

## Confined aquifer vulnerability induced by a pumping well in a leakage area

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**Abstract** Due to the pollution of shallow groundwater and the rapid development of society and economy which consume more freshwater, the exploitation of confined groundwater is steadily increasing in north China. Therefore, the rapid decline of the confined groundwater head increases the risk of confined aquifer pollution by leaky recharge from shallow aquifers. In this paper, a quantitative method for assessing confined aquifer vulnerability to contamination due to pumping has been developed. This method is based on the shallow and confined groundwater flow model and the advection and dispersion in the aquitard, including sorption. The cumulative time for the pollutant concentration at the top boundary of confined aquifer exceeding the maximum allowable level is defined as the confined aquifer vulnerability index, which can be obtained by numerically solving the solute transport equation. A hypothetical example is chosen as a case study to illustrate the whole process. The results indicate that the proposed method is a practical and reasonable assessment method of confined aquifer vulnerability.

**Key words** vulnerability; confined aquifer; leaky; solute transport

### 1 INTRODUCTION

Groundwater has served as a cheap and reliable source for water supply for a long time in many parts of the world. In the USA, for example, approximately half of the population is dependent on groundwater sources for its domestic water (US EPA 1987). This valuable groundwater is, however, vulnerable, which demands the critical protection from pollution for both shallow and confined aquifers. In some areas of the world and especially some urban areas, the shallow aquifers have, unfortunately, already been polluted due to improper water-related activities. According to the recent water resources assessment conducted by the Chinese government, for example, half of shallow aquifers in Hai River basin are polluted with the water quality grade below level IV, of which 26% is induced by human activities. The pollution of shallow aquifers forces people to exploit deep groundwater residing in confined aquifers, especially in the hyper-dry years, leading to decline of the confined groundwater head and thus an increasing trend of leakage from polluted shallow unconfined aquifers to the deeper confined aquifers. The leakage further increases the risk of confined groundwater being polluted. Under such situations, the groundwater protection should be strengthened, which requires knowledge of the groundwater vulnerability for both the shallow aquifer and the confined aquifer.

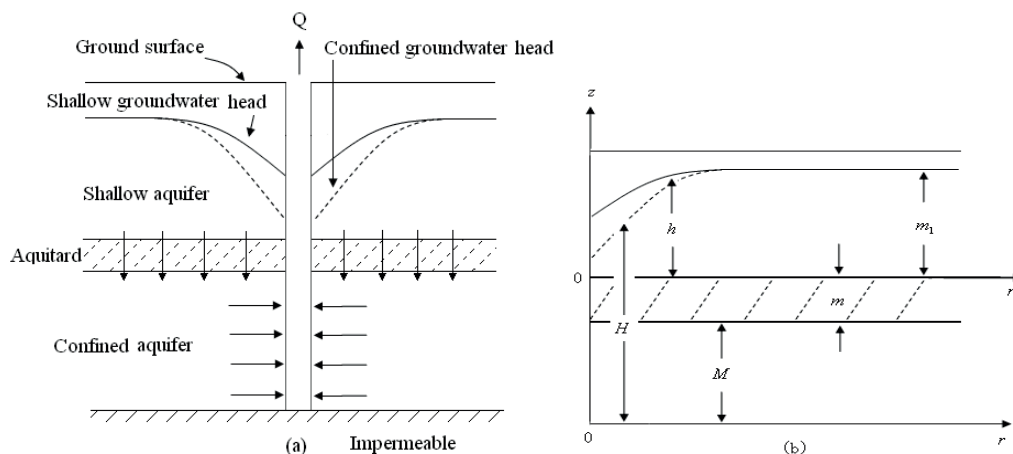
The delineation of the protection area for a pumping well or wellfield is the purpose of well vulnerability assessment. The conventional concept for pumping well protection is that of the wellhead protection area (WHPA) (US EPA 1987), which is determined commonly by the time needed for the pollutant to reach the well (time criteria). The WHPA is also known as the source protection zone in Europe (NRA 1992). For persistent chemicals with very long degradation periods, the WHPA will be similar to the well capture zone, which is defined as the zone around a well contributing water to the well. Bear and Jacobs (1965) derived an analytical solution for the boundaries of time-related capture zones in unbounded homogeneous and isotropic domains. Since then, much research has been done on capture zone delineation, among which are Javandel and Tsang (1986), Lerner (1992), Kinzelbach *et al.* (1992), Faybishenko *et al.* (1995), Bakker and Strack (1996), Bair and Lahm (1996), Zlotnik (1997), Frind *et al.* (2002), Christ and Goltz (2002), Kompani-Zare *et al.* (2005), Fienen *et al.* (2005) and Indelman *et al.* (2006). Most of these works

were based on the advective transport equation, ignoring the contaminant dispersion, and therefore, their assessments for well vulnerability are independent of any particular contaminant.

In this study, a process based method to delineate the confined aquifer vulnerability due to pumping is proposed. For the case of negligible regional flow, a simplified groundwater flow and solute transport model with a single pumping well is established. The cumulative time for the contaminant concentration at the top boundary of confined aquifer to exceed the maximum allowable level is defined as the vulnerability index and obtained by a numerical model.

## 2 GENERAL STATEMENT OF THE PROBLEM

A new confined groundwater vulnerability index is defined as the cumulative time for the pollutant concentration at the top boundary of confined aquifer to exceed the maximum allowable level, with which the protected zone for the confined aquifer pumping well can be delineated. The cumulative time is obtained by the establishment of the simplified flow and transport system consisting of a single, fully penetrating confined aquifer pumping well in an infinite flow domain with homogeneous, isotropic aquifers and an aquitard (Fig. 1). It is assumed that pumping at the well dominates the local hydraulics with a zero regional gradient and the confined and shallow groundwater heads are identical before pumping.



**Fig. 1** Schematic of a single confined aquifer pumping well shown in vertical profile (a) and its mathematical representation (b).

The impervious base of the confined aquifer, and the bases of the aquitard and shallow aquifer are presumed to be at constant elevation. For the mathematical description of confined groundwater movement, the point of coordinate origin is placed at the impervious base of the confined aquifer. And the governing equation for confined groundwater flow in cylindrical coordinates is expressed as (Bear 1979):

$$\frac{\partial^2 H}{\partial r^2} + \frac{1}{r} \frac{\partial H}{\partial r} + \frac{h + m + M - H}{B^2} = \frac{S}{T} \frac{\partial H}{\partial t} \tag{1}$$

where  $H$  and  $h$  are the confined groundwater head and shallow groundwater head respectively;  $m$  and  $M$  are the thickness of aquitard and confined aquifer respectively;  $T = KM$  is the confined aquifer transmissivity;  $S = S_s M$  is the storativity of the confined aquifer;  $S_s$  is the specific storage of the confined aquifer;  $t$  is time;  $K$  is the hydraulic conductivity of confined aquifer;  $B = \sqrt{\frac{Tm}{K_v}}$  is the leakage factor;  $K_v$  is the vertical hydraulic conductivity of aquitard; and  $r$  is the horizontal distance from the pumping well.

The initial and boundary conditions are specified as follows:

$$H(r,0) = H_0, \quad H(+\infty,t) = H_0, \quad \lim_{r \rightarrow 0} \left( 2\pi r T \frac{\partial H}{\partial r} \right) = Q \quad (2)$$

where  $Q$  is the pumping rate;  $H_0$  is the initial confined groundwater head.

For the shallow groundwater movement, the point of coordinate origin is placed at the base of the shallow aquifer. Using the Dupuit assumptions, the governing equation of shallow groundwater flow in cylindrical coordinates is expressed as (Bear 1979):

$$h \frac{\partial^2 h}{\partial r^2} + \frac{h}{r} \frac{\partial h}{\partial r} - \frac{h+m+M-H}{m} \frac{K_v}{K_1} = \frac{\mu_s}{K_1} \frac{\partial h}{\partial t} \quad (3)$$

where  $K_1$  and  $\mu_s$  are the hydraulic conductivity and the specific yield of the shallow aquifer respectively.

The initial condition and boundary conditions are specified as follows:

$$h(r,0) = H_0 - (m+M), \quad h(+\infty,t) = H_0 - (m+M), \quad \frac{\partial h}{\partial r}(0,t) = 0 \quad (4)$$

The transport equation of the stable and non-reactive pollutant in the aquitard including sorption is expressed as (Bear 1979):

$$R_1 \frac{\partial c}{\partial t} = D_{L1} \frac{\partial^2 c}{\partial z^2} + D_T \left( \frac{\partial^2 c}{\partial r^2} + \frac{1}{r} \frac{\partial c}{\partial r} \right) - u_1 \frac{\partial c}{\partial z} \quad (5)$$

where  $c$  is the pollutant concentration in the aquitard;  $z$  is the height from the base of the confined aquifer;  $D_{L1} = \alpha_1 u_1$  is the hydrodynamic dispersion coefficient;  $\alpha_1$  is the aquitard dispersion along  $z$  direction;  $u_1 = K_v \frac{h+m+M-H}{m} \frac{1}{n_1}$  is the pore water velocity;  $n_1$  is the aquitard effective porosity;

$R_1 = 1 + \frac{\rho_1}{n_1} K_{d1}$  is the aquitard retardation factor;  $\rho_1$  and  $K_{d1}$  are the aquitard bulk density and the distribution coefficient of species respectively;  $D_T$  is the aquitard dispersion coefficient along  $r$  direction; and it is assumed that  $D_T = \alpha D_{L1}$ .

To make the problem simpler, we assume that the pollutant concentration in the aquitard and confined aquifer before pumping is zero. And the pollutant concentration in the shallow aquifer is constant equal to  $c_0$ . The initial condition and boundary conditions can be expressed as follows:

$$c(r,z,0) = 0, \quad c(r,m+M,t) = c_0, \quad \frac{\partial^2 c}{\partial z^2}(r,M,t) = 0, \quad \frac{\partial c}{\partial r}(0,z,t) = 0, \quad \frac{\partial c}{\partial r}(+\infty,z,t) = 0 \quad (6)$$

The transport equation of the stable and non-reactive pollutant in the confined aquifer including sorption is expressed as (Bear 1979):

$$R_2 \frac{\partial C}{\partial t} = D_{L2} \left( \frac{\partial^2 C}{\partial r^2} + \frac{1}{r} \frac{\partial C}{\partial r} \right) + D_V \frac{\partial^2 C}{\partial z^2} - u_2 \frac{\partial C}{\partial r} - \frac{1}{r} C u_2 - C \frac{\partial u_2}{\partial r} \quad (7)$$

where  $C$  is the pollutant concentration in the confined aquifer;  $R_2 = 1 + \frac{\rho_2}{n_2} K_{d2}$  is the confined aquifer retardation factor;  $\rho_2$  and  $K_{d2}$  are the confined aquifer bulk density and distribution coefficient of species respectively;  $D_{L2} = \alpha_2 u_2$  is the hydrodynamic dispersion coefficient of confined aquifer;  $\alpha_2$  is the confined aquifer dispersion along  $r$  direction;  $u_2 = -K_2 \frac{\partial H}{\partial r} \frac{1}{n_2}$  is the pore water velocity;  $K_2$  is the hydraulic conductivity of confined aquifer;  $n_2$  is the effective porosity of confined aquifer; and  $D_V$  is the dispersion coefficient of confined aquifer along  $z$  direction; and it is assumed that  $D_V = \beta D_{L2}$ .

The initial condition and boundary conditions can be expressed as follows:

$$C(r, z, 0) = 0, \frac{\partial C}{\partial z}(r, 0, t) = 0, C(r, M, t) = C_0(t), \frac{\partial^2 C}{\partial r^2}(0, z, t) = 0, \frac{\partial C}{\partial r}(+\infty, z, t) = 0 \quad (8)$$

where  $C_0(t)$  is pollutant concentration at the base of the aquitard.

### 3 METHODOLOGY AND PROCEDURE

In this study, the approximation of the first derivative with respect to  $x_i$  of the function  $F(x_1, x_2, \dots, x_n)$  is given by:

$$\frac{\partial F}{\partial x_i} \approx \frac{F(x_1, x_2, \dots, x_i + \Delta x_i, \dots, x_n) - F(x_1, x_2, \dots, x_i - \Delta x_i, \dots, x_n)}{2\Delta x_i} \quad (9)$$

And the approximation of the second derivative with respect to  $x_i$  of the function  $F(x_1, x_2, \dots, x_n)$  is:

$$\frac{\partial^2 F}{\partial x_i^2} \approx \frac{F(x_1, x_2, \dots, x_i + \Delta x_i, \dots, x_n) - 2F(x_1, x_2, \dots, x_i, \dots, x_n) + F(x_1, x_2, \dots, x_i - \Delta x_i, \dots, x_n)}{(\Delta x_i)^2} \quad (10)$$

In the whole process, a fully implicit (first order) approximation of the temporal derivative is adopted because of its unconditional stability (Herrera and Valocchi 2005). The application to equation (1) gives the following expression that relates the confined groundwater head at time levels  $k$  and  $k+1$

$$A_1 H_{i-1}^{k+1} + B_1 H_i^{k+1} + C_1 H_{i+1}^{k+1} + D_1 h_i^{k+1} + E_1 = F_1 H_i^k \quad (11)$$

where  $i$  is the node number along  $r$  direction. The coefficients  $A_1, B_1, C_1, D_1, E_1, F_1$  are given by:

$$A_1 = -\frac{1}{(\Delta r)^2} \quad B_1 = \frac{S}{T\Delta t} + \frac{2}{(\Delta r)^2} + \frac{1}{r_i \Delta r} + \frac{1}{B^2} \quad C_1 = -\frac{1}{(\Delta r)^2} - \frac{1}{r_i \Delta r} \quad D_1 = -\frac{1}{B^2} \quad E_1 = -\frac{m+M}{B^2} \quad F_1 = \frac{S}{T\Delta t} \quad (12)$$

Application to equation (3) gives the following expression that relates the shallow groundwater head at time levels  $k$  and  $k+1$

$$A_2 h_{i-1}^{k+1} + B_2 h_i^{k+1} + C_2 h_{i+1}^{k+1} + D_2 H_i^{k+1} + E_2 = F_2 h_i^k \quad (13)$$

where the coefficients  $A_2, B_2, C_2, D_2, E_2, F_2$  are given by:

$$A_2 = -\frac{1}{(\Delta r)^2} \quad B_2 = \frac{\mu_s}{K_1 h_i^k \Delta t} + \frac{2}{(\Delta r)^2} + \frac{1}{r_i \Delta r} + \frac{K_v}{K_1 m h_i^k} \quad C_2 = -\frac{1}{(\Delta r)^2} - \frac{1}{r_i \Delta r} \quad D_2 = -\frac{K_v}{K_1 m h_i^k} \quad E_2 = \frac{K_v(m+M)}{K_1 m h_i^k} \quad F_2 = \frac{\mu_s}{K_1 h_i^k \Delta t} \quad (14)$$

Application to equation (5) gives the following expression that relates the pollutant concentration in aquitard at time levels  $k$  and  $k+1$ :

$$A_3 c_{i,j,k+1} + B_3 c_{i,j+1,k+1} + C_3 c_{i,j-1,k+1} + D_3 c_{i+1,j,k+1} + E_3 c_{i-1,j,k+1} = F_3 c_{i,j,k} \quad (15)$$

where  $i$  and  $j$  are the node numbers along  $r$  and  $z$  directions respectively. The coefficients  $A_3, B_3, C_3, D_3, E_3, F_3$  are given by:

$$A_3 = \frac{R_1}{\Delta t} + \frac{2D_{L1}}{(\Delta z)^2} + \frac{2D_T}{(\Delta r)^2} + \frac{D_T}{r_i \Delta r} - \frac{u_{i,j,k+1}}{\Delta z} \quad B_3 = -\frac{D_{L1}}{(\Delta z)^2} + \frac{u_{i,j,k+1}}{\Delta z} \quad C_3 = -\frac{D_{L1}}{(\Delta z)^2} \quad D_3 = -\frac{D_T}{(\Delta r)^2} - \frac{D_T}{r_i \Delta r} \quad E_3 = -\frac{D_T}{(\Delta r)^2} \quad F_3 = \frac{R_1}{\Delta t} \quad (16)$$

Application to equation (7) gives the following expression that relates the pollutant concentration in confined aquifer at time levels  $k$  and  $k+1$ :

$$A_4 C_{i,j,k+1} + B_4 C_{i+1,j,k+1} + C_4 C_{i-1,j,k+1} + D_4 C_{i,j+1,k+1} + E_4 C_{i,j-1,k+1} = F_4 C_{i,j,k} \tag{17}$$

where the coefficients  $A_4, B_4, C_4, D_4, E_4, F_4$  are given by:

$$A_4 = \frac{R_2}{\Delta t} + \frac{D_{L2}}{r_i \Delta r} + \frac{2D_{L2}}{(\Delta r)^2} + \frac{2D_V}{(\Delta z)^2} - \frac{u_{2,i,j,k+1}}{\Delta r} + \frac{u_{2,i,j,k+1}}{r_i} + \frac{u_{2,i+1,j,k+1} - u_{2,i,j,k+1}}{\Delta r}$$

$$B_4 = -\frac{D_{L2}}{r_i \Delta r} - \frac{D_{L2}}{(\Delta r)^2} + \frac{u_{2,i,j,k+1}}{\Delta r} \quad C_4 = -\frac{D_{L2}}{(\Delta r)^2} \quad D_4 = -\frac{D_V}{(\Delta z)^2} \quad E_4 = -\frac{D_V}{(\Delta z)^2} \quad F_4 = \frac{R_2}{\Delta t} \tag{18}$$

By solving equations (11), (13), (15), and (17), the confined aquifer vulnerability index at different distances from the pumping well is obtained.

### 4 RESULTS AND DISCUSSION

A hypothetical case is used to illustrate the procedure for confined aquifer vulnerability for a pumping well in a leakage area. Parameters involved in the groundwater flow and transport model are listed in Table 1 and the nitrate ( $NO_3^-$ ) is used as the assessment factor. Because nitrate is a highly mobile species with little sorption, the retardation coefficient is assumed to be 1 (Shamrukh et al. 2001). In this study, the highest allowable concentration of the nitrate ( $NO_3^-$ ) is 20 mg/L (to conform to the Grade III groundwater quality standards GB/T14848-93) and the classification of the confined aquifer vulnerability degree is presented in Table 2. Using equations (11)–(18), the vulnerability index ( $Tv$ ) is obtained at different distances away from the pumping well (Fig. 2).

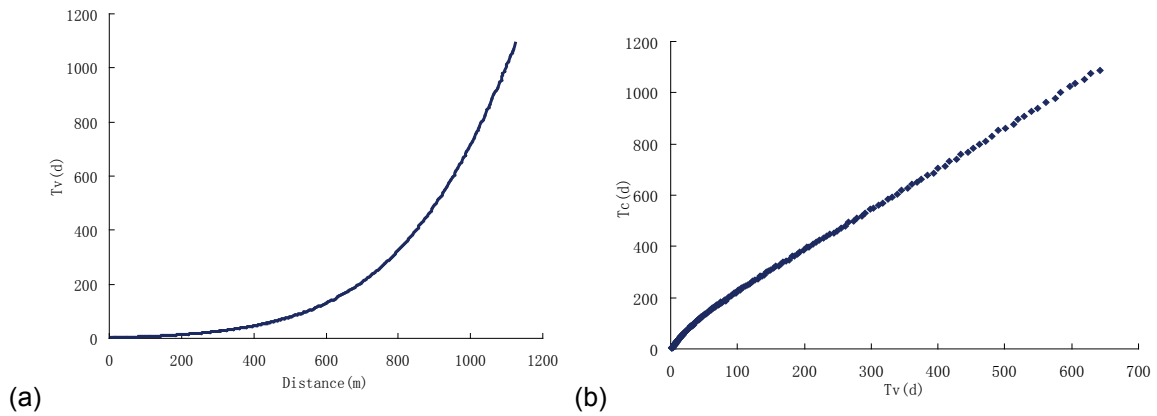


Fig. 2 (a) Index of confined aquifer vulnerability, and (b) Relationship between  $Tc$  and  $Tv$ .

Table 1 Parameter values used in the groundwater flow and transport model.

Parameter	$S_s$ ( $m^{-1}$ )	$K$ (m/d)	$K_1$ (m/d)	$K_v$ (m/d)	$M$ (m)	$m$ (m)
Value	$1 \times 10^{-5}$	10	15	0.05	50	5
Parameter	$\alpha$	$n_1$	$c_0$ (mg/L)( $NO_3^-$ )	$R_1$	$\alpha_1$ (m)	$\beta$
Value	0.1	0.03	100	1	0.05	0.2
Parameter	$R_2$	$\alpha_2$ (m)	$n_2$	$\mu_s$	$m_1$ (m)	$Q$ ( $m^3/d$ )
Value	1	1	0.2	0.15	50	5000

Table 2 Vulnerability degree classification.

$T_v$ (d)	<365	365–730	730–1095	>1095
Vulnerability degree	High	Moderate	Moderately low	Low

To further verify the reasonableness of the vulnerability index ( $Tv$ ), the nitrate concentration of confined groundwater is calculated by equations (11)–(18). The cumulative time ( $Tc$ ) for the

average nitrate concentration of the confined groundwater along direction  $z$  to exceed 5 mg/L (to conform to the Grade II groundwater quality standards GB/T14848-93) is obtained. Through drawing the relationship between  $T_c$  and  $T_v$  (Fig. 2(b)), it is found that they have a good correlation. Thus using  $T_v$  as the confined aquifer vulnerability index can reflect the contamination potential well.

## 5 SUMMARY

Because of the shallow aquifer pollution and the steady increase of confined groundwater exploitation, the risk of confined aquifer pollution is greatly increasing too. It is, therefore, very urgent to evaluate the degree of confined aquifer vulnerability to pollution and take measures to protect the confined groundwater resource.

In the present study, the concept of confined aquifer vulnerability for a single pumping well is proposed and a simplified groundwater flow and pollutant transport model is established. The cumulative time for the pollutant concentration at the top boundary of confined aquifer to exceed the maximum allowable level is defined as the confined aquifer vulnerability index. The vulnerability index can be obtained by numerically solving the groundwater flow and solute transport model mentioned above. Based on this index, different degrees of well vulnerability can be classified.

This study developed a useful method to determine the vulnerability of a confined aquifer to pollution, which is especially useful for decision makers. The vulnerability map can facilitate decision makers in pinpointing protection zones for a confined groundwater pumping well.

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