Query Optimization for Ontology-Mediated Query Answering

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ABSTRACT

Ontology-mediated query answering (OMQA) consists in asking database queries on knowledge bases (KBs); a KB is a set of facts called the KB's database, which is described by domain knowledge called the KB's ontology. A widely-investigated OMQA technique is FO-rewriting: every query asked on a KB is reformulated w.r.t. the KB's ontology, so that its answers are computed by the relational evaluation of the query reformulation on the KB's database. Crucially, because FO-rewriting compiles the domain knowledge relevant to queries into their reformulations, query reformulations may be complex and their optimization is the crux of efficiency.

We devise a novel optimization framework for a large set of OMQA settings that enjoy FO-rewriting: conjunctive queries, i.e., the core select-project-join queries, asked on KBs expressed using datalog±, description logics, existential rules, OWL, or RDFS. We optimize the query reformulations produced by state-of-the-art FO-rewriting algorithms by computing rapidly, with the help of a KB's database summary, simpler (contained) queries with the same answers that can be evaluated faster by RDBMSs. We show on a well-established OMQA benchmark that time performance is significantly improved by our optimization framework in general, up to three orders of magnitude.

CCS CONCEPTS

• **Information systems → Query optimization**; *Semantic web description languages*; • **Computing methodologies → Knowledge representation and reasoning**.

KEYWORDS

Existential rules, query optimization, data summarization

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1 INTRODUCTION

Ontology-mediated query answering [8] (OMQA) is a widely studied data management problem in Artificial Intelligence, Databases and Semantic Web. It consists in asking database-style queries on knowledge bases (KBs). A KB is a first-order (FO) theory that consists of a set of facts called a database, which models the application's data, and of a set of axioms called an ontology, which models the application's domain knowledge. The notable difference between this query answering setting and the traditional database one is that the answers to queries must be computed w.r.t. both the facts that are stored in the KB's database and the facts that can be deduced from the KB's database with the help of the KB's ontology.

There exist two main OMQA techniques in the literature. Both reduce OMQA to standard query evaluation on relational databases. The first technique is called FO-rewriting, e.g., [16]. It consists in rewriting a query asked on a KB into a so-called query reformulation, so that the query answers are obtained by evaluating the query reformulation on the KB's database. The second technique is called materialization, e.g., [1]. It consists in adding to the KB's database all the facts than can be deduced from it with the KB's ontology, so that the query answers are obtained by evaluating the (original) query on the augmented KB's database. The combination of FO-rewriting and materialization, called the combined or hybrid approach, has also been investigated, e.g., [40]. Crucially, both FO-rewriting and materialization are useful because, although there exist simple OMQA settings in which they compete, e.g., [3], there also exist more expressive OMQA settings to which only a single technique applies, e.g., [7].

In this paper, we focus on FO-rewriting. It has been mainly studied in OMQA settings consisting of (e.g., Table 1): queries expressed as conjunctive queries (CQs); KBs expressed using datalog±, description logics, existential rules, OWL, or RDFS; query reformulations expressed as unions of CQs (UCQs), unions of semi-CQs (USCQs), joins of UCQs (JUCQs), or non-recursive datalog programs (datalog^{nr}). These reformulation languages are recalled in Appendix A. We consider all these OMQA settings in this work.

Standard OMQA via FO-rewriting is illustrated in Figure 1. It consists in producing a query reformulation $q^{\mathcal{O}}$ from a query q and the ontology $\mathcal{O}$ of the KB $\mathcal{K}$, and then in evaluating $q^{\mathcal{O}}$ on the database $\mathcal{D}$ of $\mathcal{K}$ stored in an RDBMS. We point out that a query reformulation $q^{\mathcal{O}}$ may be large and complex to evaluate, e.g., [11, 33, 51]. FO-rewriting is indeed both ontology-dependent and data-independent, hence $q^{\mathcal{O}}$ must accomodate to all the possible databases and cannot be specific to the particular database $\mathcal{D}$ of $\mathcal{K}$.

So far, and similarly to semantic query optimization for deductive databases, e.g., [18], query optimization for FO-rewriting has

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Table 1: Main related works on conjunctive query answering via FO-rewriting

	Query reformulation language			
KB language	UCQ	USCQ	JUCQ	datalog^{nr}
datalog±/existential rules	[31, 32, 39]	[51]		[33, 44]
description logics/OWL	[16, 20, 45, 52]		[11]	[47]
RDF/S	[9, 30]		[10]	

focused on studying equivalent representations of query reformulations that can be evaluated faster: minimal (e.g., [20, 39]), compact (e.g., [33, 51]) or cost-based (e.g., [10, 11]) reformulations. However, because these optimizations are ontology-dependent and data-independent, optimized query reformulations remain complex to evaluate. They correspond to syntactically different but semantically equivalent variants of non-optimized query reformulations, thus they still need to accommodate to all the possible databases, and not to just the fixed database at hand.

The main contribution of this paper is a novel optimization framework for OMQA via FO-rewriting. It is illustrated in Figure 1. This framework capitalizes on the ontology-dependent and data-independent query optimization for FO-rewriting that have been studied so far in the literature (Reformulation step in Figure 1). Its originality is to include complementary data-dependent query optimization for FO-rewriting (Summarization and Optimization steps in Figure 1). Its purpose is to optimize the query reformulation $q^{\mathcal{O}}$ produced by any off-the-shelf FO-rewriting algorithm into a query reformulation $q^{\mathcal{K}}$ that is optimized for the particular database $\mathcal{D}$ of $\mathcal{K}$: $q^{\mathcal{K}}$ is simpler than $q^{\mathcal{O}}$, as it just needs to accommodate to $\mathcal{D}$, so that it can be evaluated faster; at the same time it has the same answers as $q^{\mathcal{O}}$ on $\mathcal{D}$ in order to guarantee the correctness of query answering on $\mathcal{K}$. Crucially, $q^{\mathcal{O}}$ is optimized for $\mathcal{D}$ using a summary $\mathcal{S}$ of $\mathcal{D}$, which is a typically small approximation of $\mathcal{D}$. This allows a trade-off between optimization time and the extent to which $q^{\mathcal{K}}$ is optimized for $\mathcal{D}$.

More specifically, our optimization framework builds on the following contributions.

1/ We formalize the problem of data-dependent optimization of a query reformulation using the well-known notion of query containment [1] (Section 3).

2/ We devise an optimization function Ω that rewrites a query reformulation into a simpler contained one, i.e., a simpler more specific one, with the same answers on a fixed database (Section 4.1). Containment and query answering correctness are ensured by appropriately removing useless subqueries from the query reformulation, i.e., subqueries that do not participate in producing answers on the given database while they may take time to be evaluated.

3/ We define a summary of a database, which is a (typically smaller) homomorphic database (Section 4.1). A summary can be used by our Ω optimization function in place of the original database to perform faster a sound but incomplete identification and removal of useless subqueries. Then, we adapt the quotient operation from graph theory [34] in order to build concrete summaries tailored to our needs (Section 4.2): both small summaries for fast optimization time and precise summaries to limit the incompleteness of identifying useless subqueries.

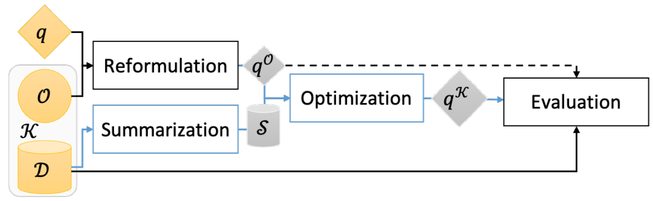

Figure 1: Standard (solid and dashed, back edges) and optimized (black and blue, solid edges) OMQA via FO-rewriting

4/ We experimentally evaluate our optimization framework on the well-established LUBM$^{\exists}$ benchmark for DL-Lite$_{\mathcal{R}}$ KBs (Section 5). DL-Lite$_{\mathcal{R}}$ is the description logic that underpins the W3C's OWL2 QL profile for OMQA on large KBs [16]. We show that our optimization framework significantly improves query answering time performance (up to 3 orders of magnitude).

The paper is organized as follows. We present OMQA and FO-rewriting in Section 2. We introduce our optimization framework and we formalize the underlying research problem in Section 3. We devise a solution (Ω optimization function and database summaries) to this problem in Section 4 and we experimentally evaluate this solution in Section 5. Finally, we conclude with related work and perspectives in Section 6. Proofs are available in the appendix.

This paper is an in-depth presentation of our optimization framework that was briefly introduced in the demonstration paper [37].

2 PRELIMINARIES

KBs. We consider FO KBs expressed using datalog± or existential rules [7, 12–15], which we call *rules* hereafter. A KB $\mathcal{K}$ is of the form $\mathcal{K} = (\mathcal{O}, \mathcal{D})$, where $\mathcal{O}$ is the KB's *ontology* and $\mathcal{D}$ is the KB's *database*. An ontology $\mathcal{O}$ is a set of rules of the form $\forall \bar{x}(q_1(\bar{x}) \rightarrow q_2(\bar{x}))$, where q_1 and q_2 are CQs (recall Appendix A) with the same set $\bar{x}$ of answer variables. Rules are used to derive entailed facts in the KB. A database $\mathcal{D}$ is a set of *incomplete facts*, i.e., whose *terms* are constants and existential variables modeling unknown values [1, 38], which we call *facts* from now. The semantics of a KB $\mathcal{K} = (\mathcal{O}, \mathcal{D})$ is that of the FO formula $\bigwedge_{\text{rule} \in \mathcal{O}} \wedge \exists \bar{v}(\bigwedge_{\text{fact} \in \mathcal{D}})$, where $\bar{v}$ is the set of variables that appear in $\mathcal{D}$.

Notation. We use small letters to denote constants, e.g., f, h, etc., and small italic letters to denote variables, e.g., x, y, etc. Also, we omit quantifiers in rules: existential variables solely appear on the right-hand side of $\rightarrow$ by virtue of FO semantics (see Appendix B).

Example 2.1 (Running example). Let us consider the following DL-Lite$_{\mathcal{R}}$ KB $\mathcal{K} = (\mathcal{O}, \mathcal{D})$, here expressed using rules [13]:

$\mathcal{O} = \{r_1 = ww(x, y) \rightarrow ww(y, x), r_2 = sup(x, y) \rightarrow ww(x, y),$
$\quad r_3 = PhD(x) \rightarrow sup(y, x)\},$

$\mathcal{D} = \{R(\mathsf{f}), R(\mathsf{h}), sup(\mathsf{f}, \mathsf{w}), sup(\mathsf{h}, \mathsf{w}), PhD(\mathsf{w}), ww(\mathsf{f}, \mathsf{h}), R(u),$
$\quad ww(u, \mathsf{c}), PhD(\mathsf{c})\}.$

The ontology $\mathcal{O}$ states that working with (ww) someone is a symmetric relation (r_1), supervising someone (sup) is a particular case of working with someone (r_2), and PhD students (PhD) are necessarily supervised (r_3). The database $\mathcal{D}$ states that f and h are researchers (R) who supervise the PhD student w, f works with h, and a researcher u works with the PhD student c. ⋄

Ontology-mediated query answering. We consider *FO queries* of the form $q(\bar{x}) = \phi$, where ϕ is an FO formula, the set of free (non-quantified) variables of which is exactly the tuple $\bar{x}$ of answer variables. The arity of a query $q(\bar{x})$ is the cardinality of $\bar{x}$; $q(\bar{x})$ is said *Boolean* if $\bar{x} = \emptyset$. A *(certain) answer to a query* $q(\bar{x})$ of arity n on a KB $\mathcal{K}$ is a tuple $\bar{\mathsf{t}}$ of n constants from $\mathcal{K}$ such that $\mathcal{K} \models q(\bar{\mathsf{t}})$, where $\models$ is the FO entailment relation and $q(\bar{\mathsf{t}})$ is the Boolean query obtained by instantiating $\bar{x}$ with $\bar{\mathsf{t}}$ in q; when q is Boolean, $\bar{\mathsf{t}}$ is the empty tuple $\langle\rangle$. From now, we denote by $ans(q, \mathcal{K})$ the *answer set* of q on $\mathcal{K}$ and we remark that if q is Boolean then the answer is true when $ans(q, \mathcal{K}) = \{\langle\rangle\}$ and the answer is false when $ans(q, \mathcal{K}) = \emptyset$.

Example 2.2 (Cont.). Let us consider the CQ asking for the supervisees who work with h that must be a researcher: $q(x) = \exists y\, R(\mathsf{h}) \wedge ww(\mathsf{h}, x) \wedge sup(y, x)$. Its answer set on $\mathcal{K}$ is $ans(q, \mathcal{K}) = \{\mathtt{w}\}$: w is obtained from $R(\mathsf{h}) \in \mathcal{D}$, $sup(\mathsf{f}, \mathtt{w}) \in \mathcal{D}$ or $sup(\mathsf{h}, \mathtt{w}) \in \mathcal{D}$, and the fact $ww(\mathsf{h}, \mathtt{w})$ entailed from $sup(\mathsf{h}, \mathtt{w}) \in \mathcal{D}$ and r_2. ⋄

Ontology-mediated query answering technique. We focus on optimizing OMQA via *FO-rewriting* [16].

FO-rewriting reduces query answering on KBs to query evaluation on relational databases in *FO-rewritable OMQA settings*. An OMQA setting is a pair $(\mathcal{L}_Q, \mathcal{L}_K)$ of query and KB languages. Such a setting is FO-rewritable if for any $\mathcal{L}_Q$ query q and any $\mathcal{L}_K$ ontology O, there exists an FO query q^O, called a *reformulation of q w.r.t. O*, such that for any KB $\mathcal{K} = (O, \mathcal{D})$: $ans(q, \mathcal{K}) = eval(q^O, \mathcal{D})$, where $eval(q^O, \mathcal{D})$ is the result of the relational evaluation of q^O on $\mathcal{D}$ from which tuples with variables are pruned away. Furthermore, each FO-rewriting algorithm computes query reformulations in a fixed FO query dialect. Recall for instance Table 1 where CQs are reformulated into UCQs, USCQs, JUCQs or datalog^{nr} programs. We therefore term *FO-rewriting setting* a triple of query language $\mathcal{L}_Q$, KB language $\mathcal{L}_K$ and query reformulation language $\mathcal{L}_R$, denoted by $(\mathcal{L}_Q, \mathcal{L}_K, \mathcal{L}_R)$, such that $(\mathcal{L}_Q, \mathcal{L}_K)$ is an FO-rewritable OMQA setting for which query reformulations are expressed in $\mathcal{L}_R$.

In this paper, we focus on FO-rewriting settings with queries expressed in the language of CQs and query reformulations expressed in the languages of UCQs, USCQs and JUCQs. These setting are widely considered in the literature on FO-rewriting (recall Table 1). We remark that datalog^{nr} reformulations may need to be unfolded into UCQs reformulations, which we consider, to be (efficiently) evaluated by RDBMSs.

A key property of the FO-rewriting settings that we consider, on which our work relies, is that a query reformulation q^O is equivalent to the CQ q w.r.t. O. In particular, q^O is equivalent, regardless of the language used to express it, to the union of all the *CQs that are maximally-contained in q w.r.t. O*, i.e., the union of all the most general CQ specializations of q w.r.t. O. We recall that (i) a query q' is *contained in a query q*, denoted by $q' \subseteq q$, if and only if for each database $\mathcal{D}$, $eval(q', \mathcal{D}) \subseteq eval(q, \mathcal{D})$ and (ii) a query q' is *contained in a query q w.r.t. an ontology O*, denoted by $q' \subseteq_O q$, if and only if for each KB $\mathcal{K} = (O, \mathcal{D})$, $ans(q', \mathcal{K}) \subseteq ans(q, \mathcal{K})$. A query q' is *maximally-contained in a query q w.r.t. an ontology O* if and only if (i) $q' \subseteq_O q$ and (ii) for any other query $q'' \subseteq_O q$, if $q' \subseteq q''$ then $q'' \subseteq q'$ (i.e., q' and q'' are equivalent).

Notation. We omit existential quantifiers in queries, as non-answer variables are existentially quantified in the query languages we consider (recall Appendix A). For instance, the CQ of Example 2.2 is now written $q(x) = R(\mathsf{h}) \wedge ww(\mathsf{h}, x) \wedge sup(y, x)$.

Example 2.3 (Cont.). Consider the following equivalent UCQ q^O_{UCQ}, USCQ q^O_{USCQ} and JUCQ q^O_{JUCQ} reformulations of q w.r.t. O, which are respectively computed by the Rapid [20], Compact [51] and GDL [11] FO-rewriting tools:

$$
\begin{aligned}
q^O_{\mathrm{UCQ}}(x) = & (R(\mathsf{h}) \wedge ww(\mathsf{h}, x) \wedge sup(y, x)) && (1)\\
& \vee (R(\mathsf{h}) \wedge ww(\mathsf{h}, x) \wedge PhD(x)) && (2)\\
& \vee (R(\mathsf{h}) \wedge sup(\mathsf{h}, x)) && (3)\\
& \vee (R(\mathsf{h}) \wedge ww(x, \mathsf{h}) \wedge sup(y, x)) && (4)\\
& \vee (R(\mathsf{h}) \wedge ww(x, \mathsf{h}) \wedge PhD(x)) && (5)\\
& \vee (R(\mathsf{h}) \wedge sup(x, \mathsf{h}) \wedge sup(y, x)) && (6)\\
& \vee (R(\mathsf{h}) \wedge sup(x, \mathsf{h}) \wedge PhD(x)) && (7)
\end{aligned}
$$

$$
\begin{aligned}
q^O_{\mathrm{USCQ}}(x) = & (R(\mathsf{h}))\\
& \wedge (ww(\mathsf{h}, x) \vee sup(\mathsf{h}, x) \vee ww(x, \mathsf{h}) \vee sup(x, \mathsf{h}))\\
& \wedge (sup(y, x) \vee PhD(x))
\end{aligned}
$$

$$
\begin{aligned}
q^O_{\mathrm{JUCQ}}(x) = (R(\mathsf{h})) \wedge \big(& (ww(\mathsf{h}, x) \wedge sup(y, x))\\
& \vee (ww(\mathsf{h}, x) \wedge PhD(x))\\
& \vee (sup(\mathsf{h}, x))\\
& \vee (ww(x, \mathsf{h}) \wedge sup(y, x))\\
& \vee (ww(x, \mathsf{h}) \wedge PhD(x))\\
& \vee (sup(x, \mathsf{h}) \wedge sup(y, x))\\
& \vee (sup(x, \mathsf{h}) \wedge PhD(x))\big)
\end{aligned}
$$

q^O_{UCQ} is the union of all the maximally-contained CQs in q w.r.t. O. q^O_{USCQ} and q^O_{JUCQ} model the same union up to the distributive property of $\wedge$ and $\vee$. The answer to q on $\mathcal{K}$ (i.e., w) results from (3) in q^{UCQ}, shown in blue, and from the logical combination of the subqueries shown in blue in q^O_{USCQ} and q^O_{JUCQ}, from which (3) can be recovered by distributing the $\wedge$'s over the $\vee$'s. ⋄

3 OPTIMIZATION PROBLEM

Motivation. The definition of FO-rewriting is data-independent: a single query reformulation q^O is able to answer the CQ q on all the KBs with ontology O. This generality of q^O follows from the fact that it is equivalent to the union of all the CQs that are maximally-contained in q w.r.t. O, which can also be regarded as all the ways databases may store answers to q according to O. As a consequence, a query reformulation may be large and complex to evaluate in practice [10, 11, 51]. For instance, the worst-case number of CQs that are maximally-contained in a CQ q w.r.t. a lightweight RDFS, DL-Lite$_{\mathcal{R}}$ or datalog$\pm_0$ ontology, is exponential in the size of the CQ q (number of atoms) [9, 16, 30, 32].

Rationale behind our optimization problem. We study the data-dependent optimization of a query reformulation for a particular KB, in order to trade its generality for more OMQA performance. When the query q is asked on a given KB $\mathcal{K} = (O, \mathcal{D})$, the database $\mathcal{D}$ is indeed fixed and just one of all the possible databases a reformulation q^O accommodates to. In particular, within the union of maximally-contained CQs to which q^O is equivalent, many CQs may be *irrelevant* to $\mathcal{D}$, because they have no answer on $\mathcal{D}$ (i.e., $\mathcal{D}$ do not store answers to q w.r.t. O this way), and translate into wasteful evaluation time.

Example 3.1 (Cont.). In q^O_{UCQ}, all the CQs except the CQ (3) are irrelevant to $\mathcal{D}$, and similarly in q^O_{USCQ} and q^O_{JUCQ}, where these CQs are present up to the distribution of the $\wedge$'s over the $\vee$'s. ⋄

Problem statement. Our goal is to devise an optimization framework for OMQA via FO-rewriting that enjoys the following properties: *generality* to be used in as many FO-rewriting settings as possible, *correctness* to compute the exact answer set of a query, and *effectiveness* to improve query answering time performance.

Our framework relies on an optimization function Ω that turns a given query reformulation $q^{\mathcal{O}}$ into an *optimized query reformulation* for a given database $\mathcal{D}$. This optimized query reformulation is hereafter denoted by $q^{\mathcal{K}}$ as it is specific to the KB $\mathcal{K} = (\mathcal{O}, \mathcal{D})$.

For the generality of our framework, the Ω function optimizes query reformulations from the language of $(\wedge,\vee)$*-combinations of CQs* (Definition 3.2 below), which includes UCQ, USCQ and JUCQ reformulations.

Definition 3.2 ($(\wedge,\vee)$-combination of CQs). A $(\wedge,\vee)$*-combination of CQs*, denoted by $(\wedge,\vee)$-CQ, is either a CQ or a conjunction or union of $(\wedge,\vee)$-CQs.

The Ω function computes an optimized query reformulation $q^{\mathcal{K}}$ contained in $q^{\mathcal{O}}$ (item 1 in Problem 1 below) since $q^{\mathcal{O}}$ is equivalent to a union of maximally-contained queries, in which we remove those irrelevant to a database $\mathcal{D}$, and removing disjuncts from a union makes it more specific. However, this containment relationship only ensures that the answers to $q^{\mathcal{K}}$ form a subset of the answers to $q^{\mathcal{O}}$ on all possible databases. For the correctness of our framework, Ω thus computes an optimized query reformulation $q^{\mathcal{K}}$ with same answers as $q^{\mathcal{O}}$ on $\mathcal{D}$ (item 2 in Problem 1 below).

Finally, for the effectiveness of our framework, the Ω function optimizes $q^{\mathcal{O}}$ for $\mathcal{D}$ using a summary $\mathcal{S}$ of $\mathcal{D}$ (item 3 in Problem 1 below). This allows for a trade-off between the number of removed irrelevant maximally-contained queries and the cost to remove them, which translates into optimization time. As we shall see in our experiments, the optimization time may be too high to improve OMQA time performance when Ω identifies irrelevant maximally-contained queries in $q^{\mathcal{O}}$ using the database $\mathcal{D}$ instead of a typically much smaller summary $\mathcal{S}$ of it.

We summarize the above discussion with the formal statement of the research problem studied in this paper.

Problem 1 (Summary-based optimization of FO-rewriting). *Let $q^{\mathcal{O}}$ be a $(\wedge,\vee)$-CQ query reformulation and let $\mathcal{D}$ be a database. Define an optimization function Ω and a summary $\mathcal{S}$ of $\mathcal{D}$ so that the optimization of $q^{\mathcal{O}}$ for $\mathcal{D}$ using $\mathcal{S}$, denoted by $q^{\mathcal{K}}$ and computed by $\Omega(q^{\mathcal{O}},\mathcal{S})$, satisfies:*

(1) $q^{\mathcal{K}} \subseteq q^{\mathcal{O}}$,
(2) $eval(q^{\mathcal{K}},\mathcal{D}) = eval(q^{\mathcal{O}},\mathcal{D})$,
(3) $c(\Omega(q^{\mathcal{O}},\mathcal{S})) + c(eval(q^{\mathcal{K}},\mathcal{D})) \le c(eval(q^{\mathcal{O}},\mathcal{D}))$ *for a given cost estimation function $c(\cdot)$ that models the cost to compute $\cdot$.*

4 OPTIMIZATION FRAMEWORK

4.1 The Ω optimization function

Rationale behind the Ω optimization function. When a query reformulation is seen as a $(\wedge,\vee)$-combination of CQs, these subCQs are parts of the maximally-contained CQs that the query reformulation models. Recall for instance Example 2.3 where the maximally-contained CQ (3) in the UCQ reformulation corresponds to the logical combinations of the subCQs shown in blue in the JUCQ and USCQ reformulations. Removing subCQs from a query reformulation seen as $(\wedge,\vee)$-combinations of CQs obviously removes all the maximally-contained queries these subCQs are part of, and crucially for us, removing such subCQs with no answer on a particular database removes maximally-contained queries that are irrelevant to this database. E.g., in Example 2.3, removing the subCQ $sup(x,\mathsf{h})$ with no answer on $\mathcal{D}$ from $q^{\mathcal{O}}_{\text{USCQ}}$ also removes from $q^{\mathcal{O}}_{\text{USCQ}}$ the two irrelevant maximally-contained CQs $R(\mathsf{h}) \wedge sup(x,\mathsf{h}) \wedge sup(x,y)$ and $R(\mathsf{h}) \wedge sup(x,\mathsf{h}) \wedge PhD(x)$: without $sup(x,\mathsf{h})$, they cannot be recovered by distributing the $\wedge$'s over the $\vee$'s. We therefore devise the Ω function to optimize a $(\wedge,\vee)$-CQ query reformulation for a given database by rewriting it from the bottom up to (i) identify subCQs with no answer on this database and (ii) propagate the effect of their removal within the query reformulation.

Identifying CQs with no answer on a database. Checking if a single CQ has no answer on a database can be done easily (e.g., using EXISTS in SQL) and efficiently in general since RDBMSs are highly-optimized for CQs, e.g., [48]. However, doing the same check for all the subCQs in a query reformulation may take significant time, especially when the database is large. To mitigate this issue, the Ω optimization function uses database summaries that are (typically small) homomorphic approximations of the databases they summarize. Using such summaries instead of the databases allows trading completeness of identifying subCQs with no answer for efficiency, while retaining soundness.

Definition 4.1 (Summary of a database). A database $\mathcal{S}$ is a *summary* of a database $\mathcal{D}$ iff (i) there exists a homomorphism σ from $\mathcal{D}$ to $\mathcal{S}$, i.e., $\mathcal{D}_\sigma = \mathcal{S}$ where $\mathcal{D}_\sigma$ is the database obtained from $\mathcal{D}$ by replacing the terms in $\mathcal{D}$ by their images in $\mathcal{S}$ through σ, such that (ii) σ maps constants in $\mathcal{D}$ to constants in $\mathcal{S}$, while it maps variables in $\mathcal{D}$ to constants or variables in $\mathcal{S}$.

In the above definition, (i) ensures that $\mathcal{S}$ is a homomorphic approximation of $\mathcal{D}$, while (ii) ensures the soundness of identifying CQs with no answer on $\mathcal{D}$ using $\mathcal{S}$ (Theorem 4.2 below). Also, we remark that a database is a particular summary of itself: $\mathcal{D} = \mathcal{S}$ holds when the database-to-summary homomorphism σ maps each term to itself, i.e., when σ is the identity function.

Theorem 4.2. *Let $\mathcal{D}$ be a database and $\mathcal{S}$ a summary of it with the homomorphism σ. Let q be a CQ q asked on $\mathcal{D}$ and q_σ the CQ obtained from q by replacing its constants with their images through σ. If q_σ has no answer on $\mathcal{S}$, then q has no answer on $\mathcal{D}$.*

We stress that, as illustrated below, if q_σ has no answer on $\mathcal{S}$ then for sure q has no answer on $\mathcal{D}$, while if q_σ has some answer on $\mathcal{S}$ then q may or may not have an answer on $\mathcal{D}$.

Example 4.3 (Cont.). Consider the summary $\mathcal{S}$ of $\mathcal{D}$ with homomorphism σ such that $\sigma(\mathsf{c}) = \sigma(\mathsf{w}) = \mathsf{p}$, $\sigma(\mathsf{f}) = \sigma(\mathsf{h}) = \sigma(u) = \mathsf{r}$:

$$\mathcal{S} = \{R(\mathsf{r}), sup(\mathsf{r},\mathsf{p}), PhD(\mathsf{p}), ww(\mathsf{r},\mathsf{r}), ww(\mathsf{r},\mathsf{p})\}.$$

Consider the CQs (1) and (5) in $q^{\mathcal{O}}_{\text{UCQ}}$, which we name q^1 and q^5 respectively. According to Theorem 4.2: $q^1_\sigma(x) = R(\mathsf{r}) \wedge ww(\mathsf{r},x) \wedge sup(y,x)$ has an answer on the summary $\mathcal{S}$ ($eval(q^1_\sigma,\mathcal{S}) = \{\mathsf{p}\}$) then q^1 may or may not have an answer on $\mathcal{D}$ (here, q^1 has no answer on $\mathcal{D}$), while $q^5_\sigma(x) = R(\mathsf{r}) \wedge ww(x,\mathsf{r}) \wedge PhD(x)$ has no answer on $\mathcal{S}$ then for sure q^5 has no answer on $\mathcal{D}$. ⋄

$(\wedge, \vee)$-CQ optimization for a database. Our Ω function builds on Theorem 4.2 to optimize a $(\wedge, \vee)$-CQ for a database $\mathcal{D}$. It rewrites a query while (i) identifying its CQs with no answer on $\mathcal{D}$ using a summary $\mathcal{S}$ of it ((1) in Definition 4.4 below) and (ii) performing a bottom-up removal of the largest subqueries with no answer on $\mathcal{D}$ that these CQs are the causes of ((2) and (3) in Definition 4.4 below).

Definition 4.4 (Optimization function Ω). Let q be a $(\wedge, \vee)$-CQ asked on a database $\mathcal{D}$ and $\mathcal{S}$ be a summary of $\mathcal{D}$ with the homomorphism σ. The *optimization of q for $\mathcal{D}$ using $\mathcal{S}$*, i.e., denoted by $\Omega(q, \mathcal{S})$, is recursively defined as follows. Below, $\varnothing$ denotes the empty relation with appropriate arity.

The optimization of a CQ q is:

$$\Omega(q, \mathcal{S}) = \begin{cases} \varnothing \text{ if } eval(q_\sigma, \mathcal{S}) = \emptyset \\ q \text{ otherwise} \end{cases} \quad (1)$$

where q_σ is obtained from q by replacing its constants by their images through σ.

The optimization of a conjunction of subqueries $\bigwedge_{i=1}^{n} q_i$ is:

$$\Omega(\bigwedge_{i=1}^{n} q_i, \mathcal{S}) = \begin{cases} \varnothing \text{ if } \exists i \in [1, n]\ \Omega(q_i, \mathcal{S}) = \varnothing \\ \bigwedge_{i=1}^{n} \Omega(q_i, \mathcal{S}) \text{ otherwise} \end{cases} \quad (2)$$

The optimization of a disjunction of subqueries $\bigvee_{i=1}^{n} q_i$ is:

$$\Omega(\bigvee_{i=1}^{n} q_i, \mathcal{S}) = \begin{cases} \varnothing \text{ if } \forall i \in [1, n]\ \Omega(q_i, \mathcal{S}) = \varnothing \\ \bigvee_{1 \le i \le n,\ \Omega(q_i, \mathcal{S}) \ne \varnothing} \Omega(q_i, \mathcal{S}) \text{ otherwise} \end{cases} \quad (3)$$

Above, the rewriting rule (1) follows from the soundness of identifying CQs with no answer using a database summary (Theorem 4.2), while the two other rewriting rules (2) and (3) follow from the semantics of the $\wedge$ and $\vee$ operators, respectively.

The next theorem establishes two semantic relationships between a $(\wedge, \vee)$-CQ and its optimization, which correspond to items 1 and 2 in Problem 1. In particular, it states the correctness of summary-based optimization of a $(\wedge, \vee)$-CQ w.r.t. relational query evaluation.

Theorem 4.5. *Let $\mathcal{D}$ be a database, $\mathcal{S}$ a summary of $\mathcal{D}$, and q a $(\wedge, \vee)$-CQ. Then $\Omega(q, \mathcal{S}) \subseteq q$ and $eval(q, \mathcal{D}) = eval(\Omega(q, \mathcal{S}), \mathcal{D})$.*

Example 4.6 (Cont.). The summary-based optimization of q^{O}_{UCQ}, q^{O}_{USCQ} and q^{O}_{JUCQ} for $\mathcal{D}$ using $\mathcal{S}$ corresponds to the following UCQ, USCQ and JUCQ, respectively. We also show in gray the subqueries that would have been additionally removed (with higher optimization cost) if Ω had used $\mathcal{D}$ instead of $\mathcal{S}$.

$$\begin{aligned} \Omega(q^{O}_{\mathrm{UCQ}}, \mathcal{S}) = & (R(\mathsf{h}) \wedge ww(\mathsf{h}, x) \wedge sup(y, x)) \quad (1) \\ & \vee (R(\mathsf{h}) \wedge ww(\mathsf{h}, x) \wedge PhD(x)) \quad (2) \\ & \vee (R(\mathsf{h}) \wedge sup(\mathsf{h}, x)) \quad (3) \end{aligned}$$

$$\begin{aligned} \Omega(q^{O}_{\mathrm{USCQ}}, \mathcal{S}) = & (R(\mathsf{h})) \\ & \wedge (ww(\mathsf{h}, x) \vee sup(\mathsf{h}, x) \vee ww(x, \mathsf{h})) \\ & \wedge (sup(y, x) \vee PhD(x)) \end{aligned}$$

$$\begin{aligned} \Omega(q^{O}_{\mathrm{JUCQ}}, \mathcal{S}) = & (R(\mathsf{h})) \wedge ((ww(\mathsf{h}, x) \wedge sup(y, x)) \\ & \vee (ww(\mathsf{h}, x) \wedge PhD(x)) \\ & \vee (sup(\mathsf{h}, x))) \end{aligned}$$

For $\mathcal{L}_R \in \{\mathrm{UCQ}, \mathrm{USCQ}, \mathrm{JUCQ}\}$, it can be easily checked that: $\Omega(q^{O}_{\mathcal{L}_R}, \mathcal{S}) \subseteq q^{O}_{\mathcal{L}_R}$ since Ω makes unions more specific by removing disjuncts, and $eval(\Omega(q^{O}_{\mathcal{L}_R}, \mathcal{S}), \mathcal{D}) = eval(q^{O}_{\mathcal{L}_R}, \mathcal{D})$ since both $q^{O}_{\mathcal{L}_R}$ and $\Omega(q^{O}_{\mathcal{L}_R}, \mathcal{S})$ model the CQ (3) that produces the sole answer w. ⋄

Finally, we provide our cost estimation function c (Definition 4.7 below) to formally characterize the efficiency of summary-based optimization of a $(\wedge, \vee)$-CQ, which corresponds to item 3 in Problem 1. It provides summable optimization and evaluation costs by (i) trivially setting the query evaluation cost to the cost of relational query evaluation [1, 46] (first item in Definition 4.7 below) and (ii) reducing the query optimisation cost to the sole cost of relational query evaluation that it induces (second item in Definition 4.7 below): Ω's simple in-memory data processing ((2) and (3) in Definition 4.4) is typically negligible w.r.t. Ω's on-disk data processing ((1) in Definition 4.4).

Definition 4.7 (Cost estimation function c). Let q be a $(\wedge, \vee)$-CQ and $\mathcal{D}$ be a database. Given a cost estimation function *cost−rel−eval* for query evaluation on relational databases, the cost estimation function c for summary-based optimization is such that:

- $c(eval(q, \mathcal{D})) = cost{-}rel{-}eval(eval(q, \mathcal{D}))$
- $c(\Omega(q, \mathcal{D})) = \sum_{i=1}^{n} cost{-}rel{-}eval(eval(cq_i, \mathcal{D}))$, where $cq_1, \ldots, cq_n$ are all the CQs in the $(\wedge, \vee)$-CQ q.

4.2 Database summarization

The concrete database summaries that we use with our Ω optimization function are defined by adapting the quotient operation from graph theory [34] to the incomplete relational databases we consider. The quotient operation has been widely used in the literature for graph database summarization [17, 41]. It offers an elegant summarization technique by decoupling the summarization method, which fuses equivalent nodes, from the high-level specification of equivalent nodes, defined by an equivalence relation, e.g., bisimilarity [2]. We recall that an equivalence relation is a reflexive, symmetric, and transitive binary relation. Assuming we have an equivalence relation between database terms (the one we use will be discussed shortly), we define a *quotient database* as follows.

Definition 4.8 (Quotient database). Let $\mathcal{D}$ be a database, $\equiv$ be some equivalence relation between terms, and let $c^1_\equiv, \ldots, c^k_\equiv$ denote, by abuse of notation, both the equivalence classes of all terms in $\mathcal{D}$ w.r.t. $\equiv$ and the terms used to represent these equivalence classes.

The *quotient database of $\mathcal{D}$ w.r.t. $\equiv$* is the database $\mathcal{D}_\equiv$ such that:
• $R(c^{\alpha_1}_\equiv, \cdots, c^{\alpha_n}_\equiv) \in \mathcal{D}_\equiv$ iff there exists $R(term_1, \cdots, term_n) \in \mathcal{D}$ with $term_i \in c^{\alpha_i}_\equiv$ and $1 \le \alpha_i \le k$, for $1 \le i \le n$,
• the term $c^j_\equiv$ in $\mathcal{D}_\equiv$, for $1 \le j \le k$, is a variable if all the equivalent terms in $\mathcal{D}$ it represents according to $\equiv$ are variables, otherwise it is a constant.

The next proposition establishes that quotient databases can be used by the optimization function Ω to identify CQs with no answer on databases. It follows from the fact that in the above definition, $\equiv$ defines an implicit function that maps the terms in $\mathcal{D}$ to the terms in $\mathcal{D}_\equiv$, which turns out to be the homomorphism σ in Definition 4.1: the first and second items in the above definition enforce respectively the conditions (i) and (ii) in Definition 4.1.

Proposition 4.9. *Quotient databases are database summaries.*

We introduce the equivalence relation $\equiv_\Omega$ used to build our summaries, i.e., how database terms are fused into summary terms. Since ontology languages [5, 7, 14, 19] are centered on *concepts* modeled by unary predicates, which are then interrelated using *relationships* modeled by n-ary predicates (with $n \geq 2$), we adopt a summarization centered on the instances of concepts stored in a KB's database: all the terms that are instances of the same concept in the KB's database are represented by a single term in the summary ((i) in Definition 4.10 below), and all the concepts with common instances in the KB's database have the same single term that represents all their instances in the summary ((ii) in Definition 4.10 below). As we shall see in our experiments, database summaries built with $\equiv_\Omega$ achieve a good tradeoff between size reduction (≥90%) and completeness of identifying CQs with no answer (92% on average).

Definition 4.10 ($\equiv_\Omega$ equivalence relation). $\equiv_\Omega$ is the equivalence relation such that two terms t_1 and t_2 are equivalent within a database $\mathcal{D}$, denoted $t_1 \equiv_\Omega t_2$, iff (i) both t_1 and t_2 are instances of the same *unary predicate*, i.e., concept, or (ii) there exists a term t_3 in $\mathcal{D}$ such that $t_1 \equiv_\Omega t_3$ and $t_2 \equiv_\Omega t_3$.

Example 4.11 (Cont.). The summary $\mathcal{S}$ in Example 4.3 is actually the quotient database of $\mathcal{D}$ w.r.t. $\equiv_\Omega$: it defines two equivalence classes, one for the researchers in $\mathcal{D}$, i.e., $\{\mathsf{f}, \mathsf{h}, u\}$, and one for the PhD students in $\mathcal{D}$, i.e., $\{\mathsf{w}, \mathsf{c}\}$; these two classes are represented in $\mathcal{S}$ by the constants r and p, respectively. ⋄

5 EXPERIMENTAL EVALUATION

We implemented our optimization framework in the OptiRef JAVA tool [37] in order to evaluate the OMQA time performance it brings.
Setup. For our KBs, we used the well-established *extended LUBM benchmark* a.k.a. LUBM$^\exists$ [42]. It is an adaptation of the Leight University benchmark a.k.a. LUBM [35] to the DL-Lite$_\mathcal{R}$ description logic [16]. We chose this benchmark for two reasons. First, DL-Lite$_\mathcal{R}$ is the most expressive KB language for which the reformulation of CQs into UCQ, USCQ and JUCQ reformulations has been studied. Second, LUBM$^\exists$ is widely-considered in the OMQA literature and provides opportunities to adapt many available queries to our needs. For the ontology $\mathcal{O}$ of all our KBs, we used the default benchmark ontology LUBM$^\exists_{20}$. It is made of 449 rules over 163 predicates: 128 unary predicates, a.k.a. concepts, and 35 binary predicates, a.k.a. roles. We used the EUGen (v0.1b) data generator provided with LUBM$^\exists$ to generate the databases of our KBs.

OptiRef relies on the open-source PostgreSQL RDBMS (v14.2) to store the generated databases and their summaries, which is commonly used in the OMQA literature. We adopted the data layout of [11] for the databases and summaries, which was found to be the most efficient for evaluating query reformulations on DL-Lite$_\mathcal{R}$ KB's database. Concepts instances are stored in unary relations, and role instances are stored in binary relations. Also, all the values are dictionary-encoded into integers; the dictionary is stored as a binary relation. Finally, for a database summary, the database-to-summary homomorphism σ, which maps the database terms to the summary terms, is stored as a binary relation. For all the above-mentioned database, summary, dictionary and homomorphism relations, each unary relation has an index on its unique attribute and each binary relation has the two two-attributes indexes.

OptiRef relies on the Rapid (v0.93) [20], Compact (v1.0b6) [51] and GDL (v1.0) [11] FO-rewriting tools that respectively compute UCQ, USCQ and JUCQ reformulations of CQs w.r.t. DL-Lite$_\mathcal{R}$ ontologies. They load and keep in memory the ontology w.r.t. which CQs are reformulated. We chose Compact and GDL because they are the only tools to respectively compute USCQ and JUCQ query reformulations, to the best of our knowledge. By contrast, there are other tools besides Rapid to compute UCQ query reformulations, e.g., Clipper [24], Graal [6], Iqaros [52], Nyaya [53], Presto [47], Requiem [45], etc. These tools differ w.r.t. query reformulation time and query reformulation minimality. We chose Rapid since it is fast, although we do not consider query reformulation cost (hence time) in our optimization problem (Problem 1), to compute minimal UCQ reformulations, i.e., within which no CQ is redundant with another.

We ran our experiments on a Ubuntu 20.04 Linux server with Intel Xeon 4215R 3.20GHz CPU, 128GB of RAM, and 7TB of HDD.
Database summarization. We generated five LUBM databases: LUBM1M, LUBM10M, LUBM50M, LUBM100M, LUBM150M. The name of a database indicates the number of stored facts in millions. Also, databases are created such that LUBM1M ⊆ LUBM10M ⊆ LUBM50M ⊆ LUBM100M ⊆ LUBM150M, where ⊆ means set inclusion, so that query answering becomes harder as data grows.

OptiRef relies on a *union-find* data structure for disjoint sets [21] for database summarization, since equivalence classes of database terms w.r.t. $\equiv_\Omega$ are disjoint sets of equivalent terms w.r.t. $\equiv_\Omega$. This data structure supports two main operations, *union* and *find*, in optimal constant amortized time complexity [49, 50], i.e., time complexity is almost constant over a sequence of union or find operations. *Union* is used to state which values must be in a same set, and results in merging the sets these values belong to. *Find* returns the representative value of the set a given value belongs to.

OptiRef first computes the homomorphism σ from the database $\mathcal{D}$ to the summary $\mathcal{S}$ (Definition 4.1) w.r.t. the $\equiv_\Omega$ equivalence relation (Definition 4.10). Given a union-find data structure for disjoint sets of integers, we use *union* to state that the (integer-encoded) terms stored in each unary relation in $\mathcal{D}$ are in a same set, as these terms are equivalent w.r.t. $\equiv_\Omega$ (condition (i) in Definition 4.10). By definition of *union*, this ensures that if unary relations share some terms, in which case all the terms of these relations are equivalent w.r.t. $\equiv_\Omega$ (condition (ii) in Definition 4.10), then these terms end up in the same set. Finally, since *find* returns a representative term for the set of equivalent terms a given term belongs to, it models the homomorphism σ from the database $\mathcal{D}$ to its summary $\mathcal{S}$ w.r.t. $\equiv_\Omega$. The computation of σ is therefore linear in the size of the data: it needs a worst-case number of calls to *union* in the size of $\mathcal{D}$, each of which is performed in constant amortized time. Then, OptiRef computes the summary $\mathcal{S}$ of the database $\mathcal{D}$ w.r.t. $\equiv_\Omega$ as per Definition 4.8: every fact in $\mathcal{D}$ leads to a fact in $\mathcal{S}$ obtained by replacing each term by its image through σ, i.e., *find*. The computation of $\mathcal{S}$ is therefore linear in the size of the data: it needs a worst-case number of calls to *find* in the size of $\mathcal{D}$ (one or two calls per fact), each of which is performed in constant amortized time.

Table 2 shows for each database $\mathcal{D}$ we generated: its size $|\mathcal{D}|$ and the size $|\mathcal{S}|$ of its summary $\mathcal{S}$, i.e., numbers of facts, the $\mathcal{D}$-to-$\mathcal{S}$ size reduction $(1 - |\mathcal{S}|/|\mathcal{D}|)$, and the summarization time with PostgreSQL (computation and storage of σ and then of $\mathcal{S}$). We observe that $\equiv_\Omega$ achieves significant size reduction (≥ 90%) and that summarization time scales linearly in the size of the data. However, we remark that it would be prohibitive to redo summarization upon database updates. OptiRef thus relies on incremental summary maintenance [37]. By definition of a summary built with $\equiv_\Omega$, in the worst case, an insertion fuses two equivalence classes and a deletion

Table 2: Characteristics of the databases and summaries, database size reduction and summarization time for PostgreSQL

Database $\mathcal{D}$	$\lvert\mathcal{D}\rvert$	$\lvert\mathcal{S}\rvert$	**size red.** (%)	**sum. time** (s)
LUBM1M	1,187k	93k	92.12	15
LUBM10M	10,794k	843k	92.18	86
LUBM50M	53,328k	4,160k	92.20	308
LUBM100M	106,596k	8,316k	92.19	699
LUBM150M	159,899k	12,474k	92.19	1,100

splits an equivalence class into two ones. Maintenance rewrites the affected summary facts, i.e., in which some term moves from an equivalence class to another, based on the updated homomorphism σ modeled with a union-find data structure that also supports the delete operation in optimal constant amortized time complexity [4].

OMQA performance. We used ten CQs adapted from [11, 42] to obtain a variety of numbers of maximally-contained CQs w.r.t. $\mathcal{O}$ that query reformulations model (recall Section 2) and of answers. The main characteristics of these CQs are shown in Table 3 (top).

For each database, OptiRef processed every query with 3 strategies per $\mathcal{L}_R$ query reformulation languages used by FO-rewriting tools: $\mathcal{L}_R$ = UCQ for Rapid, $\mathcal{L}_R$ = USCQ for Compact and $\mathcal{L}_R$ = JUCQ for GDL. The first strategy, denoted by $\mathcal{L}_R$/REF, consists in computing the $\mathcal{L}_R$ query reformulation with the FO-rewriting tool and then evaluating it with PostgreSQL; this is how OMQA is performed via FO-rewriting, hence the state-of-the-art baseline. The second strategy, denoted by $\mathcal{L}_R$/DB, departs from $\mathcal{L}_R$/REF by optimizing the query reformulation for the database $\mathcal{D}$ before evaluating it. For this strategy, our Ω function optimizes the query reformulation using the database $\mathcal{D}$. The third strategy, denoted by $\mathcal{L}_R$/S, is similar to $\mathcal{L}_R$/DB except that our Ω function optimizes the query reformulation for $\mathcal{D}$ using the summary $\mathcal{S}$ of $\mathcal{D}$.

Table 3 (bottom) shows the *optimization ratio* per query obtained with $\mathcal{L}_R$/S on LUBM100M, i.e., the percentage of CQs with no answers on LUBM100M that are identified and removed by Ω using LUBM100M's summary; the ratio is 0% with $\mathcal{L}_R$/REF and 100% with $\mathcal{L}_R$/DB. We observe that optimization ratios are high in general, 92% on average with 52.53% the lowest value (*QA*6), thus our summaries are effective to identify CQs with no answers. Similar results are obtained on LUBM1M, LUBM10M, LUBM50M and LUBM150M.

We analyze below the times we measured when our queries are processed with the above-mentioned strategies. The measured time is: optimization time + evaluation time. Each reported time is an average over 5 "hot" query runs, i.e., the first "cold" query run is discarded. For space consideration, we focus on the times measured for LUBM10M and the ten times larger LUBM100M, which are shown in Figure 2. Measured times for all our databases are in Appendix D; they gradually increase as data size grows from 1M to 150M facts.

$\mathcal{L}_R$/S ***versus the state-of-the-art baseline*** $\mathcal{L}_R$/REF**.** We observe that when query reformulations are optimized by Ω using $\mathcal{S}$:

- Performance almost always improves for UCQs (UCQ/S for all the databases except for *QA*6), often significantly and up to 3 orders of magnitude (e.g., UCQ/S for *QA*1 on LUBM10M and LUBM100M).
- Performance frequently improves for JUCQs (in half of the cases overall), up to one order of magnitude (e.g., JUCQ/S for *QA*8 on LUBM100M), otherwise performance is marginally affected. We remark that when the performance visibly degrades (e.g., *QA*9 on LUBM100M) it is just in the order of a few tens of ms.
- Performance is marginally affected for USCQs.

These observations are explained with the two following facts, and the optimization ratios obtained with our summaries (Table 3).
(1) Optimizing reformulations with Ω removes CQs with no answer from the top union in UCQs and from the unions on which the top join is performed in JUCQs; in USCQs, single-atom CQs are removed from unions on top of which joins are performed, on top of which the top union is performed.
(2) Removing CQs with no answer from a union improves its evaluation time (as it may take time for an RDBMS to find out that a CQ has no answer), while it does not change the size of its output hence the number of tuples to process after this union.

Therefore:

- Optimizing a UCQ reformulation with Ω speeds up its entire evaluation since Ω optimizes its top union. Also, because our summaries allow high optimization ratios for UCQ/S, performance is significantly improved in general. We remark that performance degrades for *QA*6 because the optimization time does not amortize with a low optimization ratio (52.53% on LUBM100M).
- Optimizing a JUCQ reformulation with Ω speeds up the evaluation of its sub-UCQs but does not affect the evaluation time of the top join (as the same tuples must be joined). JUCQ reformulations are thus more difficult to optimize than UCQ ones. This is why performance is "only" frequently improved (in half of the cases) and marginally affected otherwise, even with high optimization ratios for JUCQ/S (> 78% on LUBM100M).
- Optimizing a USCQ reformulation with Ω only removes atomic CQs from its inner unions while it does not take time for an RDBMS to figure out that these atomic CQs are empty. The optimization thus marginally affects the evaluation time of these inner unions, and the evaluation time of the subsequent joins and top union is not affected. USCQ reformulations are thus more difficult to optimize than UCQ and JUCQ ones. This is why performance is marginally affected in general, even with maximal optimization ratios for USCQ/S (100% on LUBM100M).

$\mathcal{L}_R$/DB ***versus*** $\mathcal{L}_R$/REF ***and*** $\mathcal{L}_R$/S**.** We observe that when Ω optimizes query reformulations using $\mathcal{D}$ instead of $\mathcal{S}$, performance may improve or degrade:

- Performance is overall marginally to significantly better with UCQ/DB than with the baseline UCQ/REF, although the performance with UCQ/DB is generally worse than with UCQ/S.
- Performance with JUCQ/DB is always worse than with the baseline JUCQ/REF and almost always worse than with JUCQ/S.
- Performance with USCQ/DB is always worse than with the baseline USCQ/REF and with USCQ/S.

These observations are explained by the extra-time spent by $\mathcal{L}_R$/DB w.r.t. $\mathcal{L}_R$/S in completely optimizing a query reformulation using the database (recall that optimization ratios are of 100% for $\mathcal{L}_R$/DB): optimization time with $\mathcal{L}_R$/DB is in general significantly higher than with $\mathcal{L}_R$/S, because a database is much larger than its summary, while at the same time $\mathcal{L}_R$/DB provides a moderate gain in optimization ratios because they are already very high with $\mathcal{L}_R$/S in general. This is why $\mathcal{L}_R$/DB performs worse than $\mathcal{L}_R$/S overall, and worse than $\mathcal{L}_R$/REF when optimization time is higher than the time saved when the optimized reformulation is evaluated.

Table 3: Characteristics of the queries, $\mathcal{K} = (\mathcal{O}, \mathcal{D})$ with $\mathcal{O}$ = LUBM$^{\exists}_{20}$ and $\mathcal{D}$ = LUBM100M

	Query answering									
Query	QA0	QA1	QA2	QA3	QA4	QA5	QA6	QA7	QA8	QA9
#atoms	8	5	5	6	6	8	8	8	6	8
#contained CQs w.r.t. $\mathcal{O}$	2,759	1,949	1,701	1,151	719	495	299	183	143	31
#answers in $ans(q, \mathcal{K}) = eval(q^{\mathcal{O}}, \mathcal{D})$	23,946	0	347,527	720	69	0	2	858,259	12	0
optimization ratio for UCQ/S	81.92	100	99.88	83.80	80.2	80	52.53	78.89	66.91	77.42
optimization ratio for USCQ/S	100	100	100	100	100	100	100	100	100	100
optimization ratio for JUCQ/S	100	100	100	100	100	100	100	78.89	100	100

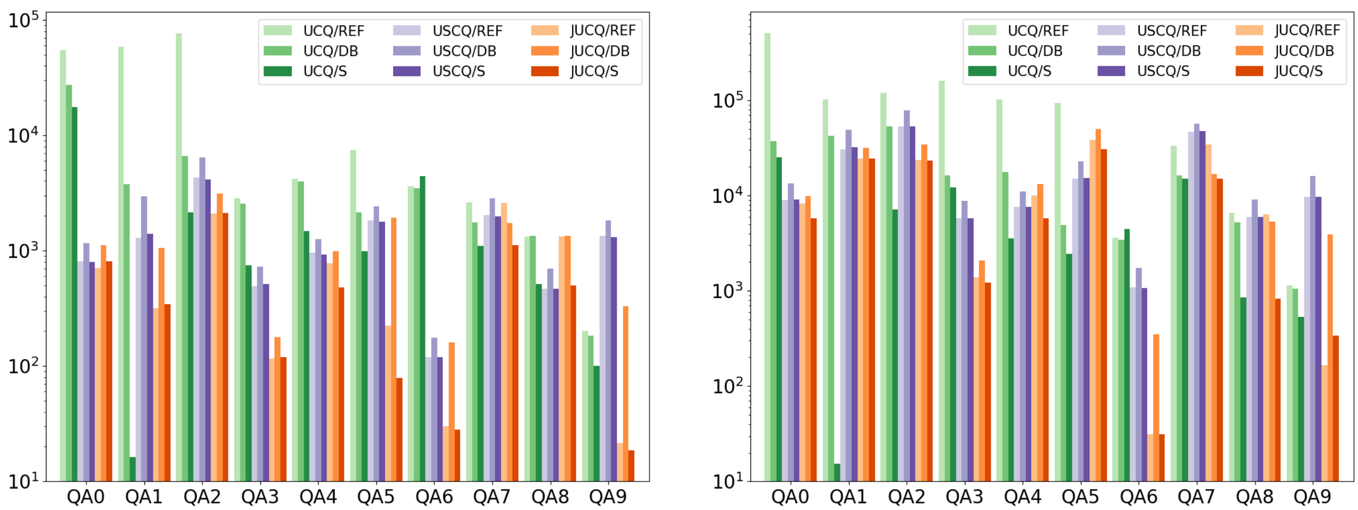

Figure 2: Query answering times (ms, logscale) with PostgreSQL on LUBM10M (left) and LUBM100M (right)

Experiment conclusion. Our summaries can be fast to compute (linear in data size), small (<10% of data size), and effective to identify CQs with no answer on a database (92% on average). Also, when they are used with our Ω optimization function, OMQA time performance can be significantly improved for UCQ reformulations in general and frequently for JUCQ reformulations, while performance is marginally affected for USCQ ones.

6 RELATED WORK AND CONCLUSION

We devised a novel optimization framework for OMQA via FO-rewriting. It is complementary to, and capitalizes on, the optimizations that have been proposed so far in the literature, e.g., [10, 11, 20, 33, 39, 51], which are both ontology-dependent and data-independent. Its novelty is to add a complementary data-dependent optimization step to query reformulations produced by state-of-the-art FO-rewriting tools, e.g., [6, 10, 11, 20, 45, 47, 51–53]. This framework is general enough to apply to a variety of FO-rewriting settings, in particular those in Table 1, and it guarantees the correctness of OMQA on the queried KBs. For the FO-rewriting settings in which it was evaluated, it significantly improves OMQA time performance for the widely-adopted UCQ query reformulations, e.g., [9, 16, 20, 30–33, 39, 44, 45, 47, 52], and for the JUCQ ones of [10, 11]. The originality of our framework is to build on the Ω optimization function that rewrites a query reformulation into a contained one, by pruning away subqueries that are useless to its evaluation on a given database. Notably, useless subqueries are identified rapidly by using database summaries, which we devised for this purpose by adapting the quotient operation [34] to databases.

Database summaries, in particular those based on the quotient operation, have been mainly investigated for graph databases, e.g., [17, 41], and description logic databases [22, 23, 25, 27], for the purpose of data exploration and of data management optimization. To the best of our knowledge, summaries have not been used for the optimization of OMQA via FO-rewriting. We adapted the quotient operation to relational databases and we defined the new equivalence relation $\equiv_\Omega$ for the special task of sound and fast identification of CQs with no answer on a database. $\equiv_\Omega$ departs from prior equivalence relations by being based on the instances of concepts that KB's databases describe with n-ary relationships between them, and not on bisimulation [36], e.g., [26, 43], or cooccurrence of relationships [28, 29]. A perspective is to study other database summaries for our framework, to improve further OMQA time performance, either via the quotient operation and other equivalence relations than $\equiv_\Omega$, or with other procedures than the quotient operation.

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A MAIN FO QUERY LANGUAGES USED FOR FO-REWRITING

Language	First-Order Logic syntax	Relational algebra syntax
CQ	$q(\bar{x}) = \exists\bar{y} \bigwedge_{i=1}^{n} atom_i$	$q(\bar{x}) = \Pi_{\bar{x}}(\bowtie_{i=1}^{n} atom_i)$
UCQ	$q(\bar{x}) = \bigvee_{i=1}^{n} CQ_i$	$q(\bar{x}) = \bigcup_{i=1}^{n} CQ_i$
JUCQ	$q(\bar{x}) = \bigwedge_{i=1}^{n} UCQ_i$	$q(\bar{x}) = \Pi_{\bar{x}}(\bowtie_{i=1}^{n} UCQ_i)$
SCQ	$q(\bar{x}) = \exists\bar{y} \bigwedge_{i=1}^{n} \bigvee_{j=1}^{m_i} atom_i^j$	$q(\bar{x}) = \Pi_{\bar{x}}(\bowtie_{i=1}^{n} \bigcup_{j=1}^{m_i} atom_i^j)$
USCQ	$q(\bar{x}) = \bigvee_{i=1}^{n} SCQ_i$	$q(\bar{x}) = \bigcup_{i=1}^{n} SCQ_i$

B EXISTENTIAL VARIABLES IN RULES

An existential rule $\forall\bar{x}(q_1(\bar{x}) \to q_2(\bar{x}))$ corresponds to an FO formula $\forall\bar{x}(\exists\bar{y} \wedge_{i=1}^{m} a_i \to \exists\bar{z} \wedge_{j=1}^{n} b_j)$ and:
$\forall\bar{x}(\exists\bar{y} \wedge_{i=1}^{m} a_i \to \exists\bar{z} \wedge_{j=1}^{n} b_j) \Leftrightarrow \forall\bar{x}(\neg(\exists\bar{y} \wedge_{i=1}^{m} a_i) \vee \exists\bar{z} \wedge_{j=1}^{n} b_j) \Leftrightarrow$
$\forall\bar{x}(\forall\bar{y}\neg(\wedge_{i=1}^{m} a_i) \vee \exists\bar{z} \wedge_{j=1}^{n} b_j) \Leftrightarrow \forall\bar{x}\forall\bar{y}(\wedge_{i=1}^{m} a_i \to \exists\bar{z} \wedge_{j=1}^{n} b_j)$

C PROOFS

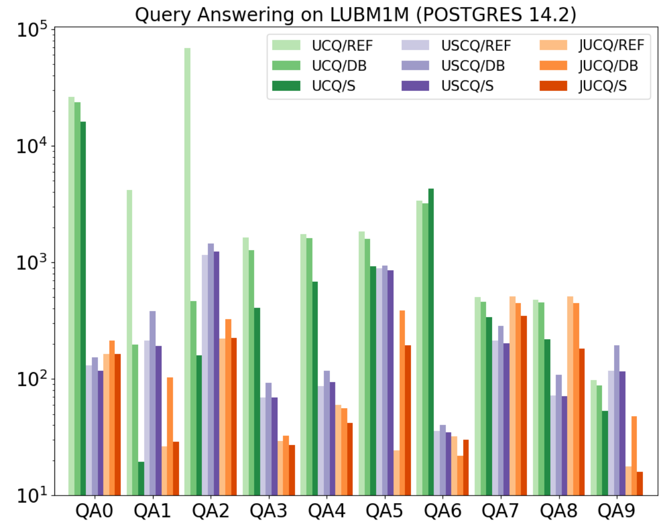
PROOF OF THEOREM 4.2. We prove the theorem by showing that its contrapositive holds, i.e., *if* q *has some answer on* $\mathcal{D}$*, then* q_σ *has some answer on* $\mathcal{S}$. If q has an answer on $\mathcal{D}$, then there exists a homomorphism h from q to $\mathcal{D}$ such that $h(q) \subseteq \mathcal{D}$ where every free variable is mapped to a constant, every existential variable is mapped to a constant or variable, and every constant is mapped to itself. Moreover, the composition $\sigma \circ h$ is a homomorphism from q to $\mathcal{S}$ such that $\sigma \circ h(q) \subseteq \mathcal{S}$ where, by definition of a database summary, every free variable is mapped to a constant, every existential variable is mapped to a constant or variable, and every constant is mapped to its image through σ. Let us now build a homomorphism g from q_σ to $\mathcal{S}$ such that $g(q_\sigma) = \sigma \circ h(q) \subseteq \mathcal{S}$: it suffices that g maps every variable exactly as $\sigma \circ h$ does, while it maps every constant to itself (constants have already been replaced by their image through σ in q_σ). Since defined this way g maps free variables to constants, q_σ has an answer on $\mathcal{S}$. ◻

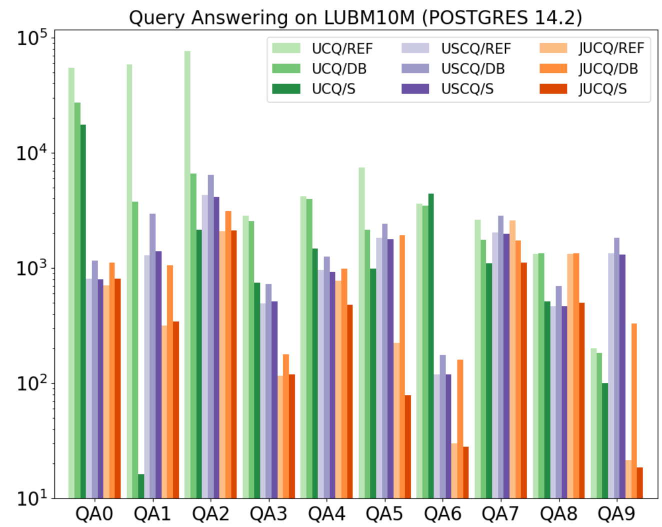
Proof of Theorem 4.5. Let us first prove $\Omega(q, \mathcal{S}) \subseteq q$. We prove this by induction on the depth d of q defined as the maximal nesting of the $\wedge$ and $\vee$ operators on top of CQs, with the *induction hypothesis* that Ω performs rewritings (rules (1), (2) and (3) in Definition 4.4) that are contained in the rewritten query. *Base case*, $d = 0$: rule (1) rewrites q either by (second case) itself or by (first case) $\emptyset$, and clearly, q is contained in itself and $\emptyset$ is contained in q. *Induction step*, $d > 0$: rule (2) rewrites a conjunction either by (second case) a contained one (induction) or by (first case) $\emptyset$ that is by definition contained in the rewritten conjunction; rule (3) rewrites a disjunction either by (second case) a contained one (induction), or by $\emptyset$ (first case) that is by definition contained in the rewritten disjunction.

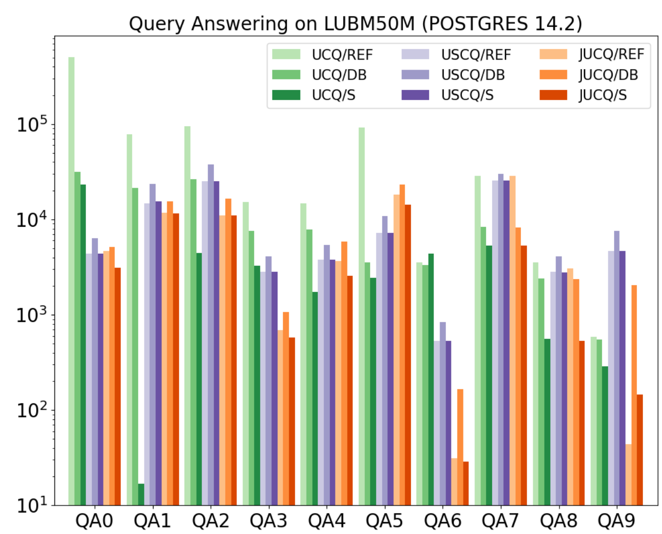
Let us now prove that $eval(q, \mathcal{D}) = eval(\Omega(q, \mathcal{S}), \mathcal{D})$. Again, we prove this by induction on the depth d of q defined as the maximal nesting of $\wedge$ and $\vee$ operators on top of CQs, with the *induction hypothesis* that Ω performs rewritings (rules (1), (2) and (3) in Definition 4.4) that are equivalent w.r.t. the database $\mathcal{D}$. *Base case*, $d = 0$: rule (1) rewrites q either by (second case) itself or by (first case) $\emptyset$ if q has no answer on $\mathcal{S}$, hence on $\mathcal{D}$ according to Theorem 4.2, i.e., q is equivalent to $\emptyset$ on $\mathcal{D}$. *Induction step*, $d > 0$: rule (2) rewrites a conjunction either by (second case) an equivalent one (induction) or by (first case) $\emptyset$ if a q_i subquery has no answer on $\mathcal{D}$ (induction), hence the conjunction is equivalent to $\emptyset$ on $\mathcal{D}$; rule (3) rewrites a disjunction either by (second case) an equivalent one (induction), or by $\emptyset$ (first case) if all its subqueries have no answer on $\mathcal{D}$, hence the disjunction is equivalent to $\emptyset$ on $\mathcal{D}$. ◻

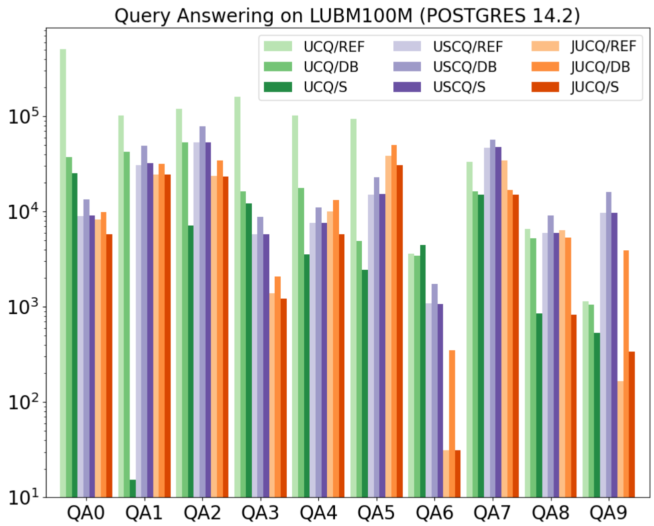
D EXPERIMENTAL RESULTS

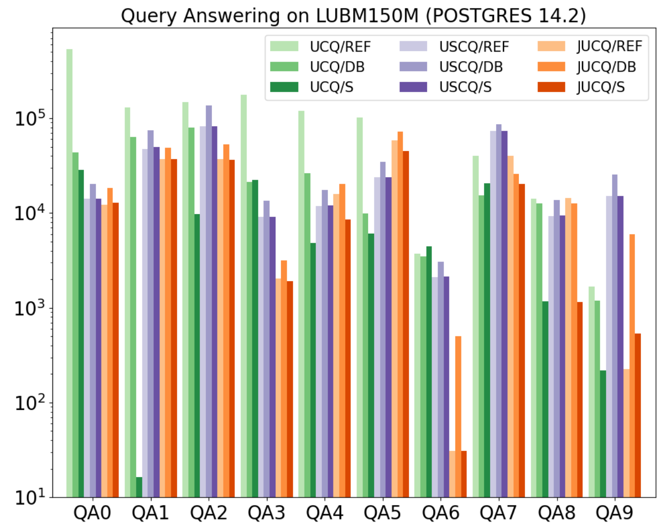
Query answering times (ms, logscale) with PostgeSQL on all our LUBM databases.








Query Answering on LUBM100M (POSTGRES 14.2)
UCQ/REF
UCQ/DB
UCQ/S
USCQ/REF
USCQ/DB
USCQ/S
JUCQ/REF
JUCQ/DB
JUCQ/S
10^1
10^2
10^3
10^4
10^5
QA0
QA1
QA2
QA3
QA4
QA5
QA6
QA7
QA8
QA9


Query Answering on LUBM150M (POSTGRES 14.2)
UCQ/REF
UCQ/DB
UCQ/S
USCQ/REF
USCQ/DB
USCQ/S
JUCQ/REF
JUCQ/DB
JUCQ/S
10^1
10^2
10^3
10^4
10^5
QA0
QA1
QA2
QA3
QA4
QA5
QA6
QA7
QA8
QA9