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Generalized Geraghty type mappings on partial metric spaces and fixed point results

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Abstract In the present paper, we introduce generalized Geraghty (Proc Am Math Soc 40:604–608, 1973) mappings on partial metric spaces and give a fixed point theorem which generalizes some recent results appearing in the literature.

Mathematics Subject Classification 54H25 · 47H10

الملخص

في هذا البحث، نقدّم راسمات جيرافتي (Geraghty) [12] المعممة على فضاءات مترية جزئية ونعطي مبرهنة نقطة ثابتة تعمِّم بعض النتائج الحالية الموجودة في دراسات سابقة.

1 Introduction

Partial metric spaces were introduced by Matthews in [16] as a part of the study of denotational semantics of dataflow networks. These spaces are generalizations of usual metric spaces where the self distance for any point need not be equal to zero.

Let us recall that a partial metric on a set X is a function $p : X \times X \to [0, \infty)$ such that for all $x, y, z \in X$: (i) $x = y \iff p(x, x) = p(x, y) = p(y, y)$ (T_0-separation axiom), (ii) $p(x, x) \leq p(x, y)$ (small self-distance axiom), (iii) $p(x, y) = p(y, x)$ (symmetry), (iv) $p(x, y) \leq p(x, z) + p(z, y) - p(z, z)$ (modified triangular inequality).

A partial metric space (for short PMS) is a pair (X, p) such that X is a nonempty set and p is a partial metric on X.

It is clear that if $p(x, y) = 0$, then $x = y$. But if $x = y$, $p(x, y)$ may not be 0.

At this point it seems interesting to remark the fact that partial metric spaces play an important role in constructing models in the theory of computation (see for instance [9,11,13]).

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Example 1.1 Let $X = [0, \infty)$ and $p(x, y) = \max\{x, y\}$ for all $x, y \in X$. Then (X, p) is a PMS.

Example 1.2 Let I denote the set of all intervals $[a, b]$ for any real numbers $a \leq b$. Let $p : I \times I \to [0, \infty)$ be the function such that $p([a, b], [c, d]) = \max\{b, d\} - \min\{a, c\}$. Then (I, p) is a PMS.

Example 1.3 Let $X = \mathbb{R}$ and $p(x, y) = e^{\max\{x,y\}}$ for all $x, y \in X$. Then (X, p) is a PMS.

Other examples of partial metric spaces may be found in [11,14,16,18].

Each partial metric p on X generates a T_0 topology τ_p on X which has as a base the family of open p-balls

$$\{B_p(x, \varepsilon) : x \in X, \varepsilon > 0\},$$

where

$$B_p(x, \varepsilon) = \{y \in X : p(x, y) < p(x, x) + \varepsilon\},$$

for all $x \in X$ and $\varepsilon > 0$.

Observe that a sequence $\{x_n\}$ in a PMS (X, p) converges to a point $x \in X$, with respect to τ_p, if and only if $p(x, x) = \lim_{n\to\infty} p(x, x_n)$.

If p is a partial metric on X, then the functions $p^s, p^w : X \times X \to \mathbb{R}^+$ given by

$$p^s(x, y) = 2p(x, y) - p(x, x) - p(y, y)$$

and

$$\begin{aligned} p^w(x, y) &= \max\{p(x, y) - p(x, x), p(x, y) - p(y, y)\} \\ &= p(x, y) - \min\{p(x, x), p(y, y)\} \end{aligned}$$

are ordinary metrics on X. It is easy to see that p^s and p^w are equivalent metrics on X.

According to [16], a sequence $\{x_n\}$ in a partial metric (X, p) converges, with respect to τ_{p^s}, to a point $x \in X$ if and only if

$$\lim_{n,m\to\infty} p(x_n, x_m) = \lim_{n\to\infty} p(x_n, x) = p(x, x).$$

A sequence $\{x_n\}$ in a partial metric (X, p) is called a Cauchy sequence if there exists (and is finite) $\lim_{n,m\to\infty} p(x_n, x_m)$. (X, p) is called complete if every Cauchy sequence $\{x_n\}$ in X converges, with respect to τ_p, to a point $x \in X$ such that $p(x, x) = \lim_{n,m\to\infty} p(x_n, x_m)$.

Finally, the following crucial facts are shown in [16]:

(a) $\{x_n\}$ is a Cauchy sequence in (X, p) if and only if it is a Cauchy sequence in the metric space (X, p^s).
(b) (X, p) is complete if and only if (X, p^s) is complete.

Matthews obtained, among other results, a partial metric version of the Banach fixed point theorem ([16] Theorem 5.3) as follows.

Theorem 1.4 *[16] Let (X, p) be a complete partial metric space and let $T : X \to X$ be a contraction mapping, that is, there exists $\lambda \in [0, 1)$ such that*

$$p(Tx, Ty) \leq \lambda p(x, y)$$

for all $x, y \in X$. Then T has a unique fixed point $z \in X$. Moreover, $p(z, z) = 0$.

Later on, Abdeljawad et al. [1], Acar et al. [2,3], Altun et al. [4–7], Karapinar and Erhan [15], Oltra and Valero [17] and Valero [23] gave some generalizations of the result of Matthews. Also, Ćirić et al. [8], Samet et al. [21] and Shatanawi et al. [22] proved some common fixed point results in partial metric spaces. The best two generalizations of it were given by Romaguera [19,20].

Theorem 1.5 *Let (X, p) be a complete partial metric space and let $T : X \to X$ be a map such that*

$$p(Tx, Ty) \leq \varphi(M(x, y))$$

for all $x, y \in X$, where

$$M(x, y) = \max\left\{p(x, y), p(x, Tx), p(y, Ty), \frac{1}{2}[p(x, Ty) + p(y, Tx)]\right\}$$

and φ satisfies one of the following:

(i) $\varphi : [0, \infty) \to [0, \infty)$ *is an upper semicontinuous from the right such that* $\varphi(t) < t$ *for all* $t > 0$ *[19].*

(ii) $\varphi : [0, \infty) \to [0, \infty)$ *is a nondecreasing function such that* $\varphi^n(t) \to 0$ *as* $n \to \infty$ *for all* $t > 0$ *[20].*

Then T *has a unique fixed point* $z \in X$*. Moreover,* $p(z, z) = 0$.

On the other hand, Dukic et al. [10] proved the following nice fixed point theorem. Before, we introduce the set S of functions $\beta : [0, \infty) \to [0, 1)$ satisfying

$$\beta(t_n) \to 1 \quad \text{implies } t_n \to 0.$$

Theorem 1.6 *Let* (X, p) *be a complete partial metric space and let* $T : X \to X$ *be a self-map. Suppose that there exists* $\beta \in S$ *such that*

$$p(Tx, Ty) \le \beta(p(x, y))p(x, y)$$

holds for all $x, y \in X$*. Then* T *has a unique fixed point* $z \in X$ *and for each* $x \in X$ *the Picard sequence* $\{T^n x\}$ *converges to* z *when* $n \to \infty$.

Remark 1.7 The function $\varphi(t) = \frac{1}{2}$ belongs to S, but it does not satisfy the condition $\varphi(t) < t$ for all $t > 0$. On the other hand, the function

$$\varphi(t) = \begin{cases} \dfrac{\arctan t}{t}, & t > 0 \\ 0, & t = 0 \end{cases}$$

belongs to S, but φ is not nondecreasing.

The purpose of this paper is to present a fixed point result in partial metric spaces by using functions belonging to S and contractions of Ciric type.

2 The main result

Our main result is the following.

Theorem 2.1 *Let* (X, p) *be a complete partial metric space and let* $T : X \to X$ *be a self-map. Suppose that there exists* $\beta \in S$ *such that*

$$p(Tx, Ty) \le \beta(M(x, y)) \max\{p(x, y), p(x, Tx), p(y, Ty)\} \tag{2.1}$$

holds for all $x, y \in X$*, where*

$$M(x, y) = \max\left\{p(x, y), p(x, Tx), p(y, Ty), \frac{1}{2}[p(x, Ty) + p(y, Tx)]\right\}.$$

Then T *has a unique fixed point* $z \in X$.

Proof We take $x_0 \in X$ and consider $x_n = Tx_{n-1} = T^n x_0$ for every $n \in \mathbb{N}$. If $x_n = x_{n+1}$ for some $n \in \mathbb{N}$, then x_n is a fixed point of T and the existence part of the proof is finished. Suppose that $x_n \ne x_{n+1}$ for every $n \in \mathbb{N}$. Then by using the contractive condition (2.1), we have for every $n \in \mathbb{N}$

$$\begin{aligned} p(x_n, x_{n+1}) &= p(Tx_{n-1}, Tx_n) \\ &\le \beta(M(x_{n-1}, x_n)) \max\{p(x_{n-1}, x_n), p(x_n, x_{n+1})\}. \end{aligned}$$

Since $\beta : [0, \infty) \to [0, 1)$, we have

$$\begin{aligned} p(x_n, x_{n+1}) &\le \beta(M(x_{n-1}, x_n)) \max\{p(x_{n-1}, x_n), p(x_n, x_{n+1})\} \\ &< \max\{p(x_{n-1}, x_n), p(x_n, x_{n+1})\}. \end{aligned}$$

This shows that

$$p(x_n, x_{n+1}) < p(x_{n-1}, x_n)$$

and so

$$p(x_n, x_{n+1}) \le \beta(M(x_{n-1}, x_n))p(x_{n-1}, x_n) < p(x_{n-1}, x_n) \tag{2.2}$$

for any $n \in \mathbb{N}$. On the other hand, since

$$\begin{aligned}\frac{1}{2}[p(x_{n-1}, x_{n+1}) + p(x_n, x_n)] &\leq \frac{1}{2}[p(x_{n-1}, x_n) + p(x_n, x_{n+1})] \\ &\leq \max\{p(x_{n-1}, x_n), p(x_n, x_{n+1})\} \\ &= p(x_{n-1}, x_n),\end{aligned}$$

then,

$$\begin{aligned}M(x_{n-1}, x_n) = \max\Big\{&p(x_{n-1}, x_n), p(x_{n-1}, x_n), p(x_n, x_{n+1}), \\ &\frac{1}{2}[p(x_{n-1}, x_{n+1}) + p(x_n, x_n)]\Big\} \\ = p(x_{n-1}, x_n)&\end{aligned}$$

and, moreover, $\{p(x_n, x_{n+1})\}$ is a nonincreasing sequence of nonnegative real numbers. Hence $\lim_{n\to\infty} p(x_n, x_{n+1}) = \gamma \geq 0$ for certain $\gamma \in [0, \infty)$.

Now we will prove that $\gamma = 0$. In the contrary case, from (2.2)

$$\frac{p(x_n, x_{n+1})}{p(x_{n-1}, x_n)} \leq \beta(M(x_{n-1}, x_n)) = \beta(p(x_{n-1}, x_n)) < 1$$

and letting $n \to \infty$

$$1 = \frac{\gamma}{\gamma} = \lim_{n\to\infty} \frac{p(x_n, x_{n+1})}{p(x_{n-1}, x_n)} \leq \lim_{n\to\infty} \beta(p(x_{n-1}, x_n)) \leq 1.$$

Consequently, $\lim_{n\to\infty} \beta(p(x_{n-1}, x_n)) = 1$ and, since $\beta \in S$, $\gamma = \lim_{n\to\infty} p(x_{n-1}, x_n) = 0$. This contradicts that $\gamma > 0$. Therefore, $\lim_{n\to\infty} p(x_{n-1}, x_n) = 0$.

Now we will prove that $\lim_{n,m\to\infty} p(x_n, x_m) = 0$. In the contrary case, $\limsup_{n,m\to\infty} p(x_n, x_m) > 0$. Using the modified triangular inequality of a partial metric, we have

$$\begin{aligned}p(x_n, x_m) &\leq p(x_n, x_{n+1}) + p(x_{n+1}, x_m) - p(x_{n+1}, x_{n+1}) \\ &\leq p(x_n, x_{n+1}) + p(x_{n+1}, x_m) \\ &\leq p(x_n, x_{n+1}) + p(x_{n+1}, x_{m+1}) + p(x_{m+1}, x_m) - p(x_{m+1}, x_{m+1}) \\ &\leq p(x_n, x_{n+1}) + p(x_{n+1}, x_{m+1}) + p(x_{m+1}, x_m).\end{aligned} \tag{2.3}$$

Since

$$\begin{aligned}p(x_{n+1}, x_{m+1}) &= p(Tx_n, Tx_m) \\ &\leq \beta(M(x_n, x_m)) \max\{p(x_n, x_m), p(x_n, x_{n+1}), p(x_m, x_{m+1})\}\end{aligned}$$

from (2.3) we get

$$\begin{aligned}p(x_n, x_m) \leq\ & p(x_n, x_{n+1}) + p(x_{m+1}, x_m) + \\ & \beta(M(x_n, x_m)) \max\{p(x_n, x_m), p(x_n, x_{n+1}), p(x_m, x_{m+1})\} \\ \leq\ & p(x_n, x_{n+1}) + p(x_{m+1}, x_m) + \beta(M(x_n, x_m)) p(x_n, x_m) \\ & + \beta(M(x_n, x_m)) \max\{p(x_n, x_{n+1}), p(x_m, x_{m+1})\} \\ <\ & p(x_n, x_{n+1}) + p(x_{m+1}, x_m) + \beta(M(x_n, x_m)) p(x_n, x_m) \\ & + \max\{p(x_n, x_{n+1}), p(x_m, x_{m+1})\} \\ \leq\ & 3\max\{p(x_n, x_{n+1}), p(x_m, x_{m+1})\} + \beta(M(x_n, x_m)) p(x_n, x_m)\end{aligned}$$

or equivalently

$$[1 - \beta(M(x_n, x_m))] p(x_n, x_m) \leq 3\max\{p(x_n, x_{n+1}), p(x_m, x_{m+1})\}$$

From the last inequality, it follows

$$p(x_n, x_m) \leq [1 - \beta(M(x_n, x_m))]^{-1} [3\max\{p(x_n, x_{n+1}), p(x_m, x_{m+1})\}].$$

Now, since

$$\lim \sup_{n,m\to\infty} p(x_n, x_m) > 0$$

and

$$\lim_{n\to\infty} p(x_n, x_{n+1}) = \lim_{m\to\infty} p(x_m, x_{m+1}) = 0,$$

the last inequality implies $\lim\sup_{n,m\to\infty}[1-\beta(M(x_n, x_m))]^{-1} = \infty$. This means that $\lim\sup_{n,m\to\infty} \beta(M(x_n, x_m)) = 1$. Since $\beta \in S$, $\lim\sup_{n,m\to\infty} M(x_n, x_m) = 0$. Since

$$M(x_n, x_m) = \max\Big\{p(x_n, x_m),\ p(x_n, x_{n+1}),\ p(x_m, x_{m+1}),\ \frac{1}{2}[p(x_n, x_{m+1}) + p(x_m, x_{n+1})]\Big\},$$

in particular, we get $\lim\sup_{n,m\to\infty} p(x_n, x_m) = 0$ and this contradicts our assumption that $\lim\sup_{n,m\to\infty} p(x_n, x_m) > 0$. Therefore, $\lim_{n,m\to\infty} p(x_n, x_m) = 0$. This means that $\{x_n\}$ is a Cauchy sequence in the complete partial metric space (X, p) and, consequently, there exists $z \in X$ such that

$$0 = \lim_{n,m\to\infty} p(x_n, x_m) = \lim_{n\to\infty} p(x_n, z) = p(z, z).$$

Now we will prove that z is a fixed point of T. For this assume $p(z, Tz) > 0$. Then, we have

$$\begin{aligned} p(z, Tz) &\le p(z, Tx_n) + p(Tx_n, Tz) - p(Tx_n, Tx_n) \\ &\le p(z, x_{n+1}) + p(Tx_n, Tz) \\ &\le p(z, x_{n+1}) + \beta(M(x_n, z)) \max\{p(x_n, z), p(x_n, x_{n+1}), p(z, Tz)\} \\ &\le p(z, x_{n+1}) + \beta(M(x_n, z)) \max\{p(x_n, z), p(x_n, x_{n+1})\} \\ &\quad +\beta(M(x_n, z))p(z, Tz) \\ &< p(z, x_{n+1}) + \max\{p(x_n, z), p(x_n, x_{n+1})\} + \beta(M(x_n, z))p(z, Tz) \end{aligned}$$

and so

$$[1 - \beta(M(x_n, z))]p(z, Tz) < p(z, x_{n+1}) + \max\{p(x_n, z), p(x_n, x_{n+1})\}$$

or equivalently

$$p(z, Tz) < [1 - \beta(M(x_n, z))]^{-1}\{p(z, x_{n+1}) + \max\{p(x_n, z), p(x_n, x_{n+1})\}\}.$$

Now, since

$$p(z, Tz) > 0$$

and

$$\lim_{n\to\infty} p(x_n, x_{n+1}) = \lim_{n\to\infty} p(x_n, z) = 0,$$

the last inequality implies $\lim_{n\to\infty}[1 - \beta(M(x_n, z))]^{-1} = \infty$. This means that $\lim_{n\to\infty} \beta(M(x_n, z)) = 1$. Since $\beta \in S$, $\lim_{n\to\infty} M(x_n, z) = 0$. This contradicts the inequality $p(z, Tz) > 0$, so that $p(z, Tz) = 0$, which implies $z = Tz$. Suppose that z and w are fixed points of T then, if $z \ne w$,

$$\begin{aligned} p(z, w) = p(Tz, Tw) &\le \beta(M(z, w)) \max\{p(z, w), p(z, z), p(w, w)\} \\ &< p(z, w) \end{aligned}$$

and this is a contradiction. This proves the uniqueness of the fixed point of T. □

We can obtain the following corollary from Theorem 2.1.

Corollary 2.2 *Let (X, p) be a complete partial metric space and let $T : X \to X$ be a self-map. Suppose that there exists $\beta \in S$ such that*

$$p(Tx, Ty) \leq \beta(M(x, y))p(x, y)$$

holds for all $x, y \in X$, where

$$M(x, y) = \max\left\{p(x, y), p(x, Tx), p(y, Ty), \frac{1}{2}[p(x, Ty) + p(y, Tx)]\right\}.$$

Then T has a unique fixed point $z \in X$.

Remark 2.3 If we take the function $\beta(t) = \lambda$, $\lambda \in (0, 1)$, which is in S, then we obtain Theorem 1.4.

Now, we give an example which illustrates our results.

Example 2.4 Let $X = [0, 1]$ and $p(x, y) = e^{\max\{x,y\}} - 1$. Then (X, p) is a complete partial metric space. Define $T : X \to X$ by

$$Tx = \begin{cases} 0, & x = 1 \\ \frac{x}{2}, & x \neq 1 \end{cases}.$$

We show that the contractive condition of Corollary 2.2 is satisfied for $\beta(t) = \frac{1}{2}$.

To this end, we have to consider the following cases.

Case 1. If $y \leq x < 1$, then

$$\begin{aligned} p(Tx, Ty) &= e^{\max\{Tx,Ty\}} - 1 = e^{\frac{x}{2}} - 1 \\ &\leq \frac{e^x - 1}{2} = \frac{1}{2}p(x, y) \\ &= \beta(M(x, y))p(x, y). \end{aligned}$$

Case 2. Let $y < x = 1$, then

$$\begin{aligned} p(T1, Ty) &= e^{\max\{T1,Ty\}} - 1 = e^{\frac{y}{2}} - 1 \\ &\leq \frac{e - 1}{2} = \frac{1}{2}p(1, y) \\ &\leq \beta(M(1, y))p(1, y). \end{aligned}$$

Case 3. Let $x = y = 1$, then

$$\begin{aligned} p(T1, T1) = 0 &\leq \frac{e - 1}{2} \\ &= \frac{1}{2}p(1, 1) \\ &\leq \beta(M(1, 1))p(1, 1). \end{aligned}$$

Hence, all conditions of Corollary 2.2 are satisfied. Therefore, T has a unique fixed point in X.

Note that if we use the ordinary metric $d(x, y) = |x - y|$ instead of p, we cannot find a function $\beta \in S$ satisfying the considered contraction condition. Indeed, in this case

$$d\left(T\frac{3}{4}, T1\right) = \frac{3}{8}, d\left(\frac{3}{4}, 1\right) = \frac{1}{4}, d\left(\frac{3}{4}, T\frac{3}{4}\right) = \frac{3}{8} \text{ and } d(1, T1) = 1$$

and so there is no any function $\beta \in S$ satisfying $\frac{3}{8} \leq \beta(M(\frac{3}{4}, 1))\frac{1}{4}$.

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