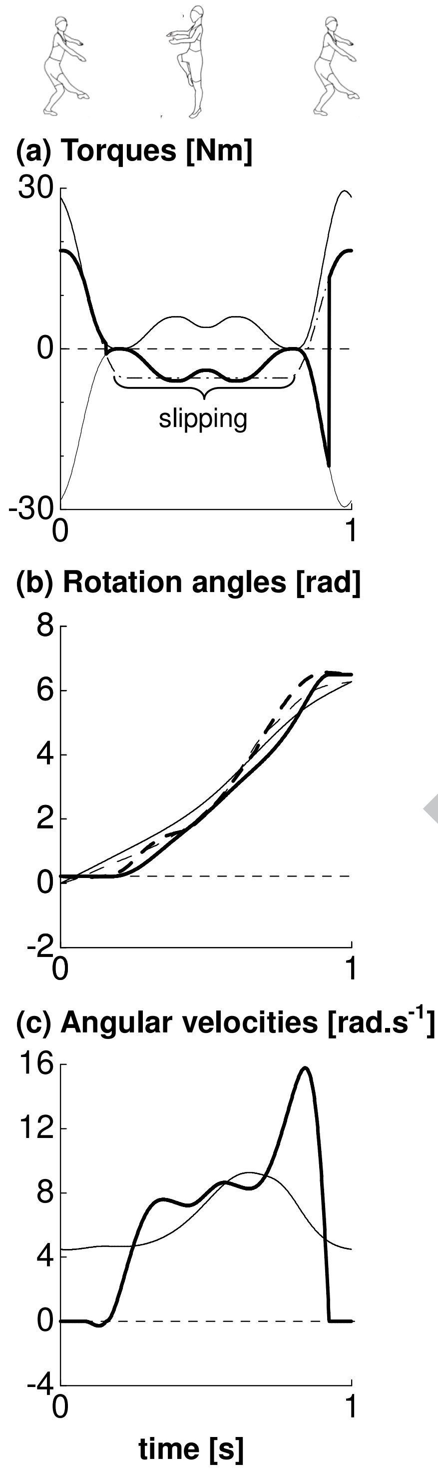
0.06
0
0
1
(b) Moment of inertia [kg.m2]
3
2
1
0
0
1
(c) Normal reaction force [N]
1400
700
0
0
1
(d) Twisting torque T [Nm]
20
0
-20
0
t1
1
Tmax
Tmin
time

Fig. 3.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-30
0
1
slipping
(b) Rotation angles [rad]
8
6
4
2
0
-2
0
1
(c) Angular velocities [rad.s-1]
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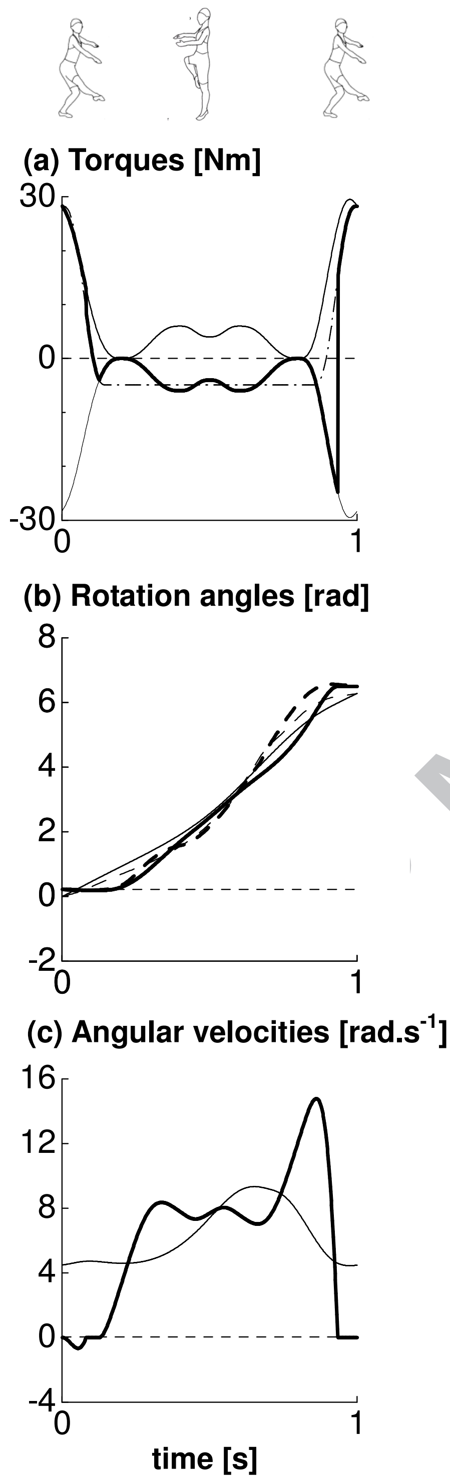
12
8
4
0
-4
0
1
time [s]

Fig. 4.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-30
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6
4
2
0
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1
(c) Angular velocities [rad.s-1]
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time [s]

Fig. 5.

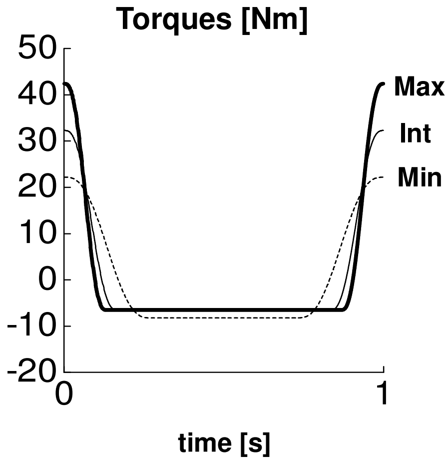

Fig. 6.