Adams, Oscar S.
Azimuths from plane coordinates.

AZIMUTHS FROM PLANE COORDINATES

OSCAR S. ADAMS

Serial No. 584

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Serial No. 584

AZIMUTHS FROM PLANE COORDINATES

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INTRODUCTION

There is one point of interest to cadastral surveyors that has not been specifically considered in any one of the publications on plane coordinates issued by the Coast and Geodetic Survey. The original survey of a given estate may specify the true azimuth of a line, but frequently only the magnetic azimuth of the line is given and it can only be reproduced by application of the magnetic variation from the true azimuth at the time of the survey. In considering the problem involved in the retracing of a property line it is necessary to determine the true azimuth of a line of the traverse instead of the plane or grid azimuth for the purpose of comparison with the value for the same line as given in the original survey. For any place there is a definite relation between the true azimuth of a line and the plane or grid azimuth of the same as determined by a given plane coordinate system. This relation and the way to determine its value for any line are discussed in this publication.

It is supposed that the line to be determined is in the midst of a traverse that is being run to connect with old property surveys. If we determine the true azimuth of the line that connects with the given line of the property survey, we shall then know the angle that it is necessary to turn to reproduce the true azimuth of the given line of the old survey. If the old value of the line is given in magnetic bearing it can be reduced to true azimuth by applying the magnetic variation for the date of the original survey. In any case, it comes to the necessity of the determination of the true azimuth of a line of the traverse. Adequate discussion of the necessary computations have already been given in previous publications, but in no case was this problem treated directly. Since we wish to make all necessary computations as plain as possible, it seems advisable to give a thorough discussion of this application to serve as an example for local surveyors.

The examples used are given in all the exactness that is possible so as to serve as samples of what can be done. As a matter of fact even the most approximate of the computations is more exact than would be justified for magnetic azimuths. These are probably never determined to greater exactness than to the nearest tenth of a minute and because of the uncertainty in the variation the minutes may be uncertain. With these facts in mind, it is easy to see that a considerable degree of approximation can be used in determining the θ angle or the $\Delta\alpha$ angle as the case may be.

COMPUTATION OF THE GEODETIC AZIMUTH OF A LINE ON THE LAMBERT GRID

We shall first discuss the method of handling the problem on the Lambert system. The amount of computation necessary depends somewhat upon the accuracy required in the result. We shall indicate the approximate method as well as the most exact method in connection with the same line of a traverse. In that way, it can be seen what reliance can be put in the approximate method. In the treatment of the subject, we shall make use of a traverse line in Nebraska, south, that was computed in the examples in Special Publication No. 194—Manual of Traverse Computation on the Lambert Grid.

On page 34 of that publication, we have given the coordinates of stations 394 and 395 and we will suppose that we want the azimuth of 395 to 394. We must first compute the grid or plane azimuth of the line in the following way:

Station	x	y
394	2, 140, 016. 50	297, 165. 35
395	2, 135, 107. 84	302, 425. 83
	$\Delta x=+4,908.66$	$\Delta y=-5,260.48$

$$\log \Delta x=3.6909629$$
$$\log \Delta y=3.7210253$$

$$\log \tan \alpha=9.9699376$$

$$\alpha= 43^\circ\ 01'\ 06''.7$$
$$\text{Grid azimuth, 395 to 394}=316^\circ\ 58'\ 53''.3$$

To get the approximate correction to this grid azimuth to reduce it to a true or geodetic azimuth, we must compute the θ angle for the station 395. This is obtained in the computation of the geodetic position from the coordinates. Since we shall want the positions of both stations for the rigid computation of the geodetic azimuth, we shall first compute these positions. (See figs. 1 and 2.) If the θ angle alone is wanted only the first part of the computation needs to be made. (See fig. 3.) In most cases it is thought that this approximate computation will be amply sufficient.

From the computation of the geodetic position of 395, the θ angle is given as $+0^\circ\ 19'\ 07''.4$ to the nearest tenth of a second. This quantity added to the grid azimuth gives the first approximation to the true azimuth.

$$\begin{array}{r}\text{Grid azimuth, 395 to 394}=316^\circ\ 58'\ 53''\!.3\\ \theta=\ \ 0^\circ\ 19'\ 07''\!.4\\ \hline \text{Approximate geodetic azimuth}=317^\circ\ 18'\ 00''\!.7\end{array}$$

A somewhat closer approximation can be obtained by computing the correction factor given by the formula,

$$\text{Correction}=\frac{(x_2-x_1)\left(y_1-y_0+\frac{y_2-y_1}{3}\right)}{2\rho_0^2 \sin 1''}$$

From the list of constants in the table given on page 187 of Special Publication No. 194, we find $y_0=486{,}221$ to the nearest foot and

$$\log \frac{1}{2\rho_0^2 \sin 1''}=0.\ 3724450-10$$

From the computation of α on page 2, we have, to the nearest foot,

$$x_2-x_1=\Delta x=+4,909$$
$$y_2-y_1=\Delta y=-5,260$$

Also,

$$\begin{array}{r}y_1=\ \ 302,426\\ y_0=\ \ 486,221\\ \hline y_1-y_0=-183,795\\ \tfrac{1}{3}\Delta y=\ \ -1,753\\ \hline y_1-y_0+\frac{y_2-y_1}{3}=-185,548\end{array}$$

$$\begin{array}{r}\log \Delta x=3.\ 69096\\ \log (185{,}548)=5.\ 26846n\\ \log\left(\frac{1}{2\rho_0^2 \sin 1''}\right)=0.\ 37244-10\\ \hline \log \text{correction}=9.\ 33186-10n\\ \text{Correction}=-0''\!.2\end{array}$$

This value must be subtracted from the approximate geodetic azimuth already found and since it is negative, it will be added arithmetically to the former value. This gives for the second approximation of the geodetic azimuth,

$$\begin{array}{r}317^\circ\ 18'\ 00''\!.7\\ +0''\!.2\\ \hline 317^\circ\ 18'\ 00''\!.9\end{array}$$

This is the best approximation that we can get by this method.

Since we have computed the geodetic positions of both stations from their coordinates, we can now determine the rigid value for the geodetic azimuth by an inverse position computation. (See fig. 4.)

In order to hold the azimuth as rigidly as possible, we have computed the geodetic positions to four decimal places. This puts the geodetic positions on a parity with the coordinates, for a unit in the fourth place of the latitude corresponds to 0.01 foot in the coordinate.

The inverse position computation checks exactly the approximate value after the correction term has been applied. For the approximate computation it is only necessary to compute the approximate θ value for one station. (See fig. 3.) Since it is not advisable to carry more than tenths of a second, seven place tables would be sufficiently accurate in this computation of θ. Of course, the full computation gives a good check on the work and the inverse position computation gives the true azimuth at both ends of the line as well as the log of the geodetic length of the line expressed in meters. By this computation we find what change has to be applied to the plane or grid azimuth to reduce it to a true azimuth.

Only a small part of the computation on the form for the computation of geodetic positions from Lambert coordinates needs to be made in order to obtain the θ angle. If a traverse runs from one control point to another an interpolation between the angles of the control points will give a fair approximation to the θ angle for any one of the stations. To illustrate the amount of computation necessary to determine the θ angle, we shall take a station in Nebraska, south, on the traverse Prosser to Shelton east base, which is published in Special Publication No. 194 on page 47 et seq. We shall then interpolate for the θ of the same station and a comparison of the two results will show the closeness of the approximation. The short computation for station 305 gives the θ angle as $+0°36'03''.6$.

To interpolate a value, we find first the difference of the θ angles for the control stations. All of the data used are found in the above-mentioned publication.

Station	θ angle	x
Prosser	+0° 35′ 51″.8	2,252,598.10
Shelton east base	+0 29 45.0	2,209,300.44
	+0 06 06.8	+43,297.66
305		2,253,816.65
Prosser		2,252,598.10
		+1,218.55

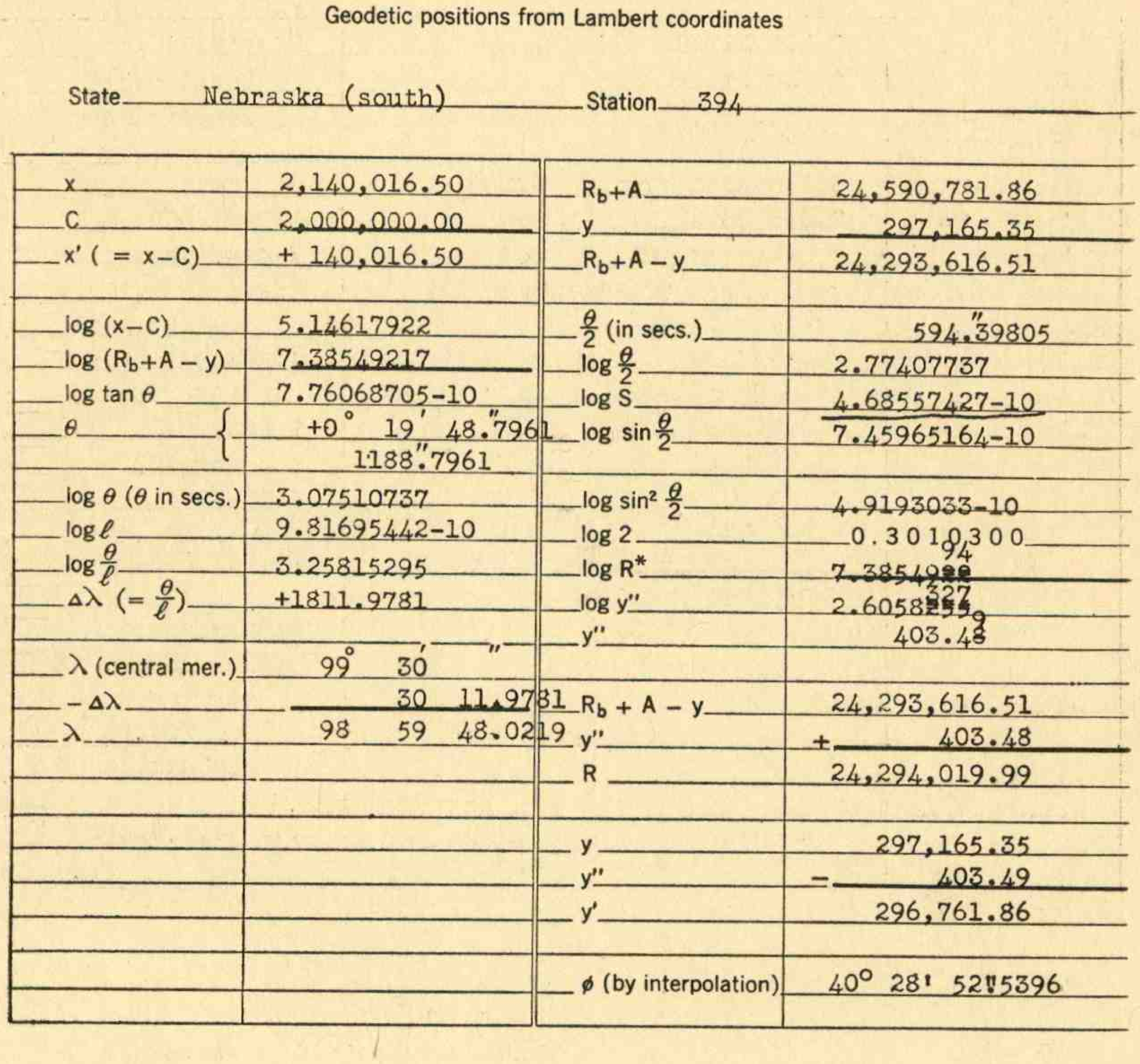
It is seen from this result that station 305 lies farther east than either of the control stations. The $\Delta\theta$ expressed in seconds is +366.8. Of this we take the part $\frac{1,218.55}{43,297.66}$ and this result must be added to the θ angle for Prosser.

$$\frac{1,218.55}{43,297.66}\times 366''.8 = 10''.3$$

Station	θ
Prosser	+0° 35′ 51″.8 + 10.3
305	+0 36 02.1

It is seen that this result differs from the computed value by 1″.5, but for most purposes this approximation would be sufficiently exact. However, this method of interpolation is not much shorter, if as short, as the abbreviated computation given in the direct computation.

After the θ angle is derived, it must be added to the grid azimuth just as was done previously. If a thorough check on the work is desired, it is best to compute the geodetic positions of the two end points of the line and compute the inverse between them. The complete computation is not difficult and one can thus be sure of the result since it gives two ways of calculation of the true azimuth.



Geodetic positions from Lambert coordinates

State Nebraska (south) Station 394

x	2,140,016.50
C	2,000,000.00
x′ (= x−C)	+ 140,016.50
log (x−C)	5.14617922
log (R_b+A − y)	7.38549217
log tan θ	7.76068705-10
θ	+0° 19′ 48.″7961 1188″.7961
log θ (θ in secs.)	3.07510737
log ℓ	9.81695442-10
log $\frac{\theta}{\ell}$	3.25815295
Δλ (= $\frac{\theta}{\ell}$)	+1811.9781
	° ′ ″
λ (central mer.)	99° 30′
− Δλ	30 11.9781
λ	98 59 48.0219

R_b+A	24,590,781.86
y	297,165.35
R_b+A − y	24,293,616.51
$\frac{\theta}{2}$ (in secs.)	594.″39805
log $\frac{\theta}{2}$	2.77407737
log S	4.68557427-10
log sin $\frac{\theta}{2}$	7.45965164-10
log sin² $\frac{\theta}{2}$	4.9193033-10
log 2	0.3010300
log R*	7.3854~~988~~ 94
log y″	2.6058~~255~~ 327 9
y″	403.4~~8~~9
R_b + A − y	24,293,616.51
y″	+ 403.48
R	24,294,019.99
y	297,165.35
y″	− 403.49
y′	296,761.86
φ (by interpolation)	40° 28′ 52″.5396

$\tan\theta = \frac{x - C}{R_b + A - y}$

$\Delta\lambda = \frac{\theta}{\ell}$

$\lambda = \lambda(\text{central mer.}) - \Delta\lambda$

$y'' = 2R\sin^2\frac{\theta}{2}$

$y' = y - y''$

C is constant added to x′ in computation of coordinates

R_b is map radius of lowest parallel

A is value of y′ for R_b; in most cases it is zero

φ is interpolated from table of y′

* Use (R_b + A − y) as an approximate value of R and later correct this value when R is obtained below.

FIGURE 1.—Computation of geodetic position of station 394 from Lambert coordinates.

Geodetic positions from Lambert coordinates

State Nebraska (south) Station 395

x	2,135,107.84	R_b+A	24,590,781.86
C	2,000,000.00	y	302,425.83
x′ (= x−C)	+ 135,107.84	R_b+A-y	24,288,356.03
log (x−C)	5.13068055	$\frac{\theta}{2}$ (in secs.)	+573″.68447
log (R_b+A-y)	7.38539812	log $\frac{\theta}{2}$	2.75867309
log tan θ	7.74528243−10	log S	4.68557431-10
θ	+0° 19′ 07″.36894	log sin $\frac{\theta}{2}$	7.44424740-10
	+1147″.36894		
log θ (θ in secs.)	3.05970309	log $\sin^2 \frac{\theta}{2}$	4.8884948-10
log ℓ	9.81695442-10	log 2	0 3010300
log $\frac{\theta}{\ell}$	3.24274867	log R*	7.385~~3981~~ 4048
Δλ (= $\frac{\theta}{\ell}$)	+1748.8343	log y″	2.57492~~39~~ 96
	° ′ ″	y″	375.7~~7~~ 8
λ (central mer.)	99° 30′		
− Δλ	29 08.8343	$R_b + A - y$	24,288,356.03
λ	99 00 51.1657	y″	+ 375.77
		R	24,288,731.80
		y	302,425.83
		y″	− 375.78
		y′	302,050.05
		φ (by interpolation)	40° 29′ 44″.7978

$$\tan\theta = \frac{x - C}{R_b + A - y}$$

$$\Delta\lambda = \frac{\theta}{\ell}$$

$$\lambda = \lambda(\text{central mer.}) - \Delta\lambda$$

$$y'' = 2R\sin^2\frac{\theta}{2}$$

$$y' = y - y''$$

C is constant added to x′ in computation of coordinates

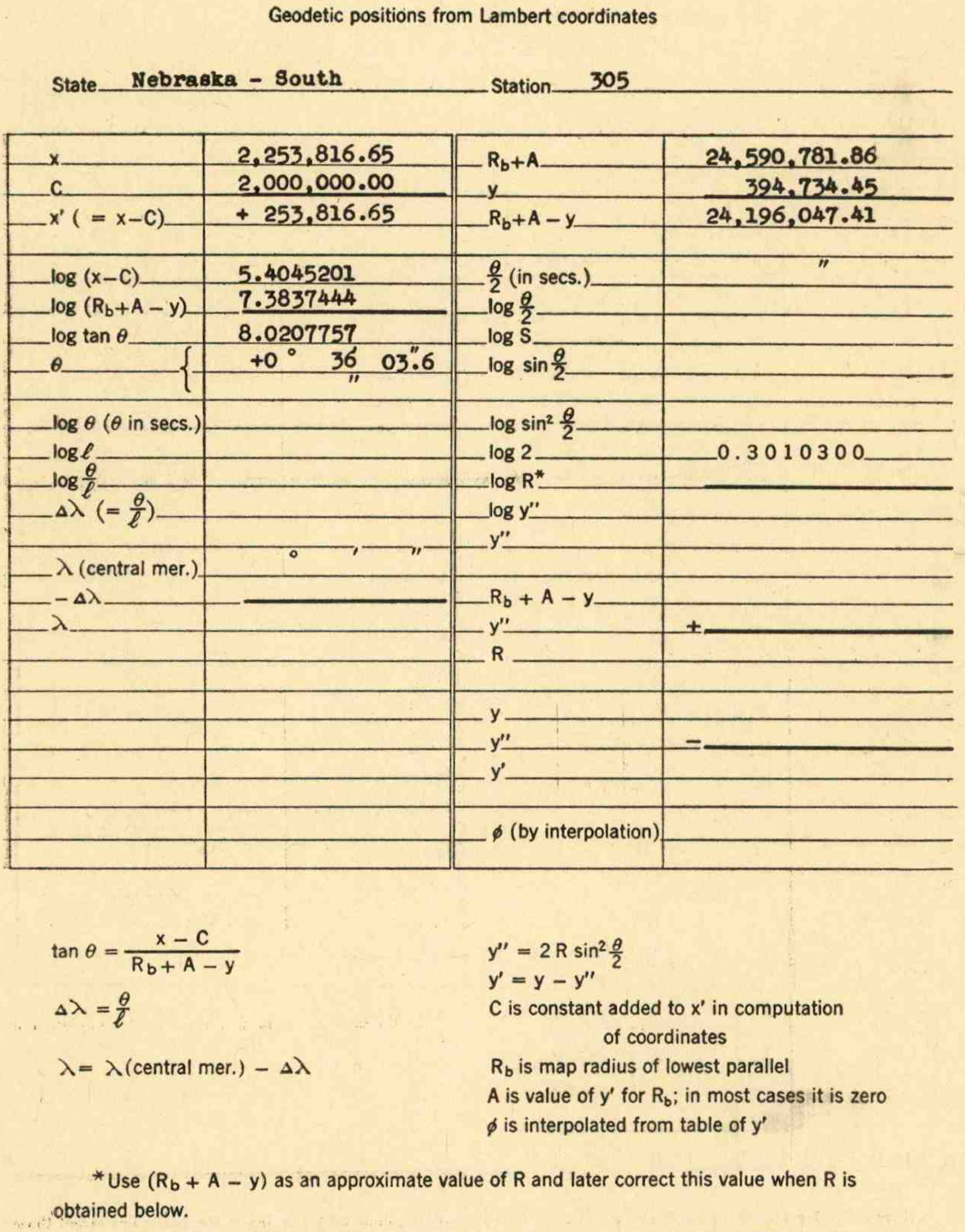
R_b is map radius of lowest parallel

A is value of y′ for R_b; in most cases it is zero

φ is interpolated from table of y′

*Use $(R_b + A - y)$ as an approximate value of R and later correct this value when R is obtained below..

FIGURE 2.—Computation of geodetic position of station 395 from Lambert coordinates.



Geodetic positions from Lambert coordinates

State **Nebraska - South** Station 305

x	2,253,816.65	R_b+A	24,590,781.86
C	2,000,000.00	y	394,734.45
x′ (= x−C)	+ 253,816.65	R_b+A − y	24,196,047.41
log (x−C)	5.4045201	$\frac{\theta}{2}$ (in secs.)	″
log (R_b+A − y)	7.3837444	log $\frac{\theta}{2}$	
log tan θ	8.0207757	log S	
θ	+0° 36′ 03″.6	log sin $\frac{\theta}{2}$	
	″		
log θ (θ in secs.)		log sin² $\frac{\theta}{2}$	
log ℓ		log 2	0.3010300
log $\frac{\theta}{\ell}$		log R*	
Δλ (= $\frac{\theta}{\ell}$)		log y″	
	° ′ ″	y″	
λ (central mer.)			
− Δλ		R_b + A − y	
λ		y″	+
		R	
		y	
		y″	−
		y′	
		φ (by interpolation)	

$$\tan\theta = \frac{x - C}{R_b + A - y}$$

$$\Delta\lambda = \frac{\theta}{\ell}$$

λ = λ(central mer.) − Δλ

$$y'' = 2R\sin^2\frac{\theta}{2}$$

$y' = y - y''$

C is constant added to x′ in computation of coordinates

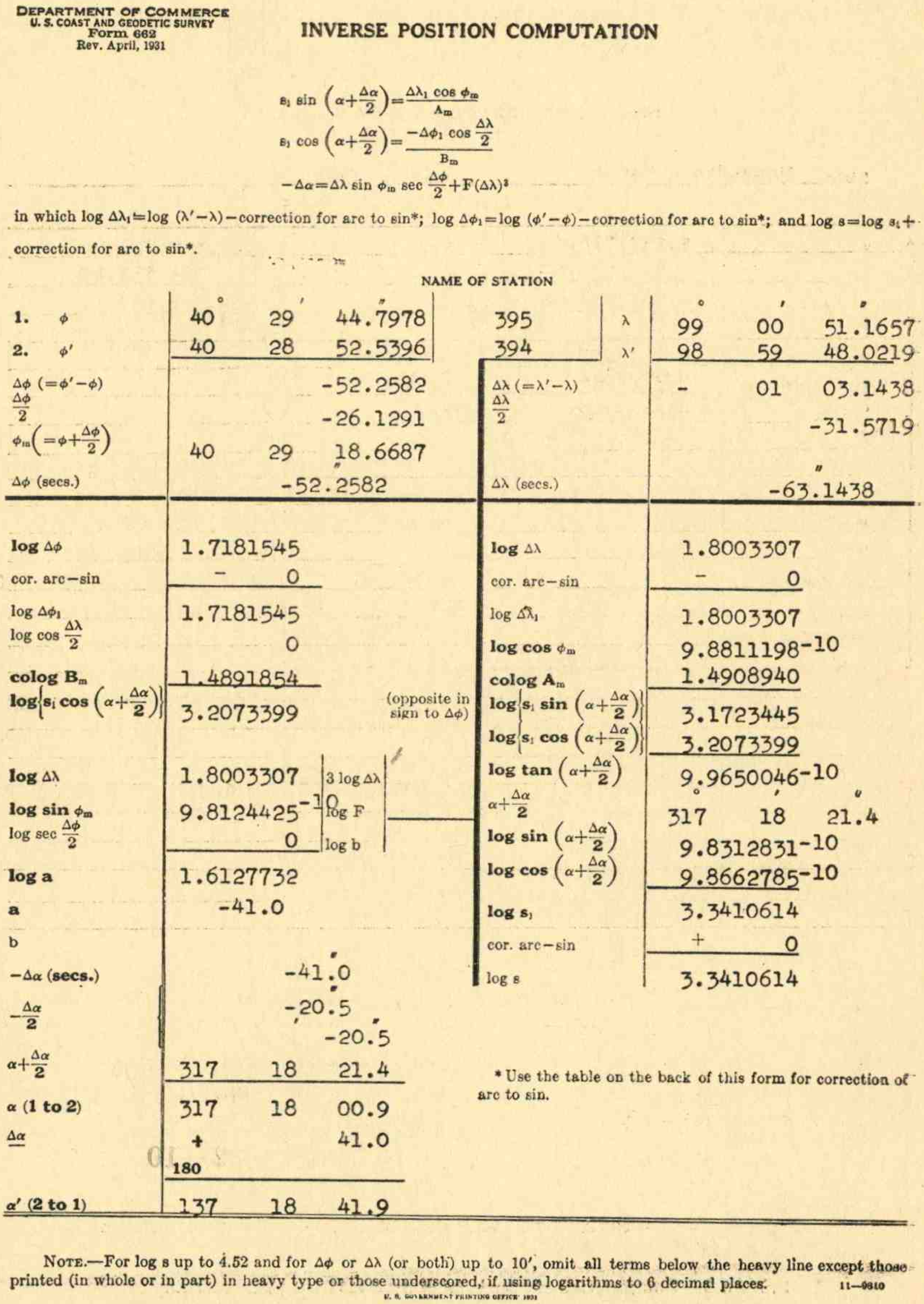
R_b is map radius of lowest parallel

A is value of y′ for R_b; in most cases it is zero

φ is interpolated from table of y′

*Use (R_b + A − y) as an approximate value of R and later correct this value when R is obtained below.

FIGURE 3.—Computation of θ angle for station 305 from plane coordinates.



DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form 662
Rev. April, 1931

INVERSE POSITION COMPUTATION

$$s_1 \sin\left(\alpha+\frac{\Delta\alpha}{2}\right)=\frac{\Delta\lambda_1 \cos\phi_m}{A_m}$$

$$s_1 \cos\left(\alpha+\frac{\Delta\alpha}{2}\right)=\frac{-\Delta\phi_1 \cos\frac{\Delta\lambda}{2}}{B_m}$$

$$-\Delta\alpha=\Delta\lambda \sin\phi_m \sec\frac{\Delta\phi}{2}+F(\Delta\lambda)^3$$

in which $\log \Delta\lambda_1=\log(\lambda'-\lambda)$ − correction for arc to sin*; $\log \Delta\phi_1=\log(\phi'-\phi)$ − correction for arc to sin*; and $\log s=\log s_1$ + correction for arc to sin*.

	°	′	″	NAME OF STATION		°	′	″
1. ϕ	40	29	44.7978	395	λ	99	00	51.1657
2. ϕ'	40	28	52.5396	394	λ'	98	59	48.0219
$\Delta\phi\ (=\phi'-\phi)$			-52.2582		$\Delta\lambda\ (=\lambda'-\lambda)$	-	01	03.1438
$\frac{\Delta\phi}{2}$			-26.1291		$\frac{\Delta\lambda}{2}$			-31.5719
$\phi_m\left(=\phi+\frac{\Delta\phi}{2}\right)$	40	29	18.6687					
$\Delta\phi$ (secs.)			-52.2582		$\Delta\lambda$ (secs.)			-63.1438″

log $\Delta\phi$	1.7181545			
cor. arc−sin	− 0			
log $\Delta\phi_1$	1.7181545			
log cos $\frac{\Delta\lambda}{2}$	0			
colog B_m	1.4891854			
log $\left\{s_1 \cos\left(\alpha+\frac{\Delta\alpha}{2}\right)\right\}$	3.2073399			(opposite in sign to $\Delta\phi$)
log $\Delta\lambda$	1.8003307			3 log $\Delta\lambda$
log sin ϕ_m	9.8124425-10			log F
log sec $\frac{\Delta\phi}{2}$	0			log b
log a	1.6127732			
a	-41.0			
b				
−Δα (secs.)			-41.0″	
$-\frac{\Delta\alpha}{2}$		-20.5″	-20.5″	
$\alpha+\frac{\Delta\alpha}{2}$	317	18	21.4	
α (1 to 2)	317	18	00.9	
Δα	+		41.0	
	180			
α′ (2 to 1)	137	18	41.9	

log $\Delta\lambda$	1.8003307		
cor. arc−sin	− 0		
log $\Delta\lambda_1$	1.8003307		
log cos ϕ_m	9.8811198-10		
colog A_m	1.4908940		
log $\left\{s_1 \sin\left(\alpha+\frac{\Delta\alpha}{2}\right)\right\}$	3.1723445		
log $\left\{s_1 \cos\left(\alpha+\frac{\Delta\alpha}{2}\right)\right\}$	3.2073399		
log tan $\left(\alpha+\frac{\Delta\alpha}{2}\right)$	9.9650046-10		
$\alpha+\frac{\Delta\alpha}{2}$	317°	18′	21.4″
log sin $\left(\alpha+\frac{\Delta\alpha}{2}\right)$	9.8312831-10		
log cos $\left(\alpha+\frac{\Delta\alpha}{2}\right)$	9.8662785-10		
log s_1	3.3410614		
cor. arc−sin	+ 0		
log s	3.3410614		

* Use the table on the back of this form for correction of arc to sin.

NOTE.—For log s up to 4.52 and for Δφ or Δλ (or both) up to 10′, omit all terms below the heavy line except those printed (in whole or in part) in heavy type or those underscored, if using logarithms to 6 decimal places. 11—9810

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FIGURE 4.—Inverse position computation between stations 395 and 394.

COMPUTATION OF THE GEODETIC AZIMUTH OF A LINE ON THE TRANSVERSE MERCATOR GRID

The most rigid method of finding the true azimuth of a line from the coordinates on the transverse Mercator grid consists in computing the geodetic positions of the two ends of the line and then in making an inverse computation from these positions. This procedure will fix the azimuth of the line with complete accuracy. We shall show this complete computation for a line in a C. W. A. traverse near Rochester, N. Y., in the system, New York, west. The coordinates for the ends of the line from station 101 to station 102 are given in Special Publication No. 195—Manual of Traverse Computation on the Transverse Mercator Grid—on page 47. The computations of the geodetic positions from the coordinates are given in full in figure 5 of the present publication. The inverse position computation is shown in figure 6. The resulting azimuth of 101 to 102 is thus found to be $21^\circ 42' 48''.7$.

An approximate computation of this value can be made by calculating the grid azimuth from the coordinates and then by computing the $\Delta\alpha$ for the initial station. This would require that the geodetic position of the initial station should be computed from the coordinates. The details of this computation are now given. After the correction term is computed and applied we find the same value for the geodetic azimuth as before. This method requires slightly less computation than had to be made for the rigid method. It would seem, however, that the rigid method might be preferred, since the result is in most cases more exact than that obtained by the other method. It happens in this case that the result of the rigid method and that of the approximate method is the same. In any case, either of the methods would give a result accurate enough for practical purposes. Any resulting difference would always be far less than the possible accuracy of observation.

Station	x	y
102	767, 779. 28	1, 130, 772. 79
101	770, 268. 54	1, 137, 250. 18
	$\Delta x = -2,489.26$	$\Delta y = -6,477.39$

log Δx $= 3.3960702$
log Δy $= 3.8114000$

log tan α $= 9.5846702 - 10$

α $= 21^\circ 01' 18''.3$
Grid azimuth, 101 to 102 $= 21\ 01\ 18.3$

COMPUTATION OF AZIMUTH, STATION 101

[For values see fig. 5]

ϕ' =43° 07′ 16″.5105
ϕ =43 07 00.3912

$\frac{1}{2}(\phi'+\phi)$ =43 07 08.4508

$\log \Delta\lambda$ =3.5615754
$\log \sin \frac{1}{2}(\phi'+\phi)$ =9.8347487

$\log \Delta\alpha_1$ =3.3963241
$\Delta\alpha_1$ (in seconds) =+2490″.72

$\log (\Delta\lambda)^3$ =10.685
$\log F$ [*] = 7.853−20

$\log b$ = 8.538−10

$\Delta\alpha_1$ (in seconds) =+2490″.72
b = +0.03

$\Delta\alpha$ (in seconds) =+2490.8
$\Delta\alpha$ =+0°41′30″.8

Grid azimuth =21° 01′ 18″.3
$\Delta\alpha$ = 0 41 30.8

Approximate geodetic azimuth=21 42 49.1

COMPUTATION OF CORRECTION TERM

[All values given to nearest foot]

x'_1 =+270, 269

$2x'_1$ =+540, 538
x'_2 =+267, 779

$2x'_1+x'_2$ =+808, 317

$\log (2x'_1+x'_2)$ =5.90758
$\log \Delta y$ =3.81140n
$\log \frac{1}{(6\rho_0^2 \sin 1'')}$ =9.89521−20

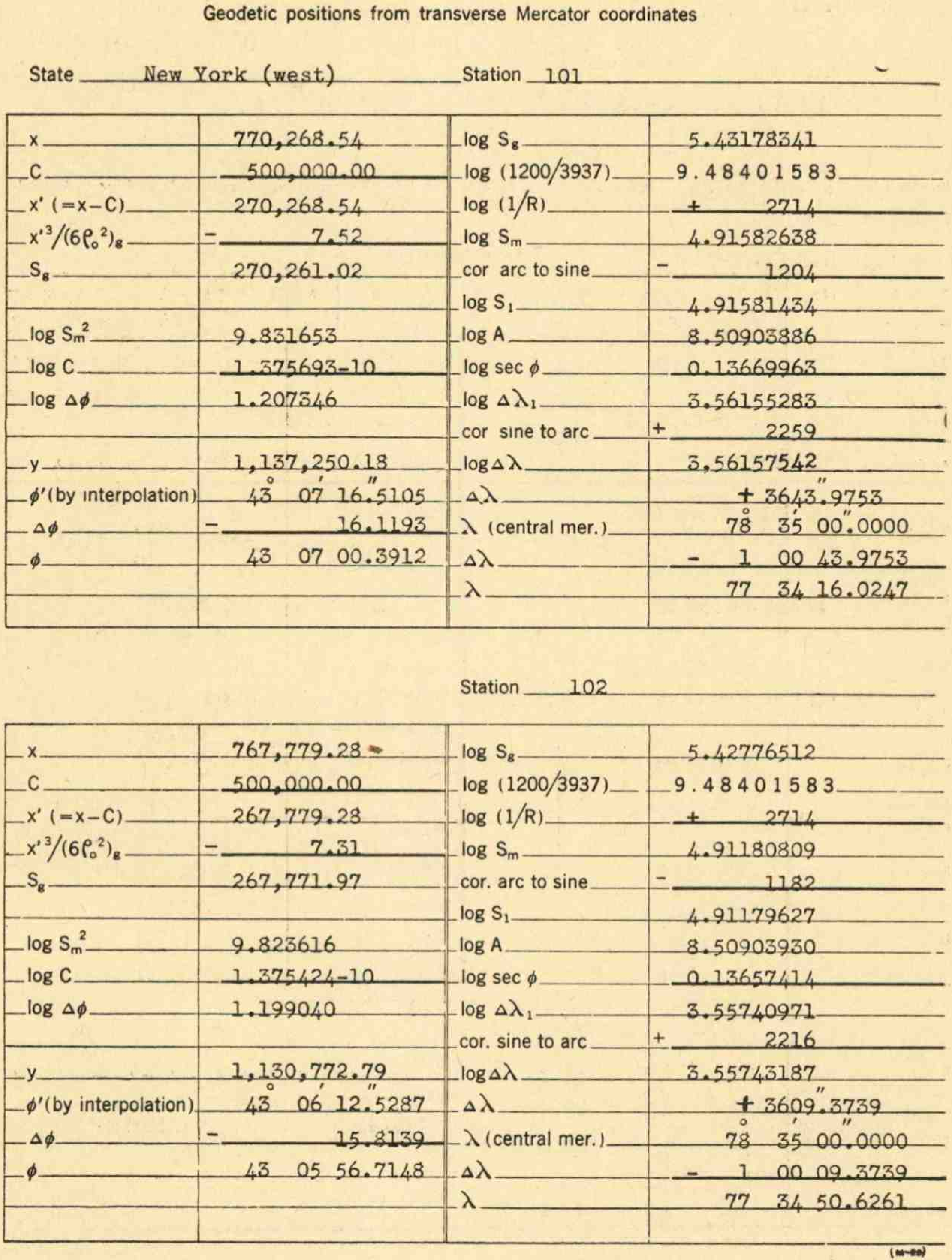
log correction =9.61419−10n
Correction =−0.4

Approximate geodetic azimuth=21° 42′ 49″.1
Correction = −0.4

Geodetic azimuth =21 42 48.7

---

[*] This is taken from U. S. Coast and Geodetic Survey Special Publication No. 8.



Geodetic positions from transverse Mercator coordinates

State New York (west) Station 101

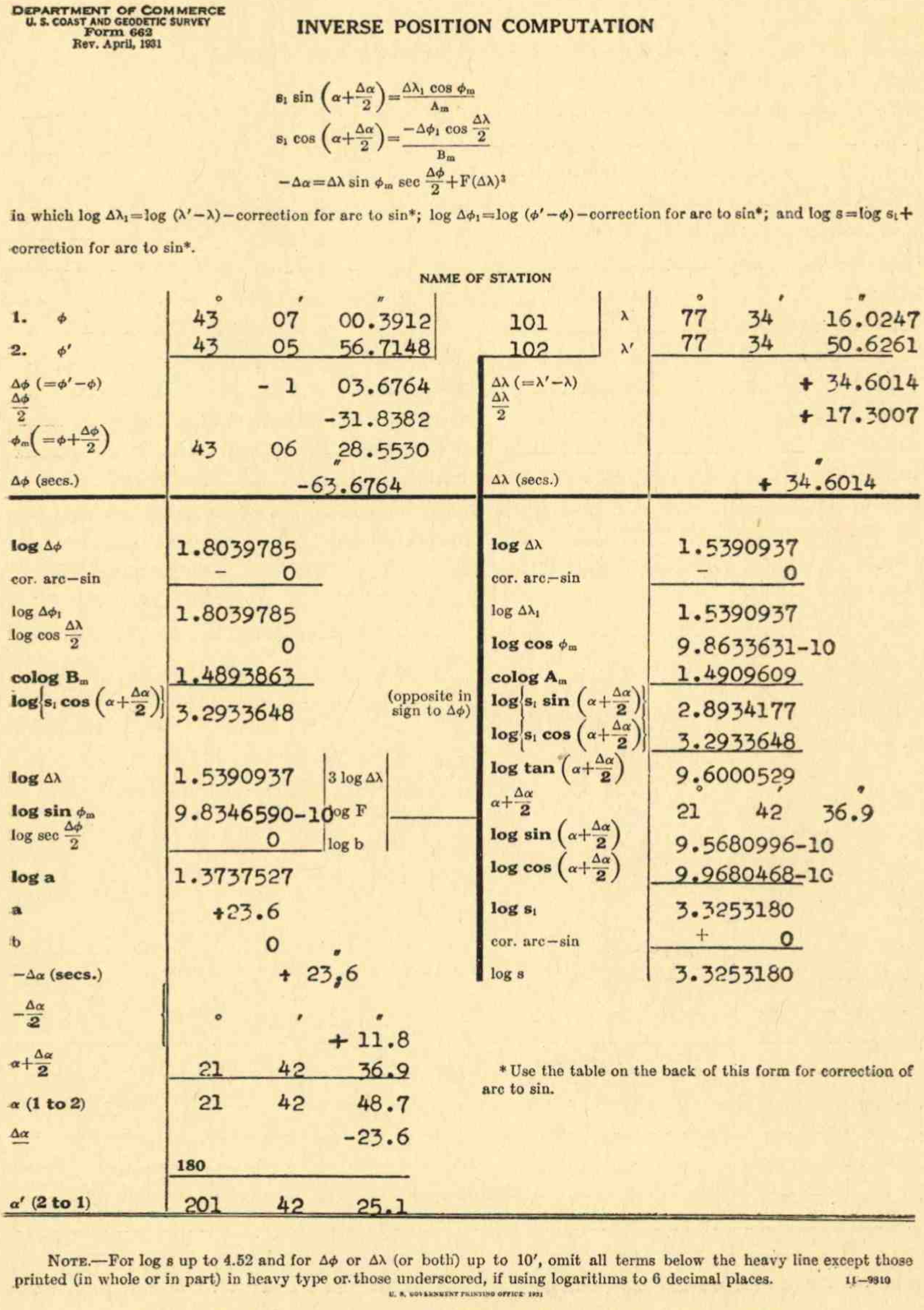
x	770,268.54	log S_g	5.43178341
C	500,000.00	log (1200/3937)	9.48401583
x' (=x−C)	270,268.54	log (1/R)	+ 2714
$x'^3/(6\rho_o^2)_g$	− 7.52	log S_m	4.91582638
S_g	270,261.02	cor arc to sine	− 1204
		log S_1	4.91581434
log S_m^2	9.831653	log A	8.50903886
log C	1.375693-10	log sec ϕ	0.13669963
log $\Delta\phi$	1.207346	log $\Delta\lambda_1$	3.56155283
		cor sine to arc	+ 2259
y	1,137,250.18	log $\Delta\lambda$	3.56157542
ϕ'(by interpolation)	43° 07′ 16″.5105	$\Delta\lambda$	+ 3643″.9753
$\Delta\phi$	− 16.1193	λ (central mer.)	78° 35′ 00″.0000
ϕ	43 07 00.3912	$\Delta\lambda$	− 1 00 43.9753
		λ	77 34 16.0247

Station 102

x	767,779.28	log S_g	5.42776512
C	500,000.00	log (1200/3937)	9.48401583
x' (=x−C)	267,779.28	log (1/R)	+ 2714
$x'^3/(6\rho_o^2)_g$	− 7.31	log S_m	4.91180809
S_g	267,771.97	cor. arc to sine	− 1182
		log S_1	4.91179627
log S_m^2	9.823616	log A	8.50903930
log C	1.375424-10	log sec ϕ	0.13657414
log $\Delta\phi$	1.199040	log $\Delta\lambda_1$	3.55740971
		cor. sine to arc	+ 2216
y	1,130,772.79	log $\Delta\lambda$	3.55743187
ϕ'(by interpolation)	43° 06′ 12″.5287	$\Delta\lambda$	+ 3609″.3739
$\Delta\phi$	− 15.8139	λ (central mer.)	78° 35′ 00″.0000
ϕ	43 05 56.7148	$\Delta\lambda$	− 1 00 09.3739
		λ	77 34 50.6261

(over)

FIGURE 5.—Computation of geodetic positions of stations 101 and 102 from transverse Mercator coordinates.



DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form 662
Rev. April, 1931

INVERSE POSITION COMPUTATION

$$s_1 \sin\left(\alpha+\frac{\Delta\alpha}{2}\right)=\frac{\Delta\lambda_1 \cos\phi_m}{A_m}$$

$$s_1 \cos\left(\alpha+\frac{\Delta\alpha}{2}\right)=\frac{-\Delta\phi_1 \cos\frac{\Delta\lambda}{2}}{B_m}$$

$$-\Delta\alpha=\Delta\lambda \sin\phi_m \sec\frac{\Delta\phi}{2}+F(\Delta\lambda)^3$$

in which $\log \Delta\lambda_1=\log(\lambda'-\lambda)-$correction for arc to sin*; $\log \Delta\phi_1=\log(\phi'-\phi)-$correction for arc to sin*; and $\log s=\log s_1+$ correction for arc to sin*.

	°	′	″	NAME OF STATION		°	′	″
1. ϕ	43	07	00.3912	101	λ	77	34	16.0247
2. ϕ'	43	05	56.7148	102	λ'	77	34	50.6261
$\Delta\phi\ (=\phi'-\phi)$		- 1	03.6764	$\Delta\lambda\ (=\lambda'-\lambda)$				+ 34.6014
$\frac{\Delta\phi}{2}$			-31.8382	$\frac{\Delta\lambda}{2}$				+ 17.3007
$\phi_m\left(=\phi+\frac{\Delta\phi}{2}\right)$	43	06	28.5530					
$\Delta\phi$ (secs.)			-63.6764	$\Delta\lambda$ (secs.)				+ 34.6014

log $\Delta\phi$	1.8039785	
cor. arc−sin	− 0	
log $\Delta\phi_1$	1.8039785	
log cos $\frac{\Delta\lambda}{2}$	0	
colog B_m	1.4893863	
log$\left\{s_1 \cos\left(\alpha+\frac{\Delta\alpha}{2}\right)\right\}$	3.2933648	(opposite in sign to $\Delta\phi$)
log $\Delta\lambda$	1.5390937	3 log $\Delta\lambda$
log sin ϕ_m	9.8346590-10	log F
log sec $\frac{\Delta\phi}{2}$	0	log b
log a	1.3737527	
a	+23.6	
b	0	
$-\Delta\alpha$ (secs.)	″ + 23.6	
$-\frac{\Delta\alpha}{2}$	° ′ ″ + 11.8	
$\alpha+\frac{\Delta\alpha}{2}$	21 42 36.9	
α (1 to 2)	21 42 48.7	
$\Delta\alpha$	-23.6	
	180	
α' (2 to 1)	201 42 25.1	

log $\Delta\lambda$	1.5390937
cor. arc−sin	− 0
log $\Delta\lambda_1$	1.5390937
log cos ϕ_m	9.8633631-10
colog A_m	1.4909609
log$\left\{s_1 \sin\left(\alpha+\frac{\Delta\alpha}{2}\right)\right\}$	2.8934177
log$\left\{s_1 \cos\left(\alpha+\frac{\Delta\alpha}{2}\right)\right\}$	3.2933648
log tan $\left(\alpha+\frac{\Delta\alpha}{2}\right)$	9.6000529
$\alpha+\frac{\Delta\alpha}{2}$	21° 42′ 36.9″
log sin $\left(\alpha+\frac{\Delta\alpha}{2}\right)$	9.5680996-10
log cos $\left(\alpha+\frac{\Delta\alpha}{2}\right)$	9.9680468-10
log s_1	3.3253180
cor. arc−sin	+ 0
log s	3.3253180

* Use the table on the back of this form for correction of arc to sin.

NOTE.—For log s up to 4.52 and for $\Delta\phi$ or $\Delta\lambda$ (or both) up to 10′, omit all terms below the heavy line except those printed (in whole or in part) in heavy type or those underscored, if using logarithms to 6 decimal places. 11—9810

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FIGURE 6.—Inverse position computation between stations 101 and 102.

GRID AZIMUTHS OF NO-CHECK POINTS AND SHORT LINES

A "no check" position is one whose determination depends upon observations from two triangulation stations whose coordinates, whether the geodetic or plane, are known. A "no check" point may be in error due to the fact that the objects sighted on from the two stations are not the same. If for example the point is a church steeple it may be that a different steeple is sighted on from the second station than that observed from the first station. On account of this possibility of error a "no check" point should, in no case, be used as a control point for local traverses. It can, however, be used as an azimuth mark at a "check" station, for it is possible that the position of a "no check" point may be erroneous and yet the grid azimuth on it from another station may be correctly determined provided the person who later uses this azimuth sights on the same object as did the original observer. It should not be expected that a grid azimuth computed from coordinates on a "no check" point will check the value that is given in the list of plane coordinates. In such a list the value given is the nearest to the true value.

In Special Publications Nos. 194 and 195 the statement is made that the grid azimuth should be computed from the coordinates whenever these values are known for both ends of the line. For the closest determination of the grid azimuth the geodetic coordinates of the stations selected should be expressed to the thousandths of a second. If the latitude and longitude are given to two decimal places only, the computed plane coordinates may be in error as much as one-half foot on either or both of the coordinates. If a line is short and such approximations are present, the grid azimuth computed from such coordinates may be incorrect by as much as 1 minute. In all such cases, the value obtained by application of the θ angle on the Lambert projection, and the $\Delta\alpha$ angle on the transverse Mercator projection should be used for such a value is about what would be obtained if the geodetic position were carried out to the usual three decimal places. The caution mentioned above would apply particularly to "no check" points whose positions are expressed only to two decimal places. As a general rule, a grid azimuth on a "no check" point should be obtained by the application of a θ angle or a $\Delta\alpha$ angle just as is done on the determination of the grid azimuth for the usual type of azimuth mark.

Even in the case of a checked intersection station or a secondary station, if the line joining it to the main station is short, the grid azimuth computed from the coordinates is not as good as that derived from application of the θ or $\Delta\alpha$ angle. The third decimal places in the geodetic positions represent approximately tenths of a foot. The neglected fourth decimal places are therefore hundredths of a foot and on a short line this neglected part may affect the azimuth unduly. For lines a mile or less in length it is therefore thought best to derive the grid azimuth by application of the θ or $\Delta\alpha$ angle to the geodetic azimuth. In both of the cases discussed above, it should not be surprising if the grid azimuth computed from the coordinates does not quite agree with the value given in the list of plane coordinates, because for the latter the computation has been made in the way to give the best possible value from the data in hand.

In a traverse, the lines are short in general but the coordinates are computed directly to the nearest hundredth of a foot and hence the adjusted grid azimuth should in all cases be computed from the final coordinates. The discussion of short lines above applies only to geodetic data and the method of computing the grid azimuths from the same.

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