Control-Oriented Power Allocation for Integrated Satellite-UAV Networks

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***Abstract*—This letter presents a sensing-communication-computing-control (SC^3) integrated satellite unmanned aerial vehicle (UAV) network, where UAVs are equipped with sensors, mobile edge computing (MEC) servers, base stations and satellite communication modules. Like a *nervous system*, this integrated network is capable of organizing multiple field robots in remote areas, so as to perform mission-critical tasks which are dangerous for humans. Aiming at activating this *nervous system* with multiple SC^3 loops, we present a control-oriented optimization problem. Different from traditional studies which mainly focused on communication metrics, we address the power allocation issue to minimize the sum linear quadratic regulator (LQR) control cost of all SC^3 loops. Specifically, we show the convexity of the problem and reveal the relationship between the optimal transmit power and intrinsic entropy rates of different SC^3 loops. For the assure-to-be-stable case, we derive a closed-form solution for ease of practical applications. After demonstrating the superiority of the control-oriented power allocation, we further highlight its difference from the classic capacity-oriented water-filling method.**

***Index Terms*—Control parameter, linear quadratic regulator (LQR), power allocation, satellite-UAV network.**

I. INTRODUCTION

FIELD robots could perform mission-critical tasks that are dangerous for humans. When being dispatched to remote areas without terrestrial cellular coverage, the operation of robots has to rely on non-terrestrial infrastructures, including satellites and unmanned aerial vehicles (UAVs) [1], [2]. For such scenarios, a UAV platform needs to have integrated functionalities to support various requirements of robots, e.g., sensing, communication, controlling and computing. For example, a UAV can be equipped with on-board sensors to collect scene information, with mobile edge computing (MEC) servers to analyze the situation and make quick decisions for robot control, with base stations to transmit control commands to robots, and with a satellite communication module to support real-time communication to the remote cloud center [3]. This leads to a sensing-communication-computing-control (**SC**^{3}) integrated satellite-UAV network, in which efficient resource orchestration for all the related functionalities is important.

In the **SC**^{3} integrated network, the whole control process, including sensing, computing, communication and control, is performed in a closed-loop manner, which is referred to as a **SC**^{3} loop. Intuitively, a **SC**^{3} loop can be regarded as a *reflex arc*, with the network as a *nervous system* [4], where multiple **SC**^{3} loops would share and compete for resources.

Existing studies on integrated satellite-UAV networks have mainly focused on communications. For example, [5] jointly optimized the channel allocation, power allocation, and hovering time of UAVs to maximize the data transmission efficiency. However, in a **SC**^{3} integrated network, control and communication are closely coupled, and one will be more concerned with the control performance, especially for mission-critical tasks where reliable control must be guaranteed.

Researchers in the control field have investigated the relationship between communication and control. It was shown that a noisy linear control system can be stabilized only if the communication throughput per cycle exceeds the intrinsic entropy rate of the control system [6]. Qiu et al. generalized this result to a multi-channel case [7]. Recently, a lower bound of the minimum data rate to achieve a certain linear quadratic regulator (LQR) cost was presented [8]. These achievements have indicated that jointly optimizing control and communication is promising and significant in the **SC**^{3} integrated satellite-UAV network. However, most of these works modeled communications as simple pipelines with simple parameters and left communication resource allocation undiscussed.

Inspired by the efforts in the control field, some recent works have considered the control part as constraints in communication design. Authors in [9] maximized the spectral efficiency of a wireless control system subject to the control convergence rate constraint. Chen et al. maximized the delay determinacy under the same constraint [10]. These studies have taken a great step toward control-oriented optimization. However, they still focused on communications as the objective. In mission-critical **SC**^{3} integrated networks, as has been discussed, the control performance is more important and should be treated as the objective rather than constraints. Compared with the convergence rate that was considered in [9], [10], the LQR cost, which has been widely used in the control theory [11], [12], is a more suitable objective for measuring the control performance, as it provides a simple form to balance the state deviation and control energy consumption.

Motivated by the above discussion, we optimize the sum LQR control cost of all **SC**^{3} loops. Particularly, we establish

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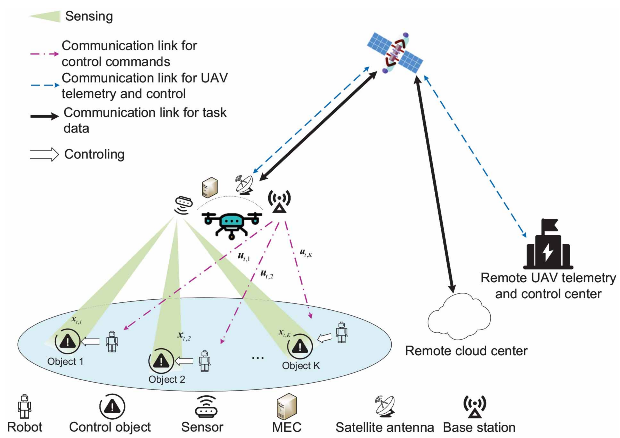

Fig. 1. Illustration of a $\mathbf{SC}^3$ integrated satellite-UAV network for organizing multiple field robots in remote areas.

a relation between the LQR cost and the communication data rate, and then formulate a control performance optimization problem which optimizes the power allocation of the UAV. We show the convexity of the problem and reveal the relationship between the optimal transmit power and the intrinsic entropy rates of different $\mathbf{SC}^3$ loops. For the assure-to-be-stable case, we derive a closed-form solution for practical applications.

II. System Model and Problem Formulation

As shown in Fig. 1, we consider a $\mathbf{SC}^3$ integrated satellite-UAV network that serves multiple field robots for mission-critical tasks. One satellite and one UAV (integrated with a sensor, a base station, a satellite communication module and a MEC server) provide sensing, communication, and computing services. The remote telemetry and control center monitors and guides the UAV through satellite. The network enables multiple $\mathbf{SC}^3$ loops simultaneously. In each loop, the sensor senses the object states. The MEC and the remote center analyze sensor data and compute control commands. Next, the UAV transmits the commands to guide the robots to properly handle the objects. The whole process is in a periodic manner.

The UAV transmits control commands to K robots through frequency orthogonal channels. Denoting the power allocated to robot k as p_k, we have $\sum_{k=1}^{K} p_k \leq P_{\max}$, where $P_{\max}$ is the maximum power. The air-to-ground channels are assumed to be dominated by line-of-sight links [13]. Therefore, the channel gain from the UAV to robot k follows the free space path loss model as $g_k = \frac{\beta_0}{d_k^2}$, where d_k is the distance between the UAV and robot k, and β_0 is the reference channel gain.

For the control part in $\mathbf{SC}^3$ loops, we model each control object as a linear time-invariant system [6], [7], [8], and hence the discrete-time system equation of the kth object is given by

$$\mathbf{x}_{k,t+1} = \mathbf{A}_k \mathbf{x}_{k,t} + \mathbf{B}_k \mathbf{u}_{k,t} + \mathbf{v}_{k,t}, \quad (1)$$

where t denotes the cycle index, $\mathbf{x}_{k,t} \in \mathbb{R}^n$ denotes the system state, $\mathbf{u}_{k,t} \in \mathbb{R}^m$ denotes the control action, $\mathbf{v}_{k,t} \in \mathbb{R}^n$ is system noise, and $\mathbf{A}_k$ and $\mathbf{B}_k$ are fixed $n \times n$ and $n \times m$ matrices denoting the state matrix and input matrix respectively.

Due to the limited computing capability of the MEC server on the UAV, some complex computing tasks will be offloaded to the remote cloud center through satellite. The queuing time is negligible as the transmission of the tasks is infrequent (only at the start of each cycle). Denoting the task data size as D_k, the transmission times from UAV to satellite and from satellite to control center can be calculated as $t_k^{\mathrm{T,U2S}} = \frac{D_k}{R^{\mathrm{U2S}}}$ and $t_k^{\mathrm{T,S2C}} = \frac{D_k}{R^{\mathrm{S2C}}}$, where R^{U2S} and R^{S2C} denote the corresponding transmission data rates. The processing time is $t_k^{\mathrm{P}} = \frac{D_k \alpha_k}{f_k}$, where α_k is the average CPU cycles to process the task per bit, and f_k is the CPU frequency. From the perspective of system robustness, we consider the maximum propagation delay based on the satellite orbit height H^{S}, the minimum elevation angle $\beta_{\min}$ and the maximum central angle $\theta_{\max}$, denoted as $\tau_{\max} = \frac{(R_e + H^{\mathrm{S}}) \sin \theta_{\max}}{c \cos \beta_{\min}}$ [15], where R_e is the Earth radius. Therefore, the total delay, when a task is conducted on the cloud, can be calculated as

$$T_{k,1} = t_k^{\mathrm{T,U2S}} + t_k^{\mathrm{T,S2C}} + t_k^{\mathrm{P}} + 4\tau_{\max}, \quad (2)$$

where the transmission time from the remote cloud center back to the UAV is omitted as the amount of the output data is small. Denoting the control period of $\mathbf{SC}^3$ loop k as T_k, the available time duration for transmitting the control commands would be shorten to $T_{k,2} = T_k - T_{k,1}$, if a task is conducted remotely.

The communication data rate between the UAV and robots is limited, which may affect the control performance. According to [6], to stabilize control object k, the data throughput transmitted in each cycle needs to satisfy the condition

$$BT_{k,2} \log_2\left(1 + \frac{g_k p_k}{\sigma^2}\right) > h_k \triangleq \log_2 |\det \mathbf{A}_k|, \quad (3)$$

where B denotes the channel bandwidth, σ^2 is the noise variance and h_k is the intrinsic entropy rate which denotes the stability of object k. A large h_k indicates an unstable object, which requires a high transmission rate to stabilize.

In control theory, the control performance can be measured by the LQR cost function [11], [12]. In this letter, we consider the worst-case long-term average LQR cost, formulated as [8]

$$l_k \triangleq \sup \lim_{N \to \infty} \mathbb{E}\left[\frac{1}{N} \sum_{t=1}^{N} \left(\mathbf{x}_{k,t}^{\mathrm{T}} \mathbf{Q}_k \mathbf{x}_{k,t} + \mathbf{u}_{k,t}^{\mathrm{T}} \mathbf{R}_k \mathbf{u}_{k,t}\right)\right], \quad (4)$$

where $\mathbf{Q}_k$ and $\mathbf{R}_k$ are semi-positive definite weight matrices. The terms $\mathbf{x}_{k,t}^{\mathrm{T}} \mathbf{Q}_k \mathbf{x}_{k,t}$ denotes the deviation of the system from the zero state, and the term $\mathbf{u}_{k,t}^{\mathrm{T}} \mathbf{R}_k \mathbf{u}_{k,t}$ denotes the input energy. These weight matrices balance the state and the energy, which can be set according to practical requirements. For example, one should set the entries of $\mathbf{Q}_k$ to be large if he expects that the state of the system converges to zero quickly.

To achieve a certain LQR cost (l_k), an additional term has to be added to the condition previously described in (3), and the data throughput in each cycle now must satisfy [8]

$$BT_{k,2} \log_2\left(1 + \frac{g_k p_k}{\sigma^2}\right) \geq h_k + \frac{n}{2} \log_2\left(1 + \frac{nN(\mathbf{v}_k)|\det \mathbf{M}_k|^{\frac{1}{n}}}{l_k - \mathrm{tr}(\mathbf{\Sigma}_k \mathbf{S}_k)}\right), \quad k = 1, 2, \ldots, K, \quad (5)$$

where $N(\mathbf{v}_k) \triangleq \frac{1}{2\pi} \exp(\frac{2}{n} h(\mathbf{v}_k))$, $h(\mathbf{v}_k)$ is the differential entropy of $\mathbf{v}_k$, $\mathbf{\Sigma}_k$ is its covariance matrix, and $\mathbf{M}_k$ and $\mathbf{S}_k$

are the solutions to control system Riccati equations shown in [8]. The right side of (5) is decreasing with l_k, which means that a lower LQR cost requires a higher data rate in a single $\mathbf{SC}^3$ loop. When multiple $\mathbf{SC}^3$ loops share resources, e.g., the power, the control-oriented and communication-oriented resource allocation schemes will have essentially different results. This will be shown in the next section.

In this letter, we focus on control-oriented resource allocation, and aim to minimize the sum long-term average LQR cost of the $\mathbf{SC}^3$ loops by optimizing power allocation $\mathbf{p} = [p_1, p_2, \ldots, p_K]$. The problem is formulated as

$$\min_{\mathbf{p},\mathbf{l}} \sum_{k=1}^{K} l_k \tag{6a}$$

$$\text{s.t.} \sum_{k=1}^{K} p_k \le P_{\max}, \tag{6b}$$

$$(5), \tag{6c}$$

where $\mathbf{l} = [l_1, l_2, \ldots, l_K]$ and (6c) is the communication constraint imposed by the control performance requirements. In the following, we will transform this problem to a convex problem and analyze its optimal solution.

III. Problem Transformation and Analysis

We first propose a lemma to show the convexity of problem (6), and further reveal the relationship between the optimal power allocation and other parameters.

Lemma 1: Problem (6) is equivalent to the convex problem

$$\min_{\mathbf{p}} \sum_{k=1}^{K} l_k(p_k) \tag{7a}$$

$$s.t. \sum_{k=1}^{K} p_k \le P_{\max}, \tag{7b}$$

$$BT_{k,2}\log_2\left(1+\frac{g_k p_k}{\sigma^2}\right) > h_k, k = 1, 2, \ldots, K, \tag{7c}$$

where

$$l_k(p_k) \triangleq \frac{n 2^{\frac{2}{n}h_k} N(\mathbf{v}_k) |\det \mathbf{M}_k|^{\frac{1}{n}}}{\left(1+\frac{g_k p_k}{\sigma^2}\right)^{\frac{2BT_k}{n}} - 2^{\frac{2}{n}h_k}} + \mathrm{tr}(\mathbf{\Sigma}_k \mathbf{S}_k). \tag{8}$$

Proof: As the right side of (5) is decreasing with l_k, the equality must hold to minimize the objective function. Therefore, we obtain the equation between l_k and p_k as (8). Correspondingly, problem (6) is recast as problem (7), where constraint (7c) ensures a positive denominator in (8).

Next, we prove the convexity of (7). The second order derivative of $l_k(p_k)$ is shown as (9) at the bottom of the page. From (9), we can find that $\frac{\partial^2 l_k}{\partial {p_k}^2} > 0$ as long as $(1+\frac{g_k p_k}{\sigma^2})^{\frac{2BT}{n}} > 2^{\frac{2}{n}h_k}$, which is equivalent to (7c). Therefore, the objective function is convex in its feasible region, which guarantees the convexity. ■

Lemma 1 transforms problem (6) into a convex problem, whose optimal solution can be obtained efficiently. Next, we first consider a special case that the LQR weight matrices are set as $\mathbf{Q}_k = \mathbf{I}_n$ and $\mathbf{R}_k = \mathbf{0}$. With this setting, the LQR cost solely describes the system deviation while the energy cost is not the focus. For this case, the relation between optimal power allocation and system stability is discussed in *Proposition 1*.

Proposition 1: When $\mathbf{Q}_k = \mathbf{I}_n$ and $\mathbf{R}_k = \mathbf{0}$, the optimal power p_k^* allocated to channel k is monotonically increasing with the intrinsic entropy rate h_k, i.e., the more unstable the control system k is, the more power should be allocated to it.

Proof: When $\mathbf{Q}_k = \mathbf{I}_n$ and $\mathbf{R}_k = \mathbf{0}$, the solutions of the Riccati equations are $\mathbf{S}_k = \mathbf{M}_k = \mathbf{I}_n$ [8]. Therefore, we have $|\det \mathbf{M}_k|^{(\frac{1}{n})} = 1$ in (8).

It is not difficult to verify that the Slater's conditions hold for problem (7), which guarantees strong duality [16]. Therefore, (7) is equivalent to its Lagrangian dual problem

$$\max_{\lambda \ge 0} \min_{\mathbf{p}} \sum_{k=1}^{K} l_k(p_k) + \lambda\left(\sum_{k=1}^{K} p_k - P_{\max}\right) \tag{10a}$$

$$s.t. \ (7c), \tag{10b}$$

where λ is the Lagrangian multiplier with respect to constraint (7b). By checking the Karush-Kuhn-Tucker (KKT) conditions, we have $\frac{\partial l_k}{\partial p_k}|_{p_k^*} + \lambda = 0, k = 1, 2, \ldots, K$, where $\{p_k^*\}$ denotes the optimal solution to problem (6). Calculating $\frac{\partial l_k}{\partial p_k}$, we obtain the following equation

$$\frac{2BT_k 2^{\frac{2}{n}h_k} N(\mathbf{v}_k) g_k \left(1+\frac{g_k p_k^*}{\sigma^2}\right)^{\frac{2BT_k}{n}-1}}{\sigma^2\left[\left(1+\frac{g_k p_k^*}{\sigma^2}\right)^{\frac{2BT_k}{n}} - 2^{\frac{2}{n}h_k}\right]^2} = \lambda. \tag{11}$$

Denoting the left side of (11) as $s_k(h_k, p_k)$, we have

$$s_k(h_k, p_k) = s_j(h_j, p_j), \quad \forall j, k = 1, 2, \ldots, K. \tag{12}$$

Similar to (9), we have $\frac{\partial s_k}{\partial p_k} < 0$. In addition, it is obvious that s_k is monotonically increasing with h_k. Therefore, when the other parameters are fixed, if h_k increases, p_k should also increase to guarantee that condition (12) still holds. ■

Remark 1: Similarly, we can prove that the optimal power allocated to $\mathbf{SC}^3$ loop k is monotonically increasing with the entropy power of $\mathbf{v}_k$. Therefore, we can draw a conclusion that one should allocate more power to unstable control systems with larger noise to improve the overall control performance.

Next, we derive a closed-form expression of the optimal power allocation without restricting the forms of $\mathbf{Q}_k$ and $\mathbf{R}_k$.

$$\frac{\partial^2 l_k}{\partial {p_k}^2} = \frac{2BT_{k,2} 2^{\frac{2}{n}h_k} N(\mathbf{v}_k) |\det \mathbf{M}_k|^{\frac{1}{n}} (1+\frac{g_k p_k}{\sigma^2})^{\frac{2BT_{k,2}}{n}} \left[\frac{2BT_{k,2}}{n} 2^{\frac{2}{n}h_k} + \frac{2BT_{k,2}}{n}(1+\frac{g_k p_k}{\sigma^2})^{\frac{2BT_{k,2}}{n}} + (1+\frac{g_k p_k}{\sigma^2})^{\frac{2BT_{k,2}}{n}} - 2^{\frac{2}{n}h_k}\right]}{(p_k + \frac{g_k}{\sigma^2})^2 \left[(1+\frac{g_k p_k}{\sigma^2})^{\frac{2BT_{k,2}}{n}} - 2^{\frac{2}{n}h_k}\right]^3} \tag{9}$$

We consider the assure-to-be-stable assumption, that the communication capability is significantly greater than the lowest capability requirement to keep the system stable in (3), i.e., $BT_k \log_2(1+\frac{g_k p_k}{\sigma^2}) \gg h_k$. This assumption means that the control systems are far from the unstable point, and we can focus on the control performance instead of the stability.

Proposition 2: Under the assured-to-be-stable assumption, if all of the $\mathbf{SC}^3$ loops have the same control period and computing delay, which means that $T_{1,2} = T_{2,2} = \cdots = T_{k,2} = T$, the optimal solution to problem (6) is obtained as (13), shown at the bottom of the page.

Proof: When $BT_k \log_2(1+\frac{g_k p_k}{\sigma^2}) \gg h_k$, we have $(1+\frac{g_k p_k^*}{\sigma^2})^{\frac{2BT}{n}} \gg 2^{\frac{2}{n}h_k}$, which means that the term $2^{\frac{2}{n}h_k}$ in the denominator of the left hand in (11) is negligible. Therefore, we can rewrite (11) as

$$p_k^* = \left[\left(\frac{2BT|\det \mathbf{M}_k|^{\frac{1}{n}} 2^{\frac{2}{n}h_k} N(\mathbf{v}_K) g_k}{\lambda\sigma^2}\right)^{\frac{n}{2BT+n}} - 1\right]\frac{\sigma^2}{g_k}. \tag{14}$$

On the other hand, due to the monotonicity of l_k with respect to p_k, the equality in (7b) must hold, i.e.,

$$\sum_{k=1}^{K} p_k = P_{\max}. \tag{15}$$

Based on (14) and (15), we obtain that the Lagrangian multiplier λ satisfies equation (16), shown at the bottom of the page.

Substituting (16) into (14), we obtain (13) immediately, which completes the proof. ■

Remark 2: From (13), we see that p_k^* is increasing with h_k and $N(\mathbf{v}_k)$, which verifies the conclusion of *Proposition 1*. In addition, it is shown that the term $\frac{n}{2BT+n}$, as the power of the control parameters, evaluates the influence of the control parameters on the allocation results. In the special case that $\frac{n}{2BT+n} \to 0$, we have $p_k^* = P_{\max}\frac{1/g_k}{\sum_{i=1}^{K}(1/g_i)}$, which is irrelevant to the control part of the system.

IV. Simulation Results

We assume 5 objects, which are randomly and evenly distributed in a circular area with a radius of 5000 m. The UAV is at the center of the circle. The UAV height, denoted as *H*, is set as 100m unless otherwise specified. Other parameters are set as $B = 5$kHz, $\beta_0 = -60$dB and $\sigma^2 = -110$dBm [17]. We assume Low Earth Orbit (LEO) satellite, with $H^{\mathrm{S}} = 1100$km, $\beta_{\min} = 50°$, and hence $\tau_{\max} = 4.56$ ms [15]. The data rates are set as $t_k^{\mathrm{U2S}} = 10$Mbps and $t_k^{\mathrm{S2C}} = 100$Mbps.

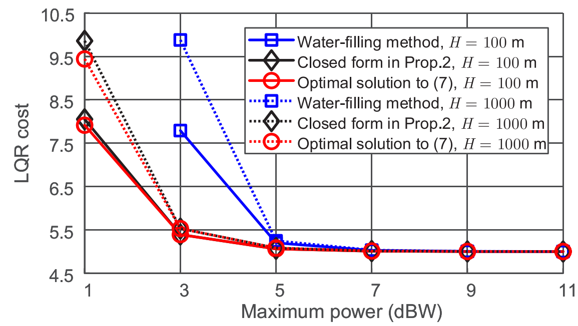

Fig. 2. LQR cost achieved by different power allocation methods.

For the control part, the intrinsic entropy rates of each system are randomly selected from the range [0, 100], the system noise is assumed to be independent Gaussian random variables with mean zero and variance 0.01, and the we set $T_1 = T_2 = \cdots = T_K = 100$ms [18] and $n = 100$. The LQR weight matrices are $\mathbf{Q}_k = \mathbf{I}_n, \mathbf{R}_k = \mathbf{0}, \forall k$. The sensor data size is set as $D_k = 200$k bits, and other parameters are set as $\alpha_k = 100$ and $f_k = 2$ GHz, for $k = 1, 2, \ldots, K$.

In Fig. 2, we compare the LQR cost achieved by the water-filling method (which is optimal for maximizing the sum rate [14, Ch. 9.4]), the closed-form allocation in *Proposition 2*, and the optimal results to (6) obtained with CVX, with different UAV heights. The figure shows the accuracy of (13), as the optimal solution and the closed form achieve nearly the same LQR cost. When $P_{\max} = 1$dBW, the LQR cost of the power allocation in *Proposition 2* is slightly higher than that of the optimal solution, because the assumption $BT\log_2(1+\frac{g_k p_k}{\sigma^2}) \gg h_k$ is not satisfied. From this figure, we can see that the LQR cost decreases with the maximum power. In addition, the LQR cost with both control-oriented methods is lower than that with the water-filling method. Particularly, when $P_{\max} = 1$dBW, the systems with the water-filling method are unstable, leading to an infinite cost. This verifies the superiority of the proposed method. It is observed that the LQR cost with $H = 100$m is lower than that achieved with $H = 1000$ m. This is because lower UAV height leads to less path loss. Notably, when the maximum power becomes large enough, the difference in LQR cost caused by different UAV heights becomes negligible. This is because the communication capability significantly exceeds control requirements and the LQR cost approaches its minimal value $\mathrm{tr}(\mathbf{\Sigma}_k\mathbf{S}_k)$, which can be observed from (8).

$$p_k^* = \frac{\left(P_{\max} + \sum_{i=1}^{K}\frac{\sigma^2}{g_i}\right)\left[|\det \mathbf{M}_k|^{\frac{1}{n}} 2^{\frac{2}{n}h_k} N(\mathbf{v}_k)\right]^{\frac{n}{2BT+n}}\left(\frac{\sigma^2}{g_k}\right)^{\frac{2BT}{2BT+n}}}{\sum_{i=1}^{K}\left[|\det \mathbf{M}_i|^{\frac{1}{n}} 2^{\frac{2}{n}h_i} N(\mathbf{v}_i)\right]^{\frac{n}{2BT+n}}\left(\frac{\sigma^2}{g_i}\right)^{\frac{2BT}{2BT+n}}} - \frac{\sigma^2}{g_k} \tag{13}$$

$$\left(\frac{1}{\lambda}\right)^{\frac{n}{2BT+n}} = \frac{P_{\max} + \sum_{k=1}^{K}\frac{\sigma^2}{g_k}}{\sum_{k=1}^{K}\left(2BT|\det \mathbf{M}_k|^{\frac{1}{n}} 2^{\frac{2}{n}h_k} N(\mathbf{v}_K)\right)^{\frac{n}{2BT+n}}\left(\frac{\sigma^2}{g_k}\right)^{\frac{2BT}{2BT+n}}} \tag{16}$$

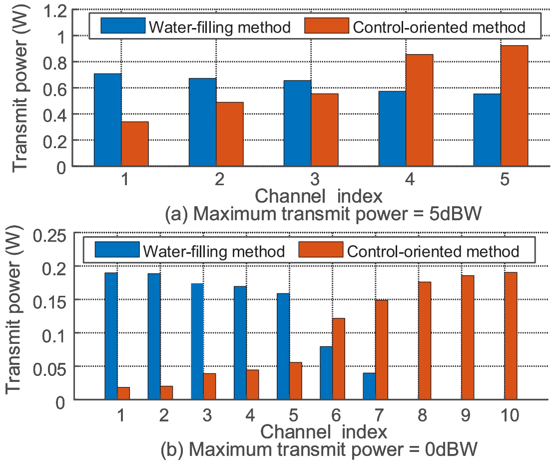

Fig. 3. Power allocated to channels with different channel gains by different methods under different maximum transmit power constraints.

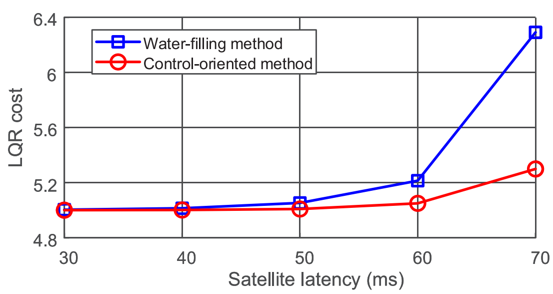

Fig. 4. LQR cost under different satellite latency.

Fig. 3a compares the power allocated to each channel obtained with different methods, with $P_{\max} = 5\text{dBW}$. The intrinsic entropy rates of each system are set as 5. The channels are sorted such that $g_1 \geq g_2 \geq \cdots \geq g_5$. In Fig. 3a, the difference between these two methods is clearly shown. The control-oriented allocation method tends to allocate more power to channels with bad conditions, while the water-filling method behaves oppositely. To show the difference more clearly, we compare the power allocation results with lower maximum power and more robots in Fig. 3b, where $P_{\max} = -10\text{dBW}$ and $K = 10$. It is seen that the water-filling method in this case will not allocate any power to some channels with poor conditions, while the control-oriented allocation method still allocates power to every channel to ensure the system stability under constraint (7c).

In Fig. 4, we show the impact of satellite latency. It is seen that the LQR cost increases as the satellite latency increases. This is because the time for command communication decreases with larger satellite latency, resulting in deterioration of the control performance. For this consideration, Geostationary Earth Orbit satellite might not be a suitable choice for $\mathbf{SC}^3$ networks, since its latency could be longer than the control period, resulting an unstable system.

V. Conclusion

In this letter, we investigated a $\mathbf{SC}^3$ integrated satellite-UAV network. A control-oriented power allocation problem was formulated. We transformed it into a convex problem and proved that more power should be allocated to the loops with higher intrinsic entropy rates to improve the control performance. We further derived the closed-form expression of the optimal power in the assure-to-be-stable case. Simulation results showed the difference between the control-oriented method and the capacity-oriented water-filling method. Specifically, the former will allocate more power to the channel with worse conditions while the latter behaves oppositely.

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