Racial Segmentation in the US Housing Market

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Abstract

This paper studies racial segmentation in the US housing market since 1960. I document large differences in housing outcomes for Black and White households. In 1960, Black households on average are 20 percentage points less likely to own a house (relative to White households with the same income); if they owned, their house values are lower by the equivalent of almost one year of annual income; and even when renting they spend less by the equivalent of one month of rental expenditures. By 2019, the rent and price gaps have declined by about half, whereas the gap in ownership rates has not changed. To interpret these facts, I use a dynamic housing assignment model with a choice to buy or rent housing. I estimate the degree of market segmentation by inferring differences in the quality of housing available to Black and White households, and the resulting differences in rents, prices, and the cost of owning a home. The model infers that Black households pay higher quality-adjusted rents and prices, especially at higher qualities, and thus sort into lower quality homes. In terms of lifetime consumption-equivalent welfare, relative to an integrated market, the average Black household is five percent worse off in 1960 and remains one percent worse off in 2019.

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1 Introduction

> *"A Realtor should never be instrumental in introducing into a neighborhood ... members of any race or nationality ... whose presence will clearly be detrimental to property values in that neighborhood."*

Code of Ethics, Article 34
National Association of Real Estate Boards (1924–1949)

Racial disparities in the US housing market are both large and persistent. And there is an extensive literature documenting how various barriers can distort the choices of Black Americans in this market. Yet, we lack an estimate of how much these disparities matter for the overall economic welfare of Black households and how much their welfare has changed over time. Nor do we know how much barriers faced by subsets of Black households, such as richer Black households, spill over onto others through equilibrium forces. Ultimately, we can think of such barriers and their equilibrium spillovers as resulting in markets that are segmented by race, and in which Black households pay different prices and thus choose different houses.

This paper quantifies the welfare consequences of racial segmentation in the housing market from 1960 to 2019. Using microdata on housing choices —what house to rent, what house to buy, and whether to buy or rent— I document large differences between Black and White households. I estimate a dynamic equilibrium model of the housing market, allowing for market segmentation by race, and use the model to map the differences in housing choices into differences in economic welfare. The main finding of the paper is that Black households pay higher quality-adjusted rents and prices, especially at higher qualities, which cause them to sort into lower quality homes. Relative to an integrated market, where Black and White households face the same rents and prices, the average Black household in 1960 is five percent worse off in terms of lifetime consumption equivalent welfare. The average White household is slightly better off (one half of a percent). Between 1960 and 2019, the welfare gaps have narrowed by four–fifths.

I begin by documenting differences in housing choices by race. In 1960, Black households are 20 percentage points less likely to own a house (relative to White households with the same income); if they owned, their house values are lower by the equivalent of almost one year of annual income; and even when renting they spend equivalent to one month less in rental expenditures. By 2019, the rent and price gaps have declined by about half, whereas the gap in ownership rates has not changed. The gaps are not explained by systemic differences in household composition or the broad location in which households live. Of course, rural households or households with bigger families may have different preferences for housing, and Black and White households differ on these and other dimensions. However, the gaps remain large when looking within individual metropolitan areas and while controlling for household characteristics such as its size and the age of its head.

The estimation results show that markets are indeed segmented by race. The composition of housing quality is worse in the segment occupied by Black households, and the resulting equilibrium rents and prices are higher than in the White segment. The gaps in rents are largest for the highest quality homes: up to 70 percent in 1960 and 15 percent in 2019. At any given quality, the gap in prices is similar to the gap in rents, suggesting that segmentation by race affects rental and owner-occupied markets to the same degree. Black households would thus gain from integrating markets, especially higher income Black households who would move into higher quality homes. Lower income Black households would remain in similar quality homes, but would also gain from lower prices due to less competition from richer Black households. White households would lose out from integration, but only marginally so, because Black households make up a small share of the total market.

I estimate a model of housing demand with data on Black and White households separately. Households' demand depends on their income, age and wealth. They choose the quality of their house and whether to buy or rent it, as well as how much to save or to spend on non-housing consumption. The estimation targets data on rental expenditures, house prices, and home ownership rates for these heterogeneous households. Under the assumption that Black and White households have the same preferences, the model implies a schedule of quality-adjusted rents and prices that best rationalizes the data. We can thus compare the estimated rents and prices for Black and White households to determine whether their markets are segmented or not.

The model includes two core features that allow me to match and interpret the data on rents, prices, and home ownership across the income distribution. Firstly, houses are indivisible and differ in quality (Sweeney, 1974a,b; Braid, 1981). Quality captures all the characteristics of the house that households value, including its physical characteristics (such as the number of bedrooms) and its location based amenities (such as the quality of schools or crime in the neighborhood). In equilibrium, higher quality homes will be more expensive because richer households will be willing to spend more on them.

Secondly, households choose whether to own or rent housing as part of a life-cycle consumption and savings decision. Households save for retirement and their portfolio choice is impacted by the relative return on housing, credit constraints, and an idiosyncratic desire to own or rent. The combination of these features results in a model that involves multidimensional matching of households to indivisible houses, where households differ in their age, income and wealth, and houses differ in their quality and whether they are available to buy or to rent. The resulting matching of households to houses determines equilibrium rents and prices.

The latent house quality is identified from the optimality conditions of households. Households prefer their own house at given equilibrium prices. The quality of their house is therefore the utility that explains the observed relationship between households and prices in the data. In a simple version of the model we can directly infer quality from the growth of rental expenditures relative to the growth of incomes. In the quantitative model, quality is an index that satisfies the optimality conditions of households that differ in multiple dimensions. In this case, quality in the rental market is primarily identified by how much rental expenditures vary across households, and likewise quality in the owner-occupied market is identified by variation in house values.

I allow for two other distortions, in addition to differences in quality, in each market segment. Firstly, discrimination may affect the attractiveness of owning a home relative to renting, for example, when houses are vandalized, when discrimination by a mortgage appraiser reduces the selling price, or when there is a higher interest rate on mortgages. I model this as a cost of home ownership that lowers the return on housing. This cost most directly maps to the higher property tax burdens identified by Avenancio-León and Howard (2022), but can also be seen as an indirect proxy for all these factors that make owning less attractive. Secondly, I introduce differences in the return on savings that depend on race, which is consistent with the overall racial gap in wealth and evidence of barriers to wealth accumulation (Boerma and Karabarbounis, 2022; Derenoncourt, Kim, Kuhn and Schularick, 2022).

I develop a test to evaluate the degree to which housing markets are racially segmented. Segmentation means that Black and White households may have different choice sets, and the resulting equilibrium outcomes —quality-adjusted prices and allocations of quality— may differ in each segment. The test thus consists of estimating the model separately with Black and White data and comparing the equilibrium outcomes in each market segment. When markets are not segmented, I will find the same quality-adjusted prices and allocations of quality. When they are segmented, I quantify the degree of segmentation in terms of differences in quality-adjusted prices, allocations of quality, and the resulting differences in economic welfare.

We can interpret the segmented equilibrium as arising either from (i) restrictions on the distribution of quality in each segment which lead to differences in equilibrium prices; or (ii) barriers that distort the effective price of some parts of the quality spectrum, and thus lead households to sort into different parts of the distribution of quality. Both interpretations are equivalent, because, in equilibrium, prices are consistent with demand. Thus, the model can be applied to data from before the Fair Housing Act 1968, when there were explicit restrictions on Black households, as well as in the more recent times when less explicit barriers may continue to distort Black household choices. Barriers to accessing housing may depend on the quality of the home (downtown apartments versus suburban single-family homes) and whether it is rented or owned (discrimination by landlords against renters versus realtors against buyers). The model captures this heterogeneity by estimating gaps in both rents and prices across the quality distribution.

To explain the differences in the data, the model finds that the Black and White segments differ substantially: the houses available to Black households were worse in quality and this resulted in higher equilibrium prices for any given quality. Black households have lower expenditure shares on housing because, despite paying higher prices for a given quality, the associated sorting into lower quality homes results in them spending less than equivalent White households. For example, in 1960 a young, median income Black household spends about $4,700 annually in rent compared to $6,100 for a similar income White household. The model finds that this $1,400 difference in expenditures arises because the Black household is renting a house that is $1600 lower in quality (evaluated at prices in the White segment) but they pay 4 percent more for that house.

Comparing price differences on the homes chosen in equilibrium underestimates the true differences in prices, because Black households substitute away from the qualities with the largest price gaps. In 1960, the average Black household pays 3 percent more than White households in the same quality homes. However, they would pay 18 percent more if they chose the same quality homes that they choose in the integrated market. In 2019, only 3 percent of Black households live in homes where the price gap is over 10 percent, though more than twice as many (7 percent) would live in these homes in the absence of the price gaps. This implies that econometric studies that focus only on the choices that are made in equilibrium may substantially underestimate the price gaps faced by Black households.

In contrast to the convergence in quality-adjusted prices, I find that the cost of owning a home remains almost as high in 2019 as it was in 1960. To account for the lower Black ownership rate, which has changed little since 1960, the model infers that Black household pay an annual cost of home ownership equivalent to 3 percent of the house price in 1960 and 2.5 percent in 2019. Ultimately, the model infers that the effective return on housing remains lower for Black households and thus they are less likely to choose to own.

To evaluate welfare, I compare outcomes relative to an integrated equilibrium where Black and White households pay the same rents and prices, which arise from matching the integrated market's supply and demand at every quality. Rents in the integrated market are up to 70 percent lower than in the Black market. However, since Black households are a small fraction of the total population, rents in the integrated market are not very different to those in the White market (up to 5 percent more expensive).

The consumption equivalent welfare loss from market segmentation is similarly larger for Black households — by ten times — relative to the welfare gain for White households: a 5 percent welfare loss versus half a percent welfare gain. The welfare losses are largest for the highest income Black households, who otherwise would sort into much better quality homes, and for the lowest income Black households, who pay higher rents due to more competition for lower quality homes. Low- to middle-income White households benefit the most from segmentation, because their housing demand would otherwise overlap most with Black households. The model infers substantial convergence in quality since 1960, as measured by the convergence in welfare. In 2019, the average welfare gap has narrowed from 5 percent to 1 percent. Yet, the welfare loss remains large for the richest quintile of Black households (2 percent).

In addition to the model's estimated measure of quality, I document that Black households live in homes that have worse observable characteristics, such older homes with fewer bedrooms. In 1960, the average Black household lived in a home that was 3.9 years older than the home of similar White households. This was almost two-thirds of the difference between the richest and poorest White households (6 years), suggesting that it was a meaningful difference in quality. The gap remains in recent data, though, consistent with the model estimates, it has narrowed to 10 percent of the gap between richest and poorest White households.

An important assumption in this analysis is that Black and White households substitute between housing and other consumption in the same way. However, this does not mean that Black and White households have identical preferences for specific types of homes. Indeed, I show that my results are robust to allowing for homophilly (i.e. a desire to live with others of the same race). Allowing for differences in substitution would only partially reconcile the data. For instance, while a lower preference for housing could partially explain the gaps in a single period, it would not be able to explain the gaps across the income distribution nor the convergence in outcomes over time. Relatedly, alternative mechanisms for explaining racial differences in consumption, such as status-signaling motives for visible spending (Charles, Hurst and Roussanov, 2009), predict that Black households would spend more on housing, not less.[1] Given the historical context that discrimination potentially segmented Black and White housing markets, and that there are few ex-ante reasons to expect time-varying differences in preferences by race, I take the approach that preferences are the same. With this assumption, I can estimate the differences in rent and price schedules that are implied by the differences in choices in the data.

Relationship to the literature. This paper contributes to four broad literatures. The first is a large literature studying racial differences in the housing market. Muth (1969) and Kain and Quigley (1975) are early examples of using expenditures shares to estimate racial differences in rents. Others have focused on documenting and explaining the large differences in ownership rates (Gyourko and Linneman, 1996; Gyourko et al., 1999; Charles and Hurst, 2002; Collins and Margo, 2011). Many papers have examined the causes of geographic racial segregation, including theoretical models of segregation (Schelling, 1971, 1978), measurement of indexes of segregation over time (Cutler, Glaeser and Vigdor, 1999; Logan and Parman, 2017), and estimation of tipping

[1] Charles, Hurst and Roussanov (2009) show that Black households spend more on conspicuous goods conditional on permanent income. I show that the housing gaps documented in this paper also hold conditional permanent income, both empirically using the methodology of Charles, Hurst and Roussanov (2009) and in a model with estimated lifecycle income processes.

points (Card, Mas and Rothstein, 2008) and of White flight (Boustan, 2010).[2] However, it is hard to map some of these outcomes —such as changes in the index of isolation or quantities of White flight — into economic welfare, especially when they interact with each other. Indeed, as King and Mieszkowski (1971) noted, the presence of segregation does not directly imply that the price of housing would differ for Black households unless there are also differences in the supply of housing in Black neighborhoods. My paper complements these prior approaches by examining the degree to which all three housing choices —what house to buy, what house to rent, and whether to own— are distorted by geographic segregation or by other barriers.[3] Furthermore, I map these distortions into differences in economic welfare across the income distribution.[4] By using data since 1960, my paper can track the extent of convergence in these outcomes and welfare since the Fair Housing Act 1968.

Secondly, I contribute to a literature on racial differences in incomes (Margo, 2016; Bayer and Charles, 2018; Chetty, Hendren, Jones and Porter, 2020), wealth (Blau and Graham, 1989; Kuhn, Schularick and Steins, 2020; Derenoncourt, Kim, Kuhn and Schularick, 2022), and welfare (Brouillette, Jones and Klenow, 2022) by quantifying the importance of gaps in the housing market for wealth gaps and welfare. Related papers have quantified the role of differences in returns and learning (Boerma and Karabarbounis, 2022) and income differences (Aliprantis, Carroll and Young, 2021). With respect to housing markets, Kahn (2021) and Kermani and Wong (2021) both empirically document lower returns on housing for Black households in recent decades, with the latter documenting that this primarily comes from distressed home sales (i.e. foreclosures and short sales). Gupta, Hansman and Mabille (2022) document that Black households tend to have higher leverage and argue that mortgage leverage restrictions could thus preclude them from moving to locations with better income opportunities. In contrast, this paper contributes by taking a long run general equilibrium approach to explain racial differences in portfolios across the income distribution. In particular, I can decompose how portfolios are effected both by distortions in relative prices of housing quality as well as the cost of home ownership.

Thirdly, I contribute to assignment models of housing markets both with a novel application to racial differences and by extending the model to include both rental and ownership markets. Early theoretical contributions were made by Sweeney (1974a,b), and Braid (1981).[5] More recently,

[2] See also Bayer, Fang and McMillan (2014) and Aliprantis, Carroll and Young (2022).

[3] Bayer, Charles and Park (2021) use 2014–2018 sample of the American Community Survey to show that Black households, relative to White households with the same income, live in neighborhoods where the median income is lower. They argue that lower neighborhood incomes reflect that these are lower quality neighborhoods.

[4] Darity, Mullen and Slaughter (2022) discuss the challenges of estimating the aggregate costs of discrimination, both for the bottom up approach of enumerating the costs of specific atrocities (such as killings or stolen property) and the top-down approach of estimating the cost of closing aggregate differences (such as the racial wealth gap). Darity and Mullen (2022) discuss the case for reparations in more detail.

[5] Arnott (1987) provides a summary of these and related theoretical contributions.

quantitative assignment models have been used to study the impact of income inequality on house prices (Maattanen and Tervio, 2014), housing markets with buyer restrictions (Landvoigt, Piazzesi and Schneider, 2014), house prices booms due to changes in credit constraints (Landvoigt, Piazzesi and Schneider, 2015), changes in housing supply (Nathanson, 2020; Wang, 2022), changes in interest rates (Hacamo, 2021), and the welfare effects of transaction taxes (Maattanen and Tervio, 2021) and of rental market evictions (Abramson, 2022).[6] Epple, Quintero and Sieg (2020) also propose a GMM based methodology to estimate such models. This paper is the first to integrate the standard components of a buy-rent decision (i.e. within a life-cycle problem with mortgages) with an equilibrium assignment model of both rental and ownership markets.[7] It also contributes a novel approach to inferring the prices and welfare effects of discrimination in the housing market by developing a test of segmentation using differences in the sorting of households by race.

Finally, this paper is similar in spirit to the literature on misallocation (Restuccia and Rogerson, 2008; Hsieh and Klenow, 2009) and related applications to the allocation of talent by gender and racial groups (Hsieh, Hurst, Jones and Klenow, 2019), and to spatial misallocation arising from zoning restrictions (Hsieh and Moretti, 2019; Babalievsky, Herkenhoff, Ohanian and Prescott, 2021), and to rent control (Glaeser and Luttmer, 2003). The price differences in this paper can similarly be viewed as quality specific gaps that arise endogenously because markets are segmented and there are differences in the supply of quality in the Black and White segments.

Roadmap. Section 2 describes the historical context. Section 3 presents the main empirical facts. Section 4 explains how I identify quality and test for market segmentation using a simplified version of the model. Section 5 presents the quantitative model and section 6 discusses how it is estimated. The results are reported in section 7.

2 Background

This section provides a brief overview of the practices that could lead to segmented housing markets by race. The 20th century had numerous explicit attempts to limit the set of housing available to Black households, as illustrated by the National Association of Realtors (1924) Code of Ethics. A number of other practices potentially limited Black households' choices including violence against Black households moving into White neighborhoods, redlining, abuses of eminent domain and

[6]Relatedly Fogli and Guerrieri (2019) and Guerrieri, Hartley and Hurst (2013) use models with sorting across locations and externalities to explain rising income segregation and endogenous gentrification respectively. Bajari and Kahn (2005) and Bayer, Ferreira and McMillan (2007) estimate housing demand models with preferences over the racial composition of the neighborhood, with the latter using restricted Census data that include geographic location.

[7]For instance, while Landvoigt, Piazzesi and Schneider (2015)'s model includes a life-cycle problem, they restrict themselves to owners only and thus do not model rental markets. Standard macro-housing models, such as Kaplan, Mitman and Violante (2020), assume that housing is divisible and study a single rent and price per unit of housing.

restrictive covenants in deeds (Massey and Denton, 1998; Rothstein, 2017).[8] Some of the country's first zoning laws explicitly segregated by race, such as the 1910 ordinance in Baltimore (Shertzer, Twinam and Walsh, 2022). After the supreme court deemed these laws unconstitutional, there is evidence that zoning remained racially biased, for example, by being more likely to classify Black neighborhoods to industrial uses (Shertzer, Twinam and Walsh, 2016).

Since the Fair Housing Act of 1968, explicitly discriminatory practices are no longer legal. However, there is more recent evidence that Black households continue to face barriers in housing and mortgage markets. For example, audit studies continue to show that realtors steer Black households to worse neighborhoods (Yinger, 1986; Christensen and Timmins, 2022). Likewise there is evidence that Black households face differences in mortgage approval rates and pay higher mortgage costs (Munnell, Tootell, Browne and McEneaney, 1996; Charles and Hurst, 2002; Woodward and Hall, 2012; Aaronson, Hartley and Mazumder, 2021; Giacoletti, Heimer and Yu, 2022; Willen and Zhang, 2022). Indeed, Albright et al. (2021) also show that there are lasting effects from events prior to the Fair Housing Act, specifically that the 1921 Tulsa Race Massacre led to lower home ownership, not only of those directly effected but also in places most exposed to newspaper coverage.

Given this historical context and prior evidence, it is likely that Black households choices were distorted. However, it is not obvious what choices were most constrained nor which households were most impacted. For instance, one might think that discrimination from landlords would increase the likelihood of owning, unless barriers to accessing mortgages and or to owner-occupied neighborhoods were more binding. One might also think that richer Black households could use their resources to avoid discrimination, or unless they were precisely the group most constrained by limited access to higher quality neighborhoods. Likewise, it is not obvious that the Fair Housing Act would equalize access on all dimensions at the same time. Thus, the goal of this paper is to estimate what choices were distorted, which households were most affected, and how these distortions changed over time.

[8] Indeed, Arrow (1998) described his experience "Residential discrimination was of course also overt, enforced primarily through voluntary choice by sellers, but also by covenants attached to the land. About 1950, I looked into joining a cooperative housing development, the members of which were primarily Stanford faculty, known liberals. I expressed my dismay on finding a clause limiting non-white participation to 10 percent of the whole. I was assured that it was considered a radical and courageous act to set the proportion above zero and that there could be no mortgage financing if they went further". Arrow's account is supported by Rothstein (2017), which documents that the Federal Housing Authority refused to insure new property developments that did not include restricted racial covenants (including one such development near Stanford).

3 Data and Empirical Evidence

3.1 Data

To test for distortions in choices, I need data on housing outcomes — ownership, and rental expenditures or house value — for comparable Black and White households (i.e., with the same income). Census microdata includes this information and it spans both before and after the Fair Housing Act 1968.

To account for other elements of household savings, I supplement Census data with a version of the Survey of Consumer Finances that was extended backwards by Kuhn, Schularick, Steins (2019). This extended version is known as the SCF+ and includes all components of household balance sheets.

The primary years of comparison are 1960 and 2019 because these are the years in which I will also estimate the quantitative model. I estimate the model in these years because 1960 is the first decade in which both Census housing data and SCF+ wealth data overlap, and 2019 is the most recent available year. Where the empirical results do not require overlap in both Census and SCF+ datasets, I also report results from prior to 1960.

Definition of race. Since this paper focuses on differences between Black and non-Black households, I group households into one of these categories in each dataset — though for ease of exposition I use the terms non-Black and White interchangeably. For much of the time series it is a reasonable approximation to use White and non-Black interchangeably, because White households (including Hispanics) make up 99% of the non-Black group until 1960 and 94% over the entire time series. When using household data, I categorize the race of a household using the race of the household head, which is also a reasonable approximation since inter-racial marriages remain a small fraction of marriages for both groups.[9]

Census microdata. I construct the main dataset as follows. I use the 1-5% samples from IPUMS USA (Ruggles, Flood, Goeken, Schouweiler and Sobek, 2022) for the decennial Census in 1940 and 1960-2000 and the 1% samples of the American Community Survey in 2010 and 2019. There is no housing data for available in the 1950 Census.[10] Data are constructed at the household level, and I exclude households in group quarters. Where household variables are not provided by IPUMS I

[9] Fryer (2007) reports that interracial marriage accounts for 5% of Black marriages and 1% of White marriages in 2000.

[10] Rents, prices and ownership are not available in 1950 because the Schedules from the 1950 Census of Housing were not microfilmed and have been destroyed (Bureau of the Census, 1984). Incomes, rents and prices were first recorded in response to the Great Depression. Rents and prices are available in Census data for 1930 (and home ownership status from 1900), however since income is not available until 1940 I cannot report gaps conditional on income and thus do not use data prior to 1940.

construct them by summing across individuals (e.g. income) or by using the the household head (e.g. age, race). Estimates are weighted using households weights.

I use the most comprehensive measure of household income that is available, for example, only wage income is available in 1940, while social security and welfare is only included from 1970 onwards. As most of the analysis compares Black and White households within a given year, these changes will only affect the results to the extent that Black and White households have different compositions of income. Where possible I account for this with related control variables, for example, the farm status indicator will correlate with the missing farm income in 1940.

Ratios are particularity sensitive to outliers and thus are windorized at the 5th and 95th percentiles. Other variables are not windsorized as IMPUS already top codes many of these.

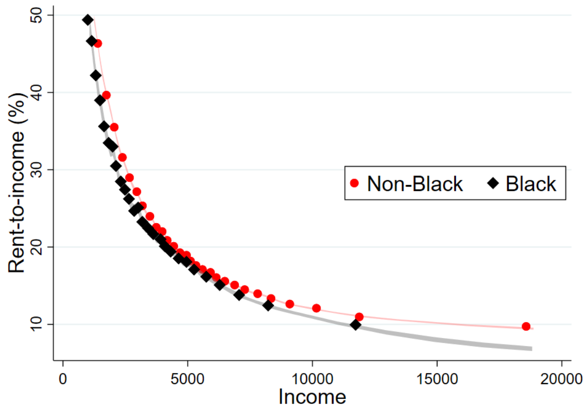
3.2 Main Fact: Racial Differences in Census microdata

In this section, I show that Black households have different housing outcomes relative to White households with the same incomes. In particular, I show that Black households (conditional on income) are less likely to own; when they own, they own houses of lower value; and even when they rent, they spend less on rental expenditures. While later sections will formally interpret these differences through the lens of a model, at this point the differences are initial evidence that Black decisions may be distorted, because standard economic models predict that, conditional on income, age, wealth and other characteristics, choices should be the same.

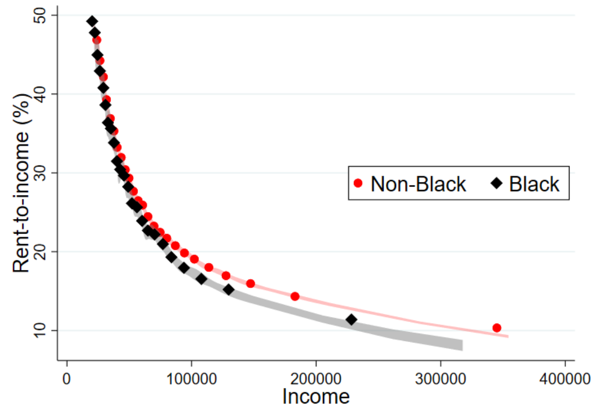
Figure 1 presents the main fact in 1960 and 2019. Panels (a) and (b) use binscatter (Cattaneo, Crump, Farrell and Feng, 2021) to show the relationship between income and the share of income spent on rental expenditures (conditional on the household renting). This relationship is downward sloping with higher income households spending a smaller share of their income on rental expenditures. Black households are poorer, thus they have a higher unconditional rent share (32 percent) relative to White households (26 percent). However, conditional on income, Black households spend less than White households at every income level. The average difference is 3 percentage points of income, which is approximately 10 percent of the unconditional share for Black households. In other words, the gap is equivalent to at least one month of rent. In 2019, the gap in rent-to-income (conditional on income) has narrowed to 1.3 percentage points.

Figure 1: Racial Differences in the Housing Market

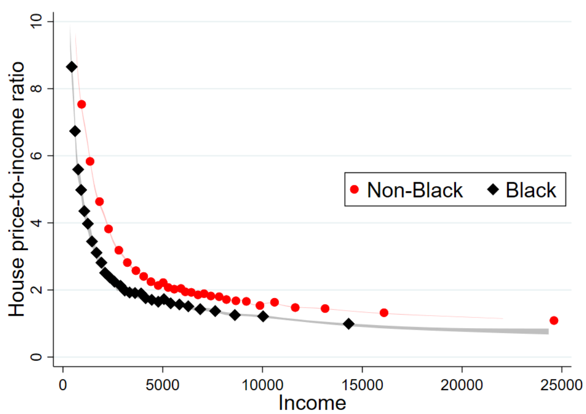
(a) Rent-to-income in 1960



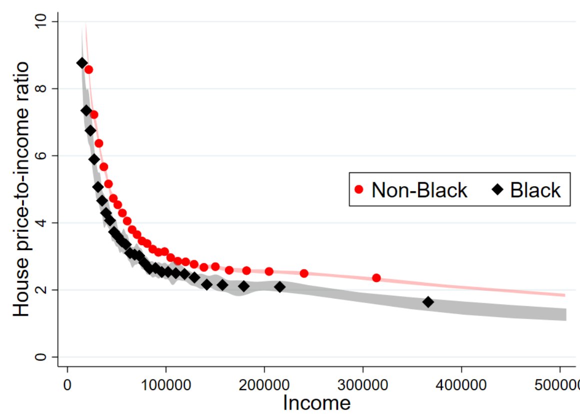
(b) Rent-to-income in 2019



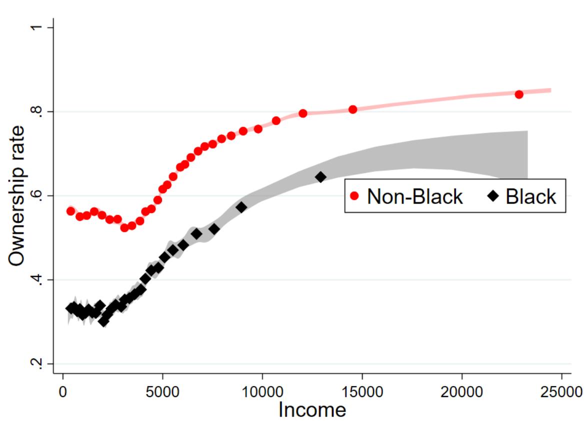
(c) Price-to-income in 1960



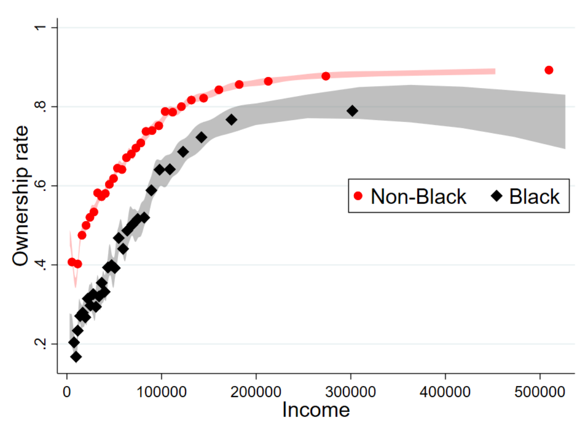
(d) Price-to-income in 2019



(e) Ownership rate in 1960



(f) Ownership rate in 1960



Notes: Top figures show the average rent-to-income share by income for Black (in black) and non-Black (red) households who rent in 1960 (left) and 2019 (right). Middle figures show the average price-to-income ratio by income for Black and non-Black households who own a home in 1960 (left) and 2019 (right). Bottom figures show the average home ownership rate by income for Black and non-Black households in 1960 (left) and 2019 (right). All plots use binscatter (Cattaneo, Crump, Farrell and Feng, 2021) without controls. Data are from IPUMS Census micro-samples. The x-axis (income) is truncated at the 1st and 99th percentiles.

Panels (c) and (d) show that a similar downward sloping relationship holds between the price-to-income ratio and income; lower income households have higher price-to-income ratios. Again, conditional on income, Black households have lower price-to-income ratios at every income. This gap was equivalent to 0.8 years of income on average for Black households in 1960. In 2019, the gap in the price-to-income ratio (conditional on income) has narrowed to 0.7 years of income. Since the average price-to-income ratio for White households has increased from 2.5 to 4 over the same time period, the relative gaps has narrowed by almost half.

In 1960, both gaps are largest for households in the lowest quintile, in which over 40 percent of Black households are. In 2019, the gaps, especially for the rent-to-income, have narrowed in the lowest income quintile but are still large at higher income quintiles. The rent-to-income share of Black households in the highest income quintile is 1.2 percentage points lower than White households (who spend 14 percent of their income on rents). Figure A2 reports these gaps by quintiles of the national income distribution.

Panels (e) and (f) show the relationship between owning and renting. Conditional on income, the average gap is 20 percentage points in 1960 and 19 percentage points in 2019. These are large gaps given that the White home ownership rate is 65 percent in 1960 and 68 percent in 2019.

To formally estimate the differences between Black and White households, and to compare the gaps across time and the income distribution, it is useful to reformulate the gaps into a linear regression. Figure 2 shows that the conditional expectation of a household's house price rank given their income rank is approximately linear. I therefore model household i's price rank as a linear function of their income rank allowing for differences in coefficients for Black and White groups

$$p_i \quad = \quad \alpha_W + \beta_W y_i + \alpha_B Black_i + \beta_B [Black_i \times y_i] + \mathbf{X}_i \Gamma + \epsilon_i, \tag{1}$$

where $p_i \in [0, 100)$ is the rank of household i in the overall price (or rent distribution), $y_i \in [0, 100)$ is the rank of the same household in the overall income distribution, and $Black_i$ is an indicator variable for the household's race. I also include a vector of controls $\mathbf{X}_i$, including the age, sex and education of the household head, the household type (couple, single), the household's location (metro area and state), and whether they live on a farm property. The intercept α_W estimates the mean house price rank of a White household in the lowest income rank ($p = 0$) and the slope β_W estimates the increase in the mean house price rank as the rank in the income distribution increases. The Black indicator and its interaction with the income rank estimate the difference in the intercept and slope coefficients for Black households. This specification builds on the use of ranks differences for estimating racial differences in incomes (Bayer and Charles, 2018) and intergenerational mobility (Chetty, Hendren, Jones and Porter, 2020). It is an attractive specification for comparing across time and space because it is insensitive to changes in the price-to-income ratios that do not affect the relative ranking of households.

The main coefficients of interest are α_W and β_B which together estimate the average gap in the price distribution at each point in the income distribution. With these coefficients I define the *rank gap* at percentile y as

$$\hat{p}_y^{gap} = \alpha_B + \beta_B \times y. \tag{2}$$

This rank gap is the estimated average difference in the price rank of Black households at the y-the rank in the income distribution relative to White households with the *same* income. I estimate this rank gap both for prices and rents. Since home ownership is a discrete choice, there is no continuous measure of the ownership rank. In this case I estimate equation (1) as a linear probability model with $p_i \in \{0, 1\}$ and refer to this gap as the ownership *rate* gap because it estimates the average gap in home ownership conditional on income.

Figure 2 presents the rank gap estimates. The right-hand panels — (b), (d) and (f)— plot the rank gaps across time for Black households with low (25th percentile), median (50th percentile) and high incomes (75th percentile). Bayer and Charles (2018) estimate that, throughout this period, the median Black households is between 22 and 30 ranks behind the median White household in the overall income distribution, thus the line representing the 25th percentile in Figure 2 is closest to the median Black household. The left-hand panels— (a), (c) and (e)— confirm that the relationship between income ranks and housing outcomes (in ranks or ownership rates) is indeed approximately linear in 1960. Figures A3, A4 and A5 present these plots for every year.

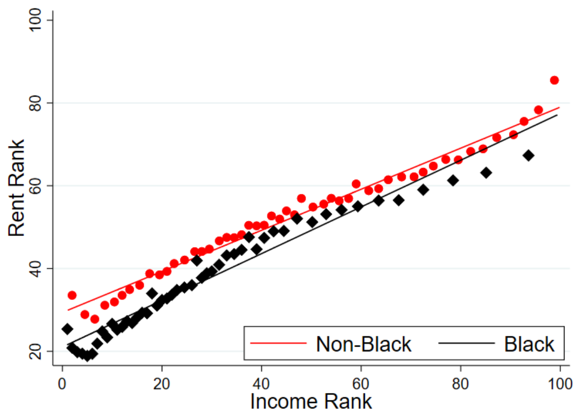
The right-hand side panels illustrate a number of patterns. Firstly, there has been a decline in both rent and house prices gaps over time. In 1960, Black households with the median overall income were behind similar income White households by 5 percentiles in the rent distribution and 11 percentiles in the price distribution. By 2019 these gaps have declined to 3.5 and 6.5 percentiles for rent and prices respectively.

Notably, the ownership gap has widened since 1960. At that time Black households were 12 percentage points less likely to own a home compared to similar income White households. However, by 2019 the ownership rate gap has increased to 13 percentage points and is growing wider. It is striking that while the ownership rate gap declined in the immediate aftermath of the the Fair Housing Act 1968, it has been widening since 1980.

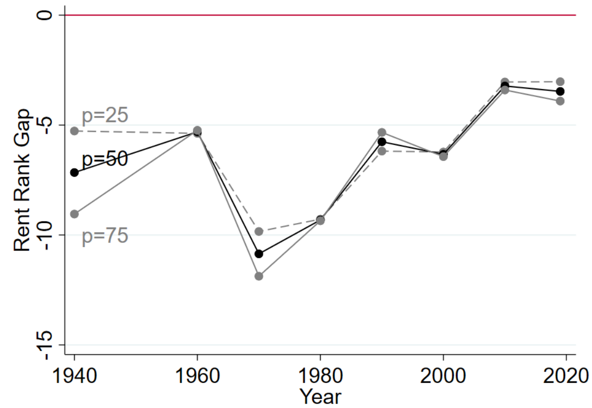
The gaps vary across the income distribution. Except in the 1940 and 1970, the gaps in terms of rents are similar across the income distribution. On the other hand, the price gaps are larger for high income households while the ownership gap is lower for these households. This might suggest that selection is important: higher income households are more likely to own but the marginal high income household buys a lower value home.

Figure 2: Racial Differences in the Housing Market over time

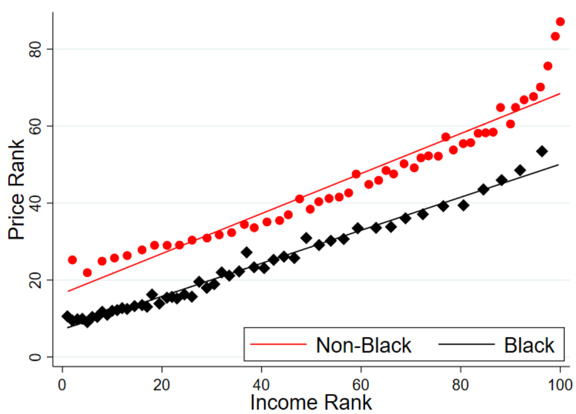
(a) Rent Rank Gap in 1960



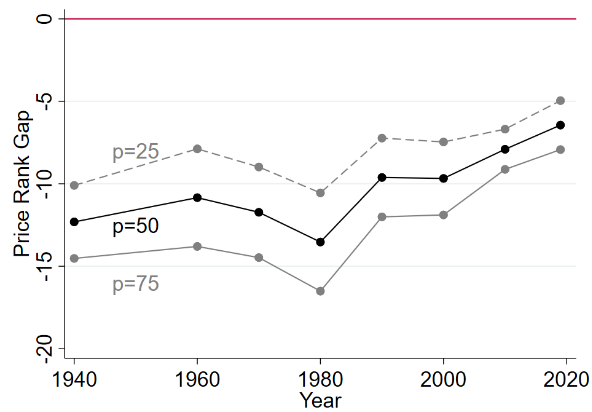
(b) Rent Rank Gap over Time



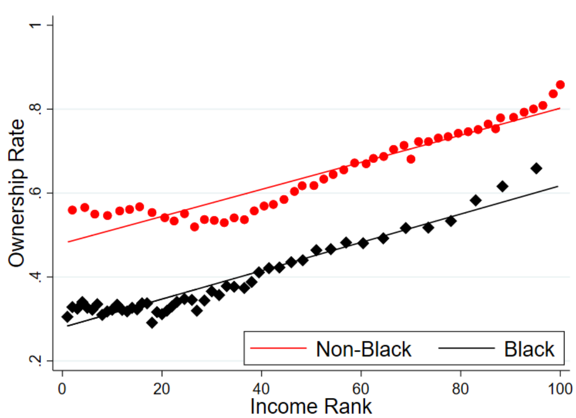
(c) House Price Rank Gap in 1960



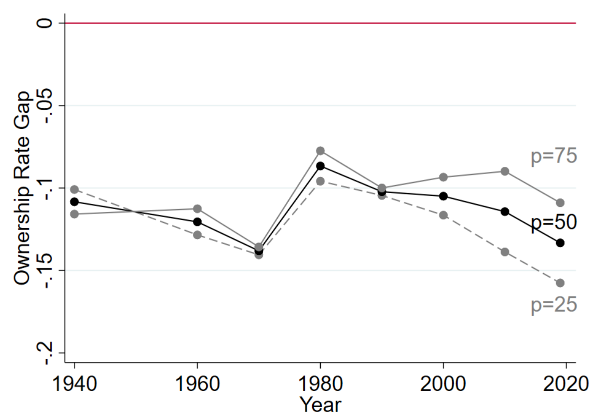
(d) House Price Rank Gap over Time



(e) Ownership Rate Gaps in 1960



(f) Ownership Rate Gap over Time



Notes: Left-hand binscatter plots show for Black and White households relationship (a) the average rank in the rent distribution, (c) the average rank in the house price distribution, and (e) the average home ownership rate, each against the households rank in the income distribution. Plots with rents are conditional on renters and likewise with house prices are conditional on owners. The right-hand plots show the estimated rent rank gap, house price rank gap and ownership rate gap, evaluated at the 25th, median and 75th percentile of the income distribution and in each decade 1940–2019. Data are from IPUMS Census micro-samples. There is no Census microdata on housing available for 1950.

3.3 Racial Differences Across Space

In this section I document how racial differences in housing outcomes evolved in the cross section of states. Racial differences were present in almost every state in 1960 and largest in Southern states at that time. While they have converged across space in the decades since, Black households continue to have worse outcomes in most states still today.

I estimate gaps using equation (1) for each state and year, $\beta_{s,t}$. Figure A7 plots the results of $\beta_{s,t}$ for each outcome in 1960 and 2019, and Figure A6 plots the associated maps. These figures show that the gaps are negative in almost every state for which there is a statistically significant estimate. Insignificant estimates are hollow in the scatter plots and gray in the maps; many of these are Western states where there were very small Black populations. The largest rent gap was in South Carolina Black households were on average 20 percentiles behind similar income White households in the distribution of rental expenditures. The largest price gap was in Nebraska, where Black households were more than 25 percentiles behind similar income White households in the distribution of house prices.

The maps illustrate that the rent gaps were largest in the Southern states, which is consistent with these states having greater restrictions on Black households (i.e. Jim Crow laws). The ownership and house price gaps are less spatially clustered in the South states with large gaps, also present in Midwestern and Northestern state. Cutler, Glaeser and Vigdor (1999) documents that these states also had high levels of segregation.

3.4 Direct Evidence on Observable Quality

This section provides direct evidence that the gaps in rents and prices are also associated with lower quality housing. I estimate the following regression equation

$$y_i = \beta \text{Black}_i + \sum_{j=1}^{10} \alpha_j \mathbf{1}\{\text{Income Decile} = \text{j}\}_i + \mathbf{X_i}\Gamma + \epsilon_i, \tag{3}$$

where y_i is a measure of housing quality for each household i, and $Black$ is an indicator for the household's race. I include dummies for each income decile to control for the non-linear relationship between income and the outcomes as shown in Figure 2, as well as a vector of controls, $\mathbf{X}$. The two best measures of quality that are observable in Census data are the age of the house and the number of bedrooms. Unfortunately the Census data do not include any measures of neighborhood amenities, such as crime or school quality. The coefficient of interest is β, which estimates the average difference between Black and White households conditional on income — I refer to it as the *gap* for each respective outcome.

Table 1 presents the results. Panel (a) shows that Black households live in homes that are older than White households. In 1960, Black households lived in homes that were 3.6 years older than White households. To give context for the magnitude, Table 1 also includes the coefficient on the highest income decile (relative to the base group of the lowest income decile). In 1960, this coefficient is -6 years, which implies that the difference between Black and White households is almost two thirds the difference between the poorest and richest households. There remains a Black-White gap of one year, though this gap is smaller than in 1960, in absolute terms and especially in relative to the gap between richer and poorer White households.

Table 1: Differences in Observable Housing Quality

Panel (a)	Age of house						
	1960	1970	1980	1990	2000	2010	2019
Black	3.85***	3.67***	2.45***	2.20***	2.22***	1.41***	0.99***
	(0.03)	(0.04)	(0.03)	(0.03)	(0.03)	(0.06)	(0.07)
Highest income decile	-5.99***	-4.38***	-6.93***	-6.32***	-7.86***	-10.18***	-10.31***
	(0.04)	(0.05)	(0.04)	(0.04)	(0.04)	(0.10)	(0.10)
Adjusted R^2	0.219	0.163	0.168	0.160	0.158	0.167	0.159
Panel (b)	No. of bedrooms						
	1960	1970	1980	1990	2000	2010	2019
Black	-0.02***	-0.08***	-0.09***	-0.10***	-0.07***	-0.04***	-0.04***
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
Highest income decile	0.12***	0.18***	0.78***	0.90***	0.97***	0.94***	0.91***
	(0.00)	(0.01)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
Adjusted R^2	0.011	0.038	0.329	0.291	0.260	0.270	0.267
Observations	2589798	1266183	3978700	4546176	5200043	1190175	1262524
Income deciles	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Controls	Yes	Yes	Yes	Yes	Yes	Yes	Yes

Notes: Table reports estimates from regression equation (3). The coefficient labeled Black reports the average difference in observable quality for Black and White households conditional on income. To interpret the magnitude, the coefficient on the highest income decile (relative to the base group of the second lowest decile) shows the average difference between rich and poor households. The table includes two measures of observable house quality: the age of the house (top panel); and the number bedrooms (bottom panel). Data are from Census.

Panel (b) shows that there are similar differences in the number of bedrooms, though the magnitudes are not quite as large. In 1960, Black households lives in houses with 0.02 fewer bedrooms, which is about one sixth the difference between the poorest and richest households. The gap remains in later decades, though it falls to between five and ten percent of the difference between the richest and poorest White households.

While this section has shown that Black households live in lower quality housing structures, it does not estimate whether the price for a given component of quality is higher. Unfortunately, it is unfeasible to get such estimates with the data that is available for much of the 20th century. While one could estimate hedonic regressions, these would likely suffer from omitted variable bias because the Census includes few neighborhood characteristics, and even the American Housing Survey does not include rich geospatial information. Without such information, if Black households live in neighborhoods with worse characteristics then this correlation will bias the estimates of the price paid by Black households for other characteristics, such as the number of bedrooms and bathrooms. Instead, I use an assignment model to infer the differences in housing quality.

3.5 Controlling for Permanent Income

I show that the housing gaps are robust to using other measures of permanent income in addition to annual income. Due to consumption smoothing, standard models predict that expenditures are another measure of permanent income, therefore I document housing gaps using data on expenditures in the Consumption Expenditure Survey 1980–2003. I follow Charles, Hurst and Roussanov (2009) and instrument for expenditures using current income to overcome concerns of endogeneity (a component of expenditure being regressed on total expenditures) and measurement error.

Tables A5–A7 confirm that the differences in housing expenditures, house values and ownership rates remain when controlling for permanent income with expenditures. Black households spend 7–18 percent less, controlling directly for expenditures, and 3–6 percent less when instrumenting for expenditures. The gap in the house value of owners (instead of the self-reported rental equivalents) is between 13 and 22 percent, controlling directly for expenditures, and between 6 and 12 percent when instrumenting for expenditures. Black households are 11–15 percentage points less likely to own a home, controlling directly for expenditures, and 10–12 percentage points less likely when instrumenting for expenditures.

In the quantitative model, I estimate life-cycle income profiles and income risk by race. Thus, the model also tests whether differences in permanent incomes can explain the gaps in the data, or whether additional frictions in prices are also needed to fit the data.

4 Simplified Model

This section uses a housing assignment model to illustrate how differences in the supply of house quality affect equilibrium outcomes — allocations and prices — and how the supply of quality can be directly estimated from observable data.

The model simplifies the quantitative model from section 5 in three dimensions. First, the simplified version is is static, wheres as the quantitative model is dynamic. Second, households only differ in income, whereas households also differ in age and wealth in the quantitative model. Third, the model only has a rental market, while the quantitative model adds a market for owner-occupied housing.

These simplifications will allow a explicit characterization of quality, and thus make clear that quality is identified from observables. The same insights will apply in the dynamic model.

4.1 Setup

There is a single housing market. The model matches heterogeneous households with houses that differ in quality. There is a mass unit mass of households who differ in income y. The income distribution has density $f(y)$ and c.d.f. $F(y)$. I assume this c.d.f. is continuous and monotonically increasing.

Houses differ in their quality, $h \in [\underline{h}, \overline{h}]$, and there is the same unit mass of houses as there are households. Quality h captures all the characteristics of the house that households value, including its physical characteristics (such as the number of bedrooms) and its location-based amenities (such as the quality of schools or crime in the neighborhood). Within each market, all houses of quality h are rented at price $\rho(h)$. The income distribution has density $g(h)$ and c.d.f. $G(h)$, and again, I assume the c.d.f. is continuous and monotonically increasing in h.

Households. Households choose consumption and a *single* house quality to maximize utility subject to a budget constraint

$$\begin{aligned} \max_{c,h} \quad & \log c + \theta \log h \\ \text{s.t.} \quad & y = c + \rho(h), \end{aligned} \tag{4}$$

where θ is the relative preference for housing. The household's preferences determine the sustainability of consumption and non-housing consumption. I assume Cobb-Douglas preferences which are consistent with the stable average expenditures shares across time and space. Expenditure shares will vary across the income distribution because housing is indivisible and rents are non-linear. The solution to this maximization is a consumption function $c^*(y)$ and housing demand function $h^*(y)$.

Equilibrium. Equilibrium is a rent function $\rho(h)$ and housing demand $h^*(y)$ such that households optimize and markets clear at every quality

$$G(h) = \int_0^\infty \mathbf{1}\left\{h^*(y) \leq h\right\} dF(y) \qquad \forall h. \tag{5}$$

Characterizing the equilibrium. I first characterize the assignment (i.e. allocations) and then prices. The assignment of households to house quality h is increasing in income y. This means that $h^*(y)$ is strictly increasing and invertible. It will be useful to work with this inverse function $y^*(h)$, which is the assignment function of incomes to house quality.

Market clearing implies that in equilibrium the mass of households assigned to qualities below h is equal to the mass of houses below h

$$F(y^*(h)) = G(h) \tag{6}$$

$$\implies y^*(h) = F^{-1}(G(h)) \tag{7}$$

where $F(\cdot)$ is invertible because it is strictly monotonically increasing.

The first order condition for households is

$$\rho'(h) = \theta \frac{y^*(h) - \rho(h)}{h}. \tag{8}$$

The left-hand side is the marginal cost for a household to increase the quality of the house, which they equate to the marginal benefit of increasing housing quality (i.e the marginal rate of substitution between housing and consumption). The marginal cost of higher quality housing is thus the slope of the price function $\rho'(h)$, which includes the special case of linear pricing (i.e. households pay $\bar{\rho} \cdot h$ for h units of housing, where $\bar{\rho}$ is a constant.).

The first order condition gives us the slope of the rent function. The rent can be found by integrating the first order condition starting from the rent of the lowest quality house, $\rho(0) = \rho_0$.

$$\rho(\tilde{h}) = \rho_0 + \int_0^{\tilde{h}} \theta \, \frac{y^*(h) - \rho(h)}{h} dh. \tag{9}$$

The equilibrium rent function is the sequence on indifference conditions for each household ordered according to their assignment.

4.2 Identification of unobserved quality

The quality h of houses is unobservable. To identify the quality of a house from observables, I define the equilibrium assignment of incomes to rents as

$$\rho^*(y) \quad := \quad \rho(h^*(y)). \tag{10}$$

The derivative is

$$\rho^{*'}(y) \quad = \quad \rho'(h^*(y)) \cdot h^{*'}(y) = \theta y - \rho^*(y)\frac{h^{*'}(y)}{h^*(y)}, \tag{11}$$

where the second equality comes from the FOC in equation (8).

Equation (11) is a differential equation in quality. Its solution is

$$\log h^*(\tilde{y}) \quad = \quad \underline{h} + \int_0^{\tilde{y}} \frac{1}{\theta}\frac{\rho^{*'}(y)}{y - \rho^*(y)} dy. \tag{12}$$

Quality is identified (conditional on $\underline{h}$ and θ) because the right-hand side of equation 12 contains only observables: income y and the relationship between rent and income $\rho^*(y)$. This equation states that we can trace out quality from its lower bound $\underline{h}$ using the change in rent as income increases. At any given income y and rent $\rho^*(y)$ we infer higher quality when the change in rent $\rho^{*'}(y)$ is larger. Intuitively, households must be receiving more quality when rent increases faster, otherwise households would choose a cheaper house instead. Conditional on the change in rent, we infer quality is higher when households already spend a large share of their income on rent, i.e. when $y - \rho^*(y)$ is small. Intuitively, households must be receiving more quality if they have already given up a lot of non-housing consumption and they continue to spend more on housing.

Finally, to get distribution of quality we use market clearing, $G(h^*(y)) = F(y)$, to find the density of quality as a function of the observable density of income and our inferred function mapping quality and observable incomes

$$g(h^*(y)) \quad = \quad \frac{f(y)}{h^{*'}(y)}. \tag{13}$$

Without the explicit relationship for quality in equation (12), I would need to use a guess and verify algorithm to find quality h. Specifically, I would have started with a guess for $h(\rho)$ and used the first order condition to find rent for each income households $\rho^*(y)$, and then iterated until these were consistent with observed expenditures. I use this type of algorithm to solve the quantitative model.

Not only is quality identified, it is also comparable across distinct market segments when $\underline{h}$ and preferences are the same. Both of these are reasonable assumptions to make: households agree on the lowest quality home that one can reasonably live in; and households substitute between housing and non-housing consumption to the same degree.

4.3 Illustrating the Mechanism with an Example

The estimation infers quality from observable choices — rent conditional on income. This section illustrates how this process works by comparing three example markets: a benchmark market, a low spending market, and a low income market. Figure 3a compares the *benchmark* market with a *low spending* market in which the rent-to-income ratio is lower, as I showed was the case for Black households in section 3. Figure 3b compares *benchmark* market with a *low income* market and shows that I can estimate the model with subsets of households (in this case, the bottom half of the income distribution), yet we will only infer distortions in quality when choices differ.

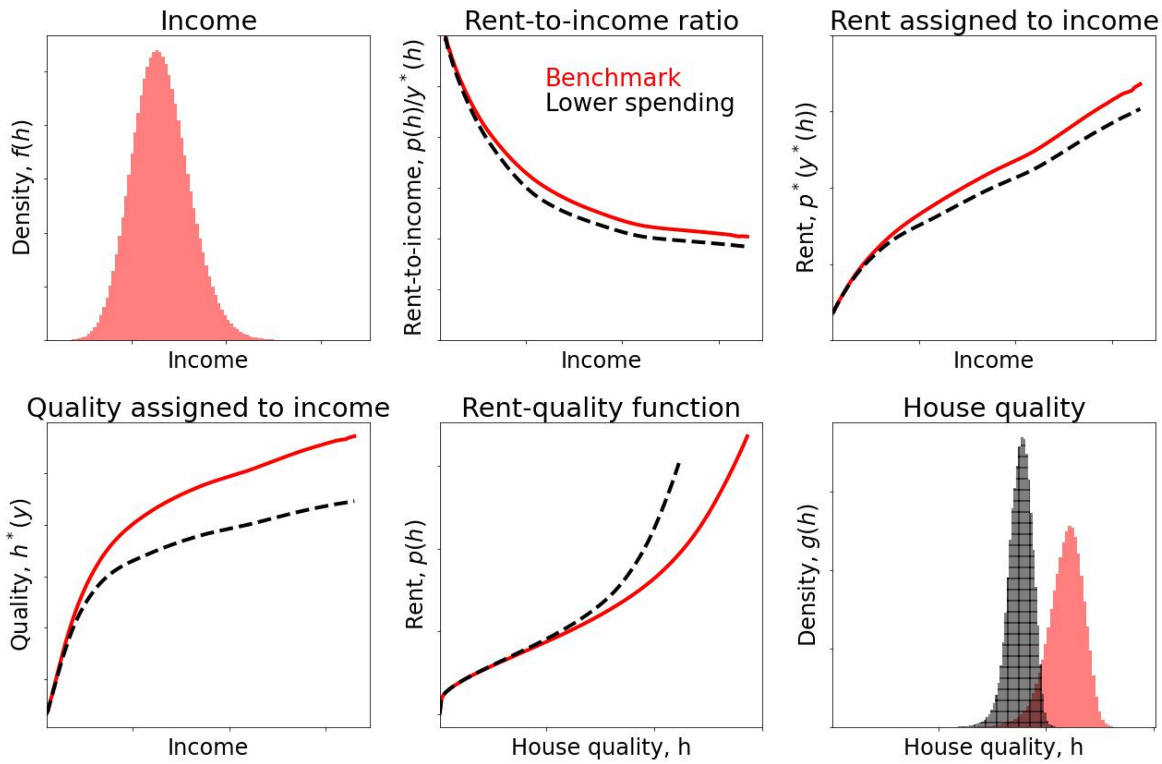
The *benchmark market* is in (solid) red in both Figures 3a and 3b. The top three panels show the observable inputs: the income distribution and the rent-to-income ratio (which is transformed into rents conditional on income in the top-right panel). The bottom three panels show the model's latent objects. The bottom-left shows quality at every income $h^*(y)$ which is inferred using equation (12). Quality increases most at lower incomes where assigned rents $\rho^*(y)$ are also increasing fastest. Once the function mapping rents and incomes $\rho^*(y)$ is identified, I use equation (9) to compute quality-adjusted rents $\rho(h)$ in the bottom middle panel and use equation (13) to construct the density of quality $g(h)$ in the bottom-right panel.

The assignment from quality to income is initially steep and then becomes flatter. Since rents rise fastest for low income households, the model infers that quality also rises fastest for low income households. For this to occur, the lower tail of the quality distribution must be more dispersed that the lower tail of the income distribution, so that, as incomes increase, households are matched to houses with larger differences in quality (relative to the changes in household incomes).

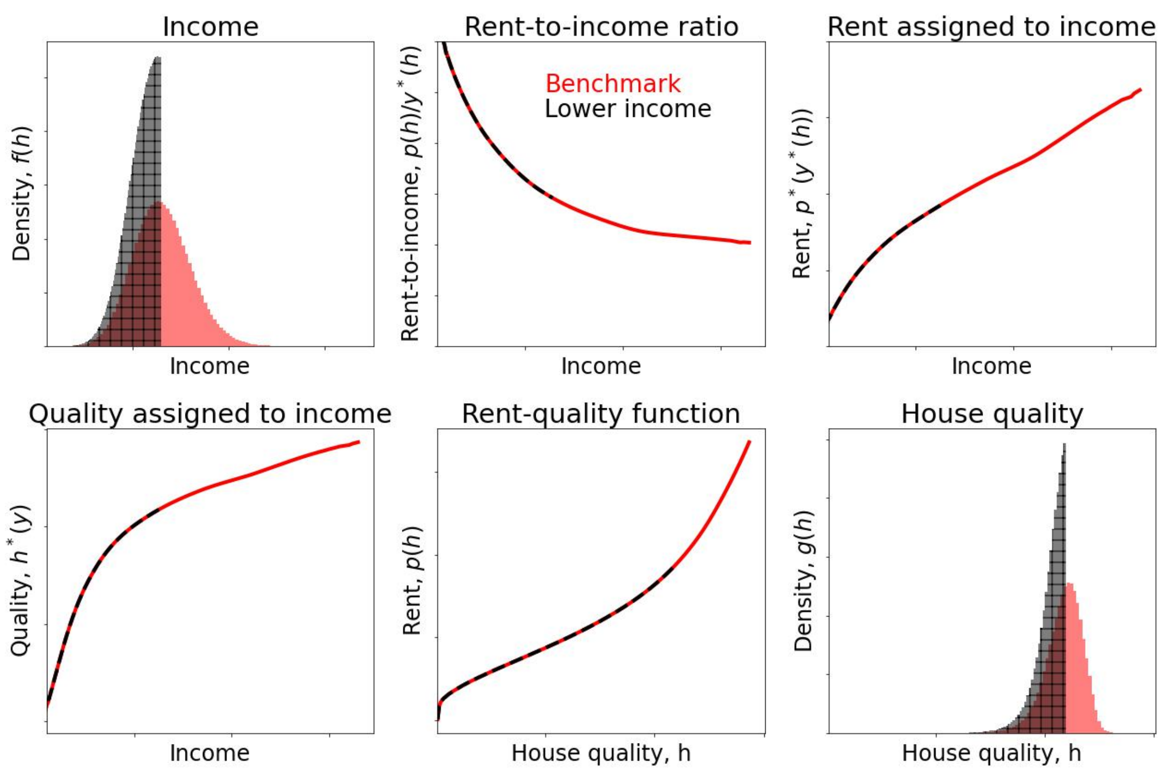
The *low spending* market is in (dashed) black in Figure 3a. The rent-to-income ratio is lower and thus rents conditional on income rise slower than in the benchmark market. The model infers that quality is lower in the lower spending market, shown by $h^*(y)$ in the bottom-left panel and the density in the bottom-right panel. The associated rent function $\rho(h)$ is steeper. From the households perspective, taking this steeper rent function as given rationalizes the substitution to lower quality and associated lower spending.

Figure 3: Example: inferring quality from observables.

(a) The *benchmark* market versus a low *spending* market



(b) The *benchmark* market versus a low *income* market



Notes: Figure compares the equilibrium outcomes in three example markets: a benchmark economy (red in both both 3a and 3b) against a low spending segment (black in 3a) and against a low income segment (red in 3b).

The steeper rent function is also consistent with market clearing in the lower spending market. The distribution of housing quality is worse in the lower spending market, thus market clearing implies that households at each income live in worse quality homes relative to the benchmark economy. Higher income households are willing to pay more for a given quality, therefore rents are higher in an equilibrium in which higher income households are all allocated to lower quality.

The *low spending* market illustrates the main mechanism of this paper. Lower expenditures by Black households can be rationalized by higher quality-adjusted rents and sorting into lower quality homes. The estimation finds that the sorting mechanism dominates, and thus rent-to-incomes are lower despite higher rents.

Figure 3b illustrates that estimating the model with subsets of households will not infer differences in the assignment of quality. One may worry that estimating the model with Black and White data separately will impose segmentation by assumption because Black households have lower incomes on average. Thus, the dashed black line shows a *lower income* market that contains only the lower half of the income distribution, but the rent-to-income ratio and thus the assigned rent conditional on income $\rho^*(y)$ are the same as the benchmark market. While the model infers that the density of housing in this market is truncated at the median, the *assignment* of quality to incomes $h^*(y)$ is unaffected. In other words, the density of quality is appropriately adjusted such that allocations in the lower income segment are the same as in the benchmark market. Since the assignment is the same, quality-adjusted rents $\rho(h)$ are also the same, as shown in the bottom-middle panel.

The simplified model provides three takeaways. First, quality can be inferred from data on rents conditional on income. Second, when evaluating the degree of segmentation, I can arbitrarily subset households and I will only infer differences in the assignment when rents conditional on income differ (or more generally when choices conditional on income and other characteristics differ). Third, I infer the distribution of quality $G(h)$ directly from the data without specifying the supply side of the market. The model does not need to specify how the house quality distribution is derived from the optimization problems of housing suppliers, because these will have to be consistent with what I measure from the data.

4.4 How Does the Expenditure Share Change with Income

The model can match the declining expenditure shares on housing despite the Cobb-Douglas preferences (which usually give rise to fixed proportions) because prices are non-linear in quality. With non-linear prices, the change in housing expenditures with respect to a change in income (the marginal propensity to consume housing) is

$$\frac{d\rho(h)}{dy} = \frac{\theta}{1+\theta+\varepsilon_{\rho,h}(h)}, \qquad \varepsilon_{\rho,h}(h) = \frac{\rho''(h)\cdot h}{\rho'(h)}, \tag{14}$$

where $\varepsilon_{\rho,h}(h)$ is the elasticity of marginal rent with respect to quality. When this elasticity is negative, households are induced to spend a higher share of their income on housing, whereas the converse is true when the elasticity is positive. See Appendix B for a derivation of equation (14), which comes from differentiating the FOC.

When prices are linear, $\rho(h) = \bar{\rho}\cdot h$, then $\varepsilon_{\rho,h}$ is zero and equation (14) has the usual Cobb-Douglas form

$$\frac{dp(h)}{dy} = \frac{\theta}{1+\theta}. \tag{15}$$

A convenient functional form is when prices have the power form, $\rho(h) = h^{\alpha}$, $\alpha \in (0,\infty)$, because the elasticity $\varepsilon_{\rho,h}$ is then a constant and the expenditure share is

$$\frac{d\rho(h)}{dy} = \frac{\theta}{\alpha+\theta} \tag{16}$$

which is less than $\frac{\theta}{1+\theta}$ when convex $\alpha > 1$ or greater than $\frac{\theta}{1+\theta}$ when concave $\alpha \in (0,1)$.

In this case concave rents induce households to spend a higher share of their income on housing. Likewise, convex prices induce households to spend less on housing because the relative price of non-housing consumption is cheapest when house price function is most convex.

To rationalize the declining expenditure shares, these equations suggest that rents should be concave at low levels of quality and convex at higher levels of quality. Intuitively, it may be cheap to increase the utility value of a low quality home (for example, with a fresh coat of paint) whereas it may be more expensive to improve a high quality home (for example, by relocating it closer to the city center or to scarce amenities like a beachfront).

4.5 Allowing for homophily in preferences

Appendix B.2 shows that my approach for inferring quality is valid even when households get utility from living near others of the same race (homophilic preferences). In this case, my approach continues to estimate the effective price of house quality, where house quality is an index that includes both a fundamental component and a homophilic component. Thus, comparisons of the effective price of house quality in the Black and White segment are still valid.

The counterfactual in which markets are integrated has a slightly different interpretation with homophily: either it is interpreted as removing the homophily in preferences (some component of which may be a legacy of discrimination) so that integration does not affect the homophilic component of quality; or the counterfactual improves the fundamental component housing quality in Black neighborhoods (while holding fixed the homophilic component) such that the effective price of quality is the same as the integrated market. The latter interpretation highlights that there is still a potential welfare gain from equalizing housing quality in the Black and White segments even when there homophily in preferences.

5 Quantitative Model

This section presents the quantitative dynamic housing assignment model. There are three main differences relative to the simplified model. The model involves a dynamic households optimization, whereas the simplified model was static. Demand for housing thus depends on each households' age, income and wealth, whereas it only depended on income in the simplified model. In addition to rental houses, there is an owner-occupied market, which means households have a discrete choice and there will be equilibrium rents and prices. These additions mean that the model now involves multidimensional assignment where households differ in their age, income and assets, and houses differ in their quality and whether they are available to buy or to rent.

5.1 Model Setup

Households. There is a continuum of households indexed by $i \in \mathcal{I}$ and their race $s \in \{w, b\}$, mass π_w of whom are White and $\pi_b = 1 - \pi_w$ are Black. I suppress the subscripts i, s unless necessary for exposition. Since the model is estimated separately for Black and White markets, I index only parameters that differ in each market and not the endogenous variables that may also differ in equilibrium.

Households live for J periods and survive to period j with cumulative probability $\tilde{\phi}_{s,j}$, or conditional probability $\phi_{s,j} = Pr_s(j|j-1)$. They choose consumption $c \in \mathbb{R}_+$, a *single* housing quality $h \in [\underline{h}, \overline{h}]$, and whether to buy or rent housing $o \in \{\mathcal{R}, \mathcal{O}\}$ to maximize the present discounted value of expected lifetime utility

$$\max_{\{c_j,h_j,m_j\}_{j=0}^{J}} \mathbb{E}_0 \left[\sum_{j=0}^{J-1} \beta^j \tilde{\phi}_{s,j} [u(c_j, h_j) + \varepsilon_j(o_j)] + \tilde{\phi}_{S,J} \beta^J v(c_J) \right], \tag{17}$$

where $v(c_J)$ is utility from bequests and $\varepsilon_j(o_j)$ is a preference shock for owning or renting. These preference shocks are i.i.d over time and drawn from a extreme value type I distribution with scale $\sigma_o \geq 0$. The preference shocks capture features outside the model that affect a household's decision to own or rent (for instance whether they own a dog).

Endowments. Each period households receive incomes that consist of an average life-cycle component $\overline{y}_{s,j}$ that is exposed to persistent shocks η_t

$$\begin{aligned} \log(y_j) &= \bar{y}_{s,j} + \eta_j \\ \eta_j &= \rho_s \eta_{j-1} + \epsilon^y_j \qquad \epsilon^y \sim \mathcal{N}(0, \sigma^y_s), \end{aligned} \tag{18}$$

where η_t is an AR(1) in logs, with persistence ρ_s and standard deviation σ^y_s. New cohorts draw an initial ϵ_0 such that each cohort is initialized at the stationary distribution of income.

Taxes. Households with labor income y pay a capital income tax τ^c, a property tax τ^h, and three taxes on labor income: a local labor tax τ^l, a payroll tax τ^{ss} (only while working-age $\mathbf{1}^w$) , and a progressive federal income tax schedule $T(\tilde{y})$ where $\tilde{y}$ is taxable income after deductions. Households can deduct a standardized deduction or itemized deductions including mortgage interest r^m, property tax τ^h, and local labor taxes τ^l

$$\begin{aligned} \tilde{y} &= \min\left\{\max(y - ID, 0), \max(y - SD, 0), y\right\} && \text{if } j < j_{ret} && (19) \\ &= \min\left\{\max(y - SD, 0), y\right\} && otherwise \\ ID &= r^m m + \tau^h p(h) + \tau^l y. && && (20) \end{aligned}$$

The tax function, as in Heathcote, Storesletten and Violante (2017), is

$$T(\tilde{y}) = \tilde{y} - \lambda \tilde{y}^{1-\tau^p} \tag{21}$$

where τ^p determines the degree of progressivity and λ determines the level. Total taxes are summarized by

$$\Gamma(y, a, h) := \tau^c r a + \tau^h p(h) + \tau^l y - \mathbf{1}^w \tau^{ss} y + T(\tilde{y}). \tag{22}$$

The structure of taxes most closely follows Karlman, Kinnerud and Kragh-Sørensen (2021) who study the tax benefit of owning a home in detail. I denote this tax benefit $TB(y, m, h)$ which depends on the households' income, and house and mortgage choices.

Liquid assets. Households can save in risk free bonds a at interest rate r_s (gross rate R_s) subject to a borrowing constraint $a \geq 0$.

Housing. Houses can be rented for rental price $\rho(h)$ or purchased for price $p(h)$. Households who own can borrow with a one period mortgage m at an interest rate $r^m > r$ where the spread on mortgages corresponds to intermediation cost.[11] Mortgages are subject to a downpayment

[11] Since there is no aggregate risk in the economy, competitive banks make zero profits and are not modeled

constraint $p(h)\lambda_s \geq m$, thus a downpayment of at least $p(h)(1-\psi_s)$ dollars must be paid in cash. The model period is ten years, so I assume that there are no adjustment costs for housing nor for mortgages. This is a reasonable assumption as such costs are small relative to ten years of income.

Owners receive the tax benefit $TB(y, b, h)$ and incur costs of owning consisting of depreciation δ, the property tax τ^h, and an additional owner cost τ_s^{UC}. The owner cost gap τ_s^{UC} represents all the features that are not modeled but may affect the relative attractiveness of owning vs renting housing for Black and white households — it will be an important margin for explaining differences in ownership rates across race. The cost most directly captures higher property taxes paid by Black households (Avenancio-León and Howard, 2022), though I consider it to be a reduced form for the many aspects of discrimination that affect the value of owner occupied housing, such as lower returns due to vandalism and discrimination in appraisals, or a lower probability of getting a mortgage due to higher rejection rates. It could also reflect aspects that do not reflect discrimination such as higher risk premia. The gross return on housing is thus

$$R_t^h = \frac{\rho_t(h) + TB(y, m, h) + p_{t+1}(h)}{(1 + \delta_t + \tau_t^h + \tau_{s,t}^{UC})p_t(h)}. \tag{23}$$

Bequests. Bequests occur accidentally when households die with positive assets, and on purpose in the final period of life because of the bequest motive. Bequests are distributed to new households at birth. They receive a share of total bequests that is proportional to their income at birth.

Supply. There is a distribution of housing quality $G_s(h)$, $\pi_s^{\mathcal{O}}$ of which is either supplied to own $G_s^{\mathcal{O}}(h)$ and $\pi_s^{\mathcal{R}} = 1 - \pi_s^{\mathcal{O}}$ is supplied to rent $G_s^{\mathcal{R}}(h)$

$$G_s(h) = \pi_s^{\mathcal{O}} G_s^{\mathcal{O}}(h) + (1 - \pi_s^{\mathcal{O}}) G_s^{\mathcal{R}}(h). \tag{24}$$

As I highlighted in the simplified model, I will measure the supply of housing using data. The decisions of suppliers has to be consistent with the distribution of rentals and owner-occupied housing that I measure in the data. However, to infer the quality of housing, I do not need to restrict the process by which $G(h)$ is produced, nor the process for whether it is supplied to own or to rent. To estimate the mapping between housing quality and observed rents and prices, I only need to specify the household side of the economy.

I do not explicitly model who owns rental companies. Other than housing, there is only one asset (the risk free bond) and since there is no aggregate risk we can think of the bond as representing the return on all capital in the model including the returns made by rental companies who own housing and rent it out.

Policy functions. A household's state is their age j, income state z and net savings w denoted by

explicitly.

$\psi = [j, z, w] \in \Psi$. The solution to the household problem (described in more detail in section 5.2) is a policy function for the probability owning $b^*(\psi) := Pr(o = \mathcal{O})$, and policy functions *conditional* on being an owner or renter $o \in \{\mathcal{O}, \mathcal{R}\}$, for consumption $c^*(o, \psi)$, house quality $h^*(o, \psi)$, savings $a^*(o, \psi)$. These policies imply a stationary distribution of agents $F(\psi)$ over the state space Ψ.

Using the choices of housing quality $h^*(0, \psi))$ and the functions for rent at each quality $\rho(h)$ and price at each quality $p(h)$, I define the rents and house prices that households choose: $\rho^*(\psi) := \rho(h^*(\mathcal{R}, \psi))$ and $p^*(\psi) := p(h^*(\mathcal{O}, \psi))$. Importantly, $p^*(\psi)$ and $\rho^*(\psi)$ are observable in data.

Equilibrium definition. Given an interest rate r_s and an exogenous supply of owner occupied $G_s^{\mathcal{O}}(h)$ and rental housing $G_s^{\mathcal{R}}(h)$, a stationary recursive equilibrium is a set of policies $b^*(\psi)$, $h^*(o, \psi)$, $c^*(o, \psi)$, $a^*(o, \psi)$, a rent function $\rho(h)$, a price function $p(h)$, and a distribution of households $F(\psi)$ such that agents optimize, the distribution of agents is stationary and markets clear at every quality

$$\pi_s^{\mathcal{O}} G_s^{\mathcal{O}}(h) = \int_\Psi \{(h^*(\mathcal{O}, \psi) \leq h) \cdot b^*(\psi)\} dF(\psi) \qquad \forall h \tag{25}$$

$$(1 - \pi_s^{\mathcal{O}}) G_s^{\mathcal{R}}(h) = \int_\Psi \{(h^*(\mathcal{R}, \psi) \leq h) \cdot (1 - b^*(\psi))\} dF(\psi) \qquad \forall h. \tag{26}$$

5.2 Household Bellman Equations

To solve the problem recursively, households keep track of the following state variables: their age, income state and savings, $\psi = [j, z, w]$. Each period households choose to buy or rent

$$\max_{o \in \{\mathcal{O}, \mathcal{R}\}} \{V(\mathcal{R}, \psi) + \varepsilon(\mathcal{R}), V(\mathcal{O}, \psi) + \varepsilon(\mathcal{O})\}, \tag{27}$$

where the respective value functions V are the solutions to the Bellman equations below. Given the assumptions on the preference shock ε, the probability of a renting or owning is given by

$$b^*(\psi) := Pr(o = \mathcal{O} | \psi) = \frac{\exp\left[\frac{V(\mathcal{O}, \psi)}{\sigma_o}\right]}{\exp\left[\frac{V(\mathcal{R}, \psi)}{\sigma_o}\right] + \exp\left[\frac{V(\mathcal{O}, \psi)}{\sigma_o}\right]}. \tag{28}$$

The expected value of next period's utility is given by the log-sum formula

$$\mathbb{E}V(\psi')] = \sigma_o \log\left(\exp\left[\frac{V(\mathcal{R}, \psi')}{\sigma_o}\right] + \exp\left[\frac{V(\mathcal{O}, \psi')}{\sigma_o}\right]\right) \tag{29}$$

(Iskhakov, Jørgensen, Rust and Schjerning, 2017).

With this expected value, the value function of a renter is

$$
\begin{aligned}
V(\mathcal{R},\psi) &= \max_{a',h}\{u(c,h)+\beta\phi_{s,j+1}\mathbb{E}V(\psi')\} \\
s.t. \quad & c+\frac{a'}{R_s}+\rho(h) = y_s(j,z)-\Gamma(y,a,h)+w \\
& a' \geq 0,
\end{aligned} \tag{30}
$$

where w is the net savings including the possible sale of any housing. Net savings tomorrow, for renters, is $w' = a'$. The solution is a set of policies for house quality $h^*(\mathcal{R},\psi)$, savings $a^*(\mathcal{R},\psi)$ and, via the budget constraint, consumption $c^*(\mathcal{R},\psi)$.

The value function of an owner is

$$
\begin{aligned}
V(\mathcal{O},\psi) &= \max_{a',h}\{u(c,h)+\beta\phi_{s,j+1}\mathbb{E}V(\psi')\} \\
s.t. \quad & c+\frac{a'}{R_s}+\mathbf{1}_{a<0}\left(\frac{a'}{R_s^m}-\frac{a'}{R_s}\right)+(1+\delta+\tau^{UC})p(h) = y_s(j,z)-\Gamma(y,a,h)+w \\
& a' \geq -\lambda_s p(h).
\end{aligned} \tag{31}
$$

where $\mathbf{1}_{a<0}$ is an indicator function for negative savings. Net savings tomorrow, for owners, is $w' = a' + p_{+1}(h)$, where $p_{+1}(\cdot)$ is the price function next period. The solution is a set of policies for house quality $h^*(\mathcal{O},\psi)$, savings $a^*(\mathcal{O},\psi)$ (which may be negative if the household borrows with a mortgage) and, via the budget constraint, consumption $c^*(\mathcal{O},\psi)$.

6 Estimation and Model Fit

The model is quantified in three steps. First, I set a number of common preferences parameters (such as the utility function) based on external evidence. Second, I estimate a number of parameters that can be cleanly identified without the structure of my model, such as mortality and income processes, and others to match aggregate moments in the model. Lastly I use the simulated method of moments to estimate the mappings between housing quality h and house prices and rents, $h(p), h(\rho)$, as well as the cost cost gap τ^{UC}, to match the observed housing choices (prices, rents and ownership rates). I estimate the model separately by race and thus uncover differences in quality-adjusted rents and prices in each market. With these, I also infer the supply of housing quality $G_s(h)$.

I estimate the model once in the pre civil rights era (1960) and again with recent data (2019). While there is housing data in the Census in 1940, I choose 1960 because it is the first year in which there is both Census housing data and microdata on wealth from the SCF+. (The SCF+ starts in 1949 but there is no Census microdata on housing for 1950.)

6.1 Common Features.

A model period is ten years. Households enter at age 25, retire at age 65 and die with certainty at age 95.

Preferences. I assume that households have Cobb-Douglas preferences over consumption an housing

$$u(c_j, h_j) = \frac{[c_j^{\alpha} h_j^{1-\alpha}]^{1-\gamma}}{1-\gamma} \tag{32}$$

where α is the share on numeraire consumption and γ is the coefficient of relative risk aversion. Cobb-Douglas is a reasonable functional form because it is consistent with average expenditure shares being constant across time and space (Davis and Ortalo-Magné, 2011). I will still be able to match the non-constant expenditure shares across the income distribution because there is indivisible housing and a non-linear price schedule for housing quality. I set $\alpha = 0.8$, which means the weight on housing $(1-\alpha)$ is 0.2. In the case of divisible housing (i.e. linear prices) this would give an aggregate expenditure share of 20%, which is consistent with the evidence in Piazzesi, Schneider and Tuzel (2007) and Piazzesi and Schneider (2016).

In the final period households sell owned housing, repay remaining mortgages and receive utility from bequests of the total remaining assets w

$$u(w) = \nu \frac{(w)^{1-\gamma}}{1-\gamma} \tag{33}$$

where ν determines the strength of the bequest motive as in De Nardi (2004).

I set $\beta = 0.95$ and $\nu = 0.1$ to match the median wealth at retirement and death for White households[12].

Tax rates. I most closely follow the parameterization in Karlman, Kinnerud and Kragh-Sørensen (2021). I set the progressivity of the federal tax system $\tau^p = 0.14$ and the average tax parameter $\lambda = 1.66$. The authors match the progressivity of the income tax system in recent decades. While this will underestimate the tax benefit from owning a home in 1960, when marginal tax rates were higher, I allow the ownership cost τ^{UC} to adjust in the estimation to match the ownership rate. The return on capital is set to $\tau^c = 0.15$. The tax rate on housing is set to one percent per year $\tau^h = 0.01$, which is consistent with the median property tax payments in the American Housing Survey. The local labor tax rate is set to $\tau^l = 0.05$. Chambers, Garriga and Schlagenhauf (2016) report that the payroll tax rate in 1960 was 1 percent so I set $\tau^{ss} = 0.01$.

[12] I choose to match White households only as the main idea of the model is to capture the distortions that are present for Black households *relative* to White households.

Table 2 summarizes the common preference parameters.

Table 2: Common preference parameters

Parameter	Description	Value
β	Discount factor	0.95
γ	Inverse EIS	2
α	Cobb-Dougas consumption share	0.8
σ_o	Variance of tenure shock	0.02
ν	Strength of bequest motive	0.1

Notes: Table reports the preference parameters that are homogeneous to Black and White households. See description in the text for more details.

6.2 Pre-estimated Parameters

Income. The income process depends on the age profile $\bar{y}_{s,j}$, and the persistence ρ_s and variance of shocks σ_s^y. I estimate the age profile $\bar{y}_{s,j}$ using a polynomial regression of age in Census data. Since Census and SCF+ do not include panel components, it is not feasible to estimate the persistence of the shocks ρ_s with these data. In the absence of estimates for Black and White households separately, I assume the persistence is 0.973 at an annual frequency, which is consistent with estimates from Heathcote, Storesletten and L.Violante (2010) and Krueger, Mitman and Perri (2016).[13] I estimate the variance of residual income in the Census data after estimating the life cycle process.

To convert the persistence to an annual frequency I set $\rho_{10yr} = (\rho_{1yr})^{10}$. For the variance of the income shock, I follow Krueger, Mitman and Perri (2016) in setting the variance of the ten year shocks to

$$\sigma_{10yr}^2 = \frac{1 - \rho_{10yr}^2}{1 - \hat{\rho}_{1yr}^2} \hat{\sigma}_{1yr}^2. \tag{34}$$

As the authors note, this achieves the goal of having the same cross-sectional distribution of income at the ten-year frequency as we see in the annual data (which is a plausible amount of dispersion). Table C3 reports the estimated life cycle processes and cross-sectional dispersion.

In retirement I assume the average income component $\bar{y}$ is a fixed replacement rate of the income in the final working period. Households continue to receive persistent shocks in retirement, which can be reinterpreted as expenditure shocks. De Nardi, French and Jones (2010) show that uncertain health expenses provide a savings motive in retirement. I set the replacement rate to

[13] For instance, Aliprantis, Carroll and Young (2021) also estimate race-specific age-profiles but report a common shock process. McKinney, Abowd and Janicki (2022) document a many components of income inequality by race in the US using Census admin data, however they do no report estimates for an AR(1) or similar shock process.

50% which is consistent with the estimates from survey data and Social Security models (Munnell and Soto, 2005).

Survival Probabilities. Conditional survival probabilities are estimated for Black and White individuals separately using the 1960 Life Tables (National Center for Health Statistics, 1964) and are reported in Table C1.

Interest rates on savings. Consistent with the evidence from Chambers, Garriga and Schlagenhauf (2016), I set the the interest rate on savings for White households to r percent, which together with the discount factor matches the median income for White households. To be consistent with lower Black wealth, but without setting a negative interest rate, I set the rate on savings for Black households 0.5 percent .

Mortgages. I pre-set the mortgage interest rate and loan-to-value (LTV) constraint to 5 percent and 0.6 respectively. While there is evidence of discrimination on mortgages, varying these parameters did not allow the model to match the ownership rate in the data. Thus in the baseline estimation, I set these parameters to be the same and allow the cost of ownership τ^{UC} (which can be seen as capturing among other things mortgage discrimination) to vary.

Calibration in 2019. For the calibration in 2019, I re-estimate the income processes and mortality risk for Black and White households. The results are reported in Tables C3 and C1 respectively.

Table 3 summarizes the race specific parameters.

Table 3: Pre-estimated parameters

	Parameter	**Description**	**Value**	**Source**
Incomes (White)	ρ_y	Persistence of shocks	0.97	Heathcote-Storesletten-Violante 10
	σ_y	Variance of shocks	0.75	Census 1960
	$\exp(\bar{y}_0)$	Intercept of age profile	1.0	Census 1960
		Replacement rate	0.5	Munnell-Soto 05
Incomes (Black)	ρ_y	Persistence of shocks	0.97	Heathcote-Storesletten-Violante 10
	σ_y	Variance of shocks	0.89	Census 1960
	$\exp(\bar{y}_0)$	Intercept of age profile	0.64	Census 1960
		Replacement rate	0.5	Munnell-Soto 05
Mortality	ϕ_j	Survival probability (White)	C1	Life tables 1960
	ϕ_j	Survival probability (Black)	C1	Life tables 1960
Saving	r	Risk free rate (White)	0.03	Median wealth
	r	Risk free rate (Black)	0.005	Median wealth
Mortgages	r^m	Mortgage rate (White)	0.05	Chambers-Garriga-Schlagenhauf 16
	r^m	Mortgage rate (Black)	0.05	Chambers-Garriga-Schlagenhauf 16
	λ	Max LTV (White)	0.60	Ownership age profile
	λ	Max LTV(Black)	0.60	Ownership age profile

Notes: Table reports parameters estimated before the Simulated Method of Moments estimation. Survival probabilities are presented in Table C1.

6.3 Simulated Method of Moments

The last step uses simulated method of moments to estimate the remaining parameters for each market segment. In the simplified model we observed rents at a given income $\rho^*(y)$ and we found the rent-quality mapping $\rho(h)$ such that the observed rents were consistent with the households' first order conditions. In the quantitative model we observe rents $\rho^*(\psi)$, prices $p^*(\psi)$, and ownership rates $b^*(\psi)$ for households who not only differ in income but also age and wealth (as summarized by ψ). We want to find a rent-quality mapping $\rho(h)$, a price-quality mapping $p(h)$, and a cost of owning τ^{UC} such that observed choices are consistent with household optimization. As in the simplified model, once we estimate $\rho(h)$ and $p(h)$, we can infer the distributions of housing quality $G^{\mathcal{O}}(h)$ and $G^{\mathcal{R}}(h)$.

Parameters. The parameters are the inverse functions $h(\rho)$ and $h(p)$, and owner cost gap τ_r^{UC}. In the baseline estimation I assume a fixed price-rent ratio for all qualities $p(h) = \bar{p}\rho(h)$, and thus estimate $h(\rho)$ and the price-rent ratio $\bar{p}$. (Estimating $\bar{p}$ is equivalent to pinning down the price of the lowest quality house $p(\underline{h})$ and assuming that the shape is the same as $\rho(h)$.) The model fit is good without needing to allow the price-rent ratio to vary over quality h.

To approximate the function $h(\rho)$, the optimization finds the value of the elasticity of the rent function with respect to quality, $\epsilon_{\rho,h} = \rho''(h)\frac{h}{\rho'(h)}$, at n_ρ points on a grid of quality. I use a cubic spline to interpolate the elasticity on a finer grid, which alongside the rent of the lowest quality can be used to find the level of the rent function on a fine quality grid.

Moments. The moments included are the average rent $\rho^*(j,z)$ and price $p^*(j,z)$ at each age j and incomes state z, and the overall ownership rate (probability of owning) $\pi^{\mathcal{O}}$.[14] Moments that only depend on age and income are the weighted average of households at these states, $\rho^*(j,z) = \int_w \rho^*(j,z,w)dF(j,z,w)$. The moments only depend on income and age because I do not observe wealth in Census data (and I do not observe rents in the SCF+ dataset). In equilibrium the share of housing in the ownership $\pi^{\mathcal{O}}$ is equal to the average ownership decision $\int_\Psi b^*(\psi)dF(\psi)$.

The weighting matrix W weights the moments by their share in the stationary distribution $dF(j,z)$. It also includes the inverse of the square of the moment, thus minimizing the percentage differences between the moment and the data. I equally weight the three sets of moments where the sum of age times income moments has the same weight as the overall ownership rate.

Objective function. To summarize, the parameter vector θ is estimated by minimizing the

[14] I also include the unconditional distribution of rents and house prices, $G_r^{\mathcal{R}}(h)$ and $G_r^{\mathcal{O}}(h)$ evaluated at n_h grid points. The weighting matrix places little weighting on these, but they help avoid local minima that can occur with all households choosing quality that is close to the bounds of $h \in [\underline{h}\bar{h}]$.

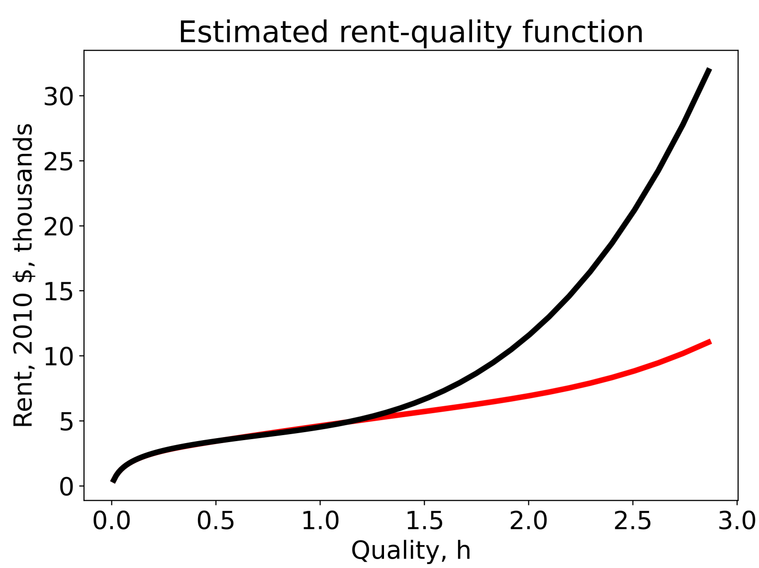
distance between data moments $\hat{m}$ and their respective simulated model moments $m(\theta)$

$$
\begin{aligned}
\theta &= \left[h(\rho), \bar{p}, \tau^{UC}\right] \\
m_r &= \left[\rho^*(j,z), p^*(j,z), \pi^{\mathcal{O}}\right] \\
\hat{\theta}_r &= argmin\left\{[\hat{m} - m(\theta)]^T W[\hat{m} - m(\theta)]\right\}.
\end{aligned}
$$

The model is solved on a grid $n_j \times n_z \times n_a$ and I use Young (2010)'s method, a fast and accurate method, to to directly simulate the distribution instead of simulating many individuals and then aggregating them. This gives $2 \times (n_j \times n_z) + 1 = 98$ moments and there are $n_\epsilon + 2 = 10$ parameters, therefore the estimation is over-identified. The estimation is performed separately using data for Black and for White households.

Simulated Method of Moments Estimates. Table 4 reports the estimates for the 1960. I find that the cost of ownership τ^{UC} is 3 percent higher in the Black segment, implying that returns on housing are 3 percent lower for Black households. Figure 4 plots the rent-quality function, which has a similar shape to that of the simplified model. The function is concave for low quality home and convex for higher quality homes. Rents are the same at low quality home but are higher in the Black segment at higher qualities.

Figure 4: Estimated Rent-Quality Function $\rho(h)$ in 1960



Notes: Figure plots the estimated rent-quality function from the simulated method of moments estimation. The estimation approximates the function with the *elasticity of rent with respect to quality* at n_ρ points on a grid of quality. The elasticity is interpolated onto a finer grid of quality and the level of the rent function is found using the exogenous initial rent $\rho(\underline{h})$.

Table 4 reports that the price-rent ratio is the same in the Black and White segments, thus differences in rental prices also translate into differences in prices in the owner-occupied segments.

Table 4: SMM Parameter Estimates in 1960

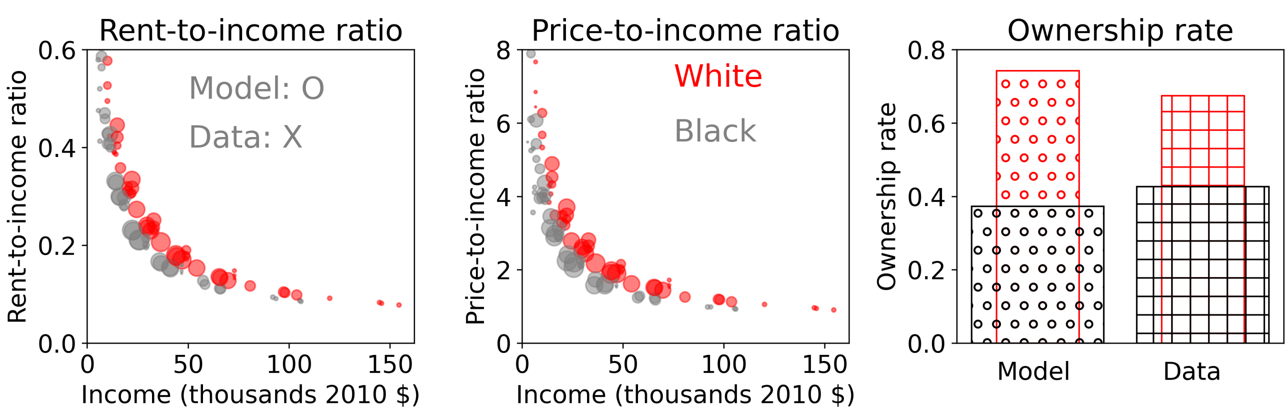
	Description	White	Black
$\rho(h)$	Rent-quality function	Figure 4	
p/ρ	Price-rent ratio	11.5	11.5
τ^{UC}	Cost of home ownership	0.0	0.03

Notes: Table reports parameters estimates from Simulated Method of Moments estimation. The cost of home ownership is normalized to zero for White households. The Rent-Quality function is plotted in Figure 4.

6.4 Model Fit

Table 5 evaluates the model fit by comparing moments in the model and data. I compare the primary outcomes rents, prices and ownership rates across both the income and age dimensions. The model provides a very good fit for the rent share, with every moment within two percentage points except for White households in the lower third of the income distribution. The second panel shows that the model also matches the hump shaped pattern in house prices over the life cycle. The model underestimates the peak in middle age, suggesting there is more smoothing than in the data. The third panel shows that the model matches the basic patterns on ownership rates: White households have higher ownership rates than Black households, and there is a lifecycle of ownership with younger households in both groups having lower ownership rates than the overall average.

Figure 5: Model Replicates Non-linear Rent- and Price-to-income ratios



Notes: Left and middle plots show the rent-to-income and price-to-income for each age-income group. The size of the bubble represents the share of the household group in the stationary distribution of agents. The right-hand plot presents the ownership rate for White and Black households.

Table 5: Model Fit in 1960

(a) Rent share by income

	White		Black	
Income group	*Model*	*Data*	*Model*	*Data*
Bottom 1/3	0.35	0.42	0.34	0.36
Middle 1/3	0.21	0.21	0.16	0.17
Top 1/3	0.14	0.13	0.12	0.12

(b) House price by age (2010 $10,000s)

	White		Black	
Age group	*Model*	*Data*	*Model*	*Data*
<35	79	86	50	52
35- 64	89	118	57	63
$\geq$ 65	80	95	48	42

(c) Ownership rate by age

	White		Black	
Income group	*Model*	*Data*	*Model*	*Data*
Overall	74%	67%	37%	42%
Age 25-34	53%	53%	13%	23%

Notes: Table reports compares the model fit using model and data moments for the estimated model in 1960. It compares fit for rents (measured in the rent-income share), house prices, and ownership rate; and does so across income and age dimensions.

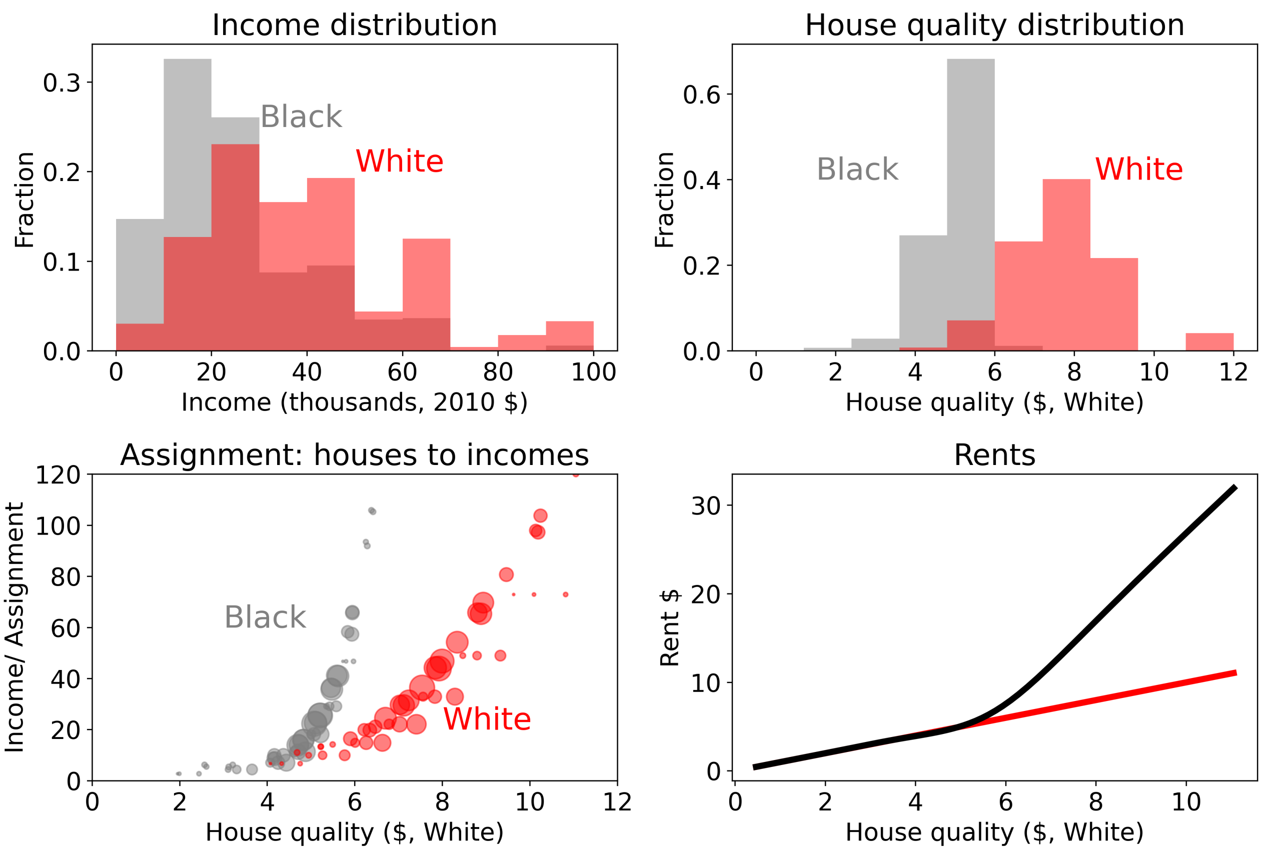
Figure 5 confirms visually that the model captures the downward sloping rent-to-income shares and price-to-income ratios, shown in the empirical section, and replicates the pattern that Black households are lower than and White households at every income. It also matches the difference in ownership rates.

7 Results

7.1 Equilibrium in 1960

Figure 6 presents the results of the estimated equilibrium in 1960. The top-left panel shows that there is considerable overlap in the distribution of incomes of Black and White households, while the top-right panel shows that there is almost no overlap in the distribution of house quality. In the plots quality is normalized to its rent in the White segment. The lack of overlap is reflected in the assignment in the bottom right panel, which shows that the assignment of Black households is steeper: lower quality homes are assigned to richer Black households relative to White households. The lower-right panel shows that rents are higher for Black households, especially for high quality homes. The Black segment has fewer medium and high quality homes and thus the rent of these homes are much more expensive relative to the White segment. Alternatively, this can be interpreted as Black households facing the largest barriers to entering the highest quality homes, and thus the effective rent is highest for these homes.

Figure 6: Equilibrium in 1960.

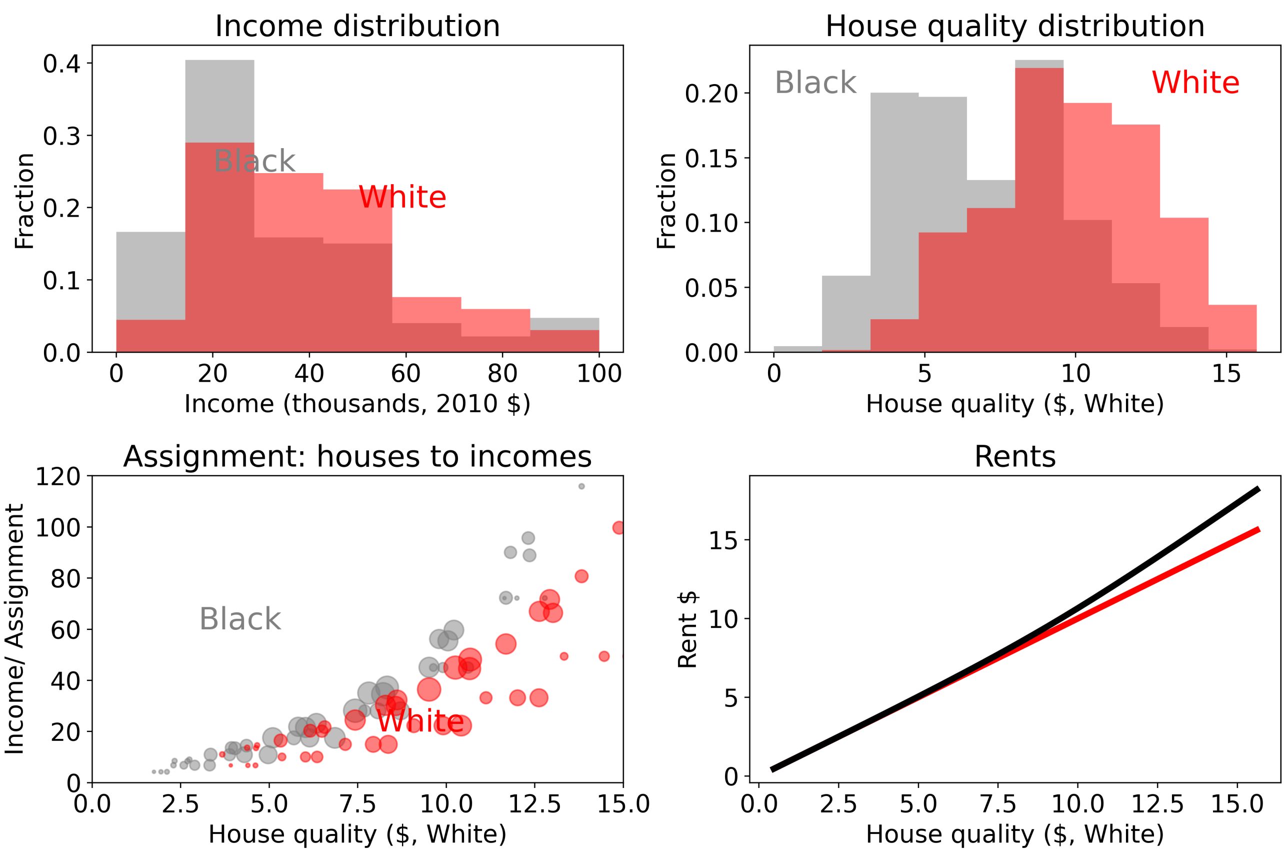


Notes: Figure plots the features of the model that was estimated using data from 1960. Top-left plot shows the income distribution for Black (in black) and White (in red) households. Top-right plot shows the distribution of housing quality in the Black (in black) and White (in red) market segments. House quality throughout is normalized to its dollar value in the White segment. Bottom-left plots shows the relationship between house quality and income in the Black and White market segments. The size of the bubbles represent the share of households in the stationary distribution. The lower-right plots show the rent-quality function that was estimated using SMM.

7.2 Equilibrium in 2019

Figure 7 presents the results of the estimated equilibrium in 2019. The top-left panel shows that there is considerable overlap in the distribution of incomes of Black and White households. The top-right panel shows that there is substantially more overlap, relative to 1960, in the distribution of house quality for Black and White households. Again, quality is normalized to its rent in the White segment. Relative to 1960, the assignment of quality to incomes in the bottom-left panel is much flatter for Black households. Yet, it is still steeper than the assignment for White households. The convergence in assignment is reflected in the differences in rents, which have also converged relative to 1960.

Figure 7: Equilibrium in 2019.



Notes: Figure plots the features of the model that was estimated using data from 2019. Top-left plot shows the income distribution for Black (in black) and White (in red) households. Top-right plot shows the distribution of housing quality in the Black (in black) and White (in red) market segments. House quality throughout is normalized to its dollar value in the White segment. Bottom-left plots shows the relationship between house quality and income in the Black and White market segments. The size of the bubbles represent the share of households in the stationary distribution. The lower-right plots show the rent-quality function that was estimated using SMM.

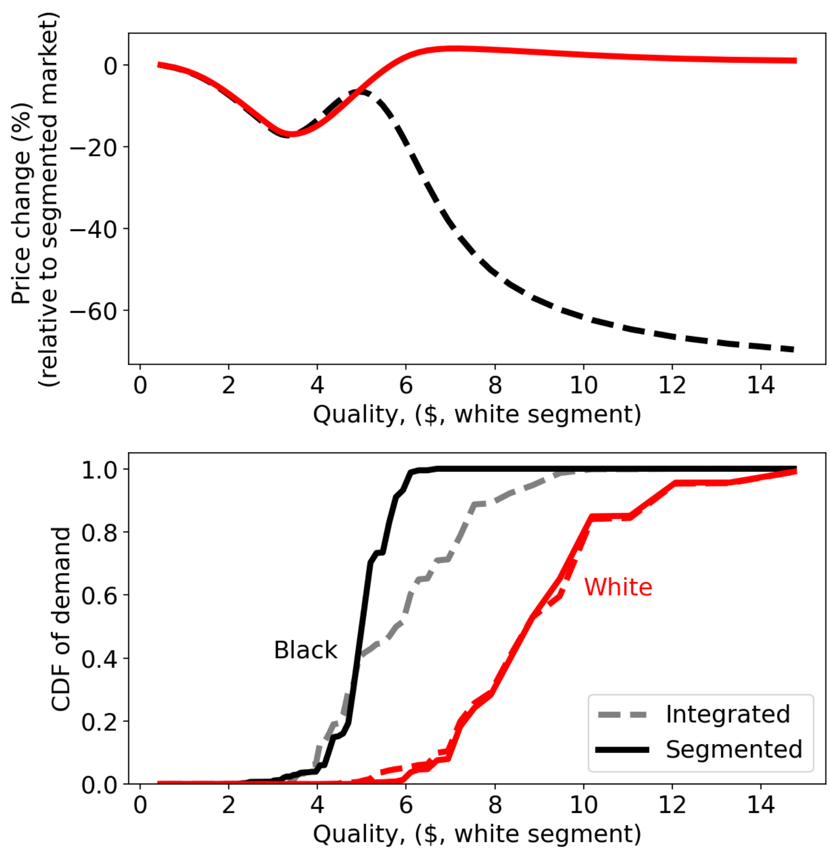
7.3 Market Integration and Welfare

I solve for the counterfactual equilibrium in which the Black and White markets are integrated and evaluate the welfare gains and losses of market segmentation relative to this integrated equilibrium.

The supply of housing is the sum of the supply in each segment $G_I(h) = \pi_w G_w(h) + \pi_b G_b(h)$. I solve for the rent function $\rho(h)$ that clears the integrated market, assuming that the price-rent ratio remains constant in the counterfactual. This is equivalent to assuming that rental companies can frictionlessly convert owner occupied homes into rental homes (and vice versa).

Consumption equivalent welfare is the percentage change in non-housing consumption in every state and time period (in the integrated equilibrium) that is needed to equate welfare in the segmented equilibrium. It can be interpreted as the welfare losses or gains from segmentation.

Figure 8: Changes in Demand and Rents After Market Integration in 1960



Notes: The top panel plots the percentage change in price in the integrated market relative to the price in the each segmented market. Since the price-to-rent ratio is held constant, this plot shows the percentage change in both prices and rents. The dashed black line is the price change relative to the Black segmented market, and the solid red line is the price change relative to the White segmented market. Quality is normalized to the rent in the White segment. The bottom panel plots the c.d.f of housing demand for Black and White households in the segmented and integrated markets.

Figure 8 shows the change in rents and demand when markets are integrated (the rent functions in levels are reported in Figure D1). The top panel shows that rents are lower at all quality levels relative to the Black segment, and fall most at higher qualities. Rents in the White segment fall in the lowest qualities (where few White households live) and rise modestly (less than 5%) at higher qualities. Intuitively, since Black households are a small share of the overall population (10 percent), rents in the integrated market are closest to those in the White market.

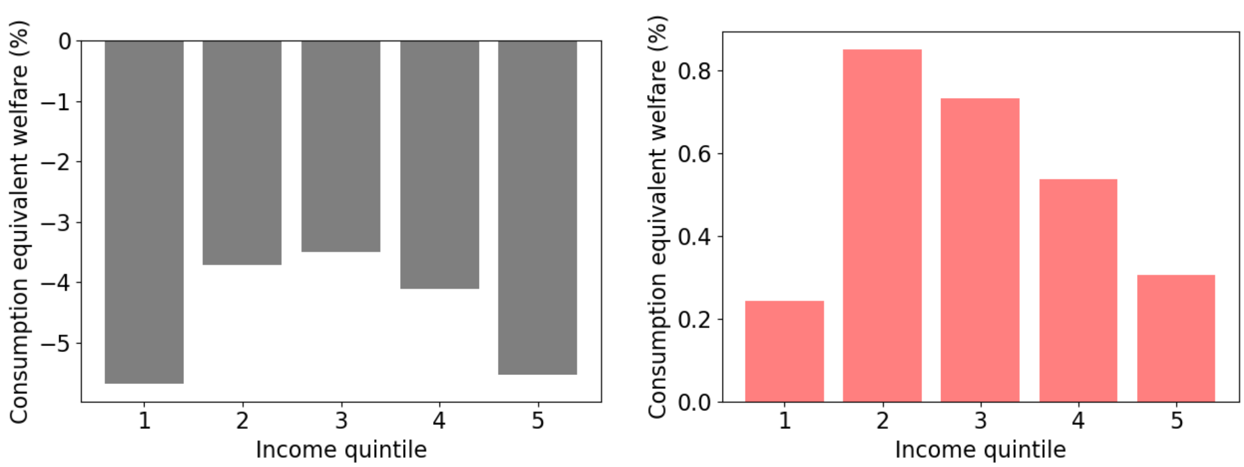
The bottom panel shows that when markets are integrated, richer Black households sort into much higher quality homes. For markets to clear, the higher sorting by Black households means that low and middle income White households have to sort into worse quality homes. However, White households are only moderately affected because Black households make up a small share of the overall market.

The change in sorting from the segmented to the integrated market has implications for what the average difference in prices between the Black and White segments is. Black households pay 3.2 percent more on average when evaluated at Black households' choices in the segmented market. However, the average gap is 18.8 percent when evaluated at the choices that Black households would have made in the integrated market. Thus, while Black households do not pay substantially higher prices that White households in equilibrium, this is because they sort away from the qualities that have the biggest price gaps. Thus, an econometrician who could estimate the price gaps between the Black and White segments would need to also estimate the price gaps on houses not chosen in equilibrium to fully understand the differences in the price schedules faced by Black and White households. Indeed, while price gaps are informative, an advantage of the welfare gaps is that they account for both the differences in prices paid as well as the differences in quality.

Figure 9: Welfare Gains and Losses Relative to Integrated Market in 1960.

(a) Black households

(b) White households

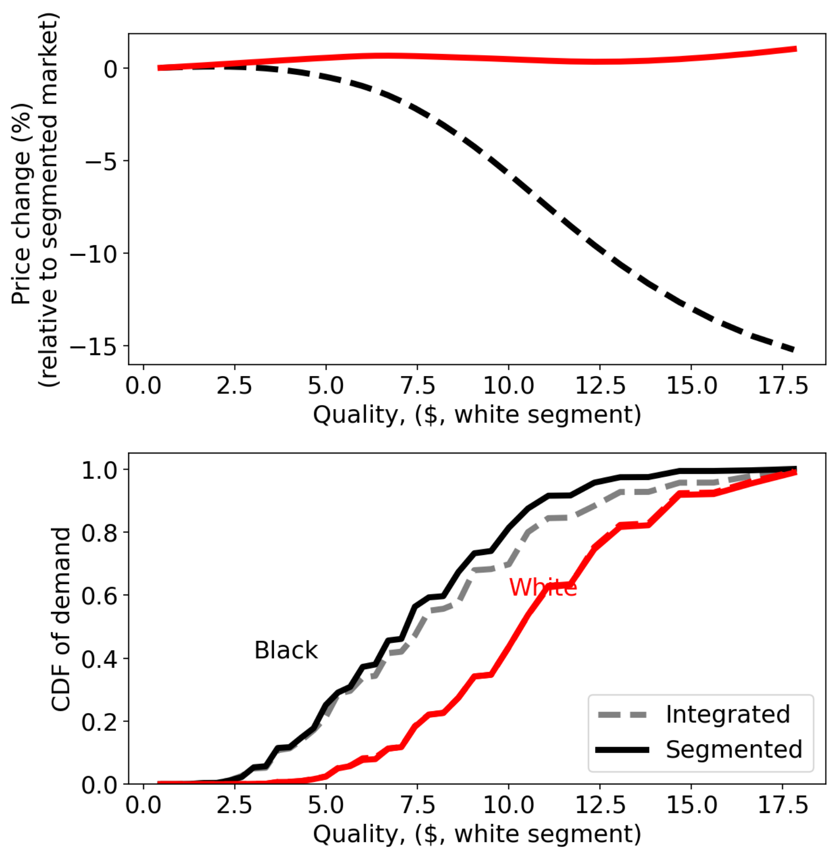


Notes: Figure plots the consumption equivalent welfare of households in the segmented equilibrium relative to the integrated equilibrium. Consumption equivalent welfare is the percentage change in non-housing consumption in every state and period (in the integrated equilibrium) that is needed to equalize expected welfare at birth in the segmented equilibrium. The income quintile is the income quintile of the household at birth.

Figure 9 reports the welfare gains and losses in the segmented market relative to the integrated market. The average welfare loss for Black households (4.5 percent) is ten times as large as the the gain for White households (0.5 percent). The biggest welfare loss is for Black households in the highest income quintile, who otherwise would have sorted into much better quality homes, and the lowest income household who are affected by higher income Black households competing for low quality homes.

Figure 10 shows the change in rents and demand when markets are integrated in 2019. Again, the rents are closest to those in the larger White segment, and Black households sort into higher quality homes. However, since the Black market was not too different to the White one, the change in sorting is smaller than in 1960. Thus, the sorting of White households is only marginally affected by the small changes in Black demand when the markets become integrated.

Figure 10: Changes in Demand and Rents After Market Integration in 2019



Notes: The top panel plots the percentage change in price in the integrated market relative to the price in the each segmented market. Since the price-to-rent ratio is held constant, this plot shows the percentage change in both prices and rents. The dashed black line is the price change relative to the Black segmented market, and the solid red line is the price change relative to the White segmented market. Quality is normalized to the rent in the White segment. The bottom panel plots the c.d.f of housing demand for Black and White households in the segmented and integrated markets.

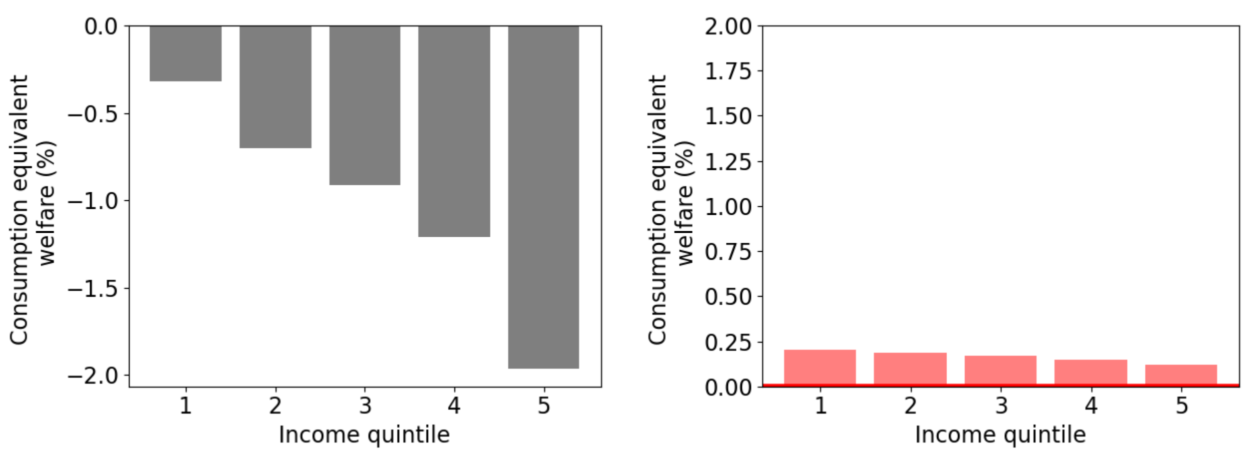
In 2019, the average price gaps continue to be smaller on the homes that are chosen in equilibrium (3.3 percent) than on the homes that otherwise would have been chosen in the integrated equilibrium (4.4 percent). Indeed, though few Black households pay the largest price differences, such large price differences distort many of their choices. In equilibrium, less than 3 percent of Black households live in homes that are more than 10 percent more expensive than in the White segment; though over 7 percent would live in these homes if there were no price gaps. Less than half a percent of Black households pay 15 percent or more than White households, though more than two percent of Black households' choices are distorted price gaps of this size. This suggests that econometric studies that focus only on choices that are made in equilibrium may substantially underestimate the impact of price gaps.

Figure 11 shows that in 2019 the welfare losses for Black households are still 1 percent on average and 2 percent for the highest income households. The welfare gain for White households are negligible (less than one quarter percent), because integrating the market has minimal impact on equilibrium rents.

Figure 11: Welfare Gains and Losses Relative to Integrated Market in 2019.

(a) Black households

(b) White households



Notes: Figure plots the consumption equivalent welfare of households in the segmented equilibrium relative to the integrated equilibrium. Consumption equivalent welfare is the percentage change in non-housing consumption in every state and period (in the integrated equilibrium) that is needed to equalize expected welfare at birth in the segmented equilibrium. The income quintile is the income quintile of the household at birth.

8 Conclusion

This paper has documented that there are large gaps in housing outcomes for Black households relative to White households. Using a segmented markets approach, I quantify both the sources of the gaps, as well as their impact on household welfare. In 1960, there were considerable differences in the Black and White segments, which meant that Black households paid higher quality-adjusted prices and sorted into worse quality homes. All Black households are worse off under segmented markets, but especially higher income households (who otherwise would have sorted into much higher quality homes) and also lower income households (who face higher prices because richer Black households compete for low quality homes). The heterogeneous welfare effects highlight the benefit of using a model that can capture these rich differences across households. In recent data, Black households are sorting into higher quality homes but barriers to home ownership appear to be as large as they were almost a century ago. This suggests that future research should focus on the sources of these barriers to home ownership, as well as finding ways to mitigate their impact.

The framework developed in this paper is an exciting laboratory in which to study a broader set of questions where heterogeneity is important. A key innovation of quantitative assignment models is that they allow us to model how changes that affect some households spill over to others through the equilibrium. They also allow us to match new features of the data that vary across households, such as the high housing expenditures shares of low-income households. Assignment models are thus are a natural environment to study how shocks — such as changes in interest rates, borrowing constraints, or housing supply — differentially impact renters versus owners, young versus old, and rich versus poor households.

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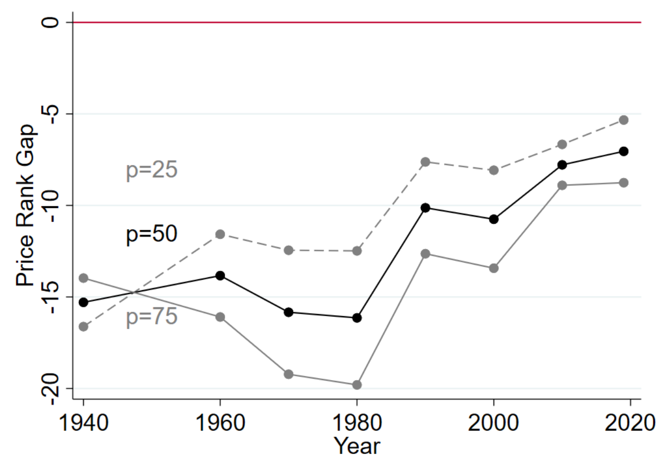
A Appendix to Section 3: Data and Empirical evidence

This appendix presents additional empirical results that are cross referenced in the main body.

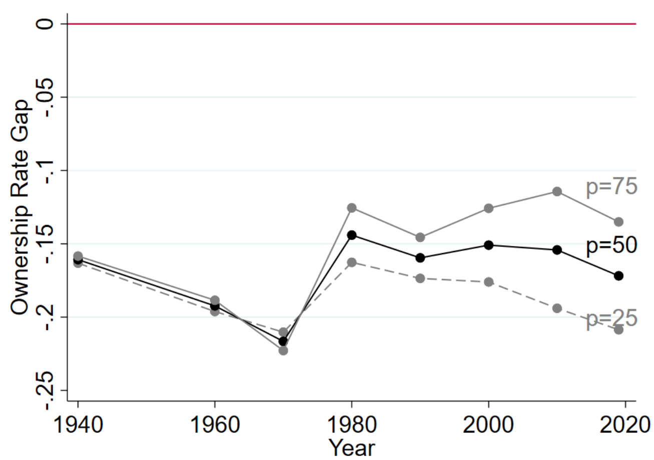
Figure A1: Racial Differences in the Housing Market over Time (without controls)

(a) Rent Rank Gap over time

(b) House Price Rank Gap over time



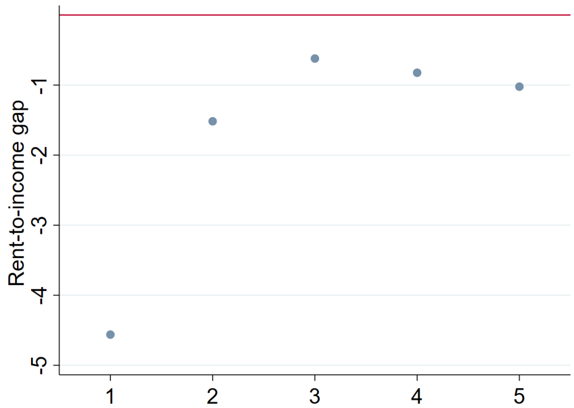
(c) Ownership Rate Gap over time



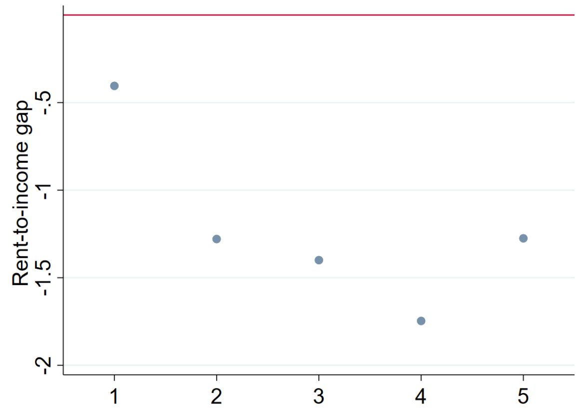
Notes: data are from IPUMS Census micro-samples. There is no Census data on housing for 1950. Black line is the gap for the median income household. The light solid line is for the 75th percentile and the light dashed line is for the 25th percentile.

Figure A2: Racial Differences in Rents and Prices by Income Quintile

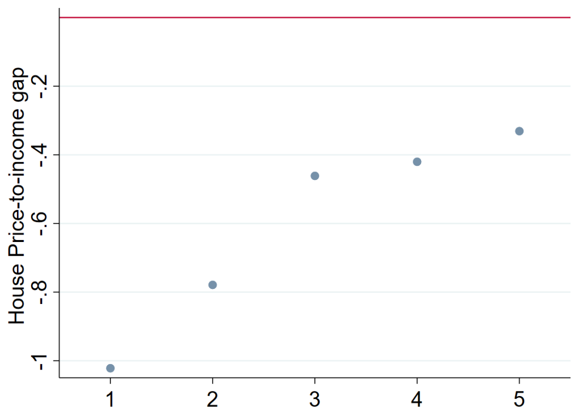
(a) Rent-to-income in 1960



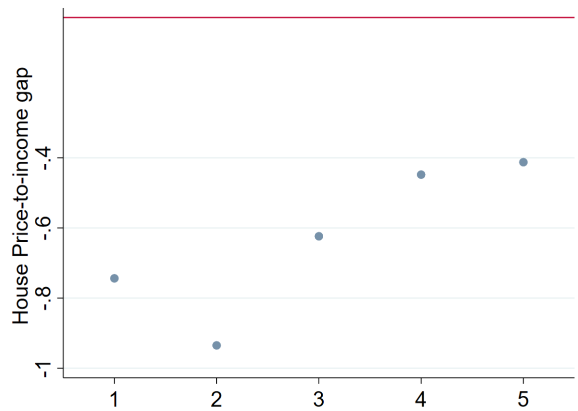
(b) Rent-to-income in 2019



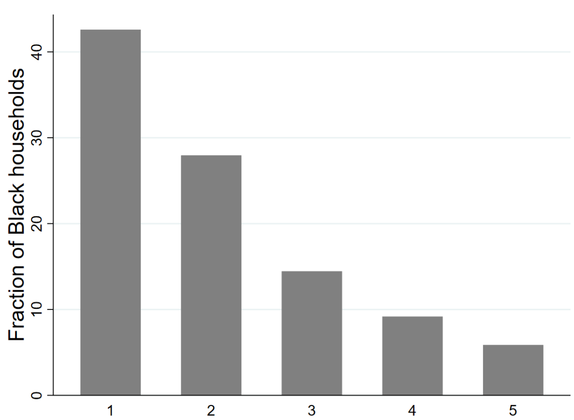
(c) Price-to-income in 1960



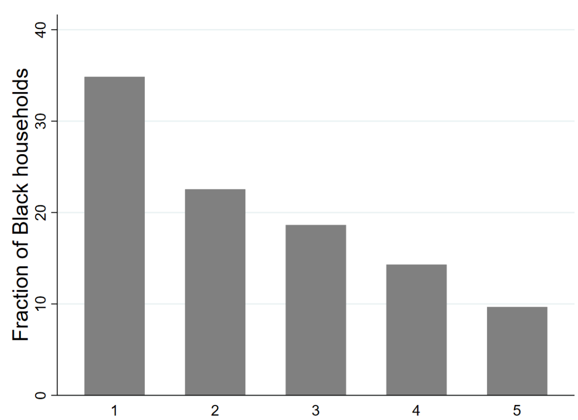
(d) Price-to-income in 2019



(e) Black income distribution in 1960



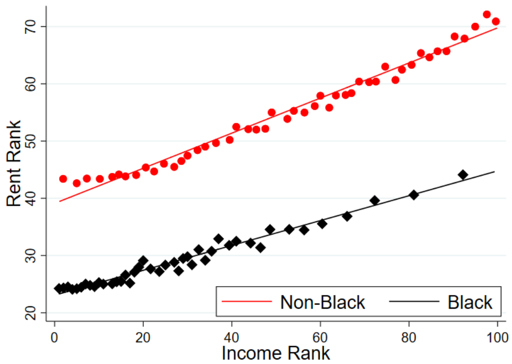
(f) Black income distribution in 2019



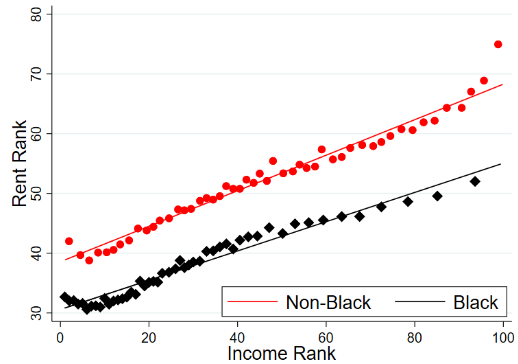
Notes: The top and middle panels show the gaps in rent-to-income share (top) and price-to-income ratio (middle) by income quintile (1 is the lowest quintile). The bottom panel shows the share of Black households in each quintile of the national income distribution. The left panels report 1960 and the right panels report 2019. Estimates of the gaps in rent-to-income share and price-to-income ratio are from a regression on dummies for each income quintile. Data are from Census.

Figure A3: Rent Rank Gap over time with controls

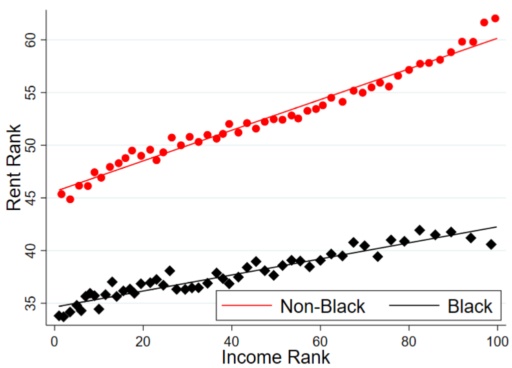
(a) 1940



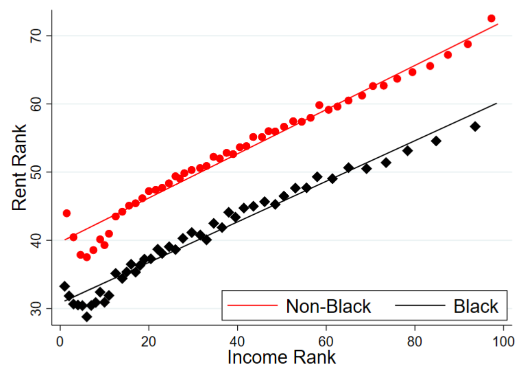
(b) 1960



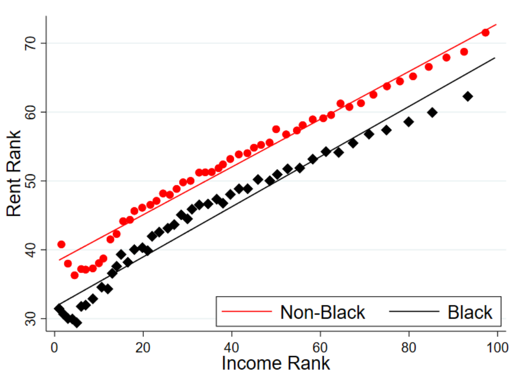
(c) 1970



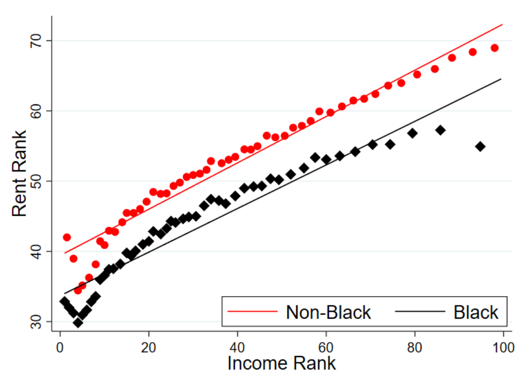
(d) 1980



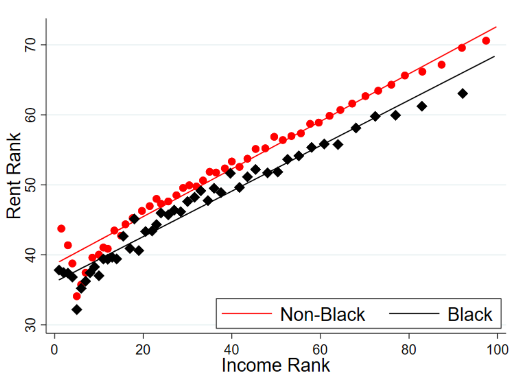
(e) 1990



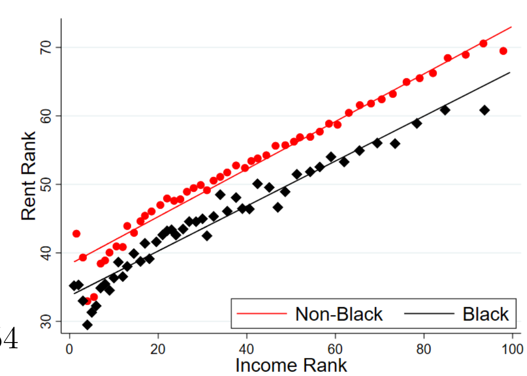
(f) 2000



(g) 2010



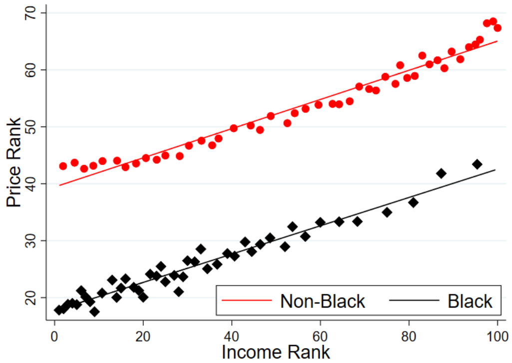
(h) 2019



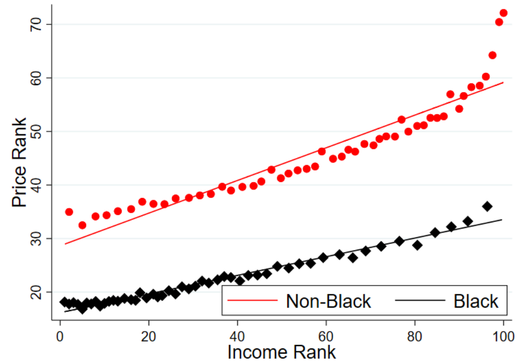
Notes: Data are from Census. There is no Census data on housing for 1950.

Figure A4: House Price Rank Gap over time with controls

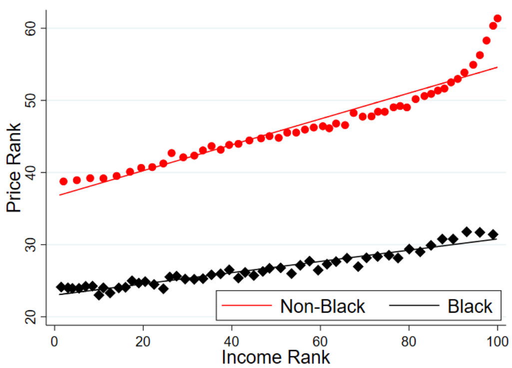
(a) 1940



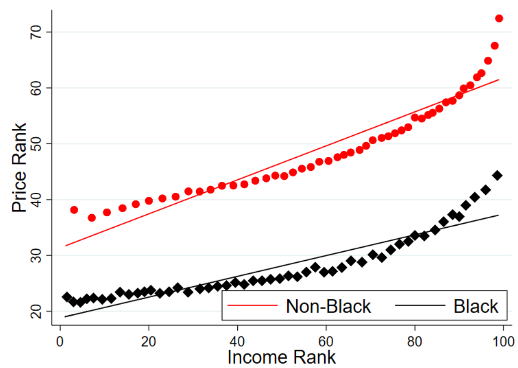
(b) 1960



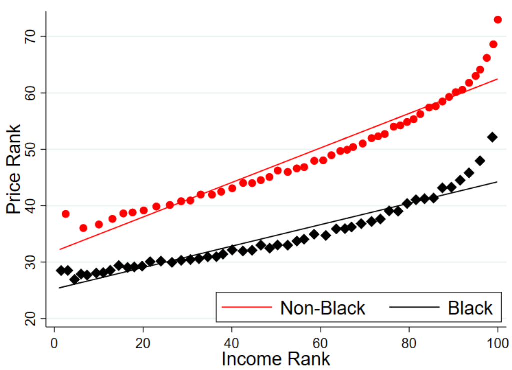
(c) 1970



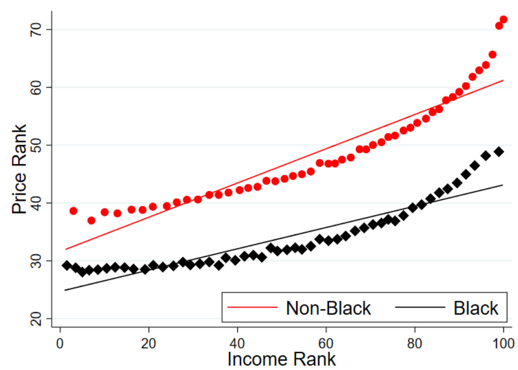
(d) 1980



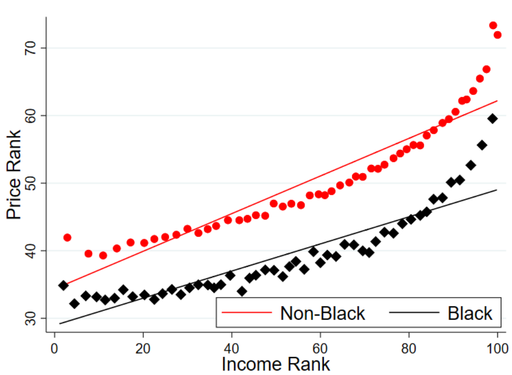
(e) 1990



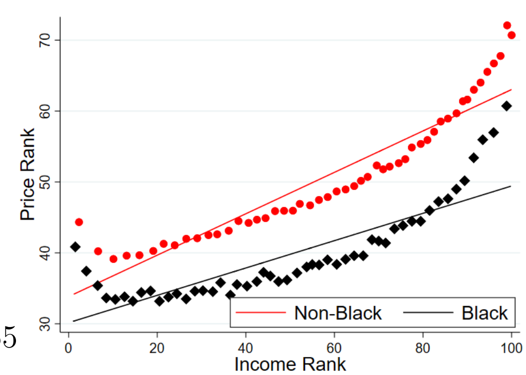
(f) 2000



(g) 2010



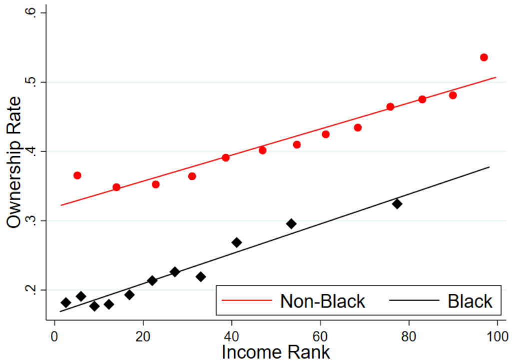
(h) 2019



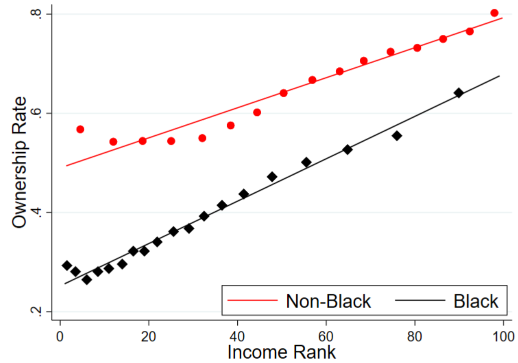
Notes: Data are from Census. There is no Census data on housing for 1950.

Figure A5: Ownership Rate Gap over time with controls

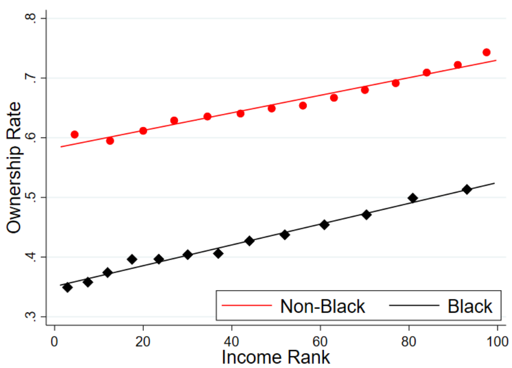
(a) 1940



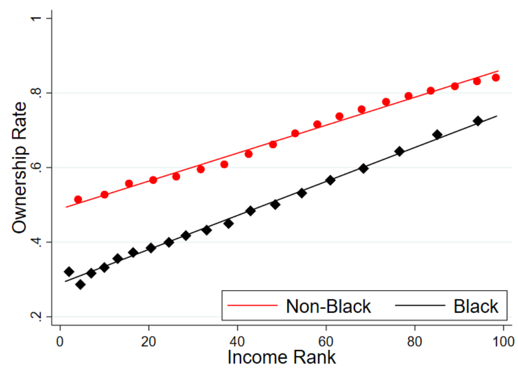
(b) 1960



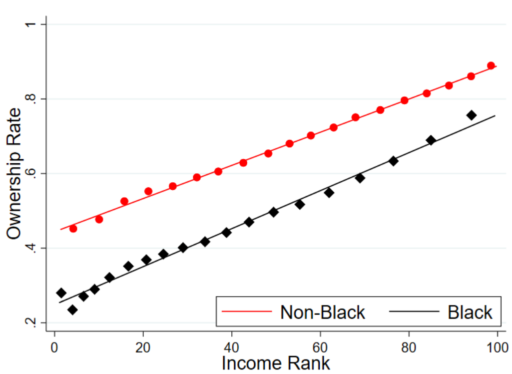
(c) 1970



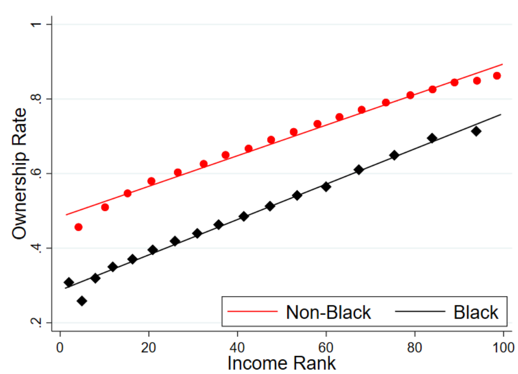
(d) 1980



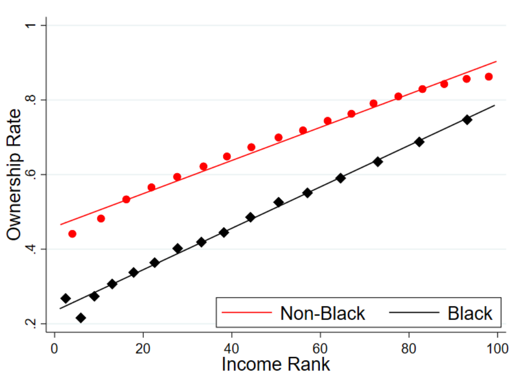
(e) 1990



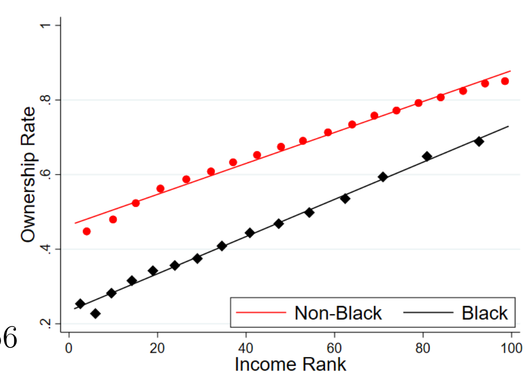
(f) 2000



(g) 2010



(h) 2019



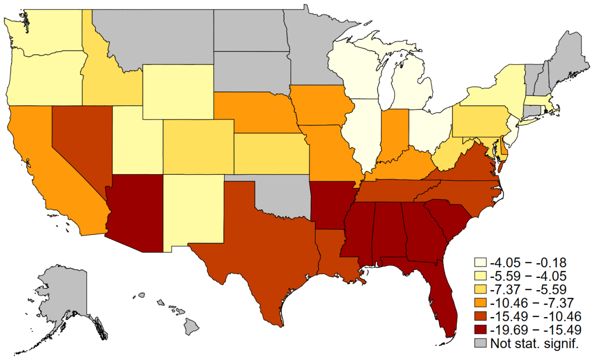
Notes: Data are from Census. There is no Census data on housing for 1950.

A.1 Cross-sectional estimates

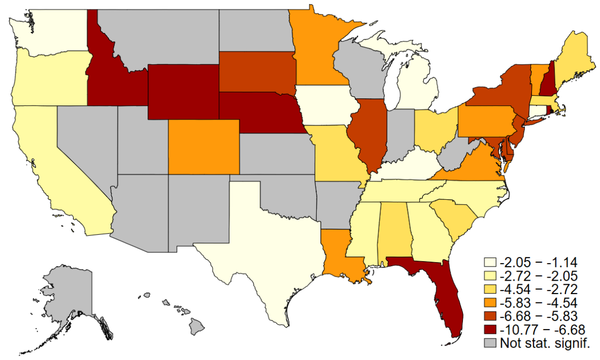
Figure A6: Rent, Price and Ownership Gaps, $\beta_{s,t}$, maps in 1960 and 2019

(a) Rent gap in 2019

1960

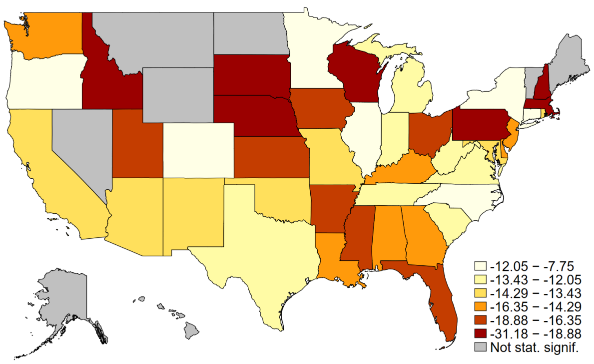


2019

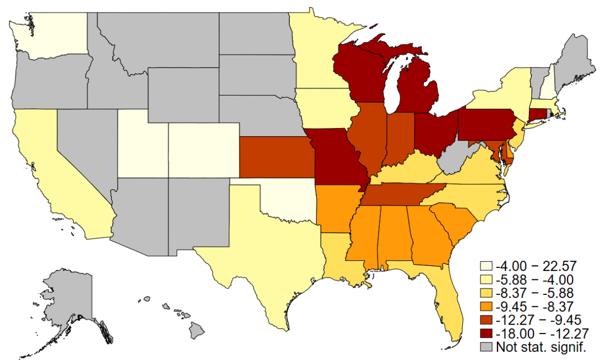


(b) Price gap

1960

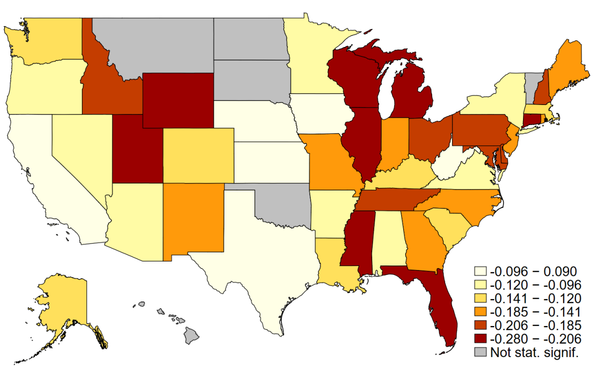


2019

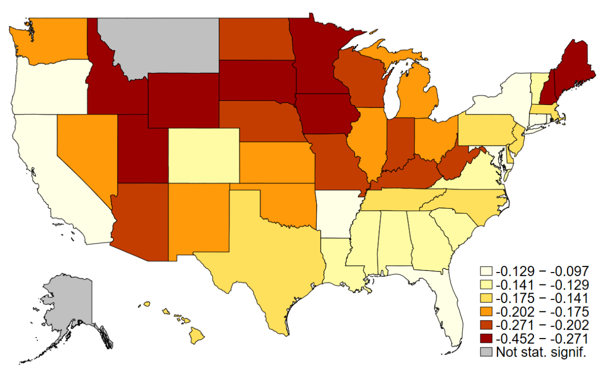


(c) Ownership gap

1960



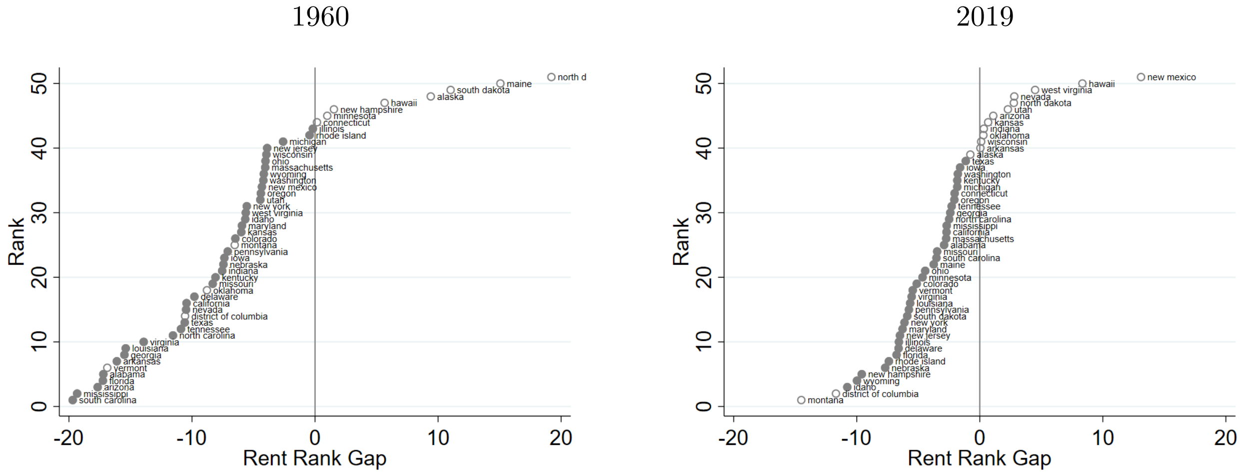
2019



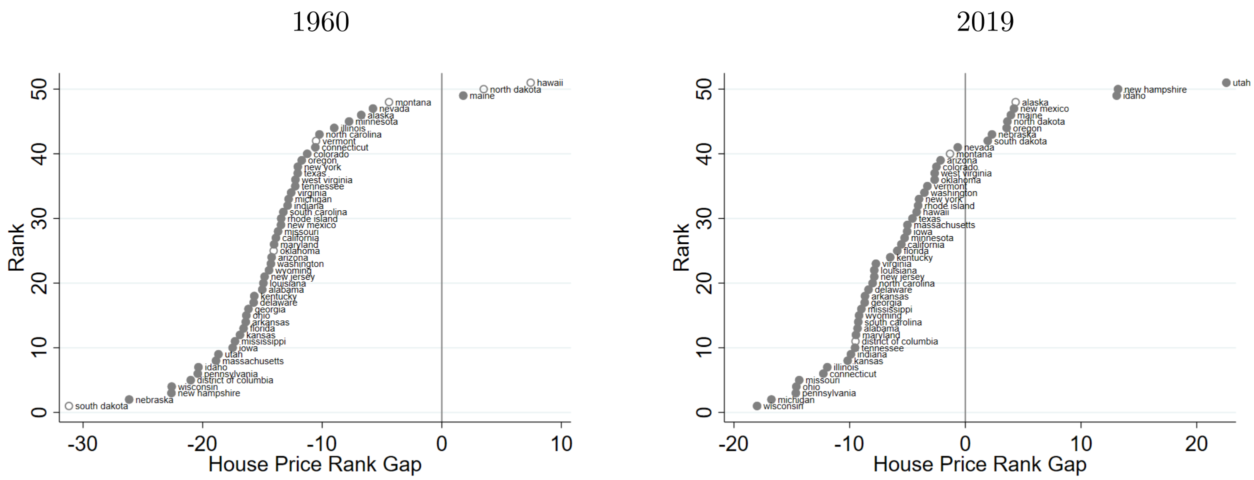
Notes: Figure shows the Rent Rank Gap (top) Price Rank Gap (middle) and Ownership Rate Gap (bottom) for each state in 1960 (left) and 2019 (right). The Rank Gaps are β in equation (1). Estimates that are not statistically significant are gray in the maps.

Figure A7: Rent, Price and Ownership Gaps, $\beta_{s,t}$, across states in 2019

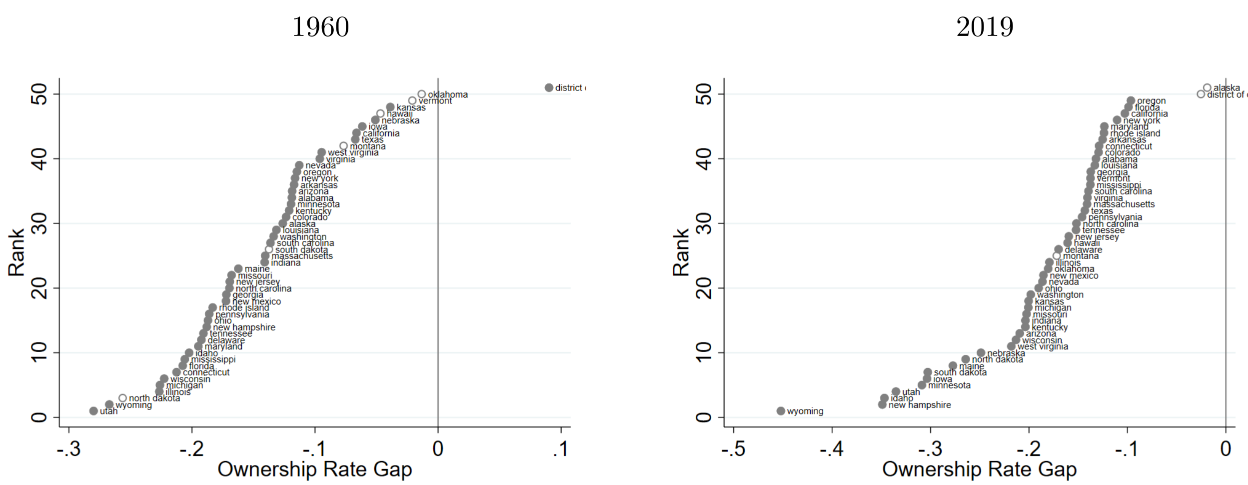
(a) Rent Rank Gap



(b) Price Rank Gap



(c) Ownership Rank Gap



Notes: Figure shows the Rent Rank Gap (top) Price Rank Gap (middle) and Ownership Rate Gap (bottom) for each state in 1960 (left) and 2019 (right). The Rank Gaps are β in equation (1). Estimates that are not statistically significant are hollow dots in the figures.

A.2 Segmentation by Gender and other Racial Groups

Table A1: Gender Differences in the Housing Market

	Rent Rank		Price Rank		Owner Rate	
	1960	2019	1960	2019	1960	2019
Female	4.89***	0.62***	1.24***	-0.74***	-0.01***	-0.00
	(0.07)	(0.10)	(0.08)	(0.06)	(0.00)	(0.00)
Black	-5.57***	-3.35***	-9.51***	-6.89***	-0.13***	-0.15***
	(0.08)	(0.14)	(0.08)	(0.12)	(0.00)	(0.00)
Observations	859596	332216	1460950	902581	2561643	1256319
Adjusted R^2	0.393	0.500	0.459	0.511	0.210	0.291

Notes: Table reports average difference in housing outcomes for Black and female households. All regressions include controls for income rank, household size, farm status, age bins, education and location (state and metropolitan area). Data are from Census.

Table A2: Racial Differences in the Housing Market: Asian American and Pacific Islanders

	Rent Rank		Price Rank		Owner Rate	
	1960	2019	1960	2019	1960	2019
Asian American and PI	-9.97***	-1.88***	-0.03	3.34***	-0.08***	-0.02***
	(0.34)	(0.21)	(0.38)	(0.13)	(0.00)	(0.00)
Black	-5.43***	-3.29***	-9.44***	-6.32***	-0.13***	-0.14***
	(0.08)	(0.14)	(0.08)	(0.12)	(0.00)	(0.00)
Observations	859596	332216	1460950	902581	2561643	1256319
Adjusted R^2	0.405	0.503	0.461	0.516	0.216	0.301

Notes: Table reports average difference in housing outcomes for Asian American and Pacific Islander households, and Black households. All regressions include controls for income rank, household size, farm status, age bins, education and location (state and metropolitan area). Data are from Census.

A.3 Direct estimates of housing quality

Table A3: Differences in Observable Housing Quality

	Age of house						
	1960	1970	1980	1990	2000	2010	2019
Black	3.85***	3.67***	2.45***	2.20***	2.22***	1.41***	0.99***
	(0.03)	(0.04)	(0.03)	(0.03)	(0.03)	(0.06)	(0.07)
Sixth income decile	-1.76***	-1.65***	-2.86***	-1.99***	-2.53***	-3.67***	-3.87***
	(0.04)	(0.05)	(0.04)	(0.04)	(0.04)	(0.09)	(0.09)
Observations	2561642	1245990	3915785	4504267	5148591	1182818	1256319
Adjusted R^2	0.219	0.163	0.168	0.160	0.158	0.167	0.159
Income deciles	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Controls	Yes	Yes	Yes	Yes	Yes	Yes	Yes

Notes: Table reports estimates from regression equation (3). The coefficient labeled Black reports the average difference in observable quality for Black and White households conditional on income. To interpret the magnitude, the coefficient on the sixth income decile (relative to the base group of the second lowest decile) shows the average difference between middle class and poor households. The table includes one measures of observable house quality: the age of the house. Data are from Census.

A.4 Controlling for Permanent Income

This section uses expenditures as a measure of permanent income. Black and White households have different life-cycle age profiles and income risk, therefore I wish to compare households with the same *permanent* income. I follow Charles, Hurst and Roussanov (2009) who estimate the following equation

$$\log(\text{Expenditure Category})_i = \alpha + \beta_1 Black_i + \beta_2(\text{Permanent Income})_i + \Gamma\mathbf{X}_i + \varepsilon_i \tag{35}$$

where the dependent variable is the expenditure of household i in a given category, and X includes a vector of controls (bins for age, households size, the number of children, sex of the household head and year). Due to consumption smoothing, standard models predict that expenditures are better proxy for permanent income than income in a given period, thus the tables below include total expenditures and total non-durable expenditures as proxies for permanent income[15]. However, including a component of expenditures on the left hand side and total expenditures on the

[15] I include non-durable expenditures, in addition to total expenditures both because it is not obvious which is a better measure of permanent income. There is evidence that durable consumption responds more than non-durable consumption to changes in transitory income Souleles (1999); Jappelli and Pistaferri (2010).

righthand side leads to an issue of endogeneity and measurement error. I follow Charles, Hurst and Roussanov (2009) who propose instrumenting for expenditures using income (in logs and a cubic in levels). I use data from the Consumption Expenditure Survey 1980–2003.

Table A4 replicates the finding of Charles, Hurst and Roussanov (2009) that Black households spend more on visible expenditure categories.

Tables A5– A7 confirm that the differences in housing expenditures, house values and ownership rates remain while controlling for permanent income using expenditures. Black households spend 7–18 percent less when conditioning directly on expenditures, and 3–6 percent when using instruments. The gap in the house value of owners (instead of the self-reported rental equivalents) are between 13 and 22 percent when conditioning directly on expenditures, and 6–12 percent when using instruments. The gap in ownership rates is 11–15 percent when conditioning directly on expenditures, and 10–12 percent when using instruments.

Table A4: Conspicuous Consumption Conditional on Permanent Income using CEX data

	Log Visible Expenditures						
Black	-0.05*	0.07***	0.24***	0.17***	0.31***	0.31***	0.23***
	(0.021)	(0.018)	(0.019)	(0.019)	(0.015)	(0.016)	(0.016)
Observations	53412	53412	53412	53412	53412	53412	53412
Adjusted R^2	0.12	0.35	0.28	0.30	0.54	0.47	0.53
Income		Yes	Yes	Yes			
Non-durable expenditures					Yes	Yes	Yes
Total expenditures			Yes	Yes		Yes	Yes
IV		Yes		Yes	Yes		Yes

Notes: Regressions of log permanent income on log visible consumption. Following Charles et al. (2009), visible consumption includes expenditure categories *clothing/jewelry*, *personal care*, and *vehicles*. Permanent income is proxied by income in column 1, non-durable expenditures in columns 2-4 and total expenditures in columns 5-7. Columns 3, 4, 6, and 7 instrument for expenditures using log income, a cubic in the level of income and a dummy for missing income. Controls include bins for age, households size, the number of children, sex of the household head and year. Regressions are weighted using sample weights. Data are from the Consumer Expenditure Survey 1980-2003, using the sample from Aguiar and Hurst (2013).

Table A5: Housing Expenditures Conditional on Permanent Income using CEX data

	Log Housing Expenditures						
Black	-0.26***	-0.18***	-0.10***	-0.06***	-0.07***	-0.07***	-0.03**
	(0.012)	(0.011)	(0.009)	(0.008)	(0.009)	(0.010)	(0.009)
Observations	53412	53412	53412	53412	53412	53412	53412
Adjusted R^2	0.17	0.33	0.58	0.64	0.50	0.46	0.49
Income	Yes						
Non-durable expenditures		Yes	Yes	Yes			
Total expenditures					Yes	Yes	Yes
IV			Yes	Yes		Yes	Yes
Controls		Yes		Yes	Yes		Yes

Notes: Regressions of log permanent income on log housing expenditures. Housing expenditures are the rent paid for renters or the self-reported rental equivalent of the respondent's house for homeowners, as in Aguiar and Hurst (2013). Permanent income is proxied by income in column 1, non-durable expenditures in columns 2-4 and total expenditures in columns 5-7. Columns 3, 4, 6, and 7 instrument for expenditures using log income, a cubic in the level of income and a dummy for missing income. Controls include bins for age, households size, the number of children, sex of the household head and year. Regressions are weighted using sample weights. Data are from the Consumer Expenditure Survey 1980-2003, using the sample from Aguiar and Hurst (2013).

Table A6: Ownership Rates Conditional on Permanent Income using CEX data

	Home ownership dummy						
Black	-0.21***	-0.15***	-0.14***	-0.12***	-0.12***	-0.14***	-0.11***
	(0.008)	(0.007)	(0.008)	(0.007)	(0.007)	(0.008)	(0.007)
Observations	53412	53412	53412	53412	53412	53412	53412
Adjusted R^2	0.08	0.26	0.19	0.29	0.30	0.20	0.30
Income	Yes						
Non-durable expenditures		Yes	Yes	Yes			
Total expenditures					Yes	Yes	Yes
IV			Yes	Yes		Yes	Yes
Controls		Yes		Yes	Yes		Yes

Notes: Regressions of log permanent income on home ownership. Permanent income is proxied by income in column 1, non-durable expenditures in columns 2-4 and total expenditures in columns 5-7. Columns 3, 4, 6, and 7 instrument for expenditures using log income, a cubic in the level of income and a dummy for missing income. Controls include bins for age, households size, the number of children, sex of the household head and year. Data are from the Consumer Expenditure Survey 1980-2003, using the sample from Aguiar and Hurst (2013).

Table A7: House Values Conditional on Permanent Income using CEX data

	Log Housing Value						
Black	-0.27***	-0.22***	-0.15***	-0.12***	-0.13***	-0.11***	-0.06***
	(0.020)	(0.018)	(0.017)	(0.017)	(0.017)	(0.018)	(0.018)
Observations	40514	40514	40514	40514	40514	40514	40514
Adjusted R^2	0.09	0.28	0.34	0.42	0.35	0.31	0.32
Income	Yes						
Non-durable expenditures		Yes	Yes	Yes			
Total expenditures					Yes	Yes	Yes
IV			Yes	Yes		Yes	Yes
Controls		Yes		Yes	Yes		Yes

Notes: Regressions of log permanent income on log house values. House values are the self-reported house values for owners only. Permanent income is proxied by income in column 1, non-durable expenditures in columns 2-4 and total expenditures in columns 5-7. Columns 3, 4, 6, and 7 instrument for expenditures using log income, a cubic in the level of income and a dummy for missing income. Controls include bins for age, households size, the number of children, sex of the household head and year. Regressions are weighted using sample weights. Data are from the Consumer Expenditure Survey 1980-2003, using the sample from Aguiar and Hurst (2013).

B Appendix to Section 4: simplified Model

B.1 Additional details on model characterization

Assignment. To show that $h^*(y)$ is (strictly) increasing in y it is sufficient to show that $U(h, c, y)$ has the (strict) Mirlees-Spence single crossing condition, i.e. that for all (h, c), the ratio $U_h(h, c, y)/U_c(h, c, y)$ is (strictly) increasing in w.

$$U = \log(y - \rho(h)) + \theta \log h \tag{36}$$

$$\frac{U_h}{U_c} = \frac{\theta(y - \rho(h))}{h} \tag{37}$$

$$\frac{d\frac{U_h}{U_c}}{dy} = \frac{\theta}{h} \geq 0 \tag{38}$$

This is strictly positive given the restriction on θ, except where it is undefined at $h = 0$.

An alternative way to see that the assignment is increasing in y. Consider an equilibrium with a strictly increasing price function $\rho(h)$. Take two qualities h and h' with $h' > h$ and thus

$p(h') > \rho(h)$. If h' is preferred for some y then the following inequality holds $\log(y - p(h')) + \theta \log h' > \log(y - \rho(h)) + \theta \log h$

$$\theta(\log h' - \log h) > \log \frac{y - \rho(h)}{w - p(h')} \tag{39}$$

Since $p(h') > \rho(h)$, the right hand side is decreasing in y, thus the inequality also holds for all $y' > y$. Note that the right hand side is also increasing in θ, thus the inequality also holds for all $\theta' > \theta$.

Characterizing the equilibrium. Households choose a single house quality and consumption to solve

$$\begin{aligned} \max\nolimits_{c,h} \quad & \log c + \theta \log h \\ s.t. \quad & y = c + \rho(h) \end{aligned} \tag{40}$$

The Lagrangian is

$$\mathcal{L} = \log c + \theta \log h + \lambda[c + \rho(h) - y] \tag{41}$$

and the first order conditions are

$$\frac{1}{c} = \lambda \qquad\qquad \frac{\theta}{h} = \lambda \rho'(h). \tag{42}$$

Re-arranging and subbing in the budget constraint yields the FOC in the main text

$$\rho'(h) = \theta \frac{y^*(h) - \rho(h)}{h}. \tag{43}$$

B.2 homophily in preferences

This section shows that the baseline model, and inference of quality, is robust to preferences that include homophily. If there is homophily in preferences, the exercise still infers the effective price of quality, where the quality of the home includes both a fundamental component and a component from homophily.

Households choose a single house and consumption to solve

$$\begin{aligned} \max\nolimits_{c,h} \quad & \log c + \theta(\theta_h \log \tilde{h} + \theta_k \log k) \\ s.t. \quad & w = c + \rho(h, k) \end{aligned} \tag{44}$$

where a house is indexed by both its quality $\tilde{h}$ and the share of households that are the same race as the household, k. The weights placed on quality and homophily are θ_h and θ_k.

Within a given racial group, households care about the combined utility from quality and homophily so we can define the combined index as $h = \tilde{h}^{\theta_h} k^{\theta_k}$. Houses with combined quality h provide the same utility, regardless of the composition of underlying combination of $\tilde{h}$ or k. Thus, the rent of the house will only depend on h, $\rho(h)$. The optimization of the household can thus be restated as

$$\begin{array}{rl} \max_{c,\tilde{h}} & \log c + \theta \log h \\ s.t. & w = c + \rho(h). \end{array} \tag{45}$$

This is the same problem as in the main text and thus inference of quality is the same as before. Thus, the addition of homophily leaves the inference exercise unchanged. The counterfactual is different however, and needs to be interpreted as removing the homophily in preferences while leaving the total combined quality of homes h unchanged. An alternative interpretation is that the counterfactual in the paper improves the quality of homes in Black neighbohoods such that the effective price of quality is the same as in the integrated market. This alternative shows that there is still a potential welfare improvement when there is homophily in preferences.

B.3 How does demand change with income

Assuming that the second order condition holds (which will be the case in equilibrium that we consider), then the following statement holds.

Proposition 1. *The change in quality with respect to a change in income is*

$$\frac{dh}{dy} = \frac{\theta}{p''(h)h + (1+\theta)p'(h)} \tag{46}$$

which is equal to $\frac{\theta}{(1+\theta)p}$ *when prices are linear.*

Proof. Totally differentiate the first order condition (8) and rearranging yields the result

$$\begin{aligned} h\rho'(h) &= \theta y - \rho(h) && (47) \\ dh[\rho''(h)h + \rho'(h)] &= \theta dy - \theta\rho'(h)dh && (48) \\ dh[\rho''(h)h + (1+\theta)\rho'(h)] &= \theta dy && (49) \\ \frac{dh}{dy} &= \frac{\theta}{\rho''(h)h + (1+\theta)\rho'(h)} && (50) \end{aligned}$$

□

With linear housing we get the usual Cobb-Douglas relationship, there is a constant expenditure share so h increases at a constant rate. With non-linear prices, the change in h depends on the shape of the non-linearity. When prices are convex, (conditional on having the same slope) additional quality costs more than with linear pricing and thus households increase quality at a slower rate. Conversely when prices are concave, the marginal cost of quality is lower than under linear pricing, and thus households increase quality by more.

Note that these previous statements hold conditional on the same slope; otherwise there are two countervailing forces, the slope and the second derivative.

B.4 How does the expenditure share change with income

Proposition 2. *The change in expenditure with respect to a change in income is*

$$\frac{dp(h)}{dy} = \frac{\theta}{\frac{\rho''(h)\cdot h}{\rho'(h)} + (1+\theta)} \tag{51}$$

which has the usual Cobb-Douglas for when prices are linear, $p(h) = p \cdot h$

$$\frac{dp(h)}{dy} = \frac{\theta}{1+\theta} \tag{52}$$

When $p(h)$ is constant elasticity of the form $p(h) = h^\alpha$, $\alpha \in (0, \infty)$

$$\frac{dp(h)}{dy} = \frac{\theta}{\alpha+\theta} \tag{53}$$

which is less than $\frac{\theta}{1+\theta}$ when convex $\alpha > 1$ or greater than $\frac{\theta}{1+\theta}$ when concave $\alpha \in (0, 1)$.

Comment. The first equation (14) gives the change in price with respect to the change in income. When prices are linear the term $\frac{\rho''(h)\cdot h}{\rho'(h)}$ is zero and we get the usual Cobb-Douglas constant expenditure share (i.e. the change in $p(h)$ with respect to w is constant).

The term $\frac{\rho''(h)\cdot h}{\rho'(h)}$ is the elasticity of marginal price $p'(h)$ with respect to quality. When prices have the power form $p(h) = h^\alpha$ this elasticity is constant, so this is a convenient functional form to focus on.

With the power form and concave prices, households are induced to spend more on housing because the the elasticity of marginal price with respect to quality is lower than linear prices (i.e. they get more bang per buck). To match the declining expenditure shares in the data, we thus expect the price function to be concave where expenditures shares are high (i.e. low quality housing that low income households are consuming) and convex where expenditure shares are lowest (i.e.

higher qualities that high income households are consuming).

Proof. The result obtains from differentiating the first order condition. Expenditures are given by $p(h)$. Recall the FOC is

$$p(h) = w - \frac{1}{\theta} h p'(h) \tag{54}$$

Taking the derivative of the FOC with respect to w

$$\begin{aligned}
\frac{dp(h)}{dy} &= 1 - \frac{1}{\theta}\frac{dh}{dy} p'(h) - \frac{1}{\theta}\frac{dh}{dy} p''(h) h && (55)\\
\frac{dp(h)}{dy} &= 1 - \frac{1}{\theta}\frac{dh}{dy} \left(p'(h) - p''(h) h\right) && (56)
\end{aligned}$$

Plugging in $\frac{dh}{dy}$ from Equation (46) and re-arranging

$$\begin{aligned}
\frac{dp(h)}{dy} &= 1 - \frac{1}{\theta}\frac{\theta}{\rho''(h)h + (1+\theta)\rho'(h)} \left(p'(h) - p''(h)h\right) && (57)\\
\frac{dp(h)}{dy} &= 1 - \frac{p'(h) - p''(h)h}{\rho''(h)h + (1+\theta)\rho'(h)} && (58)\\
\frac{dp(h)}{dy} &= \frac{\theta p'(h)}{\rho''(h)h + (1+\theta)\rho'(h)} && (59)\\
\frac{dp(h)}{dy} &= \frac{\theta}{\rho''(h)\frac{h}{\rho'(h)} + (1+\theta)} && (60)\\
\frac{dp(h)}{dy} &= \frac{\theta}{\varepsilon_\rho + (1+\theta)} && (61)
\end{aligned}$$

where $\varepsilon_\rho = \rho''(h)\frac{h}{\rho'(h)}$.

Let's specialize to constant elasticity function $p(h) = h^\alpha$ with $\alpha \in (0, \infty)$ so it is increasing and concave for $\alpha \in (0,1)$, convex for $\alpha > 1$, and linear for $\alpha = 1$ [16].

$$\begin{aligned}
p(h) &= h^\alpha && (62)\\
p'(h) &= \alpha h^{\alpha-1} && (63)\\
p''(h) &= \alpha(\alpha-1)h^{\alpha-2} && (64)\\
p''(h)\cdot\frac{h}{p'(h)} &= \frac{\alpha(\alpha-1)h^{\alpha-1}}{\alpha h^{\alpha-1}} = \frac{\alpha(\alpha-1)}{\alpha} = \alpha - 1 && (65)
\end{aligned}$$

[16] To see the constant elasticity take logs, $\log p(h) = \alpha \cdot h$, and thus $\frac{d\log p(h)}{d\log} = \alpha h$.

Sub the last line in to Equation (61) gives

$$\frac{dp(h)}{dy} = \frac{\theta}{\alpha + \theta} \tag{66}$$

When linear

$$\frac{dp(h)}{dy} = \frac{\theta}{1 + \theta} \tag{67}$$

When convex $\alpha > 1$

$$\frac{dp(h)}{dy} = \frac{\theta}{\alpha + \theta} < \frac{\theta}{1 + \theta} \tag{68}$$

When concave $\alpha \in (0, 1)$

$$\frac{dp(h)}{dy} = \frac{\theta}{\alpha + \theta} > \frac{\theta}{1 + \theta} \tag{69}$$

$\square$

C Appendix to Section 6: Estimation and Model Fit

C.1 Computation

Solved on a grid with $n_j = 7$ ages, $n_y = 7$ income states, $n_a = 200$ asset grid points. Choices are made on a grid of $n_h = 75$ house quality points and a finer grid of $\tilde{n}_a = 500$ asset grid points. I parallelize the computation of value functions and choices across the income and asset grid points using a graphics processing unit (GPU) (Aldrich, Fernández-Villaverde, Ronald Gallant and Rubio-Ramírez, 2011; Fernández-Villaverde and Valencia, 2018). It takes approximately 10 seconds to solve the steady state on an laptop computer (2020 ThinkPad X1 Extreme with a Nvidia GeForce GTX 1650 GPU), or 2 seconds on a node with 16 cores and a Nvidia Telsa A100 GPU.

C.2 Survival Probabilities

Table C1: Estimated Conditional Survival Probability, $Pr(j+1|j=j)$

	1960		2019	
Age	**White**	**Black**	**White**	**Black**
25	0.99	0.97	0.99	0.98
35	0.97	0.94	0.98	0.97
45	0.93	0.87	0.96	0.94
55	0.85	0.76	0.91	0.87
65	0.68	0.62	0.83	0.77
75	0.38	0.44	0.61	0.57
85	0.08	0.12	0.22	0.25

Notes: Source: Life Tables 1960 and 2019. The model assumes that all agents die before age 95 so the final conditional probability is set to zero in the model.

Table C2: Estimated Total Survival Probability, $Pr(j|j=0)$

	1960		2019	
Age	**White**	**Black**	**White**	**Black**
25	1.00	1.00	1.00	1.00
35	0.99	0.97	0.99	0.98
45	0.96	0.91	0.97	0.95
55	0.90	0.80	0.93	0.90
65	0.76	0.60	0.85	0.78
75	0.52	0.37	0.71	0.61
85	0.20	0.17	0.43	0.35
95	0.02	0.02	0.10	0.09

Notes: Source: Life Tables 1960 and 2019.

C.3 Income profiles

I estimating lifecycle income processes for Black and White households using Census data.

$$\log(income)_i = \alpha_t + \beta_0 age_i + \beta_1 age_i^2 \tag{70}$$

$$+\beta_2(Black_i \times age_i) + \beta_3(Black_i \times age_i^2) + \Gamma X_i + \epsilon_i \tag{71}$$

Controls include fixed effects for education, sex of household head, household type, a farm indicator, and year. I do not observe a panel so we cannot include individual fixed effects. Robust standard errors are reported.

Table C3: Lifecycle income profile, pooled

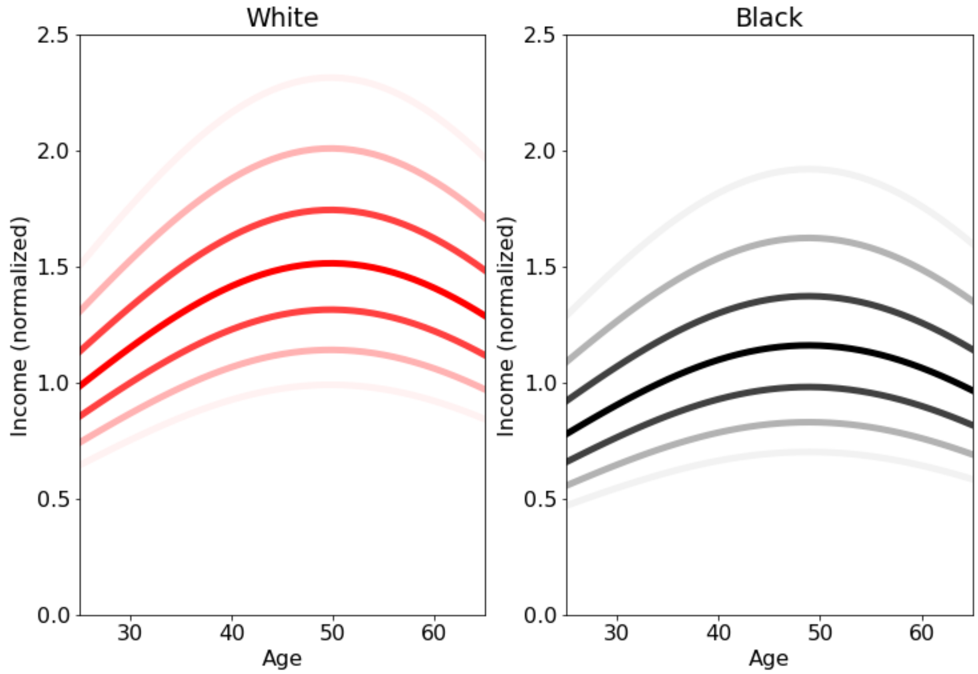
	1940-1960	1970-1990	2000-2019	1940-2019
	Log income	Log income	Log income	Log income
Constant	7.03408***	8.30511***	9.25354***	6.49865***
	(0.053314)	(0.005481)	(0.009900)	(0.058217)
Age	0.06140***	0.07105***	0.07149***	0.06982***
	(0.000658)	(0.000257)	(0.000450)	(0.000244)
Age squared	-0.00062***	-0.00072***	-0.00073***	-0.00070***
	(0.000007)	(0.000003)	(0.000005)	(0.000003)
Black=1	-0.33693***	-0.29629***	-0.13807***	-0.19417***
	(0.049338)	(0.020153)	(0.033576)	(0.019046)
Black=1 × Age	-0.00233	0.00600***	-0.00400**	-0.00136
	(0.002335)	(0.000960)	(0.001537)	(0.000890)
Black=1 × Age squared	-0.00004	-0.00008***	0.00005**	0.00000
	(0.000026)	(0.000011)	(0.000017)	(0.000010)
St.dev residuals (White)	.8234128937404255	.6670915566495002	.7888817576533299	.7549780391459388
St.dev residuals (Black)	.9082839037487705	.8465072876743432	.8940252011238707	.8902765405831011
Observations	2453924	7208779	5576812	15239515
Adjusted R^2	0.509	0.336	0.350	0.704
Controls	Yes	Yes	Yes	Yes

Table C4: Lifecycle income profile, estimated decade by decade

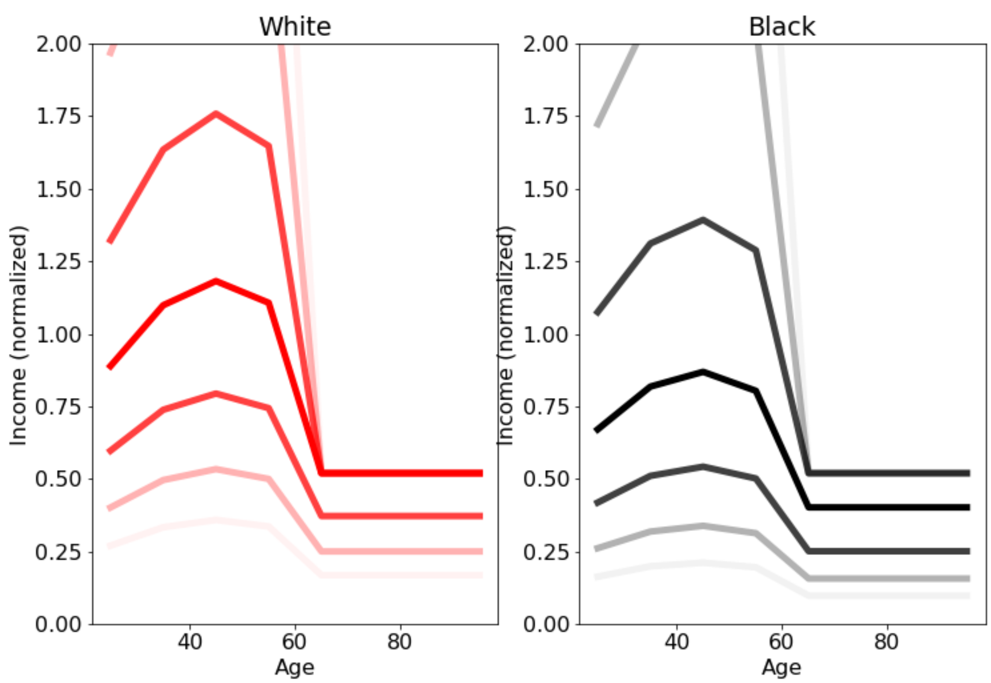
	1940	1950	1960	1970	1980	1990	2000	2010	2019
Constant	6.06071***	7.38947***	8.04731***	8.69451***	8.08506***	8.75468***	9.22339***	9.40212***	9.67677***
	(0.030231)	(0.042443)	(0.052848)	(0.010372)	(0.006598)	(0.007471)	(0.007617)	(0.018991)	(0.020851)
Age	0.06037***	0.02982***	0.07353***	0.05118***	0.08658***	0.07897***	0.07309***	0.07112***	0.07042***
	(0.001455)	(0.002028)	(0.000384)	(0.000492)	(0.000314)	(0.000349)	(0.000352)	(0.000852)	(0.000943)
Age squared	-0.00059***	-0.00029***	-0.00075***	-0.00052***	-0.00089***	-0.00080***	-0.00075***	-0.00072***	-0.00072***
	(0.000017)	(0.000023)	(0.000004)	(0.000006)	(0.000004)	(0.000004)	(0.000004)	(0.000009)	(0.000010)
Black=1	-0.32531**	-0.50521**	-0.15844***	0.17705***	-0.22247***	-0.79814***	0.04982	-0.18302**	-0.14817*
	(0.104630)	(0.158655)	(0.032529)	(0.038635)	(0.024114)	(0.029444)	(0.026271)	(0.060895)	(0.069714)
Black=1 × Age	-0.00902	0.01053	-0.00813***	-0.01192***	0.00211	0.02304***	-0.01388***	-0.00277	-0.00250
	(0.005034)	(0.007452)	(0.001540)	(0.001842)	(0.001154)	(0.001378)	(0.001233)	(0.002765)	(0.003168)
Black=1 × Age squared	0.00005	-0.00021*	0.00002	0.00009***	-0.00005***	-0.00022***	0.00017***	0.00005	0.00002
	(0.000058)	(0.000084)	(0.000017)	(0.000021)	(0.000013)	(0.000015)	(0.000014)	(0.000030)	(0.000034)
St.dev residuals (White)	.8640457024375866	.9849114708352739	.6813765944201086	.6049458807127912	.6757384990506278	.7149153341216834	.7604325302675732	.7910327326630097	.8102425530330527
St.dev residuals (Black)	.8894858126580275	1.011967609615802	.8510948712143706	.7381188588875631	.8568733477491958	.9056953418776702	.9173731903675859	.8687219558329433	.8968936313467618
Observations	231472	170434	2052018	944585	2913850	3350344	3865316	862321	849175
Adjusted R^2	0.254	0.199	0.301	0.164	0.309	0.329	0.315	0.335	0.337
Controls	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes

Figure C1: Lifecycle Income Profiles

(a) Estimated Lifecycle Income Profiles, annual frequency



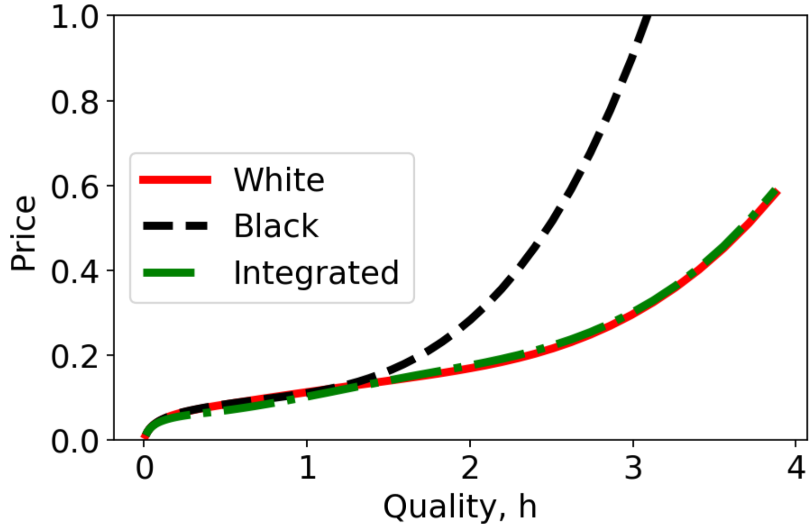
(b) Model Lifecycle Income Profiles with Retirement, ten year frequency



Notes: Income is normalized so that the income of the youngest (age 25) White household in the central income state equals one. Shading represents the share of each income state in the stationary income distribution.

D Appendix to Section 7

Figure D1: Rents in Black, White and Integrated Markets



Notes: Figure plots the rent-quality function in the Black, White and integrated market . The dashed black line is the price in the Black segmented market, the solid red line is the price in the White segmented market, and the dot-dashed green line is the price in the integrated market.