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Simultaneous optimization of controlled structures*

M. Salama, J. Garba and **L. Demsetz**
Applied Technologies Section, Jet Propulsion Laboratory, California Institute of Technology, Pasadena, CA, USA

F. Udwadia
Civil Engineering, Mechanical Engineering and Decision Systems, University of Southern California, Los Angeles, CA, USA

Abstract. A formulation is presented for finding the combined optimal design of a structural system and its control by defining a composite objective function as a linear combination of two components; a structural objective and a control objective. When the structural objective is a function of the structural design variables only, and when the control objective is represented by the quadratic functional of the response and control energy, it is possible to analytically express the optimal control in terms of any set of "admissible" structural design variables. Such expression for the optimal control is used recursively in an iterative Newton-Raphson search scheme, the goal of which is to determine the corresponding optimal set of structural design variables that minimize the combined objective function. A numerical example is given to illustrate the computational procedure. The results indicate that significant improvement of the combined optimal design can be achieved over the traditional separate optimization.

1 Introduction

The optimal design of structural systems whose response to disturbances must be controlled to meet certain design objectives has traditionally proceeded along two separate but sequential paths. First, the structure is optimized by selecting an optimal set of structural design variables $\boldsymbol{a}^*$, which minimize a structural objective function J_S – often taken as the mass of the structure – subjected to a set of predetermined behavioral constraints $h_j(\boldsymbol{a}) \geq 0$ on deformations, stresses, frequencies, etc.:

$$J_S(\boldsymbol{a}^*) = \min_{\boldsymbol{a}} J_S(\boldsymbol{a}); \quad h_j(\boldsymbol{a}) \geq 0; \quad \boldsymbol{a}^* \in \boldsymbol{a} \tag{1}$$

During the structural optimization, the external loads are taken to be design-invariant, regardless of whether they are due to external disturbances or due to actions for controlling the response of interest.

Second, having completely specified the optimal structural design $\boldsymbol{a}^*$, optimal control theory is used to determine an optimal set of control variables $\boldsymbol{u}^*$ that minimize a control objective function J_C – frequently taken as a quadratic cost functional of the response and control energy:

$$J_C(\boldsymbol{a}^*, \boldsymbol{u}^*) = \min_{\boldsymbol{u}} J_C(\boldsymbol{a}^*, \boldsymbol{u}); \quad \boldsymbol{u}^* \in \boldsymbol{u} \tag{2}$$

During the minimization in (2), $\boldsymbol{a}^*$ is not allowed to change. Performing the two minimization problems (1) and (2) sequentially, is mathematically equivalent to finding the linear sum, $J(\boldsymbol{a}^*, \boldsymbol{u}^*)$, of two separate minima:

$$J(\boldsymbol{a}^*, \boldsymbol{u}^*) = \min_{\boldsymbol{a}} J_S(\boldsymbol{a}) + \min_{\boldsymbol{u}} J_C(\boldsymbol{a}^*, \boldsymbol{u}) \tag{3}$$

During each minimization, the design space is artificially decoupled into a structure design space and a control design space, and searching for the optimal $\boldsymbol{a}^*$ and $\boldsymbol{u}^*$ is carried out in their respective spaces, separately.

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The question then arises as to whether or not it is possible to attain a more superior optimum by determining the minimum of a single objective function which linearly combines the structure and control objectives, instead of linearly combining their separate minima as in (3). The former approach is referred to herein as "simultaneous" optimization, while the latter is the traditional "separate" optimization.

The paper is focussed on developing a methodology for performing the simultaneous optimization problem and contrasting the results with those from the traditional separate optimization. In a simultaneous optimization approach, one is interested in selecting values of the set of structural and control design variables, $\boldsymbol{a}^{**}$ and $\boldsymbol{u}^{**}$, which together, must minimize a combined objective criterion $J(\boldsymbol{a}^{**}, \boldsymbol{u}^{**})$, subject to certain behavioral constraints $h_j(\boldsymbol{a})$. Thus;

$$J(\boldsymbol{a}^{**}, \boldsymbol{u}^{**}) = \min_{\boldsymbol{a},\boldsymbol{u}} [J_S(\boldsymbol{a}) + J_C(\boldsymbol{a}, \boldsymbol{u})]; \qquad h_j(\boldsymbol{a}) \geq 0 \tag{4}$$

Since the "min" is a nonlinear operator which decreases the value of the operand, it follows from (3) and (4) that

$$J(\boldsymbol{a}^{**}, \boldsymbol{u}^{**}) \leq J(\boldsymbol{a}^{*}, \boldsymbol{u}^{*}) \tag{5}$$

i.e., the minimum of the sum is less than or equal to the sum of the minima, and the simultaneous optimum is expected to be superior to the separately obtained optimum.

The basic motivating idea embodied by Eq. (5) has been recently recognized by a number of investigators (Hale et al. 1983; Hanks and Skelton 1983; Komkov 1983; Messac and Turner 1984; Venkayya and Tischler 1984). In this paper, we present a methodology for performing the simultaneous optimization when the structural objective function does not depend on the control variables and a quadratic control objective function is assumed. Structure-control systems belonging to this class are shown here to lead to a simple formulation that enjoys the same theoretical guarantees regarding stability and controllability of the system as for the conventional state regulator problem. A suitable computational scheme is developed, and the results of the method are illustrated by a numerical example.

2 Formulation

2.1 Optimization objective

Consider a structural system subject to known initial conditions $\boldsymbol{v}(t_0) = \boldsymbol{v}_0$, $\dot{\boldsymbol{v}}(t_0) = \dot{\boldsymbol{v}}_0$. The discrete system equation which include a force vector $\boldsymbol{u}(t)$ for controlling the dynamic response $\boldsymbol{v}(t)$ is:

$$\mathbf{M}(\boldsymbol{a})\,\ddot{\boldsymbol{v}} + \mathbf{D}(\boldsymbol{a})\,\dot{\boldsymbol{v}} + \mathbf{K}(\boldsymbol{a})\,\boldsymbol{v} = \mathbf{B}_0\,\boldsymbol{u} \tag{6}$$

where $\mathbf{M}(\boldsymbol{a})$ is $(n_s \times n_s)$ symmetric positive definite mass matrix, n_s being the number of dynamic degrees-of-freedom of the second order system in (6); $\mathbf{K}(\boldsymbol{a})$ is $(n_s \times n_s)$ symmetric definite or semi-definite stiffness matrix; $\boldsymbol{v}$ is n_s-dimension vector of physical coordinates; and $\boldsymbol{u}$ is n_c-dimension vector of control forces whose points of application are mapped onto the structure by the $(n_s \times n_c)$ control influence matrix $\mathbf{B}_0$. Additionally, the damping matrix $\mathbf{D}(\boldsymbol{a})$ is congruent to a diagonal matrix through the modal transformation:

$$\boldsymbol{v} = \boldsymbol{\Phi}\boldsymbol{\eta}; \qquad \boldsymbol{\Phi}^T\mathbf{M}\boldsymbol{\Phi} = \mathbf{I}; \qquad \boldsymbol{\Phi}^T\mathbf{K}\boldsymbol{\Phi} = \mathrm{diag}\,(\omega_{n_s}^2); \qquad \boldsymbol{\Phi}^T\mathbf{D}\boldsymbol{\Phi} = \mathrm{diag}\,(2\zeta_{n_s}\omega_{n_s}) \tag{7}$$

The first order form of Eq. (6) may be expressed in $2n_s$ coordinates $\boldsymbol{x}^T = (\boldsymbol{v}, \dot{\boldsymbol{v}})$, and leads to:

$$\dot{\boldsymbol{x}} = \mathbf{A}(\boldsymbol{a})\boldsymbol{x} + \mathbf{B}(\boldsymbol{a})\boldsymbol{u} \tag{8}$$

where

$$\mathbf{A}(\boldsymbol{a}) = \left(\begin{array}{c|c} \mathbf{0} & \mathbf{I} \\ \hline -\mathbf{M}^{-1}\mathbf{K} & -\mathbf{M}^{-1}\mathbf{D} \end{array}\right); \qquad \mathbf{B}(\boldsymbol{a}) = \left(\begin{array}{c} \mathbf{0} \\ \hline \mathbf{M}^{-1}\mathbf{B}_0 \end{array}\right) \tag{9}$$

The structural system described by (6), (7), and (8) is assumed to consists of n_a independent design variables, $\boldsymbol{a}$, whose magnitudes may be adjusted to create designs having various degrees of efficiency.

For example, $\boldsymbol{a}$, may designate member sizes of bars or beams, or thicknesses of plates or membranes. As such, **M**, **D**, **K**, **A**, **B**, and ω_{n_s} are all function of the design variables $\boldsymbol{a}$.

We assume an output measurement vector z of dimension n_m,

$$z = \mathbf{c}\,\boldsymbol{x} \tag{10}$$

where $\mathbf{c} = [\mathbf{c}_1, \mathbf{c}_2]$ is $(n_m \times 2n_s)$ observation matrix.

In accordance with (4), the simultaneous optimization problem may be stated as follows:

Find the optimal set of variables $[\boldsymbol{a}^{**}, \boldsymbol{u}^{**}] \in [\boldsymbol{a}, \boldsymbol{u}]$ that minimize the objective

$$J(\boldsymbol{a},\boldsymbol{u}) = \left[J_S(\boldsymbol{a}) + \frac{1}{2}\varrho^2 \int_{t=0}^{\infty} (\boldsymbol{x}^T\mathbf{Q}\,\boldsymbol{x} + \boldsymbol{u}^T\mathbf{R}\,\boldsymbol{u})\,\mathrm{d}t \right] \tag{11}$$

subject to the state Eq. (8), along with any n_h number of compatible behavior constraints $h_j(\boldsymbol{a})$ on frequencies, deformations, stresses ... etc.

$$h_j(\boldsymbol{a}) = \bar{U} - U_j(\boldsymbol{a}) \geq 0; \quad j = 1, 2, \ldots n_h \tag{12}$$

and upper bound $\bar{a}$ or lower bound $\underline{a}$ on the i-th design variable a_i:

$$\bar{a} \geq \underline{a}_i \geq \underline{a}; \quad i = 1, 2, \ldots n_a \tag{13}$$

In the present simultaneous structures-control problem, the composite objective function in (11) is selected as a linear combination of two parts; a structural objective, and a control objective. The assumption is made further that the first part $J_S(\boldsymbol{a})$ representing the structural measure of optimality is dependent only upon the structural design variables $\boldsymbol{a}$, while the second part $J_C(\boldsymbol{a},\boldsymbol{u})$ representing the control measure of optimality – here taken as the traditional quadratic performance index – is dependent upon both $\boldsymbol{a}$ and $\boldsymbol{u}$. Since the two parts of the objective function do not necessarily have the same units or magnitudes, one may choose the scalar coefficient ϱ so as to control the relative importance of the two objectives during computations.

2.2 Optimality conditions

Since the control objective in (11) is dependent upon both $\boldsymbol{a}$ and $\boldsymbol{u}$, while the structural objective is dependent only on $\boldsymbol{a}$, Eq. (11) may be restated as:

$$J(\boldsymbol{a}^{**},\boldsymbol{u}^{**}) = \min_{\boldsymbol{a}} \left[J_s(\boldsymbol{a}) + \min_{\boldsymbol{u}} \left[\frac{1}{2}\varrho^2 \int_{t=0}^{\infty} (\boldsymbol{x}^T\mathbf{Q}\,\boldsymbol{x} + \boldsymbol{u}^T\mathbf{R}\,\boldsymbol{u})\,\mathrm{d}t \right]\right] \tag{14}$$

For a specified initial condition $\boldsymbol{x}(0) = \boldsymbol{x}_0$, it is known that (Athans 1966):

$$\min_{\boldsymbol{u}} \left[\frac{1}{2} \int_{t=0}^{\infty} (\boldsymbol{x}^T\mathbf{Q}\,\boldsymbol{x} + \boldsymbol{u}^T\mathbf{R}\,\boldsymbol{u})\,\mathrm{d}t \right] = \frac{1}{2}\boldsymbol{x}_0\mathbf{P}(\boldsymbol{a})\boldsymbol{x}_0 \tag{15}$$

and that the corresponding optimal control $\boldsymbol{u}^{**}$ is obtained from:

$$\boldsymbol{u}^{**} = -\mathbf{R}^{-1}(\boldsymbol{a})\mathbf{B}^T(\boldsymbol{a})\mathbf{P}(\boldsymbol{a})\boldsymbol{x} \tag{16}$$

where $\mathbf{P}(\boldsymbol{a})$ is the positive definite solution of the algebraic Ricatti equation:

$$\mathbf{A}^T(\boldsymbol{a})\,\mathbf{P}(\boldsymbol{a}) + \mathbf{P}(\boldsymbol{a})\,\mathbf{A}(\boldsymbol{a}) + \mathbf{Q}(\boldsymbol{a}) - \mathbf{P}(\boldsymbol{a})\,\mathbf{B}(\boldsymbol{a})\,\mathbf{R}^{-1}(\boldsymbol{a})\,\mathbf{B}^T(\boldsymbol{a})\,\mathbf{P}(\boldsymbol{a}) = \mathbf{0} \tag{17}$$

From the above, it is seen that the optimal control $\boldsymbol{u}^{**}$ and the expression for the minimum in (15) are implicit functions of the design variables $\boldsymbol{a}$. The stability of the closed loop system is assured by the positive definiteness of $\mathbf{P}(\boldsymbol{a})$ for all physically meaningful values of $\boldsymbol{a}$. It is assumed here that conditions for the existence of a positive definite solution to the Ricatti equation are satisfied.

In deriving conditions (15), (16) and (17), use was made of the state Eq. (8), but the behavior and side constraints (12) and (13) were not enforced. The minimization of (11) is thus reduced to selecting an optimal set of structural design variables $\boldsymbol{a}^{**}$ that minimize $\mathbf{F}(\boldsymbol{a})$:

$$F(\boldsymbol{a}) = \left[J_S(\boldsymbol{a}) + \frac{1}{2} \varrho^2 \boldsymbol{x}_0^T \mathbf{P}(\boldsymbol{a}) \boldsymbol{x}_0 \right] \tag{18}$$

subject to constraints (12) and (13). The relationship between the optimal control variables $\boldsymbol{u}^{**}$ and optimal structural variables $\boldsymbol{a}^{**}$ is implicitly preserved by satisfying Eqs. (12), (13), (16), (17) and (18). The constrained problem (18), (12) and (13) may be converted to an unconstrained one of the form:

$$L = F + \sum_{j=0}^{n_h} \lambda_j h_j + \sum_{i=1}^{n_a} [\mu_i (a_i - \underline{a}) + v_i (\bar{a} - a_i)] \tag{19}$$

where λ_j, μ_i and v_i respectively are unknown multipliers, one for each constraint. A local optimum of (19) must necessarily satisfy the following first order Kuhn-Tucker optimality conditions (Hadley 1964):

$$F_{,a_i} + \sum_{j=1} \lambda_j h_{j,a_i} + \mu_i + v_i = 0; \qquad \lambda_j h_j = 0; \qquad \mu_i (a_i - \underline{a}) = 0; \qquad v_i (\bar{a} - a_i) = 0 \tag{20}$$

where the multipliers λ_j, μ_i, v_i must be non-negative for all $j = 1, 2 \ldots n_h$, and $i = 1, 2 \ldots n_a$.

3 Computational aspects

3.1 Recursive relations

A first order minimization based on Fletcher-Powell method was first attempted to perform the optimization numerically. However, the convergence rate was extremely poor. This led us instead, to employing a second order minimizaton based on a modified Newton-Raphson scheme to insure satisfaction of the optimality condition (20). The method is relatively general so as to allow various forms of dependence of $\mathbf{Q}(\boldsymbol{a})$, $\mathbf{R}(\boldsymbol{a})$, $\mathbf{B}(\boldsymbol{a})$, and various types of constraints h_j. The set of design variables $\boldsymbol{a}^{**}$ and multipliers λ that satisfy (18) are obtained iteratively from the recursive relations:

$$\left\{ \begin{array}{c} \boldsymbol{a} \\ \hline \lambda \end{array} \right\}_{r+1} = \left\{ \begin{array}{c} \boldsymbol{a} \\ \hline \lambda \end{array} \right\}_{r} - \alpha \left[\begin{array}{c|c} F_{,\boldsymbol{aa}} + \sum_j \lambda_j h_{j,\boldsymbol{aa}} & h_{j,\boldsymbol{a}} \\ \hline h_{j,\boldsymbol{a}} & 0 \end{array} \right]_r^{-1} \left\{ \begin{array}{c} F_{,\boldsymbol{a}} + \sum_j \lambda_j h_{j,\boldsymbol{a}} \\ \hline h_j \end{array} \right\}_r \tag{21}$$

Rather than including the multipliers μ_i and v_i for the side constraints in (21), these are dealt with indirectly through the parameter α which limit the step size during an iteration so that none of the design variables go outside their range $\underline{a}$ and $\bar{a}$.

3.2 Expression for gradients

The implementation of Eq. (21) requires the availability of the first and second derivatives of the Ricatti solution $\mathbf{P}$, structural cost function, and constraints, all with respect to the design variables $\boldsymbol{a}$. The first derivatives $\mathbf{P}_{,a_i}$, $i = 1, 2, \ldots n_a$ are obtained from Eq. (17), and are governed by the Lyapunov equation:

$$\mathbf{c}_1 + \mathbf{c}_2 \mathbf{P}_{,a_i} + \mathbf{P}_{,a_i} \mathbf{c}_2^T = 0 \tag{22}$$

where

$$\mathbf{c}_1 = \mathbf{A}_{,a_i}^T \mathbf{P} + \mathbf{P} \mathbf{A}_{,a_i} - \mathbf{P} \mathbf{E}_{,a_i} \mathbf{P} + \mathbf{Q}_{,a_i}; \qquad \mathbf{c}_2 = \mathbf{A}^T - \mathbf{P} \mathbf{E}; \qquad \mathbf{E} = \mathbf{B} \mathbf{R}^{-1} \mathbf{B}^T \tag{23}$$

Similarly, the second derivatives $\mathbf{P}_{,a_i a_k}$ $(i = 1, 2, \ldots n_a,\ k = 1, 2, \ldots n_a)$ also obey the Lyapunov equation:

$$\mathbf{c}_3 + \mathbf{c}_2 \mathbf{P}_{,a_i a_k} + \mathbf{P}_{,a_i a_k} \mathbf{c}_2^T = 0 \tag{24}$$

where

$$\mathbf{c}_3 = \mathbf{c}_4 + \mathbf{c}_4^T + \mathbf{c}_5 + \mathbf{c}_5^T$$

$$\mathbf{c}_4 = (\mathbf{A}_{,a_i}^T - \mathbf{P}_{,a_i}\mathbf{E} - \mathbf{P}\mathbf{E}_{,a_i})\mathbf{P}_{,a_k}$$

$$\mathbf{c}_5 = \mathbf{A}_{,a_i a_k}^T \mathbf{P} + \mathbf{A}_{,a_k}^T \mathbf{P}_{,a_i} - \mathbf{P}_{,a_i}\mathbf{E}_{,a_k}\mathbf{P} - \frac{1}{2}\mathbf{P}\mathbf{E}_{,a_i a_k}\mathbf{P} + \frac{1}{2}\mathbf{Q}_{,a_i a_k} \tag{25}$$

Other gradient information for the structural objective function and constraints have been dealt with in the literature (Fleury 1979; Fox and Kapoor 1968; Nelson 1976; Plaut and Huseyin 1973). For example, if a lower bound is placed on the lowest open-loop frequency $\omega_1^2 \geq \omega^{*2}$, then

$$h_1 = \omega_1^2 - \omega^{*2} \geq 0 \tag{26}$$

According to Eq. (20), both ω_{1,a_i}^2 and $\omega_{1,a_i a_k}^2$ are needed, and may be computed in a number of ways as discussed in (Fox and Kapoor 1968; Nelson 1976).

3.3 Minimization algorithm

The algorithm begins with a feasible initial design with which an unconstrained minimization is carried out. This is accomplished by performing a line search using the gradient information to move down in the direction of the negative gradient of the combined objective function. The unconstrained minimization is continued until the minimum is reached, or until the constraints become binding. If the latter occurs, constrained minimization is employed, during which all emergent designs are kept in the feasible domain. If a design moves in the unfeasible domain, the step length is continually reduced until it becomes feasible again. If this caused the step length to be unduly small, an automatic restart is initiated with a new feasible design that lies in the neighborhood of the lastly calculated design. At each step in the constrained minimization procedure, the emergent design is checked to ascertain the extent to which the constraints are satisfied. When the design moves away from the constraints, unconstrained minimization is reverted to.

Thus, the minimization process alternates between iterations which involve unconstrained minimization using the gradient search direction, and iterations which involve constrained minimization utilizing the Hessian matrix. Generally, it was not required in the examples studied to compute the Hessian matrix or the gradient direction at every iteration. Updates were performed every fifth iteration. This reduced the computational effort substantially while providing for rapid convergence with negligible loss of accuracy. As indicated by Eq. (21), the iterative Newton method requires an initial estimate of λ each time a constrained minimization is reverted too. The initial estimate used in this study was taken to be.

$$\lambda = -[\mathbf{H}^T\mathbf{H}]^{-1}\mathbf{H}^T F_{,\mathbf{a}} \tag{27}$$

where

$$\mathbf{H} = [h_{1,\mathbf{a}}, h_{2,\mathbf{a}}, \ldots, h_{n_h,\mathbf{a}}]$$

3.4 Numerical example

The analytical and computational methodology developed in the preceding sections is illustrated by the following numerical example. Consider a cantilever beam modeled by three finite bending elements of equal fixed length l. A lateral control force is applied at the free tip. An optimal design of the structure and control is desired such that

(1) The total structural mass $J_S = \sum_{i=1}^{3} l_i\, a_i\, m_i$ is minimized.

(2) The control energy $J_C = \int_{t=0}^{\infty} (\boldsymbol{x}^T\mathbf{Q}\boldsymbol{x} + \mathrm{u}^T\mathbf{R}\boldsymbol{u})\,\mathrm{d}t$ is minimized.

(3) The fundamental open loop frequency $\omega \geq 0.10$ rad/sec.

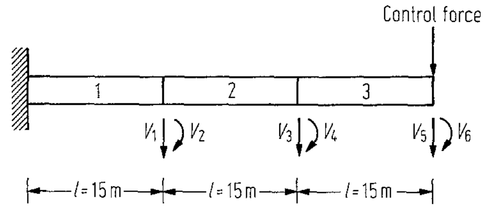


Fig. 1. Structural model; mass density = 1660 kg/m³; elastic modulus = 9.56×10^{10} N/m²

The structural model is shown in Fig. 1. The element mass matrices are assumed diagonal with the mass linearly dependent upon the cross section area a_i. The element bending stiffness is represented by a quadratic function of a_i such that $EI_i = b_1 a_i + b_2 a_i^2$. For a tubular cross section with fixed inner diameter d_i, $b_1 = (\mathrm{E}d_i^2/8)$ and $b_2 = (1/4\pi)$. Damping is assumed independent of the design variables and equal 0.5% of the critical damping of modes of the uniform design. An initial uniform design $a_i = 0.001$ m², $i = 1, 2, 3$ with a corresponding first open loop frequency $\omega = 0.1185$ rad/sec. is used.

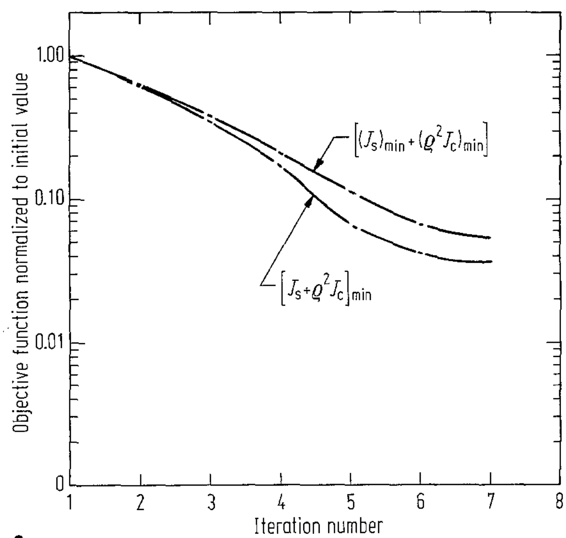
We assume that the beam is given an initial displacement vector $\boldsymbol{x}_0$ of the form:

$$\boldsymbol{x}_0 = (0.011,\ 1.35 - 03,\ 0.037,\ 0.002,\ 0.0688,\ 0.00216)$$

The $\boldsymbol{x}_0$ values correspond to V_1 to V_6 degrees-of-freedom in the fundamental mode of a uniform beam. The associated initial velocities are assumed zero. The control weighting matrices **Q** and **R** are arbitrarily taken to be identity matrices. We note that since J_S is a linear function of $\boldsymbol{a}$, its gradient is constant and its Hessian is zero. In the following, comparison is made between the two optimization approaches discussed. A minimization of J_S and J_C separately leads to an objective function value equal to $[(J_S)_{min} + (\varrho^2 J_C)_{min}]$; while the simultaneous optimization minimizes the sum $[J_S + \varrho^2 J_C]$. Figure 2 gives the iteration histories and final results of the two approaches, and Fig. 3 shows the associated iteration histories of the individual contributing parts. All iteration histories in Figs. 2 and 3 are normalized to their own starting values. Table 1 gives the numerical details. The following observations can be made from the results:

(1) The minimum value of the objective function when both the structural and the control cost functions are simultaneously minimized is less than that obtained when the two are separately minimized. In the case considered, the simultaneous minimization yielded 50% lower value than the separate minimization, Fig. 2.

(2) The simultaneous optimization of the structural and control costs of Eq. (11) may be thought of as a minimization of the structural cost subject to a "control penalty". It would be expected, therefore, that $J_S(\boldsymbol{a}^*) \leq J_S(\boldsymbol{a}^{**})$, where $J_S(\boldsymbol{a}^{**})$ is the structural cost evaluated for the design $\boldsymbol{a}^{**}$ob-



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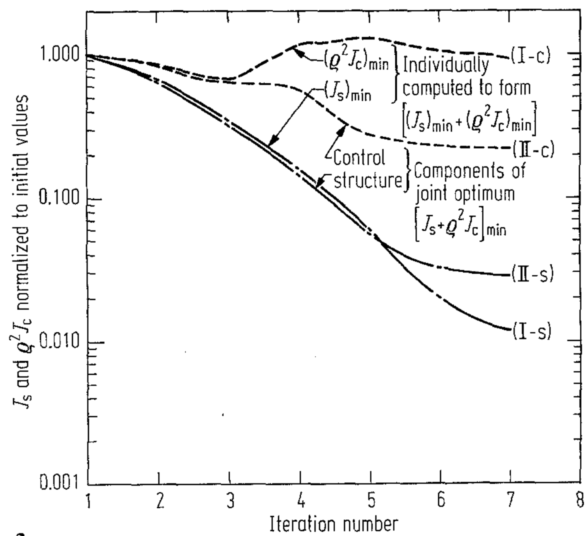

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Figs. 2 and 3. Iteration histories; **2** for the "separate" and "simultaneous" optimization; **3** iteration histories of components of the "separate" and "simultaneous" optimization

Table 1. Numerical detail of iteration histories. ( ) = normalized to initial value

Iteration	Approach I					Approach II				
	Optimum structure (J_S)			Opt. Control for (J_S)	Sum of	Simultaneous optimization				
No.	Areas	ω	$(J_S)_{min}$	$(\varrho^2 J_C)_{min}$	$(J_S)_{min}+(\varrho^2 J_C)_{min}$	Areas	ω	$(J_S+\varrho^2 J_C)_{min}$	(J_S) Component	$(\varrho^2 J_C)$ Component
1	1.000-3 1.000-3 1.000-3	0.118525	74.700 (1.0)	3.525 (1.0)	78.255 (1.0)	1.000-3 1.000-3 1.000-3	0.118525	78.255 (1.0)	74.700 (1.0)	3.525 (1.0)
2	0.667-3 0.667-3 0.667-3	0.100054	49.840 (0.667)	3.096 (0.878)	52.936 (0.667)	0.647-3 0.637-3 0.622-3	0.100136	50.484 (0.645)	47.460 (0.635)	3.025 (0.858)
3	0.441-3 0.425-3 0.194-3	0.112005	26.381 (0.353)	2.344 (0.665)	28.725 (0.367)	0.457-3 0.318-3 0.216-3	0.108975	26.962 (0.345)	24.668 (0.330)	2.294 (0.651)
4	0.261-3 0.227-3 0.163-4	0.138112	12.553 (0.168)	4.114 (1.16)	16.667 (0.213)	0.280-3 0.153-3 0.409-4	0.128120	13.805 (0.176)	11.803 (0.158)	2.002 (0.568)
5	0.109-3 0.740-4 0.437-5	0.123045	4.661 (0.062)	4.583 (1.300)	9.243 (0.118)	0.152-3 0.139-4 0.209-5	0.159051	5.146 (0.066)	4.180 (0.056)	0.966 (0.274)
6	0.451-4 0.101-4 0.133-5	0.133043	1.407 (0.019)	3.746 (1.063)	5.153 (0.065)	0.887-4 0.594-5 0.209-5	0.125635	3.238 (0.041)	2.408 (0.032)	0.831 (0.236)
7	0.318-4 0.211-5 0.133-5	0.100415	0.877 (0.012)	3.284 (0.932)	4.161 (0.053)	0.849-4 0.625-6 0.149-6	0.147986	2.917 (0.037)	2.134 (0.029)	0.783 (0.222)

tained from the simultaneous optimization, and $J_S(\boldsymbol{a}^*)$ is the structural cost obtained without regard to the control cost. The extent of the above inequality would indeed depend on the relative contribution of the control penalty cost to the total objective function $(J_S + \varrho^2 J_C)$. This observation is born out in Fig. 3 curves I-S and II-S, where during the last few iterations, the ratio $J_S(\boldsymbol{a}^*)/J_S(\boldsymbol{a}^{**})$ is of the order 1/(2.3). That $J_S(\boldsymbol{a}^*) \not\leq J_S(\boldsymbol{a}^{**})$ during intermediate iterations is due to the fact that the iteration histories of the two approaches follow different paths of their respective minima.

(3) As a consequence to the preceeding two observations;

namely $[J_S(\boldsymbol{a}^{**}) + \varrho^2 J_C(\boldsymbol{a}^{**}, \boldsymbol{u}^{**})]_{min} \leq [J_S(\boldsymbol{a}^*)_{min} + \varrho^2 J_C(\boldsymbol{a}^*, \boldsymbol{u}^*)_{min}]$

and $J_S(\boldsymbol{a}^*)_{min} \leq J_S(\boldsymbol{a}^{**})$

it follows that: $J_C(\boldsymbol{a}^{**}, \boldsymbol{u}^{**}) \leq J_C(\boldsymbol{a}^*, \boldsymbol{u}^*)_{min}$

This is evident from curves I-C and II-C of Fig. 3. Here again, the extent of this inequality depends on the relative contribution of the control cost to the total cost function. In turn, the control cost depends upon the choice of the initial design, the weighting factor ϱ, the matrices **Q** and **R**, and the nature of the control force. Numerically, the ease with which the minimum could be located depends on several factors such as the initial trial deisgn, step size, frequency, and accuracy of computing the gradient and Hessian, and the criteria for determining the convergence.

4 Conclusions

Being at the the intersection of two relatively complex and computationally demanding problems, a successful simultaneous optimization of the structure-control system crucially depends upon simplicity of the formulation and the strength of its theoretical foundation. The composite objective function introduced here as a linear combination of a structural objective (which is function of the structural design variables, $\boldsymbol{a}$) and a control objective (which is function of both $\boldsymbol{a}$ and the control design variable $\boldsymbol{u}$) allowed a simple and computationally tractable solution. By assuming the usual quadratic performance for the control objective, it is possible to solve analytically for the optimal control variables $\boldsymbol{u}^{**}(\boldsymbol{a})$. As such, one is able to carry over without modification all mathematical bases for solution existence, stability, and robustness, readily available in the optimal control literature. Being valid for all feasible structural design variable $\boldsymbol{a}$, the analytical expression for the optimal control $\boldsymbol{u}^{**}$ is easily encapsulated within an iterative numerical search scheme to determine a corresponding optimal set of structural variables $\boldsymbol{a}^{**}$, without increasing the dimensionality of the design space being searched. This is an important consideration for computatinal efficiency, especially as one seeks to solve practical problems having larger number of variables $\boldsymbol{a}$ and $\boldsymbol{u}$, and constraints h_j.

The example discussed in this paper illustrated the numerical results for a set of structural and control parameters. It was shown that the separate optimization of $(J_S)_{min}$ and $\varrho^2(J_C)_{min}$ produced a final (structure + control) design which is inferior by 50% to the joint optimum $(J_S + \varrho^2 J_C)_{min}$ produced by the simultaneous optimization approach outlined in this paper.

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